INTELLIGENT ROBOTS

CHAPTER 8: PARTICLE FILTER AND MONTE CARLO LOCALIZATION

Outline

- Sample-based Localization Problem
- Importance Sampling
- Particle Filter
- Monte Carlo Localization

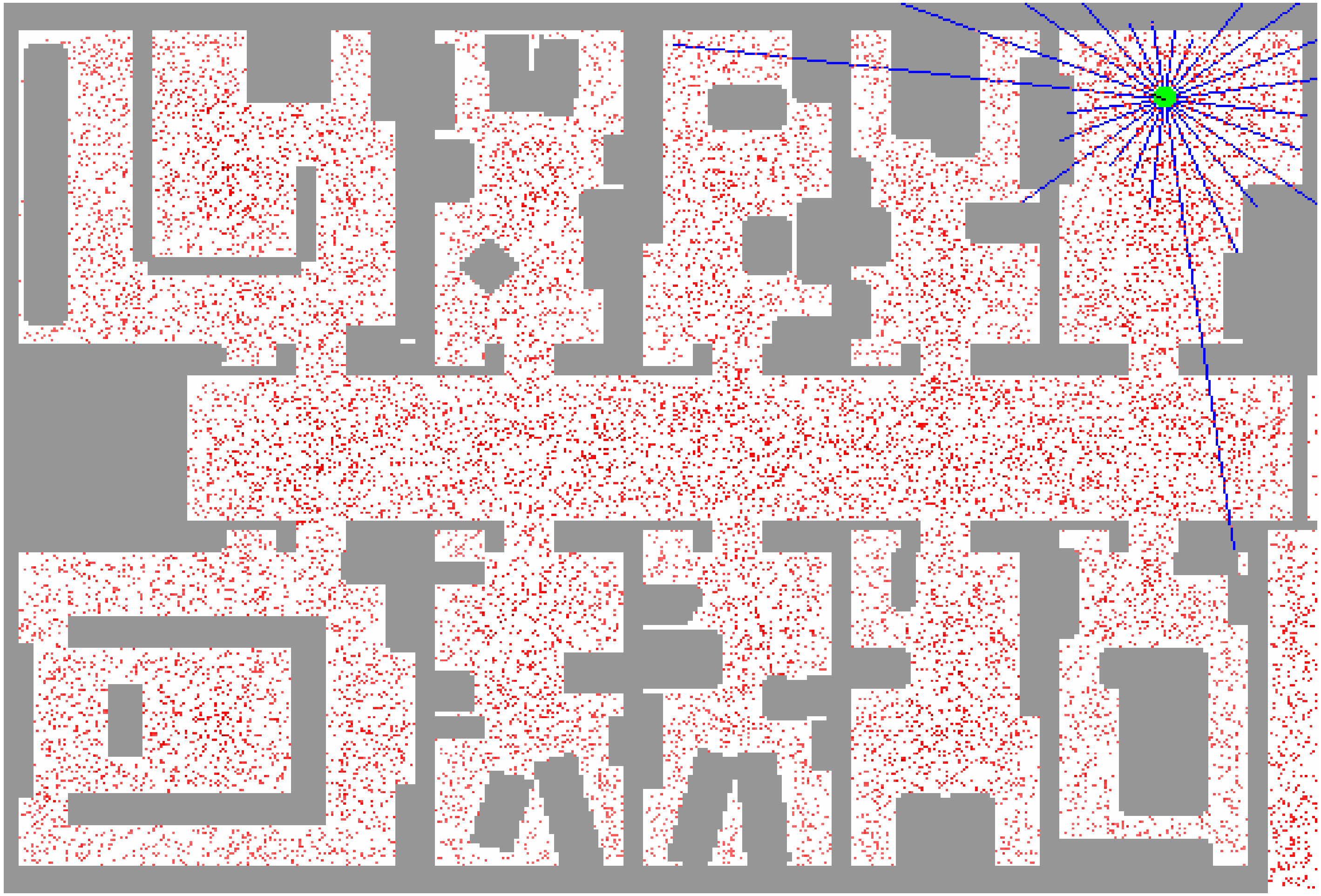
Motivation

- Discrete filter
  - Discretize the continuous state space
  - High memory complexity
  - Fixed resolution (does not adapt to the belief)

