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Determining the refractive index of liquids using a modified Michelson interferometer

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ABSTRACT

An accurate method for measuring the refractive index of liquids is proposed, modeled, and experimentally verified. The experimental setup is a modified Michelson interferometer employing a novel liquid chamber that allows the optical pathlength to be continuously varied without moving a mirror. This experiment allows determining the refractive index of a given liquid to high precision, with an accuracy limited only by the normal random variables encountered in interferometric measurements and ultimately the accuracy to which the wavelength of the laser light is known.

1. Introduction

The refractive index is not only useful in optics related fields; it is a metric used in pharmaceuticals, food production, and other chemical-based industries to monitor purity of the end product. In many instances, refractive index data for liquids cannot be found in reference books and must be measured as needed. Thus a cost effective method that also provides high precision (if not accuracy as well) would find application in a wide range of industries. Detecting a small difference in the refractive index of a solution is often more important than the absolute value of the index itself and these differences cannot be measured precisely using traditional methods [1].

There are various techniques for determining the refractive index of liquids. Among them, measuring the minimum angle of deviation of a light beam passing through the liquid contained in a triangular cell is often used [2]. This method has been found to be a relatively simple way of obtaining the refractive index of liquid solutions when neither high precision nor accuracy is required. A telescope must often be used to locate the position changes in the refracted beam. The probability of an error occurring in determining the angle is high. An automatic refractometer based on laser beam displacement through the liquid has been developed; however, two measurements must be made—one with the cell full and one with it empty [3]. Other methods require measurement of the critical angle at the boundary between the liquid and the cell containing it [4,5]. In this method, an accurate value of the refractive index of the cell at the laser wavelength is needed. In recent years, interferometric measurement techniques have been widely applied in determining the refractive index of fluids. An interferometric refractometer that is basically a combination of Mach–Zehnder and Michelson interferometers has been built and tested [6].

In the present paper a modified Michelson interferometer is described which has been employed for measuring the refractive indices of liquids. Two techniques that utilize the same hardware were developed and each has advantages. The FWHM (full width at half maximum) method requires only the analysis of a single fringe, making it very fast, although not very accurate or precise. The fringe-counting method relies on counting hundreds of fringes as the optical pathlength through the liquid is changed by a substantial amount, making for a very precise measurement (and potentially accurate as well), but requiring more time. Image processing software was created for the task and used to record, analyze, and accurately count the interference fringes and determine the FWHM.

2. Experimental setup

The experimental setup is a modified Michelson interferometer as sketched in Fig. 1. A small 2.5 mW He–Ne laser of nominally 632.8 nm wavelength was used as the light source. The spatially filtered and collimated beam was split by a beamsplitter into two beams that travel perpendicular to each other. The reference beam travels to a fixed mirror (M1), where it is reflected back to the beamsplitter and to the detector. The probe

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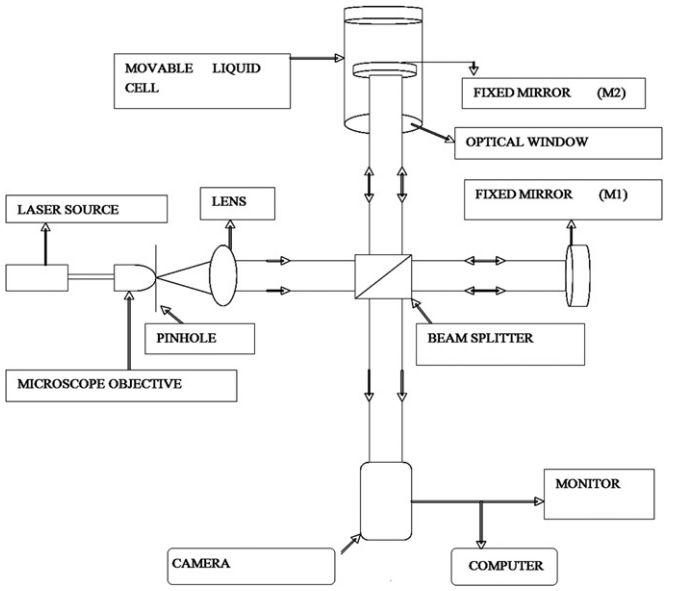

Fig. 1. Schematic diagram of the experimental setup.

