Tree Formula List

1) In a binary tree, maximum no. of nodes on level l is 2^l, where $l \geq 0$.

It is also true for level = $i+1$

max no. of nodes at level $i+1$ is 2×2^i

$= 2^{i+1}$

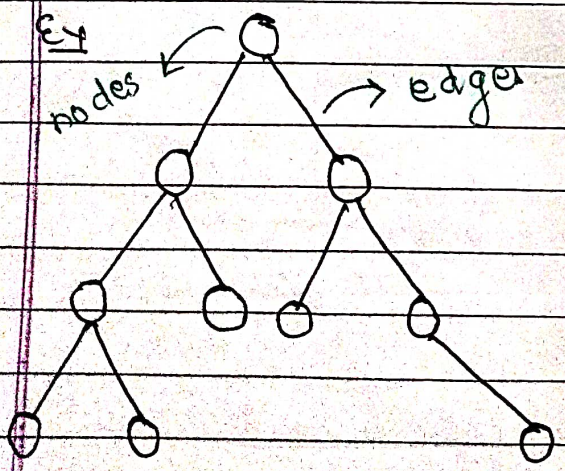
2) $\boxed{n_{max} = 2^{h+1} - 1}$ where h is the height of the tree.

(max no. of possible nodes in a binary tree.

3) $\boxed{n_{min} = h+1}$

(min. no. of nodes possible in a binary tree.)

4) $\boxed{n = e + 1}$ where n is the no. of nodes & e is the no. of edges.



Total edges = 9

So, Total no. of nodes = 9+1 = 10

works only when we make a tree in which every node has either zero or two child.

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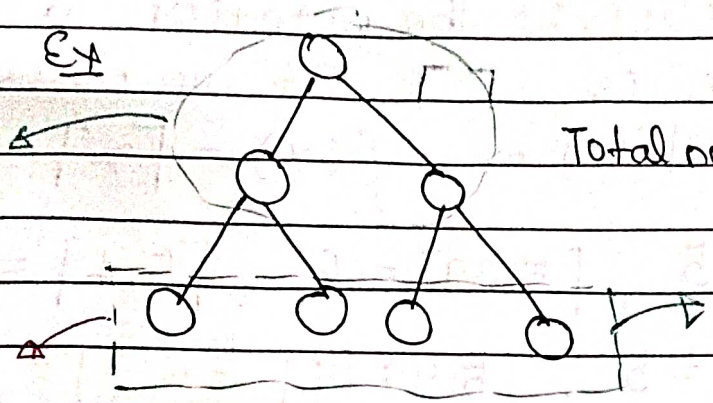
5) $L = I + 1$

Where L is for leaf nodes & I is for internal nodes

Ex



Total no. of internal nodes = 3

Leaf nodes :- nodes that further dont have any child

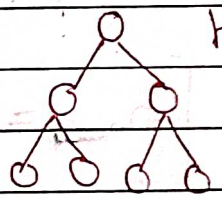
So, Total no. of leafes = 3+1 = 4

internal nodes :- All nodes except Leaf Node are the internal Node. Root node is also counted as a internal node.

6) $h_{min} = \log_2(n+1) - 1$

This formula only works when this portion comes out to be even

Ex



h_{min} with max. no. of nodes

$n_{max} = 7$

so, $h_{min} = \log_2(7+1) - 1$

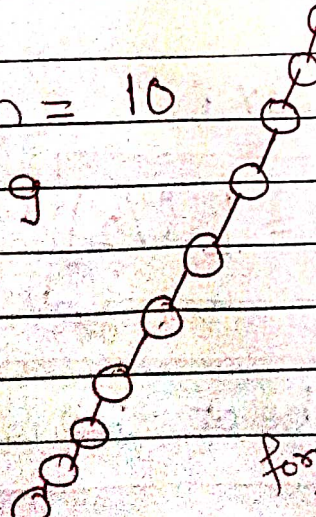
$= 3 - 1 = 2$

$h_{max} = n - 1$

Ex for max. height make a tree having least no. of childs.

