Assignment 1 SPFII Derivatives

The goal of this assignment is two-fold:

- To price Interest Rate Swaps, and
- To delve deeper into Mean Reversion models.

So here are the two parts of the assignment.

Part 1. IRS

Create an environment for pricing Interest Rate Swaps. This environment must allow for:

- Bootstrapping the relevant zero reference curves
- Two curve situation: reference rate is LIBOR/EURIBOR and discount rate is OIS
- Swap can be at initiation (so here you are essentially determining the swap rate) or an already existing swap (and here you are determining swap's value).
- It should also allow for amortizing principal amount (so not fixed principal).

Note that this is one if these rare assignments which is possible to do in Excel (but of course you should do it in something more advanced like Python or Matlab).

With your "IRS engine", you will price a few swaps. It is pretty much up to you to define the swap characteristics. For example, you can choose yourself what is your reference rate (EURIBOR, US dollar LIBOR, GBP Libor or any other currency rate – download the interest rate curves relevant for you from Internet, they are available from all Central Banks or Treasury departments).

For long maturity swaps, you have to bootstrap the curve to obtain long-maturity reference rates. Also, you have to specify other swap characteristics yourself, but make sure they are realistic (for example, no point in assuming fixed rate of, say, 10% or even 5% - these are long gone). For all swaps, assume that the payments are exchanged every three months. Also assume that for all your swaps you pay fixed and receive floating rate and, for all already issued swaps, the swap rate is so high that they are currently out of the money for you.

You have to consider three specific situations:

- An already issued swap with the remaining lifetime of 1 to 2 year. See examples in Hull (exercises for Chapter 7) for inspiration.
- A newly issued mid-maturity swap (say, 5 years) with principal amortizing according to some simple predetermined schedule (for example, fixed repayments per time interval) – for this swap, you have to determine the swap rate that you would offer to your client.
- An already issued long-maturity swap. Assume that the remaining lifetime of this swap is still quite long, so longer than 5 years. Determine the swap value and investigate how sensitive it is to 1% and 2% changes (up or down) in the current zero curve.

Part 2. Mean reversion.

The goal of this exercise is to set up both simulation and binomial tree for mean reversion process (rather than GBM, which you already have done in SPF1 – feel free to use your existing code and modify it for MR situation).

It turns out that the Binomial tree method can be extended to accommodate MR process. The paper of Nelson and Ramaswamy (1990) explains how to do that, but there are other, later papers on this issue that explain it even better. Investigate how CRR Binomial tree can be amended for Mean Reverting process and incorporate it in your code.

Amend your GBM Monte Carlo simulation engine to simulate Mean Reversion. Everywhere assume that log-price follows MR process, so MR is in log-price and not in price itself (to keep the log-normality of prices).

With both of these engines (Binomial and MC) you now will investigate how option prices in MR model compare to those in GBM.

Price exactly the same options as you did in Assignments in SPF1, with the same volatility parameters, but now assuming MR for the log of underlying. Do it by both Binomial method and MC. Choose realistic parameters for long term mean (take e.g., long term mean equal to the current value of the underlying) and a realistic mean reversion speed (not too fast and not too slow – half time of, say, a couple of weeks to a couple of months). Compare the option pricing results to those obtained with GBM model.

Investigate the sensitivity of option prices to mean reversion speed, with all other parameters being fixed. Present this sensitivity in the form of an appropriate graph.

Investigate how the difference between MR and GBM option prices depends on the maturity of the option, with all other parameters being fixed. Again, present a graph summarizing your findings. Explain the observed results.