Interstellar medium 23-09-2019

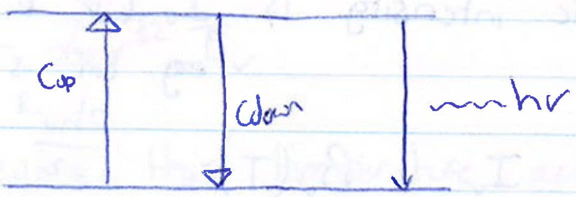
Physical conditions and radiative processes

ISM is not in thermodynamic equilibrium. Eg stars emit much more energy than they take in. In TD eq. every process has to be counter balanced by the reverse process.

! Detailed balance : every process is matched exactly by the reverse process (this is the case in TD eq.)

Are our normal distributions valid in ISM?

- Maxwell distribution is valid in the ISM
  Elastic collisions are ~~frequently~~ sufficiently frequent to thermalize velocity distributions.
  Usually $T_{kinetic} \equiv T_{electrons} = T_{ions} = T_{neutrals}$
  (nb. exceptions exist)
  → Detailed balance, but locally
- Planck energy distribution is, naturally, not valid
- Boltzmann distribution is generally not valid



As long as collisions dominate, Boltzmann works.

$$\frac{n_i}{N} = \frac{g_i \exp(E_i/kT)}{\sum_j g_j \exp(E_j/kT)}$$

If densities are low, spontaneous decays become more important.

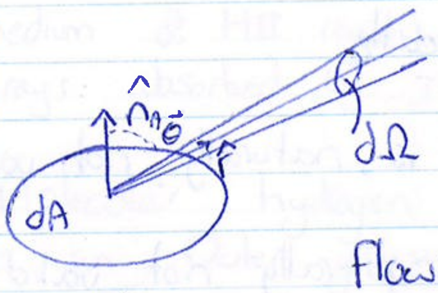
→ excitation temperature

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} \exp\left(-(E_u - E_l)/kT_{ex}\right)$$

In general, $T_{ex} \neq T_{kin}$. If thermalized, $T_{ex} = T_{kin}$. In the case of subthermal excitation, $T_{ex} < T_{kin}$.

Statistical Equilibrium

In the absence of TD equilibrium, (hence detailed balance) in the ISM the weaker condition of statistical equilibrium is valid.
Sum of rates of all processes placing species in level ~~upper~~ j equals the sum of rates out of level j.



$$dE_\nu = I_\nu \cos(\theta)\, d\nu\, d\Omega\, dA\, dt$$

What is the energy flowing through this surface?

- effective area is $\cos(\theta)\, dA$
- specific intensity is $I_\nu(\nu, \hat{n}, \vec{r}, t, \ldots)$
  erg cm^{-2} s^{-1} sr^{-1} Hz^{-1}

$$I_\nu \underset{\uparrow \text{LTE}}{=} B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

- B_λ fine as well but need spectral density ($d\lambda$ or $d\nu$).

n_γ is the photon occupation number.

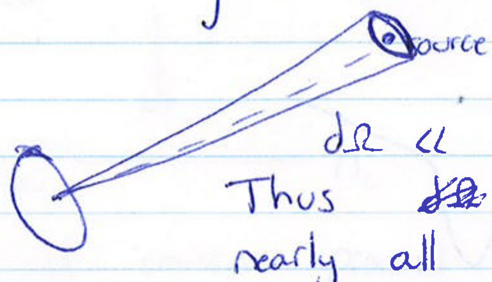
$$\rightarrow n_\gamma = n_\gamma(\nu, \vec{r}, t, \hat{n}) = \frac{c^2}{2h\nu^3} I_\nu(\ldots)$$

All equations become much easier if we use the photon occupation number.

! Flux density

$$dS_\nu = I_\nu \cos(\theta)\, d\Omega \quad [\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}]$$

$$S_\nu = \int I_\nu \cos(\theta)\, d\Omega$$



$d\Omega \ll 1$ sr.

