ISM 26-09-2019

Know difference between statistical and thermodynamic equilibrium

TDE : detailed balance ; each process is reversed by the inverse process

SE : sum of all processes into one level is equal to the sum of all processes out of that level

$$\boxed{\frac{n_u}{n_e} = \frac{g_u}{g_e} \exp\left(-h\nu_{ul} / kT_{ex}\right)}$$

We often simplify by considering systems as two-level systems. Given Einstein A coefficient we can calculate all other coefficients.

$[A_{ul}] = s^{-1} \longleftrightarrow A_{ul}^{-1} = t$ (average lifetime)

$$-\left(\frac{dn_e}{dt}\right) = \left(\frac{dn_u}{dt}\right) = B_{lu}\, n_l \int u_\nu \varphi_\nu \, d\nu$$

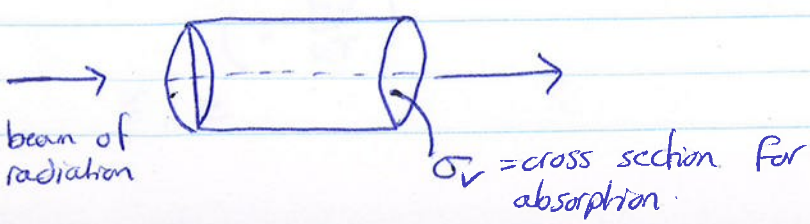
density of photons (as by Draine)
Lightman / Rybicki uses I_ν

RL : $B_{RL} = \frac{4\pi}{c} B_{DRAINE}$

Now we will look at line profile

$$\int \varphi_\nu \, d\nu = 1 \quad \leftarrow \text{line is somewhere}$$

Cross-section $\sigma_{lu}(\nu)$



$$R = k_{UL}\, n_e\, n_b \quad ; \text{ collision} \quad [cm^{-3}\, s^{-1}]$$

$$\propto \sigma_{LU}(\nu)\, c\, dt$$

$$\left(\frac{dn_U}{dt}\right) = n_L \int \sigma_{LU}(\nu)\, c\, \frac{U_\nu}{h\nu_{eu}}\, d\nu$$

because of narrow spectral line

$$\approx n_L \frac{c U_\nu}{h\nu} \int \sigma_{LU}(\nu)\, d\nu$$

$$= B_{LU}\, n_L\, U \int \phi_\nu\, d\nu$$

$$B_{LU} \Leftrightarrow B_{UL} \qquad B_{UL} \Leftrightarrow A_{UL}$$

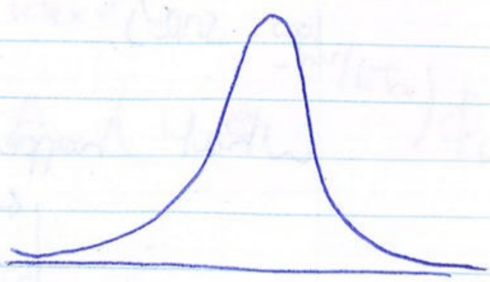
$$\int \sigma_{LU} \underset{\uparrow \text{variable}}{(\nu)}\, d\nu = \frac{g_U}{g_L} \frac{c^2}{8\pi \underset{\uparrow \text{constant}}{\nu_{UL}^2}} A_{UL}$$

$$\sigma_{LU}(\nu) = \frac{g_U}{g_L} \frac{c^2}{8\pi \nu_{UL}^2} A_{UL}\, \phi_\nu$$

$$\sigma_{UL}(\nu) = \frac{g_L}{g_U}\, \sigma_{LU}(\nu)$$

Harmonic oscillator line profile

$$\phi_\nu = \frac{4\gamma_{UL}^2}{16\pi^2(\nu-\nu_0)^2 + \gamma_{UL}^2}$$



$$\Delta\nu_{FWHM} = \frac{\gamma_{UL}}{2\pi} \rightarrow [s^{-1}]$$

$$\Delta E \Delta t \geq \hbar/2$$

$$\gamma_{ul} = \sum_{j<u} A_{uj} + \sum_{j<l} A_{lj}$$

Some numeric values

$\rightarrow \Delta v \sim 0.1 \text{ km s}^{-}$ in optical

All ISM lines are much wider than this so in most cases we can ignore the natural broadening. In X-ray, $\Delta v \sim 10 \text{ km s}^{-1}$, comparable to ISM linewidth. In X-ray, due to T the thermal linewidth is still dominating the broadening

