Interstellar Medium 2019
Problem Set 2

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1 Problem 1: Radii of Strömgren spheres

Consider a spherical H II region ionized by an O9V star. Relevant parameters for a star of this type are given in Draine, Table 15.1. Assume that the He/H abundance ratio is 10% by number. What will be the ratio $R_{\mathrm{HeII}}/R_{\mathrm{HII}}$, where R_{HII} is the radius out to which hydrogen is ionized and R_{HeII} is the radius out to which helium is singly ionized? An answer to $\sim 10\%$ accuracy is OK. You will also need Draine Table 14.7. Don't worry over details but do state your assumptions.

2 Problem 2: Radio recombination lines

In class we briefly discussed radio recombination lines. The interpretation of these lines is complicated by stimulated emission. However, if you take line ratios such as for instance He166α/H166α (which we use in this problem), the interpretaion becomes very simple. This is because at these high quantum levels the electron is so far away from the inner part of the atom that this inner part is effectively just a point source with charge +1. As a result all relevant atomic paramaters and corrections for stimulated emission are the same for the hydrogen line and the helium line. In the line ratio these factors then cancel out. The only difference between the hydrogen line and the corresponding helium line is a small frequency difference, resulting from the more massive nucleus of the He atom. We will now use such a line ratio to estimate the temperature of the central star of an H II region. So suppose we use a radio telecope to observe the H166α and He166α recombination lines from an H II region. The line ratio (integrated over the H II region) is found to be He166α/H166α=0.032.

(a) Use Draine Tables 14.7 and 15.1 to estimate the temperature of the exciting star of the H II region, assuming it to be of luminosity class V. Assume that all photons above the ionization edge of helium are absorbed by helium. Assume case B recombination at a temperature $T_{\mathrm{e}} = 10^4$ K.

(b) The observed emission lines are measured to have Gaussian line profiles with a full width at half maximum (FWHM) of 23.5 and 15.3 km s^{-1} for H and He respectively. The spectrometer with which the lines are detected has a instrumental line width (FWHM) of 5.0 km s^{-1}. Assume that the only motions are thermal motions due to the temperature of the gas, and turbulence (constant throughout the H II region) with an unknown velocity dispersion. Assume that instrumental line profile of the spectrograph and the thermal and turbulent velocity distributions are all Gaussians. What is the one-dimensional turbulent velocity dispersion in the nebula? What is the kinetic temperature T_{kin} in the nebula?

3 Problem 3: An obscured H II region

We observe Hα ($\lambda = 6563$ Å) and Hβ ($\lambda = 4861$ Å) emission from an H II region with $T_e = 10^4$ K at a distance of $D = 450$ pc. The observed flux densities integrated over the lines are: $S(\mathrm{H}\alpha) = 1.0 \cdot 10^{-7}$ erg s^{-1} cm^{-2} and $S(\mathrm{H}\beta) = 2.0 \cdot 10^{-8}$ erg s^{-1} cm^{-2}. The H II region is observed through a foreground H I cloud, which may be dusty in which case it obscures Hβ more strongly than Hα, with $\tau_{\mathrm{H}\beta}/\tau_{\mathrm{H}\alpha} = 1.60$. What is the spectral type of the ionizing star (assumed to be of luminosity class V) of this H II region?

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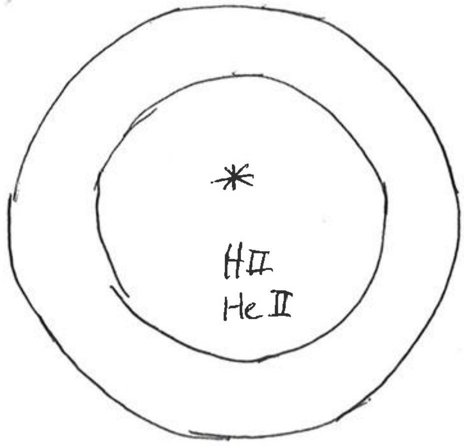
① Radii of Strömgen Spheres

O9V star

Draine table 15.1

$\frac{He}{H} = 10\ \%$ by number



$$\frac{R_{He\,II}}{R_{H\,II}}$$

$$Q_0\big|_{O9V} = 10^{48.06}\ s^{-1}$$

$$Q_1\big|_{O9V} = 0.0145 \cdot 10^{48.06}\ s^{-1}$$

$$\frac{Q_1}{Q_0} = 0.0145$$

$$\cancel{R_S = 32\ pc \left(\frac{Q_0}{10^{49}\,s^{-1}}\right)^{1/3} \left(\frac{n_H}{100\ cm^{-3}}\right)^{-2/3}}$$