- Particle filters are a way to **efficiently** represent **non-Gaussian distribution**

- Basic principle
  - Set of state hypotheses ("particles")
  - Survival-of-the-fittest

Sample-based Localization (Sonar)



Outline

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Mathematical Description

- Set of weighted samples

$$S = \{\langle s^{[i]}, w^{[i]} \rangle \mid i = 1, \ldots, N\}$$

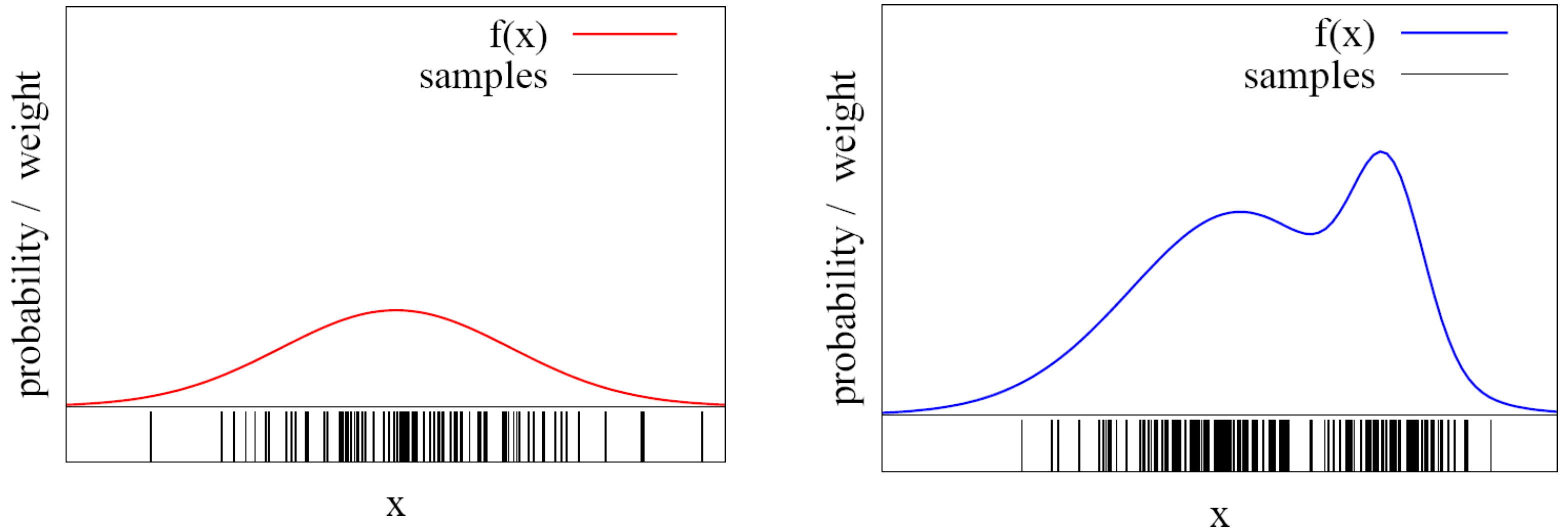
State hypothesis Importance weight

- The samples represent the posterior

$$p(x) = \sum_{i=1}^{N} w_i \cdot \delta_{s^{[i]}}(x)$$

Function Approximation

- Particle sets can be used to approximate functions



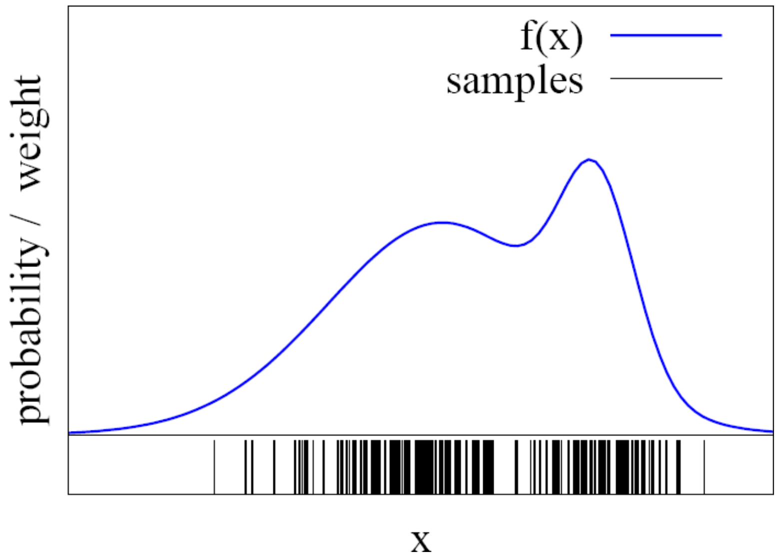
- The more particles fall into an interval, the higher the probability of that interval

