Interstellar Medium 18-11-2019

Molecular Clouds

- radiative trapping
- optically thick mol. emission lines
- measuring molecular gas mass

Mass is dominated by -unobservable- H_2, but we have CO as proxy. → Can we use line strength to probe mass? Yes, ~~at~~ even if the molecular cloud is optically thick.

~~Easy~~
Molecular clouds are self-gravitating objects. CO has spectral lines with upper level $E_{up} \propto J^2$ and critical density $n_{crit} \propto J^3$. Thus higher J lines probe denser / hotter gas. Works for other simple linear molecules as well, but have higher energy levels

CO(1-0): the probe for coolish / low density gas

$$\nu = 115 \text{ GHz}$$
$$\lambda = 2.6 \text{ mm}$$
$$A_{10} = 6.78 \cdot 10^{-8} \text{ s}^{-1}$$

Typical molecular cloud:

$$n_H \sim 10^3 \text{ cm}^{-3}$$
$$R \approx 10^{19} \text{ cm} \quad (3 \text{ pc})$$
$$\frac{n_{CO}}{n_H} \approx 7 \cdot 10^{-5}$$ (for 25% freely available CO, rest is locked up in dust grains)

$$\tau_\nu = \int \kappa_\nu \, dl = \kappa_\nu R$$

$$\hookrightarrow = n_0 \left(1 - \frac{n_1}{n_0} \frac{g_0}{g_1}\right) \frac{\lambda^2}{8\pi} \frac{g_1}{g_0} A_{10} \underbrace{\frac{1}{\sqrt{\pi}} \frac{\lambda}{\sqrt{2}\,\sigma} e^{-\frac{v^2}{2\sigma^2}}}_{\text{line profile}}$$

Peak optical depth $\tau_0 = \kappa_0 R = n_0 R \underbrace{\left(1 - \frac{n_1}{n_0} \frac{g_0}{g_1}\right)}_{\text{assuming } T_{ex}} \frac{\lambda^3}{8\pi^{3/2} \underbrace{\sqrt{2}\,\sigma}_{\text{assuming } \sigma}} \frac{g_1}{g_0} A_{10}$

For cold molecular cloud, $T_{ex} \approx 8\ K$.

$$\tau_0 = 15 \left(\frac{n_H}{1000\ cm^{-3}}\right)\left(\frac{R}{10^{19}\ cm}\right)\left(\frac{n_{CO}/n_H}{7\cdot 10^{-5}}\right)\left(\frac{1.4\ km\,s^{-1}}{\sigma}\right)$$

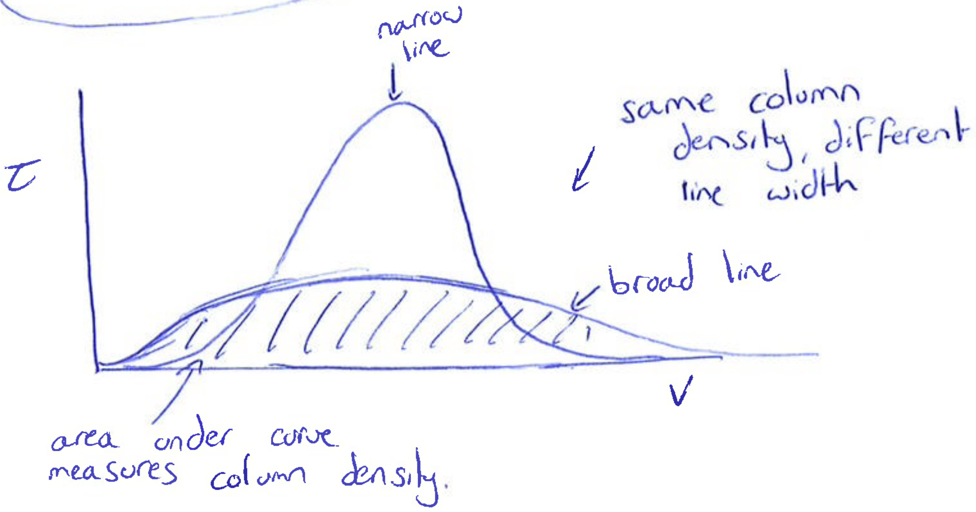
So in general we have optically thick lines. For rare molecules, we have the same problem. To get optically thin lines, we can only use ~~optically thin~~ ~~tn~~ rare isotopes. eg. ^{13}CO instead of $^{12}C^{16}O$

$^{13}C^{17}O$

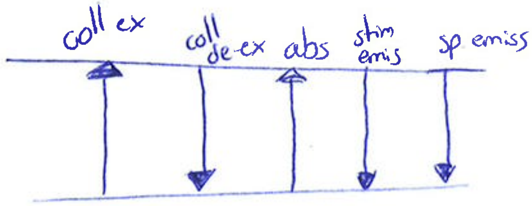
$^{13}C^{18}O$

etc

$$\tau_0 \propto \frac{N(CO)}{\sigma}$$



$$\frac{dn_U}{dt} = \left(n_c k_{LU} + \bar{n}_\gamma \frac{g_U}{g_L} A\right) n_L - \left[n_c k_{UL} + (1+\bar{n}_\gamma) A_{UL}\right] n_U$$



$$= n_c k_{LU} n_L - n_c k_{UL} n_U - A_{UL} n_U + n_L \frac{g_U}{g_L} A_{UL} \bar{n}_\gamma \left(1 - \frac{n_U g_L}{n_L g_U}\right)$$

In molecular cloud, $\bar{n}_\gamma$ will change from place to place

↳ conditions at every point are coupled to conditions at every other point. That is not the case for ~~t~~ optically thin emission.