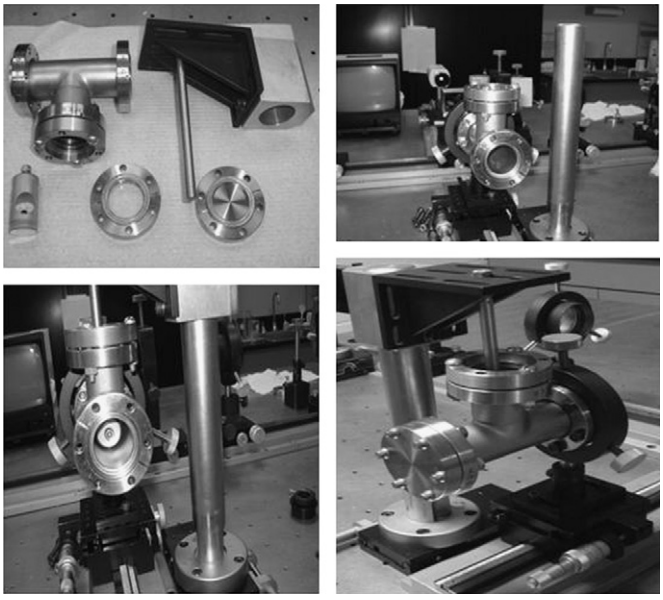

Fig. 2. Photographs of the liquid cell.

beam enters the liquid cell, travels through the liquid, and is reflected by the fixed mirror (M2) inside the liquid cell, back through the beamsplitter to the detector. The two beams interfere at the detector and circular fringes are observed when the mirrors are aligned without tilt between the beams. A central bright fringe was observed when the optical path difference between the beams was (assumed to be) zero. Alternating central bright and dark fringes were observed by translating the cell containing the liquid.

A manual micrometer having an overall linear travel range of 2.5 mm with a 0.0001 mm (100 nm) minimum increment was used to move the translation stage and thus accurately change the pathlength difference. A standard digital camera was used to detect the interference pattern and a video monitor allowed visualization of the camera's output. The camera output was routed to a computer/framegrabber to record the interference intensity in digital format. A dedicated software program was written in LabVIEW® and could be modified based on the requirements of the experiment. The software was able to produce a near real-time picture of the fringe patterns, acquire intensities (grey scale pixel values) of the fringe patterns almost continuously, and store the information in a text file format. Basic data processing, noise reduction, and fringe counting are additional features useful in this research.

2.1. The liquid cell

The liquid cell described in Fig. 1 was cylindrically shaped with an optically flat quartz window on one end and a blank plate covering the other. Both were easily removed for cleaning. The cell had an opening in the top which allowed the mirror (M2) to be inserted into the liquid. The mirror was attached to a steel rod that could be fixed rigidly to the optical bench. Fig. 2 is a photograph of the liquid cell, which might be recognized as being constructed of common high vacuum plumbing hardware.

The liquid cell was filled with the liquid for which the refractive index was being measured. It was then mounted on a linear translation stage which effectively changed the amount of liquid between the quartz window and the fixed mirror (M2) without moving the mirror. The cell was aligned on the linear translation stage such that the window of the liquid cell and the mirror inside it were normal to the incident beam. Translating the cell while keeping the mirror fixed proved to be much more stable than translating the mirror, which is the usual procedure followed in Michelson interferometer-based experiments. Fig. 3 shows the final experimental setup.