- How to draw samples form a function/distribution?

Rejection Sampling

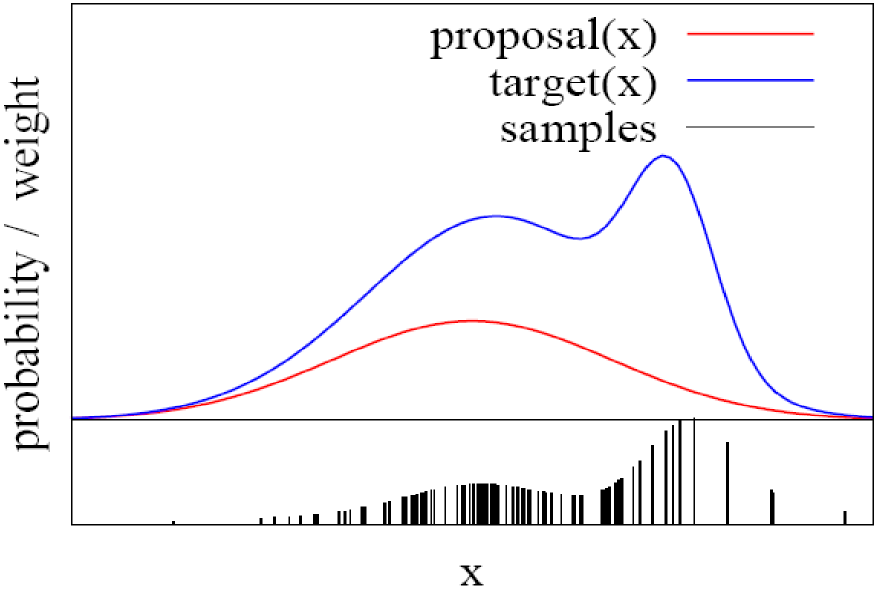
- Let us assume that $f(x)<1$ for all x
- Sample x from a uniform distribution
- Sample c from [0,1]
- if $f(x) > c$ keep the sample