$\cancel{n_e = n_H}$

$$\cancel{\frac{R_{He\,II}}{R_{H\,II}} = \left(\frac{Q_1}{Q_0}\right)^{1/3} \left(\frac{n_{He\,II}}{n_{H\,II}}\right)^{-2/3}}$$

$$\cancel{\frac{R_{He}}{R_H} = 1.13}$$

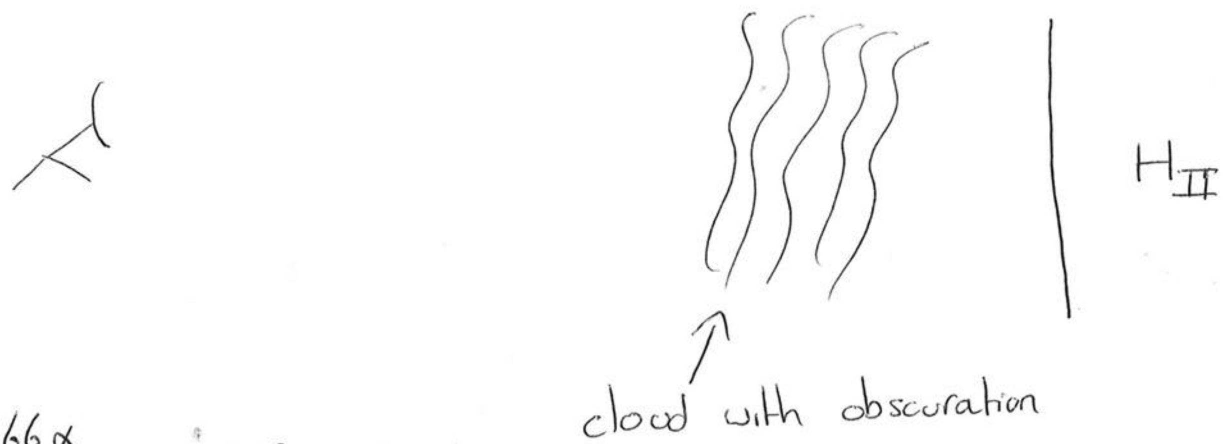
$n_e = n_H$

$$R_S = \left(\frac{3\,Q_0}{4\pi\,\alpha_B\,n_e\,n_p}\right)^{1/3} = \left(\frac{3\,Q_0}{4\pi\,\alpha_B\,n_H^2}\right)^{1/3}$$

$$He:\ R_S = \left(\frac{3\,Q_1}{4\pi\,\alpha_B\big|_{He}\,(0.1)^2\,n_H^2}\right)^{1/3}$$

$$\frac{R_S\big|_{He}}{R_S\big|_{H\,II}} = \left(\frac{Q_1}{Q_0}\right)^{1/3} \left(\frac{\alpha_{H\,II}}{\alpha_{He}}\right)^{1/3} \left(\frac{n_{H\,II}}{n_{He}}\right)^{2/3}$$

2. Radio Recombination Lines



$$\frac{He\ 166\alpha}{H\ 166\alpha} = \cancel{0.02}\ 0.032$$

For high levels, the nucleus is a point source, so structure of atomic nucleus does not make a difference (only net charge)

a)

$$\frac{S(He\ 166\alpha)}{S(H\ 166\alpha)} = \frac{\int dV \frac{h\nu}{4\pi} n_{e} n_{p}\, \alpha_B(T)\big|_{He}}{\int dV \frac{h\nu}{4\pi} n_e n_p\, \alpha_B(T)\big|_{H}}$$

Since we use such a high n, we may assume $\alpha_B\big|_{He}$ equals $\alpha_B\big|_{H\,II}$

$$\cancel{\int \nu V} \quad \hookrightarrow = \frac{\left(\frac{4}{3}\pi R_s^3\right)\big|_{He} n_e\, n\, \alpha_B\big|_{He}}{\left(\frac{4}{3}\pi R_s^3\right)\big|_{H} n_e\, n\, \alpha_B\big|_{H}} = 0.032$$

$$\hookrightarrow \frac{Q_{He}}{Q_H} = 0.032 \frac{\alpha_{He}}{\alpha_H} = 0.034$$