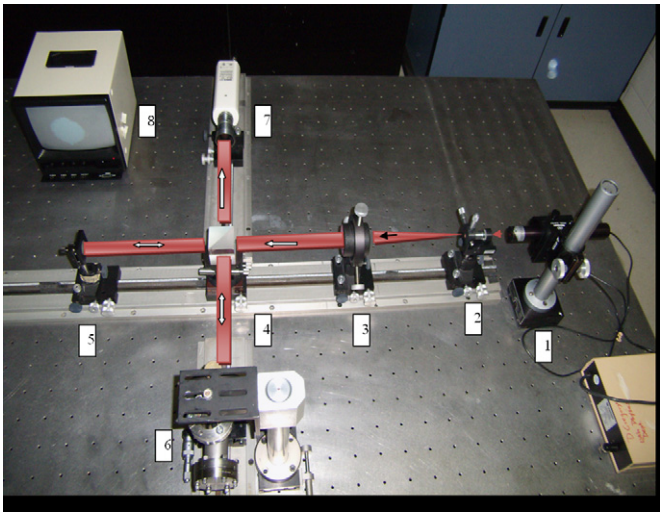

Fig. 3. Photograph of the final experimental setup: (1) Laser Source; (2) Microscope objective; (3) Collimating lens; (4) Beamsplitter; (5) Fixed mirror M1; (6) Liquid cell with fixed mirror M2; (7) Camera; and (8) Monitor.

3. Modeling

Two methods were used in evaluating the Interferometric data: the full width half maximum (FWHM) method and the fringe-counting method. Fig. 4 shows the pathlengths in the experimental setup.

The interference irradiance can be expressed as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi), \tag{1}$$

where I_1 and I_2 are the irradiances of the beams following the two paths and the pathlengths of the transmitted and reflected beams are:

$$l_T = 2Y + Y' \tag{2}$$

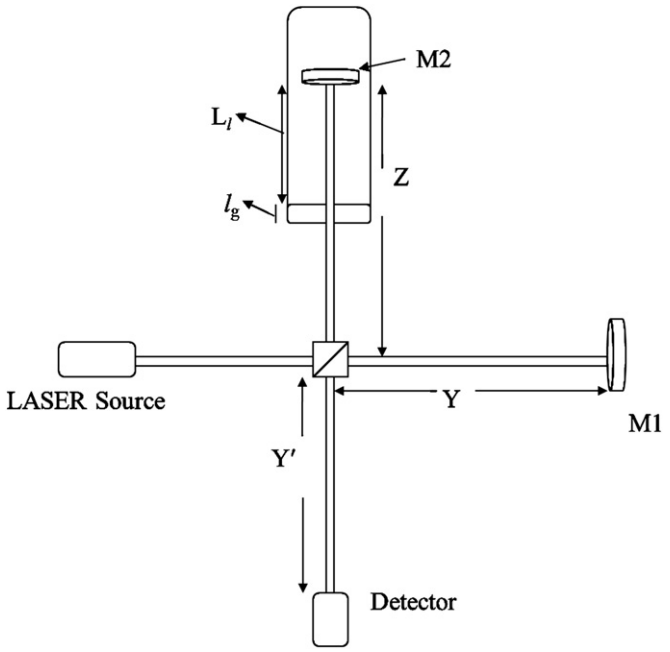

Fig. 4. Experimental setup showing pathlengths.

or

$$l_R = 2Z - 2l_g - 2L_l + 2n_g l_g + 2n_l L_l + Y' + R_1 + R_2. \quad (3)$$

The pathlength difference is then:

$$\Delta l = l_R - l_T. \quad (4)$$

The phase difference between these two waves is given by $\Delta\Phi = k\Delta l$, where $k = 2\pi/\lambda$ and λ is the wavelength of the laser beam. $\Delta\Phi$ is thus written as

$$\Delta\Phi = (2kZ - 2kl_g - 2kL_l + 2kn_g l_g + 2kn_l L_l + kY' + R_1 + R_2) - (2kY + kY') \quad (5)$$

or

$$\Delta\Phi = 2k(Z - Y) + 2kL_l(n_l - 1) + 2kl_g(n_g - 1) + R_1 + R_2 \quad (6)$$

where l_g is the thickness of the quartz window, which is a constant; n_g is the refractive index of the quartz window, which is also a constant; and n_l is the refractive index of the liquid. L_l is the thickness of the liquid between the mirror (M2) and the quartz window. The change in L_l provides the optical pathlength difference (Δl). R_1 and R_2 are the reflectivities of the first and second surfaces of the quartz window respectively. For normal incident (as in the present case) the reflectivity depends only on the difference in indices of the two media. In this case the refractive index of the quartz is similar to that of the liquids measured therefore the reflectivity between the window and the liquid is very small (no more than 1%) and the reflectivity between the window and air is not more than 4%. Light from these two surfaces can interfere and produce fringes, however the fringe contrast will be poor due to the difference in irradiances of the beams and if the pattern does exist at all, it is fixed and will not change as the liquid cell is translated, therefore this does not contribute to the measured phase difference. Experimentally, the contribution of R_1 and R_2 (and any other internal reflections) to the fringe shift is a negligible background. Y and Z are the lengths of the arms and when they are equal, $\Delta\Phi$ becomes (with R_1 and R_2 ignored):