  otherwise reject the sampe



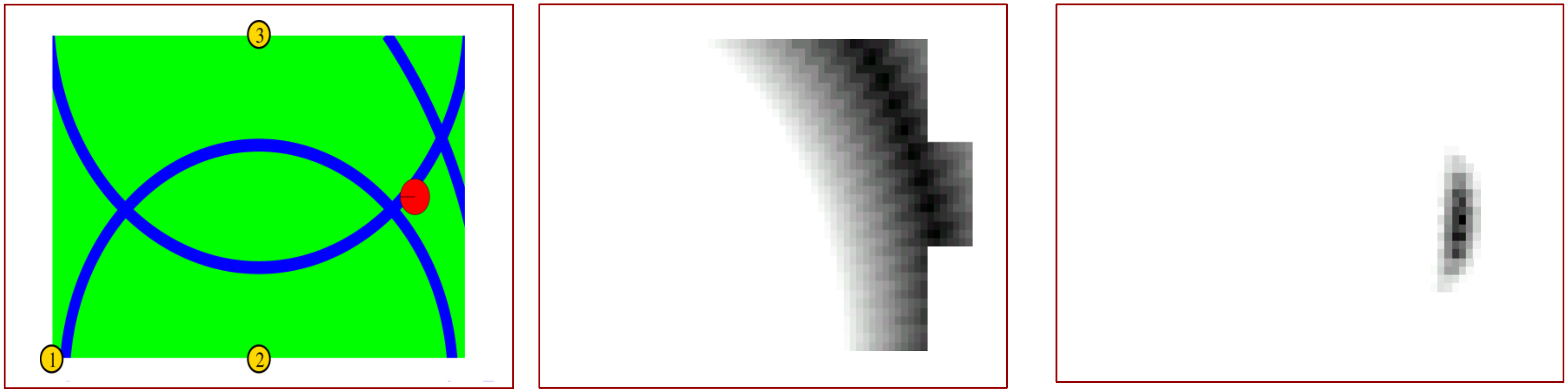
Importance Sampling Principle

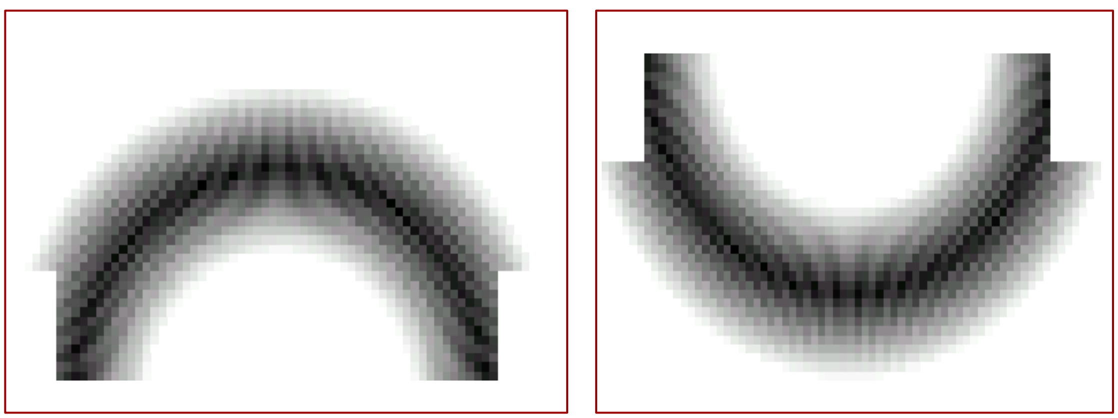
- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f"
- $w = f / g$
- f is often called target
- g is often called proposal
- Pre-condition: $f(x) >= 0 \qquad g(x) > 0$



Landmark Example

Example

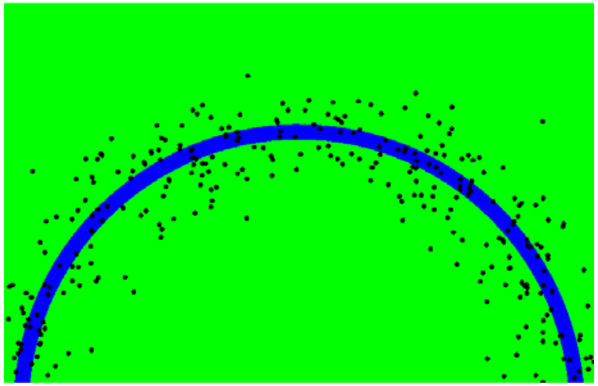


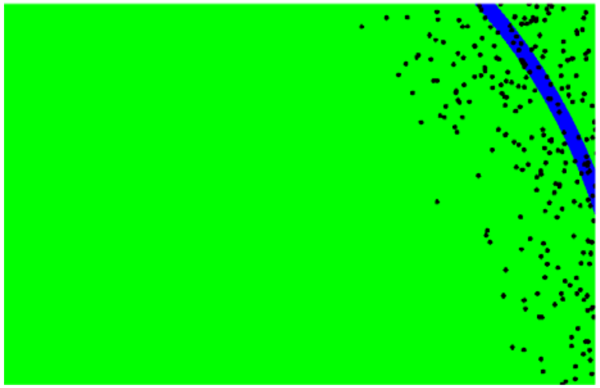


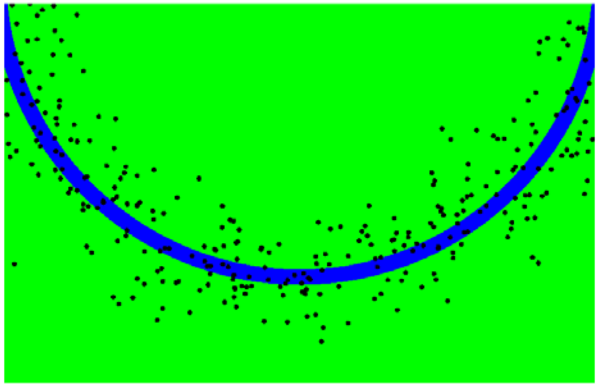
Wanted: samples distributed according to $p(x \mid z_1, z_2, z_3)$

Example

We can draw samples from $p(x|z_l)$ by adding noise to the detection parameters.







