Case 1: Binomial Trees and Geometric Brownian Motion

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1 Simulation of normally distributed stock prices

Suppose we have the following model for monthly stock price returns:

$$\ln S_{t+1} - \ln S_t = \mu + \sigma \varepsilon_{t:t+1},$$

where

$$\varepsilon_{t:t+1} \sim NID(0,1).$$

(a) Simulate 1,000 paths for the **1-year stock return**. Plot the distribution of the simulated 1-year stock returns. Report the average and standard deviation of the simulated distribution. Check if the average of the 1-year simulated stock returns is in the 99% confidence interval of the 1-year expected stock return (i.e. confidence interval of the theoretical distribution). Model parameters are: $\mu = 0.5\%$ and $\sigma = 5\%$.

(b) Increase the number of paths to 5,000. Again, plot the distribution of the simulated 1-year stock returns. Report the average and standard deviation of the simulated distribution. Check if the sample average is in the 99% confidence interval of the 1-year expected stock return (i.e. confidence interval of the theoretical distribution). Model parameters are still $\mu = 0.5\%$ and $\sigma = 5\%$. Use the same seed for your simulation as in (a) and comment on the difference in results with (a).

(c) Execute 1,000 runs of the simulation of question (b). For each run, store the average 1-year stock return. Plot the distribution of the 1,000 average stock returns. Comment on the average and standard deviation of this distribution.

(d) Suppose we would have formulated a model for daily stock price returns instead of monthly stock price returns. Would this change/have an impact on the standard deviation you have calculated in (c), i.e. would you find that your data points are closer to the mean? Please motivate your answer. Calculations are not required.

2 Binomial Trees

A commonly used approach to compute the price of an option is the so-called binomial tree method. In this approach, option prices are computed through a well-known backward induction scheme, which was explained in one of the first lectures and can be found in all option pricing literature.

Consider an at-the-money European call option on a non-dividend-paying stock with a maturity of three months. Suppose that the underlying of this option is the S&P-500 index. Let the 3-month annualized interest rate be equal to 0.5% (with quarterly compounding). You are not allowed to use a package for the Binomial Tree

Remark: 1% per annum with quarterly compounding means that the interest over 3 months is 0.25%. The per annum continuously compounded rate is:

$$r^{c,a} = \ln\left(\left(1 + \frac{1\%}{4}\right)^4\right).$$

(a) Download historical data of the S&P-500 index. Construct a time series of monthly returns with end date August 31, 2020. The number of months to cover equals the sum of your months of birth times 12 with a minimum of 60 and a maximum of 360. Show a graph of the monthly returns and report a table with the main summary statistics.

(b) Build a binomial tree for the S&P-500 index in MatLab (or package of your preference) with starting date August 31, 2020. The purpose is to price a call-option with strike price equal to USD 3,300 and maturity three months. Use the sample standard deviation as an estimate for volatility and use three steps for the tree. Tips: Use the logarithm of the returns (log-returns). Also think about how to set **u** & **d** when using log-returns. For convenient programming make a function of the Binomial tree and use the main script for initialisation and plotting. Furthermore, begin calculating the **V**'s at the end and go backwards.

Suppose the risk premium on investing in the S&P-500 index is 1% over a period of 3 months.

(c) Calculate the upward probability p that ensures that the expected value of the S&P-500 index is consistent with the assumption on the risk premium.

(d) Calculate the price of a European call option with maturity 3 months and strike price USD 3,300 by the Black-Scholes formula. Compare the results.

(e) Study the convergence of the method for the call option of the previous question (b), increasing the number of steps in the tree (3, 10, 50, 100, 150, 200, 250 steps). Illustrate the convergence by means of graphs of the corresponding quantity vs the number of steps.

(f) Change the code such that it can compute the price of a European put option with strike price USD 3,300 and maturity three months. Calculate the price of the put option for the same parameters as mentioned above and verify whether your result satisfies the put-call parity relationship.

(g) Calculate call and put option prices with maturity 3 months and different strikes USD 2,900, USD 3,100, USD 3,200, USD 3,300, USD 3,400, USD 3,500 and USD 3,600. Compare these with the Black-Scholes prices and plot both results.

(h) Now suppose that the option is American. Change the code such that it can handle early exercise opportunities. What are the values of the American put and call (strike USD 3,300 and maturity 3 months) for the same initial parameters? Which one should be exercised early? Also comment on the difference between the European and the American option.

General information cases

The two cases of this course should be done in groups of two students. For each case, you have to write a short report where you must include the results, a short discussion/concluding section and the appendix with the codes. The reports should be submitted electronically, before the corresponding deadlines.

The preferred implementation tool is MatLab. During the allocated computer sessions, you will be able to get support and ask questions regarding your MatLab implementations. However, it is advised that you work on the assignments outside teaching hours and not only during the allocated sessions, as they will require significant time and effort.

In principle, you are not bound to MatLab and are free to choose the programming language/environment in which you would like to write your computer programs (so another alternative could be, e.g., R or Python). However, in this case we cannot guarantee implementation support.

Reporting requirements

- Report should be written in English. This means that you also should use English decimal notation in the text and in graphs.
- There is no maximum to the number of words or pages but please try to be concise but please make sure that it is clear to me what you did and what your interpretation of the results is.
- Figures and graphs should have captions such that they can be read independently from the text.
- Use your spelling checker.
- Put names and VU student numbers of all group members on the front page.
- Final reports should be uploaded on Canvas before 23:59pm CET on the deadline day.

Deadline

The deadline for the first case is as follows:

- Case 1: Monday September 21, 2020 at 23:59pm CET

Last but not least:

ENJOY AND GOOD LUCK!!