$$\Delta\Phi = 2kL_l(n_l - 1) + 2kl_g(n_g - 1). \quad (7)$$

3.1. The Full Width Half Maximum (FWHM) method

During simulations of the normalized intensity (V) versus the optical path difference (Δl) it was observed that the FWHM of the plot changed with the refractive index of the liquid chosen. A typical simulated plot for water is shown in Fig. 5.

When constructive interference occurs, the phase difference ($\Delta\Phi$) is zero. Therefore, Eq. (7) becomes

$$0 = 2kL_l(n_l - 1) + 2kl_g(n_g - 1).$$

Then an expression for the optical path difference (L_l) can be written as

$$L_l = \frac{-2kl_g(n_g - 1)}{2k(n_l - 1)} = L_{max}, \quad (8)$$

where L_{max} indicates the position of the micrometer at the interference intensity maximum. From Eq. (1), which is the interference irradiance equation, the value at the FWHM is

$$I_{1/2} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\Phi_{1/2}), \quad (9)$$

where $\Phi_{1/2}$ is the phase difference at the FWHM point and I_1 (in this case) is the intensity of the transmitted beam, which is measured with the reflected beam arm of the interferometer blocked. I_2 represents the intensity of the reflected beam, and is measured when the interferometer's other arm is blocked. Finally, $I_{1/2}$ is the irradiance at the FWHM points. The phase difference at the FWHM can be written as

$$\Delta\Phi_{1/2} = \cos^{-1}\left(\frac{I_{1/2} - I_1 - I_2}{2\sqrt{I_1 I_2}}\right).$$

Substituting the phase difference ($\Delta\Phi$) from Eq. 7:

$$2kL_l(n_l - 1) + 2kl_g(n_g - 1) = \cos^{-1}\left(\frac{I_{1/2} - I_1 - I_2}{2\sqrt{I_1 I_2}}\right),$$

or

$$L_l = \frac{\cos^{-1}\left(\frac{I_{1/2} - I_1 - I_2}{2\sqrt{I_1 I_2}}\right) - 2kl_g(n_g - 1)}{2k(n_l - 1)} = L_{1/2}, \quad (10)$$

where $L_{1/2}$ is the position of the micrometer at the FWHM. From Eqs. (8) and (10), the FWHM can be expressed as

$$\frac{FWHM}{2} = (L_{1/2} - L_{max}) = \frac{\cos^{-1}\left(\frac{I_{1/2} - I_1 - I_2}{2\sqrt{I_1 I_2}}\right) - 2kl_g(n_g - 1)}{2k(n_l - 1)} + \frac{2kl_g(n_g - 1)}{2k(n_l - 1)}$$

and solving for the index of refraction:

$$n_l = \frac{\cos^{-1}(I_{1/2} - I_1 - I_2/2\sqrt{I_1 I_2})}{k(FWHM)} + 1. \quad (11)$$

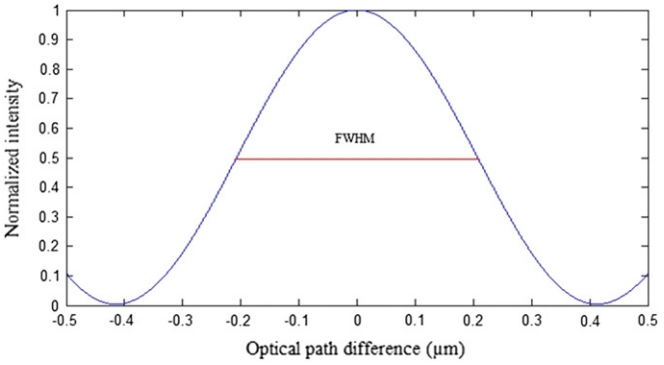

Fig. 5. A simulated plot for water, showing the optical path difference ($\Delta l = L_l$) versus normalized intensity.