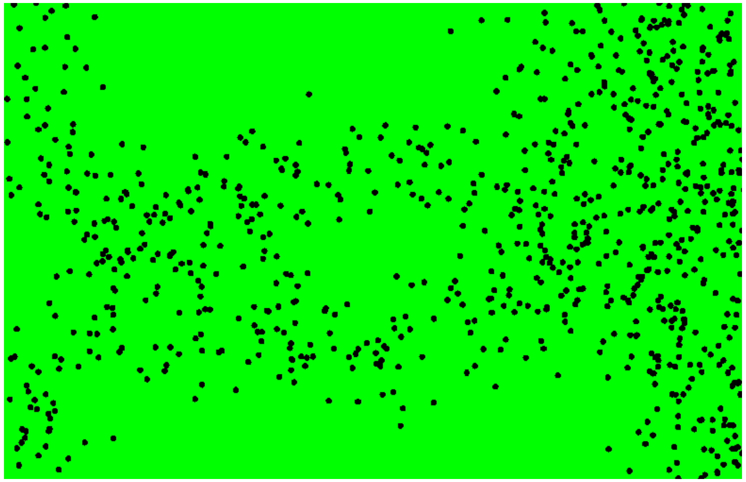
Importance Sampling

Target distribution f : $p(x \mid z_1, z_2, \ldots, z_n) = \dfrac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$

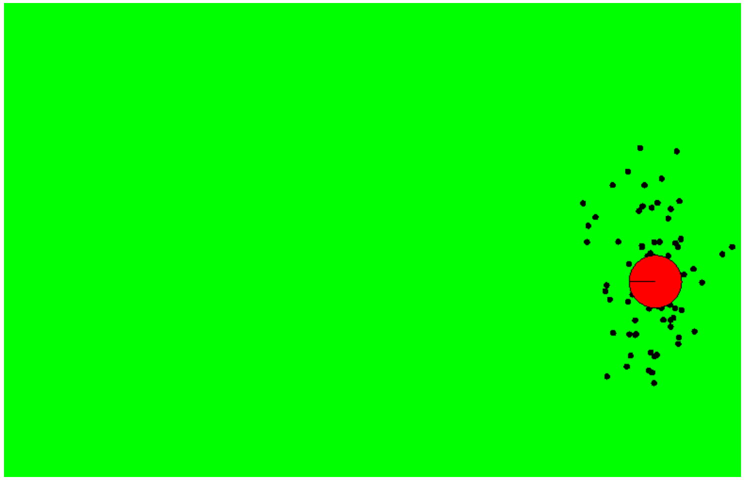
Sampling distribution g : $p(x \mid z_l) = \dfrac{p(z_l \mid x) p(x)}{p(z_l)}$

Importance weights w : $\dfrac{f}{g} = \dfrac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \dfrac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$