$\hookrightarrow$ O8.5V star.

b) FWHM : 23.5 and 15.3 $km\,s^{-1}$ for H and He

instrumental line width = 5 $km\,s^{-1}$

$$FWHM = 2\sqrt{2\ln 2}\ \sigma = \sqrt{8\ln(2)}\ \sigma$$

$$T_e = 10^4\ K \quad \leftrightarrow \quad \sigma = \cancel{\sqrt{\frac{2kT}{m}}} \quad \sqrt{\frac{kT}{m}}$$

$$\sigma_{meas}^2 = \sigma_{turb}^2 + \sigma_{inst}^2 + \sigma_{thermal}^2$$

$$\sigma_{meas}^2 = \frac{kT}{m} + \sigma_{turb}^2 + \sigma_{inst}^2$$

$$\hookrightarrow \sigma_{turb}^2 = \sigma_{obs}^2 - \frac{kT}{m} - \sigma_{inst}^2$$

$$\sigma_{turb}^2 + \frac{kT}{m} = \left(\frac{FWHM_{obs}}{2.355}\right)^2 \cancel{\frac{kT}{m}} - \left(\frac{FWHM_{inst}}{2.355}\right)^2$$

This gives us a set of two equations

$$(1)\ H: \quad \sigma_{turb}^2 + \frac{kT_H}{m_H} = 101.3 - \cancel{4.0}\ 4.5 = 96.78\ (km\,s^{-1})^2$$

$$(2)\ He: \quad \sigma_{turb}^2 + \frac{kT}{m_{He}} = 43 - 4.5 = 38.43\ (km\,s^{-1})^2$$

$$(1) - (2): \quad \frac{kT}{m_H} - \frac{kT}{4m_H} = 96.78 - 38.43$$

$$\frac{3kT}{4m_H} = 59.35$$

$$\hookrightarrow T = \cancel{2125}\ 9321 - 273 = \cancel{1852}\ 9048\ K$$

$$\sigma_{turb} = 4.3\ km\,s^{-1}$$

③ Obscured H II region

Hα and Hβ $T_e = 10^4$ K $D = 450$ pc

$S(H\alpha) = 1 \cdot 10^{-7}$ erg s^{-1} cm^{-2}

$S(H\beta) = 2 \cdot 10^{-8}$ erg s^{-1} cm^{-2}

$\cancel{\tau = 1.6}$ $\frac{\tau_{H\beta}}{\tau_{H\alpha}} = 1.60$

Hβ is more attenuated by dust than Hα

$$\frac{S(H\alpha)}{S(H\beta)} = 5 \qquad \text{theoretical ratio : } \frac{S(H\alpha)}{S(H\beta)} = 2.86$$

$$S_{obs}(H\alpha) = S_{intr.}(H\alpha) \exp(-\tau_{H\alpha})$$

$$S_{obs}(H\beta) = S_{intr}(H\beta) \exp(-\tau_{H\beta})$$

$$5 = 2.86 \exp(-\tau_{H\alpha} + \tau_{H\beta})$$

$$\rightarrow -\tau_{H\alpha} + \tau_{H\beta} = 0.5586$$

$$\frac{\tau_{H\beta}}{\tau_{H\alpha}} = 1.6$$

$$-\tau_{H\alpha} + 1.6\,\tau_{H\alpha} = 0.5586$$

$$\tau_{H\alpha} = 0.93$$

$$\tau_{H\beta} = 1.49$$

Spectral type :

$$S_{H\alpha|true} = \frac{1 \cdot 10^{-7} \text{ erg } s^{-1} cm^{-2}}{e^{-0.93}} = 2.53 \cdot 10^{-7} \text{ erg } s^{-1} cm^{-2}$$

$$= \cancel{1.362 \cdot 10^{27}\ W} \quad 6.13 \cdot 10^{29}\ W$$

$$L(H\alpha) = \frac{\gamma(T)}{\alpha_B} h\nu Q_0$$

$$\hookrightarrow Q_0 = \frac{6.13 \cdot 10^{29}\ W\ \alpha_B}{h\nu\ \ \gamma(T)} = 4.4 \cdot 10^{48}\ s^{-1}\ \alpha_{eff}(H\alpha) \approx O7.5V$$