

Multiples of 3 and 5

If we list all the natural number below 10 that are multiples of 3 or 5, we get 3, 5, and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

Solution

A simple way to do this is to go through all numbers from 1 to 999 and test whether they are divisible by 3 or by 5. But if we want to do the same for numbers less than 1,000,000,000 that is going to take a long time.

To get a more efficient solution you could also calculate the sum of the numbers that are divisible by 3, plus the sum of numbers divisible by 5. But as you summed numbers divisible by 15 twice you would have to subtract the sum of the numbers divisible by 15.

Lets write out some of the terms for $n = 3$.

$$3 + 6 + 9 + 12 + \cdots + 999 = 3(1 + 2 + 3 + 4 + \cdots + 333)$$

We also not that $1 + 2 + 3 + \cdots + p = \frac{1}{2}p(p + 1)$. Using these two facts, we are able to calculates the sum of numbers divisible by n less than t like so:

$$p = t/n \quad sum = n(p(p + 1))/2$$

Then the final solution will have the persudo code of

```
target=999
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```
Function SumDivisibleBy(n)
  p=target / n
  return n*(p*(p+1)) / 2
EndFunction
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Output SumDivisibleBy(3)+SumDivisibleBy(5)-SumDivisibleBy(15)
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