Importance Sampling with Resampling



Weighted samples



After resampling

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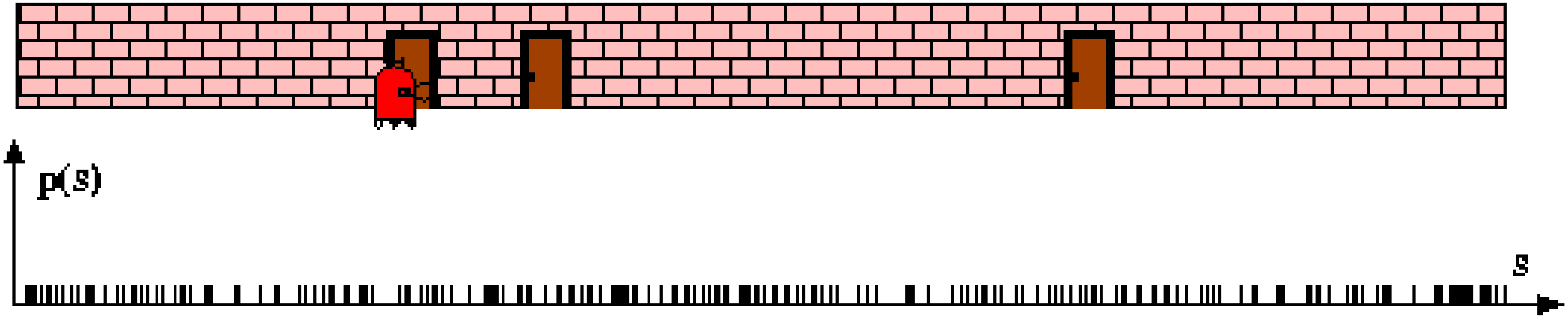
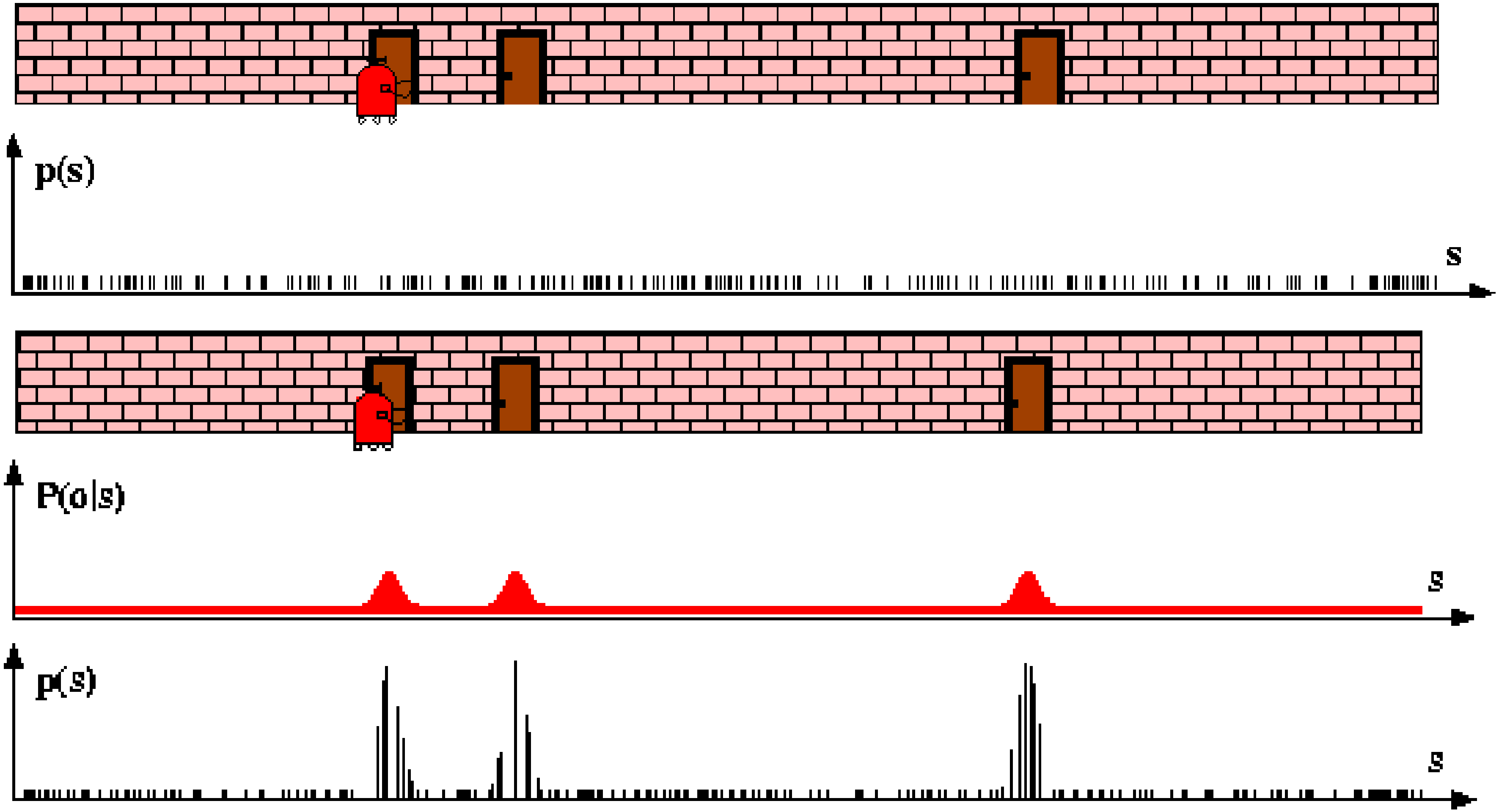
Particle Filter