Thus ~~$d\Omega$~~ $\cos(\theta) \approx 1$ in ~~all cas~~ nearly all cases. This is only important if detector is sensitive to emission from all directions (e.g. Lofar?))

Direction averaged intensity $\bar{I}_\nu$

$$\hookrightarrow \frac{1}{4\pi}\int I_\nu\, d\Omega = \frac{S_\nu}{4\pi}$$

$$u_\nu = \frac{1}{c}\int I_\nu(\nu, \hat{n}, \vec{r}, t)\, d\Omega = \frac{4\pi}{c}\bar{I}_\nu$$

! Rayleigh-Jeans brightness temperature. Drain calls this T_A, but that is wrong!

$$T_B(\nu) = \frac{c^2}{2k\nu^2} I_\nu$$

In radio regime, this temperature actually relates to the true bb temperature.

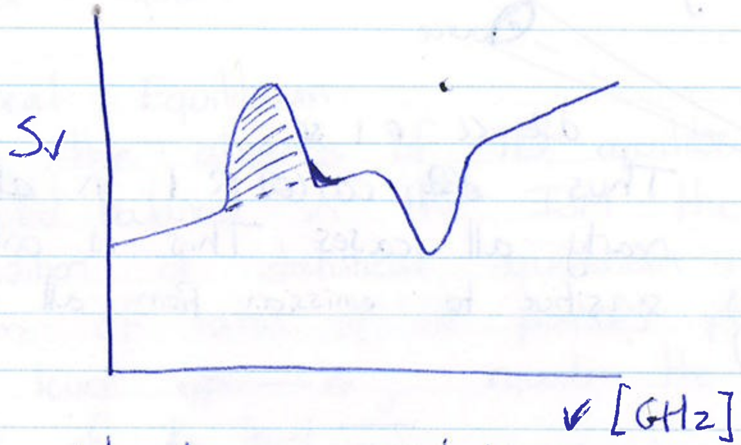
→ Taylor exp of B_ν (or n_γ)

$$\hookrightarrow \frac{2h\nu^3}{c^2}\frac{1}{h\nu/kT} \leftrightarrow \frac{2kT\nu^2}{c^2}$$

In practice we mostly work with flux units
→ Jy. = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1} Flux density is integrated over solid angle.

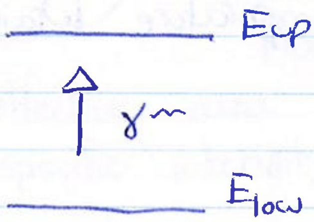
For extended sources, we use Jy sr^{-1}, or Jy pix^{-1}, Jy $beam^{-1}$, etc.

Example : spectrum



When integrating a spectral feature, we get something else than flux density. Eg Jy km s^{-1}
→ Flux integral = flux integrated over line

Excitation and de-excitation of particles



$$W_\gamma = E_{up} - E_{low}$$

Densities n_L and n_{up}. Consider we double n_L → this doubles the number of absorptions of photons.

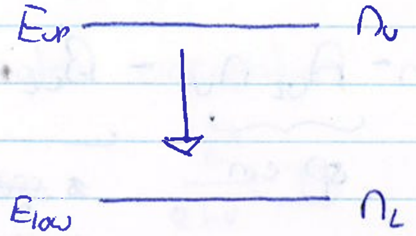
$$\left(\frac{dn_u}{dt}\right)_{\substack{L\to U\\ \text{absorptions}}} = -\left(\frac{dn_L}{dt}\right)_{\substack{L\to U\\ \text{absorp}}} = n_L \, u \, B_{LU}$$

B_{LU} is the Einstein B coefficient for absorption
In reality we have more something like a line

profile : $\left(\frac{dn_u}{dt}\right) = B_{LU}\, n_e \int u_\nu\, \phi_\nu\, d\nu$

where $\int d\phi_\nu\, d\nu = 1$

Consider inverse / spontaneous emission



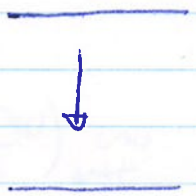
$X_{up} \rightarrow X_{low} + h\nu$

$h\nu = E_{up} - E_{low}$

Rate of emission process (depopulating upper level) will depend on density of upper level.