Using Eq. (11), the refractive index of a given liquid can then be calculated from experimental data.

3.2. The fringe-counting method

In Eq. (7), the refractive index of the quartz window, n_g, is a constant for the wavelength of the source light. The thickness of the quartz window, l_g is also a constant. So, the entire second term can be written as a constant (C). Therefore, the phase difference can be expressed as

$$\Delta\Phi = C + 2kL_l(n_l - 1). \tag{12}$$

Let $\Delta\Phi = 2m\pi$ be the condition of the phase difference for a peak in irradiance so that

$$2m\pi = C + 2kL_l(n_l - 1)$$

$$m = \frac{C}{2\pi} + \frac{kL_l(n_l - 1)}{\pi},$$

or

$$m = C' + \frac{2L_l(n_l - 1)}{\lambda},$$

where C' is just $C/2\pi$. Taking the derivative with respect to L_l:

$$\frac{dm}{dL_l} = \frac{2}{\lambda}(n_l - 1),$$

and, thus, the index of refraction is expressible as

$$n_l = 1 + \frac{\lambda}{2}\frac{dm}{dL_l}, \tag{13}$$

where the index of refraction of the liquid is n_l and dm is the number of fringe peaks counted in the total pathlength translation dL_l.

4. Results

4.1. Refractive index of water using the FWHM method

Distilled water was chosen to validate the measurement techniques discussed above. Mirrors M1 and M2 were adjusted precisely to obtain good superposition of transmitted and reflected waves. Circular interference fringes of high visibility were then observed on the video monitor. The available manual micrometer with a minimum increment of 100 nm was used to translate the liquid cell. A zero (or multiple of λ) pathlength difference condition was obtained by adjusting the micrometer, and then the irradiance was recorded using the image processing software for every 100 nm increment. The irradiances of both the transmitted beam, I_1, and the reflected beam, I_2, were measured by blocking alternate arms of the interferometer. The experimental data plotted as normalized intensity versus the optical pathlength difference is given in Fig. 6.

A representative value of the FWHM of this curve is 0.542 μm ± 0.05 μm; the uncertainty for a single measurement here being solely the resolution of the micrometer. This was substituted into Eq. (11) to obtain the refractive index of water. The measured refractive index of the water sample, n_w, using this method is 1.35 ± 0.01; the error being determined by choosing maximum and minimum values of the parameters with their associated errors. The same procedure was repeated a number of times in order to determine the precision (standard deviation) which was found to be ± 0.1. A high standard deviation occurred because of the difference in the FWHM of each data set, which was a result of only having seven data points for each curve using the micrometer available.

4.2. Refractive index of water using the fringe counting method

For the fringe counting method, the liquid cell was translated manually with the micrometer to obtain a total pathlength difference of 1 mm. The interference intensities were simultaneously recorded continuously using the image processing software. The total number of central order fringe cycles in the 1 mm pathlength difference was counted in two different ways: using the image processing software, and manually by recording the experimental data in a spreadsheet and counting the number of peaks. The fringe count data was then used in Eq. (13) to calculate the refractive index of the water sample. The same procedure was repeated ten times and the standard deviation was calculated. The measured refractive index of the water, n_w, using the fringe counting method was 1.331903, and the standard deviation was better than 1×10^{-6}. The refractive indices of different liquid samples were also measured and the results are shown in Table 1.

Choosing maximum and minimum extremes of measured parameters in Eq. (13), the error in the index is calculated to be ± 0.0002 for water. This absolute error is representative of that expected throughout the research. It is interesting to note that even if the wavelength error is only ± 0.00001 μm, the error in the index is basically the same. This illustrates the fact that while the precision of the measurement in the present investigation may be limited by the number of significant digits the wavelength is known to, the accuracy of the index measurement is considerably less.