$$P_v \, dv = \frac{1}{\sqrt{2\pi}\,\sigma_v} \exp\left(\frac{-(v-v_0)^2}{2\sigma_v^2}\right)$$

e.g. the fraction of particles with velocity between v and $v+dv$.

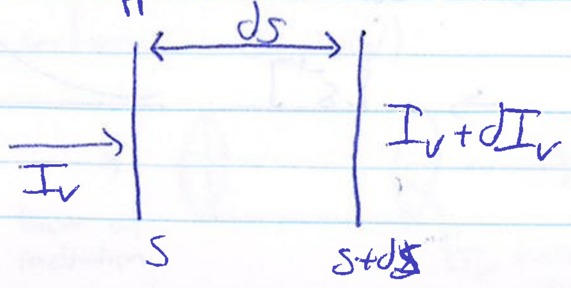
Combining natural and gaussian line profile from ~~th~~ turbulent ~~as~~ broadening (convolution) is the Voigt profile

$\rightarrow$ Voigt = Natural $\otimes$ Gaussian (Draine 6.37)

Profile has wings more prominent than Gaussian. At some point the natural broadening wings will thus start to dominate. Cross-over point happens at $v - v_0 \geq 4.5\,\sigma_v$

Normally we do not measure signal here (due to low SNR).

What happens with an incoming beam of radiation



$$dI_\nu = -I_\nu \underbrace{\kappa_\nu}_{\substack{\text{fractional absorption \&} \\ \text{per unit length } [cm^{-1}]}} ds + \overbrace{j_\nu}^{\text{emissivity } [\cancel{cm^{-1}}]} ds \quad [erg\ s^{-1} cm^{-3} Hz^{-1} sr^{-1}]$$

$$\frac{\text{power}}{\text{volume}} = h\nu_{ul} n_u A_{ul}$$

$$\text{power}/\text{volume}/sr = \frac{h\nu_{ul}}{4\pi} n_u A_{ul}$$

$$\text{power}/\text{volume}/sr/\text{freq} \;\&\; \underbrace{j_\nu}_{\text{emissivity}} = \frac{h\nu_{ul}}{4\pi} n_u A_{ul} \phi_\nu$$

absorption

$$\kappa_\nu = \underbrace{\underbrace{n_l \sigma_{lu}(\nu)}_{\text{pure absorption}} - \underbrace{n_u \sigma_{ul}(\nu)}_{\text{stim emission}}}_{\text{netto absorption}}$$

$$= n_l \sigma_{lu}(\nu) \left(1 - \frac{n_u/n_l}{g_u/g_l} \right)$$

$$= n_l \sigma_\nu(\nu) \; 1 - \exp\left(-h\nu_{ul}/kT_{ex} \right)$$

$$\Rightarrow n_l \kappa_\nu = n_l \frac{g_u}{g_l} \frac{c^2}{8\pi \nu_{ul}^2} A_{ul} \left(1 - e^{-h\nu/kT_{ex}} \right) \phi_\nu$$

Optical depth : $d\tau_\nu = \kappa_\nu ds$

We can now write equation of radiative transfer in terms of optical depth.