$$\left(\frac{dn_l}{dt}\right)_{uL} = -\left(\frac{dn_u}{dt}\right)_{ul} = A_{ul}\, n_u$$

Einstein A coefficient



Stimulated emission

$X_u + h\nu \rightarrow X_L + 2h\nu$

$$-\left(\frac{dn_u}{dt}\right)_{\substack{uL\\SE}} = \left(\frac{dn_L}{dt}\right)_{\substack{uL\\SE}} = \overset{\text{stem}}{B_{uL}}\, n_u \int u_\nu\, \phi_\nu\, d\nu$$

Einstein $B_{(uL)}$ coefficient for stimulated emission.

$$\frac{n_u}{n_L} = \frac{g_u}{g_L} \exp\left(-(E_u - E_L)/kT\right)$$

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$$u_\nu = \frac{4\pi}{c} B_\nu(T) = \frac{8\pi\nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

For the total population, the change must be zero (netto)

$$\rightarrow \left(\frac{dn_u}{dt}\right)_{abs} + \left(\frac{dn_u}{dt}\right)_{em} + \left(\frac{dn_u}{dt}\right)_{st\ em} = 0$$

$$\underbrace{B_{LU}\, n_L \cdot \frac{8\pi h\nu^3}{c^3}\, \frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}}_{\text{absorption}} - \underbrace{A_{UL}\, n_U}_{\text{sp. em}} - \underbrace{B_{UL}\, n_U \frac{8\pi h\nu^3}{c^3} \frac{1}{-}}_{\text{~~sponta~~ stimulated emission}}$$

Must be valid for all temperatures → take limiting cases.

CASE 1 $\frac{h\nu}{kT} \ll 1$: $\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1} \simeq \frac{kT}{h\nu}$

$$\hookrightarrow \cancel{B_{LU} n_L \frac{8\pi kT\nu^2}{c^3}} - A_{UL}\, n_U - B_{UL}\, n_U \frac{8\pi kT\nu^2}{c^3} = 0$$

insignificant because high T

$$\rightarrow B_{LU}\, n_L - B_{UL}\, n_U = 0$$

$$\rightarrow \boxed{B_{LU} = \frac{g_U}{g_L} B_{UL}}$$

True at all T's, since this result is independent of temperature. Thus not only for high T.

CASE 2 $\frac{h\nu}{kT} \gg 1$ (low T)

$$\frac{1}{e^{\frac{h\nu}{kT}}-1} \simeq e^{\left(-\frac{h\nu}{kT}\right)}$$

$$\boxed{B_{UL} = \frac{c^3}{8\pi h\nu^3} A_{UL}}$$

↑ st. emis.

If you know A_{UL}, we can calculate all others!

Photon occupation number.

$$\begin{cases} n_\gamma = \dfrac{c^2}{2h\nu^3} I_\nu \\ \bar{n}_\gamma = \dfrac{c^2}{2h\nu^3} \bar{I}_\nu = \dfrac{c^3}{8\pi\nu^3 h} U_\nu \end{cases}$$

Makes expressions much more easy.

$$\begin{cases} \left(\dfrac{dn_L}{dt}\right)_{U\to L} = n_U A_{UL} (1+\bar{n}_\gamma) \\ \left(\dfrac{dn_U}{dt}\right)_{L\to U} = n_L \underbrace{\dfrac{g_U}{g_L} A_{UL} \bar{n}_\gamma}_{\text{absorption}} \end{cases}$$

↑ sp. em   ↖ st. em

We can now see when stimulated emission is important.

$$\bar{n}_\gamma = \frac{1}{e^{h\nu/kT}-1}$$

$$\begin{cases} n_\gamma \ll 1 & h\nu \gg kT \\ n_\gamma \gg 1 & h\nu \ll kT \end{cases}$$