We observe photons escaping from the cloud. So what we want to use is a photon escape probability.

Probability for photon to escape is

$$p(\vec{r}, \hat{n}) = e^{-\tau_\nu(\vec{r},\hat{n})}$$

$$= \beta(\vec{r}, \hat{n})$$

Averaging over directions ;

$$\bar{\beta}(\vec{r}) = \frac{1}{4\pi} \int \beta(\vec{r}, \hat{n})\, d\Omega$$

$$\langle \bar{\beta}(\vec{r}) \rangle = \frac{\int \bar{\beta}_\nu(\vec{r})\, \phi_\nu\, d\nu}{\int \phi_\nu\, d\nu} = \int \bar{\beta}_\nu(\vec{r})\, \phi_\nu\, d\nu$$

So escape probability averaged over direction ~~an~~ and frequency.

Now

1) Assume excitation temperature is not function of temperature. (This will not be the case if mol. cloud is very inhomogeneous

2) Use on-the-spot approximation → if photon does not escape, it is absorbed immediately again.

Now we have a local problem, which makes things much easier

$$\begin{cases} I_\nu = I_\nu(0)\, e^{-\tau_\nu} + B_\nu(T_{ex})(1 - e^{-\tau_\nu}) \\ n_\gamma = \dfrac{c^2}{2h\nu^3} I_\nu \qquad \dfrac{n_u}{n_L} = \dfrac{g_u}{g_L}\, e^{-\frac{h\nu_{ex}}{kT_{ex}}} \end{cases}$$

$$\rightarrow \quad n_\gamma(\nu) = n_\gamma^{(0)} e^{-\tau_\nu} + \frac{1 - e^{-\tau_\nu}}{\dfrac{n_L g_u}{n_u g_L} - 1}$$

We assume fixed β, which is good approximation except for edge of cloud, but ~~there~~ that has very small ~~occupation~~ volume filling / occupation.

$$n_\gamma(\nu) = n_\gamma^{(0)} \bar{\beta}_\nu + \frac{1-\bar{\beta}_\nu}{\frac{n_L g_U}{n_U g_L} - 1}$$

~~Averg~~

Average over line profile

$$\langle n_\gamma(\nu) \rangle = \langle \bar{\beta} \rangle n_\gamma^{(0)} + \frac{1-\langle \bar{\beta} \rangle}{\frac{n_L g_U}{n_U g_L} - 1}$$

~~d~~

$$\frac{dn_U}{dt} = n_c k_{LU} n_L - n_c k_{UL} n_U - \langle \bar{\beta} \rangle A_{UL} n_U$$

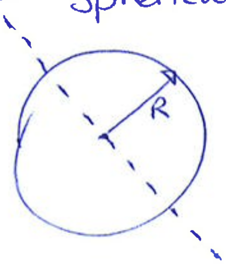
$$+ n_L \frac{g_U}{g_L} \langle \bar{\beta} \rangle A_{UL} n_\gamma^{(0)} \left(1 - \frac{n_U g_L}{n_L g_U}\right)$$

Because of on-the-spot approximation, we have no internally generated radiation field.

We have to calculate the escape probability $\langle \bar{\beta} \rangle$

The energy that would normally be in the radiation field ~~is~~ stays now inside the molecule.

Consider spherical cloud with radius R



$$\langle \bar{\beta} \rangle = \frac{1}{1+0.5\tau_0}$$ ← numerical fit !

Escape probability is quite small, at order of 0.04. An ~~an~~ numeric fit is also possible for uniformly expanding cloud

↳ $v \propto R$, e.g. Hubble flow

Every layer / radius in the cloud has its own velocity. Thus line is wide and peak optical depth is low.

↳ $\frac{dv}{dR} \approx \frac{\Delta v}{R}$

Real molecular clouds are turbulent, therefore an ~~LGG~~ LGV (large velocity gradient) model is a much better approximation. (as above)

$n_{crit} = \frac{A_{UL}}{k_{UL}} \rightarrow \langle \bar{\beta} \rangle \frac{A_{UL}}{k_{UL}}$ and thus lower.

CO (1-0) : $\tau \ll 1$ $\quad n_{crit} \sim 1100 \left(\frac{T}{10k} \right)^{-0.2} cm^{-3}$

Temperature dependend because collision-coefficient is T-dependent.

$\tau \gg 1$ $\quad n_{crit} = \langle \bar{\beta} \rangle \, 1100 \left(\frac{T}{10k} \right)^{-0.2} cm^{-3} \approx 50 \left(\frac{T}{10k} \right)^{-0.2} cm^{-3}$

Photon trapping

So we <u>can</u> do radiative transfer in optically thick lines