↳ $dI_\nu = -I_\nu \, d\tau_\nu + \dfrac{j_\nu}{k_\nu} \, d\tau_\nu$

$\dfrac{j_\nu}{k_\nu} = S_\nu$ = source function

$\rightarrow dI_\nu = (S_\nu - I_\nu) \, d\tau_\nu$

$$e^{\tau_\nu}(dI_\nu + I_\nu \, d\tau_\nu) = e^{\tau_\nu} S_\nu \, d\tau_\nu$$

$$d(e^{-\tau_\nu} I_\nu) = e^{\tau_\nu} S_\nu \, d\tau_\nu$$

$$\Rightarrow e^{\tau_\nu} I_\nu(\tau_\nu) - \cancel{e} \, I_\nu(0) = \int_0^{\tau_\nu} e^{\tau_\nu'} S_\nu \, d\tau_\nu'$$

multiply by $e^{-\tau_\nu}$

$$\Rightarrow \cancel{I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu')} S_\nu \, d\tau_\nu'}$$

$$I_\nu(\tau_\nu) = \underbrace{I_\nu(0)}_{\text{source intensity}} \underbrace{e^{-\tau}}_{\text{attenuation}} + \underbrace{\int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu')} S_\nu \, d\tau_\nu'}_{\text{emission in slab, but corrected for attenuation}}$$

$$S_\nu = \frac{j_\nu}{k_\nu} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_{ex}} - 1} = B_\nu(T_{ex})$$

↳ Kirchhoff's law.

$$I_\nu = I_\nu(0)\, e^{-\tau_\nu} + B_\nu(T_{ex})\,(1 - e^{-\tau_\nu})$$

~~τ → τ → inf~~

Limiting cases : ~~Absorption only~~

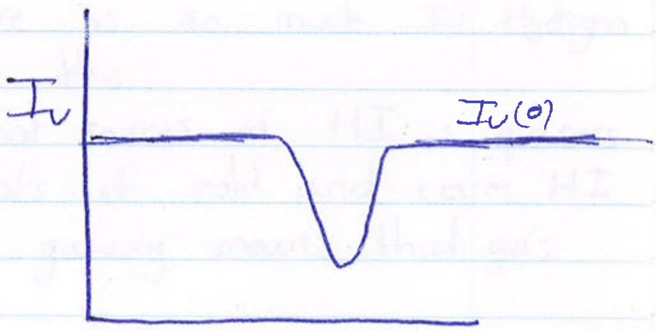
1) $\tau \to \infty$

$$\left.\begin{aligned} & I_\nu(0)\, e^{-\tau_\nu} \to 0 \\ & 1 - e^{-\tau_\nu} \to 1 \\ \to\ & I_\nu = B_\nu(T_{ex}) \end{aligned}\right\} \text{Planck}$$

2) $\tau \to 0$ ~~for~~ with source function outside medium.

$$\left.\begin{aligned} & 1 - e^{-\tau_0} \to 0 \\ & I_\nu = I_0(0) \end{aligned}\right\} \text{see right through}$$

3) Abs only : $I_\nu = I_\nu(0)\, e^{-\tau_\nu}$



3) Emission only

$\tau \ll 1$

$$I_\nu = B_\nu(T_{ex})(1 - e^{-\tau_\nu})$$

$$I_\nu \doteq B_\nu [\tau_\nu]$$

$$= \tau_\nu B_\nu(T_{ex})$$ emission proportional to optical depth

$$d\tau_\nu = n_L \frac{g_U}{g_L} \frac{c^2}{2\pi \nu_{UL}^2} A_{UL} \left(1 - \exp(h\nu_{UL}/T_{ex}k)\right) \phi_\nu\, ds$$

$$= \underbrace{N_L}_{\text{col. dens}} \frac{g_U}{g_L} \frac{c^2}{2\pi \nu_{UL}^2} A_{UL} \left(1 - e^{\left(h\nu_{UL}/kT_{ex}\right)}\right) \phi_\nu$$

$$\tau_\nu = N_L \frac{c^2}{2\pi \nu_{uL}^2} A_{uL} \left(e^{h\nu/kT_{ex}} - 1 \right) \phi_\nu$$

To solve, we need to know excitation temperature

- additional measurements
- use guesses

Tex can become negative if $\frac{n_u}{n_L} > \frac{g_u}{g_L}$, as T is just an expression for level values, this is no problem.

✓ Know something of Masers

- Tex <0
- Rotation
- etc