4.3. The refractive index of sugar solutions

As an application example, the refractive indices of different concentrations of sugar solutions were measured using the fringe counting method. Sugar and distilled water were used to make 5%, 10%, 15%, 20%, and 25% sugar solutions (by weight). The refractive index of each sugar solution was then measured. A linear relationship between refractive index and the concentration has been observed by other researchers and it was observed

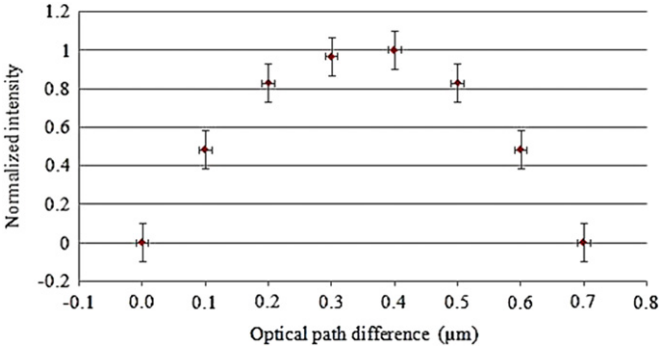

Fig. 6. Experimental data for the distilled water sample.

Table 1
The refractive indices of different liquid samples. ± indicates the standard deviation in the measurements. Absolute accuracy is no better than 0.0002.

Liquid	Refractive index (This work)	Refractive index (Others)
Water	1.331903 ± 0.000001	1.330 ± 0.01 [3]
RP-1 (liquid rocket fuel)	1.454350 ± 0.000001	unknown
Glycerin	1.471436 ± 0.000001	1.472 ± 0.01 [8]
Alcohol (ethyl alcohol)	1.360696 ± 0.000001	1.360 ± 0.01 [3]
Cooking oil	1.47460 ± 0.000001	1.47 ± 0.01 [9]

Table 2
The refractive indices of different concentrations of sugar solutions. ± indicates the standard deviation in the measurements. Absolute accuracy is no better than 0.0002.

Concentration of the sugar solution (%)	**Refractive index of the sugar solution**
5	1.338548 ± 0.000001
10	1.341712 ± 0.000001
15	1.348040 ± 0.000001
20	1.354368 ± 0.000001
25	1.363860 ± 0.000001

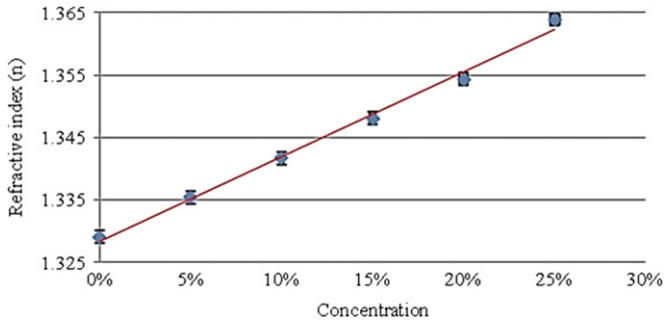

Fig. 7. Index of refraction of sugar/water solution versus concentration (by weight).

here as shown in Table 2 and Fig. 7 [7]. The slope of the line in Fig. 7 is 0.0015 which agrees well with the 0.002 slope other researchers have observed [7]. Since the maximum concentration was well below the normal 80% used to demonstrate gradient index effects in sugar water, no bending of the beams incident on and reflected from the mirror were observed [10]. The system can tolerate some bending as long as the incident and reflected beams still overlap. The phase contribution due to the bending would be a constant and not affect the fringe counting. Normal precautions were taken to ensure the sugar was dissolved completely before taking any data.

5. Conclusions

A modified Michelson interferometer employing a novel liquid chamber was developed for precisely and economically determining the refractive index of transparent liquids. The novel aspect of the interferometer was the design and construction of a liquid cell that allowed changes to be made in the optical pathlength without translating a mirror. Image processing software was used to record, analyze, and accurately count the interference fringes. The refractive indices of a collection of liquids including a sugar/water solution were measured to illustrate the technique. No consideration was given to the temperature dependence of the index which is about 0.00045/°C for most organic liquids and about 0.0001/°C for water [11]. Measurements typically took only a few minutes in a closed laboratory at 22 ± 1 °C and any temporal change in the index would likely have shown up in the standard deviation.

Analyzing the shape of a single fringe was found to provide a quick measure of the index if the associated error can be tolerated, with the fringe counting method providing much lower error and better precision. The accuracy of both methods would greatly benefit from higher resolution translation stages—which would also allow automation of the measurement for real-time monitoring applications.

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