Particle Filter: Sensor Information

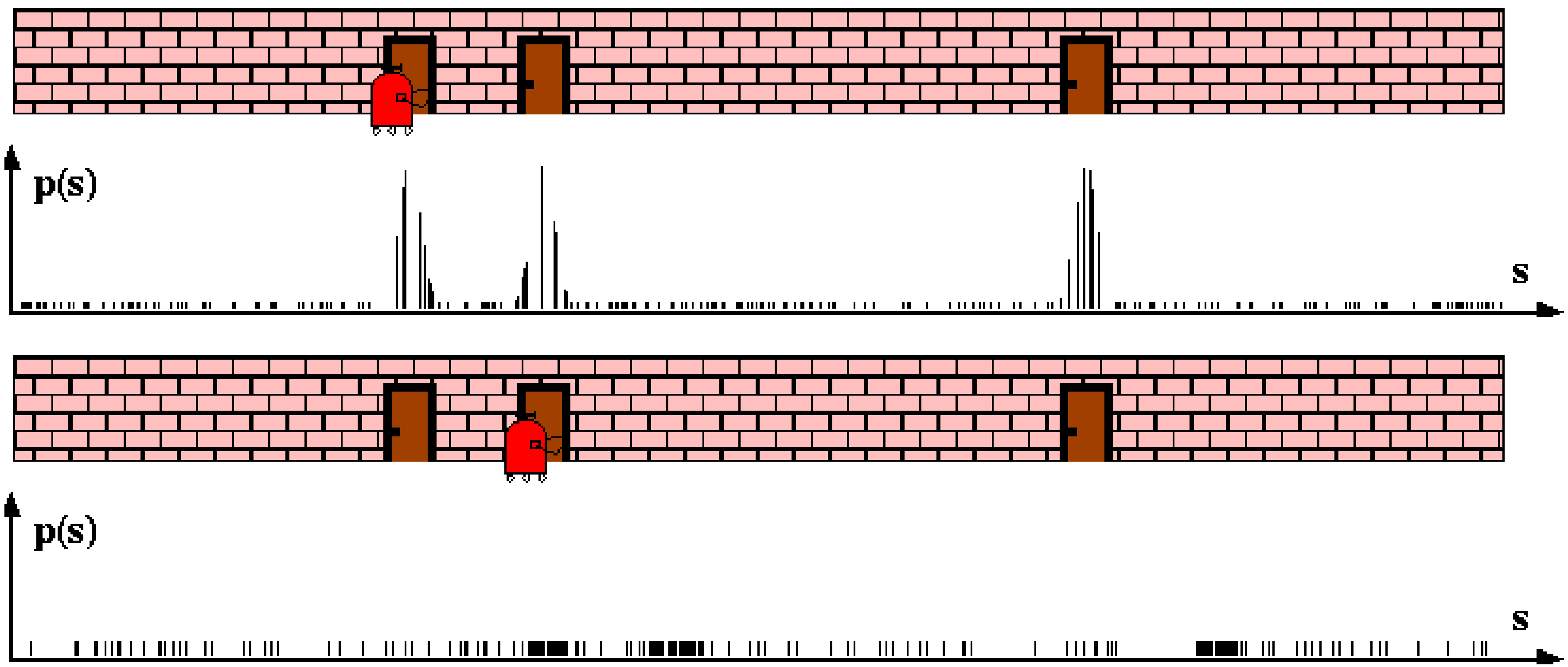
$$
\boxed{
\begin{aligned}
Bel(x) \quad &\leftarrow \quad \alpha\ p(z \mid x)\, Bel^{-}(x) \\
w \quad &\leftarrow \quad \frac{\alpha\ p(z \mid x)\, Bel^{-}(x)}{Bel^{-}(x)} \quad = \quad \alpha\ p(z \mid x)
\end{aligned}
}
$$