for $n = 10$

$h_{max} = 9$



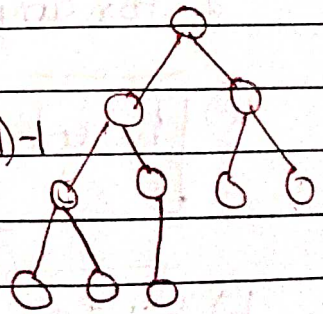
for $n = 10$

$h_{min} = \log_2(10+1) - 1$

$= 3.45 - 1$

$= (2.45)$

formula fails here



So, right formula for h_{min} after applying ceiling function :- The notation to represent this function is $\lceil \rceil$. It can be used as $\lceil x \rceil$ or ceil(x) or $f(x) = \lceil x \rceil$

Eg $\lceil 4.5 \rceil = 5$, $\lceil 5.39 \rceil = 6$, $\lceil 5.1 \rceil = 6$

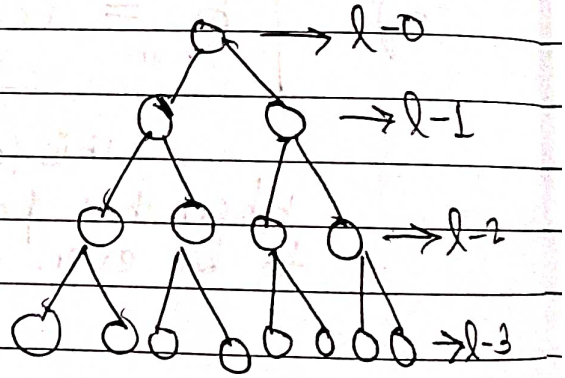
$\lceil 5 \rceil = 5$, $\lceil 5.9999 \rceil = 6$, $\lceil 5.00001 \rceil = 6$

Hence, $\boxed{h_{min} = \lceil \log_2(n+1) - 1 \rceil}$

Note:- $\boxed{\text{height of a tree} = \text{height of a root}}$

7) $\boxed{h_{max} = l_{max}}$ for $n = 'l_{max} = 3$

max height of a binary tree — level



So, h = 3

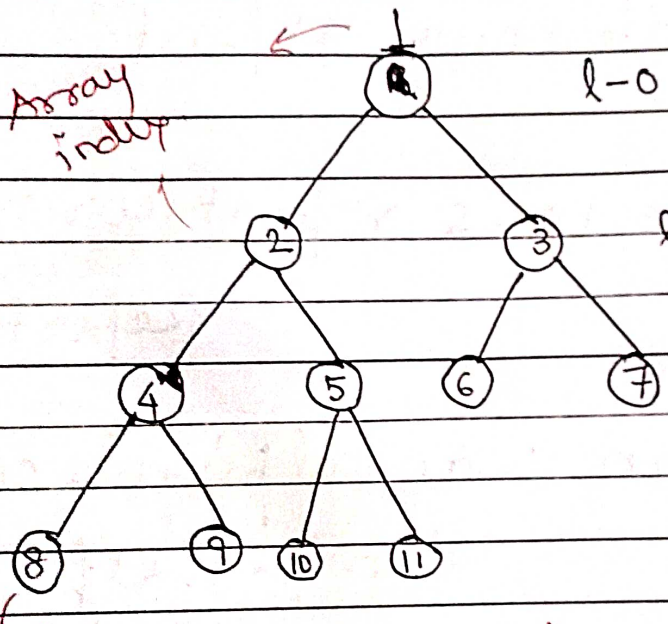
8) The root node is at location 1

2. For any node with index i,

a) $\boxed{\text{Parent of a child is at} = \lfloor i/2 \rfloor}$ — i: index of a child; floor function

b) $\boxed{\text{Childs of a root will be at } 2i \text{ \& } (2i+1)}$ — i: index of the root

Ex



↳ parent of 8 is at = $\lfloor 8/2 \rfloor = 4$

Child of Roots 2 & 3 are 2×3 & 2×3+1 ⇒ 6 & 7

↑ the having index = 3

6 ↓ LC

7 ↓ RC