ISM Problem set I

① Observing HI emission
We observe the CMB at $T = 2.7255\,k$, with ~~an~~ a cloud (atomic hydrogen) in front at $T_s = 50k$. Optical depth of $\tau = 0.1$
What is observed brightness?

In temperature units, we have

$$T_{obs} = T_{CMB}\, e^{-\tau_0} + T_s\,(1 - e^{-\tau_0})$$

$$= 2.7255\, e^{-0.1} + 50(1 - e^{-0.1})$$

$$= 7.224\ k$$

In intensity units, we get

$$I_\nu = B_\nu(T_{CMB})\, e^{-\tau_0} + B_\nu(T_s)(1 - e^{-\tau_0})$$

$$= \frac{2k}{\lambda^2}\left[T_{CMB}\, e^{-\tau_0} + T_s\, \cancel{e}(1 - e^{-\tau_0})\right]$$

$$= \frac{2k}{\lambda^2} \cdot 7.224\ k$$

With $\lambda = 21.11$ cm, we find

$$I_\nu = 4.476 \cdot 10^5\ Jy\ sr^{-1}.$$

(2) We consider a warm and cold neutral medium, with $T_c \sim 70k$ and $T_w \sim 5000k$. We consider two cases:

(1) warm layer located between us and cold layer
(2) cold layer located between us and warm layer

Both layers have velocity dispersion σ_v. Background sky is I_v^{sky} such that we get a flux density $I_v \Omega$. S_0 is the flux density of a background radio source

a) (1) $S_v^* = [I_v^{sky}\Omega + S_0]\, e^{-\tau_w - \tau_c} + \Omega[B_v\, T(w)\,(1-e^{-\tau_w})$
$+ B_v(T_c)\, e^{-\tau_w}(1-e^{-\tau_c})]$

(2) $S_v^* = [I_v^{sky}\Omega + S_0]\, e^{-\tau_w - \tau_c} + \Omega[B_v(T_c)(1-e^{-\tau_c})$
$+ B_v(T_w)\, e^{-\tau_c}(1-e^{-\tau_w})]$

b) Now for blank sky

(1) $S_v^{*off} = I_v^{sky}\Omega\, e^{-\tau_w - \tau_c} + \Omega[B_v\, T(w)(1-e^{-\tau_w}) + B_v(T_c)\, e^{-\tau_w}$
$(1-e^{-\tau_c})]$

(2) $S_v^{off} = I_v^{sky}\Omega\, e^{-\tau_w - \tau_c} + \Omega[B_v(T_c)(1-e^{-\tau_c}) + B_v(T_w)\, e^{-\tau_c}(1-e^{-\tau_w})]$

c) By subtracting $S_v^* - S_v^{off}$ we get $S_0\, e^{-\tau_w - \tau_c}$
rewriting the above
$\tau_c + \tau_w = \ln\left(\frac{S_v^* - S_v^{off}}{S_0}\right)$

d) How can I_ν^{sky} be determined?

If $\tau_c \ll 1$ and $\tau_w \ll 1$, we have

$$S_\nu^{off} = I_\nu^{sky}\,\Omega + \Omega\left[B_\nu(T_w)\tau_w + B_\nu(T_c)\tau_c\right]$$

From Draine eq 8.11

$$\tau = 2.190 \cdot 10^{-19}\,\mathrm{cm}^2\, n(HI) \left(\frac{k}{T_{spin}}\right)\left(\frac{\mathrm{km\,s^{-1}}}{\sigma_v}\right) e^{-\frac{u^2}{2\sigma^2}}$$

$$= \frac{CN}{T\sigma}\, e^{-u^2/2\sigma^2}$$

$B_\nu(T) = \dfrac{2kT}{\lambda^2}$ since we are in the RJ regime

$$\rightarrow \frac{S_\nu^{off}}{\Omega} \cancel{+} - I_\nu^{sky} = \frac{2Ck}{\sigma_v \lambda^2}\left[T_w \frac{N_w}{T_w} + T_c \frac{N_c}{T_c}\right]$$

So $$N_w + N_c = \frac{\sigma\lambda^2}{2Ck}\left[S_\nu^{off}/\Omega - I_\nu^{sky}\right]$$

e) $$\tau = \tau_w + \tau_c$$

$$= \frac{C}{\sigma_v}\left(\frac{N_w}{T_w} + \frac{N_c}{T_c}\right)$$

For the effective single spin temperature we have

$$\frac{CN}{\sigma_v T_{eff}} = \frac{C}{\sigma_v}\left[\frac{N_w}{T_w} + \frac{N_c}{T_c}\right]$$

$$T_{eff} = \frac{N_c + N_w}{\dfrac{N_c}{T_c} + \dfrac{N_w}{T_w}}$$

3. Conditions for maser emission

Maser emission can occur when the process of populating a certain energy level is easier than that of others, lower levels. We consider a molecule with

$$E_0 < E_1 < E_2$$

And a radiation field $h\nu = E_2 - E_0$

We have A_{20}, A_{21}, A_{10} and S_{02} the absorption probability.

a) $\cancel{\frac{dn_1}{dt} = A_{10} A_{21}}$

$$\cancel{\frac{dn_{02}}{dt} = 0 = A_{20}\, n}$$

$$\cancel{\frac{dn_1}{dt} = 0 = A_{10}\, n_1}$$

The probability for stimulated emission is given by

$$\frac{g_0}{g_2} S_{02}$$

$$\frac{dn_2}{dt} = -n_2 A_{20} - n_2 A_{21} \cancel{+} \frac{g_0}{g_2} S_{02}\, n_{\cancel{0}2} \cancel{+} + n_0 S_{02} = 0$$

$$n_0 S_{02} = n_2 A_{20} + n_2 A_{21} + {}^{g_0}/_{g_1} S_{02}\, n_2$$

$$\frac{dn_1}{dt} \Rightarrow n_2 A_{21} = n_1 A_{10}$$

$$\hookrightarrow n_2 = n_1 \frac{A_{10}}{A_{21}}$$

$$\frac{n_2}{n_0} = \frac{S_{02}}{A_{20} + A_{21} + {}^{g_0}/_{g_1} S_{02}} \longrightarrow \frac{n_1}{n_0} = \frac{S_{02} A_{21}}{A_{10}\left[A_{20} + A_{21} + {}^{g_0}/_{g_1} S_{02}\right]}$$

b) To have maser emission, we need

$$\frac{n_1}{n_0} > \frac{g_1}{g_0}$$

Thus $S_{02} > \dfrac{g_1}{g_0} \dfrac{A_{10}(A_{21}+A_{20})}{A_{21} - g_1/g_2 A_{10}}$

since

$$\frac{S_{02} A_{21}}{A_{10}(A_{21}+A_{20}+ g_0/g_2 S_{02})} > \frac{g_1}{g_2}.$$

c) To have a maser in $2 \to 1$, we need

$$\frac{n_2}{n_1} > \frac{g_2}{g_1}$$

$$\hookrightarrow \frac{n_2}{n_1} = \frac{A_{10}}{A_{21}} > \frac{g_2}{g_1}$$

$$\rightarrow \frac{A_{10}}{A_{21}} > \frac{g_2}{g_1}$$

This is thus a pure property of the levels and species.