Particle Filter: Sensor Information



Particle Filter: Robot Motion

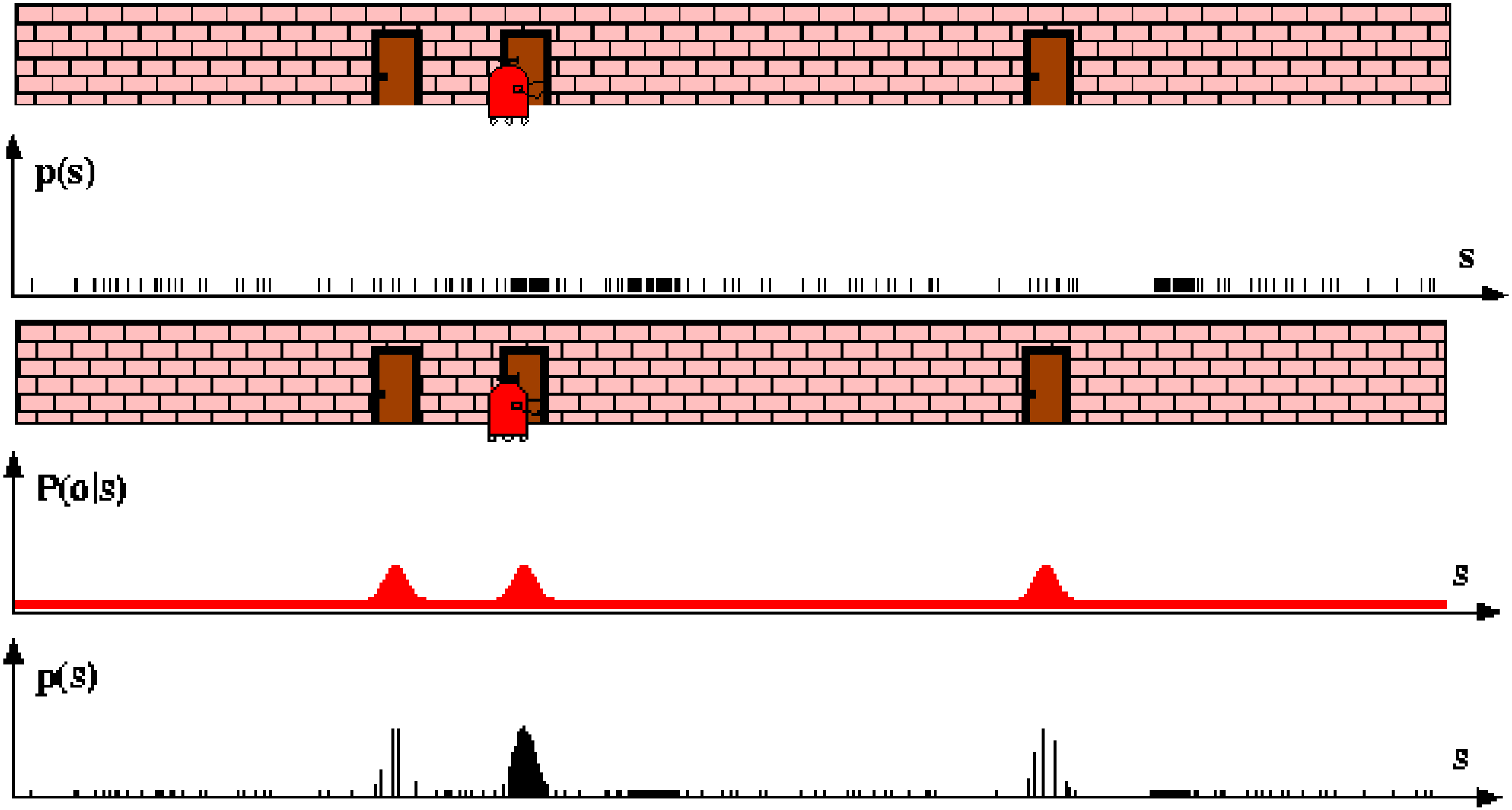
$$Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x') \quad \mathrm{d}x'$$



Particle Filter: Sensor Information

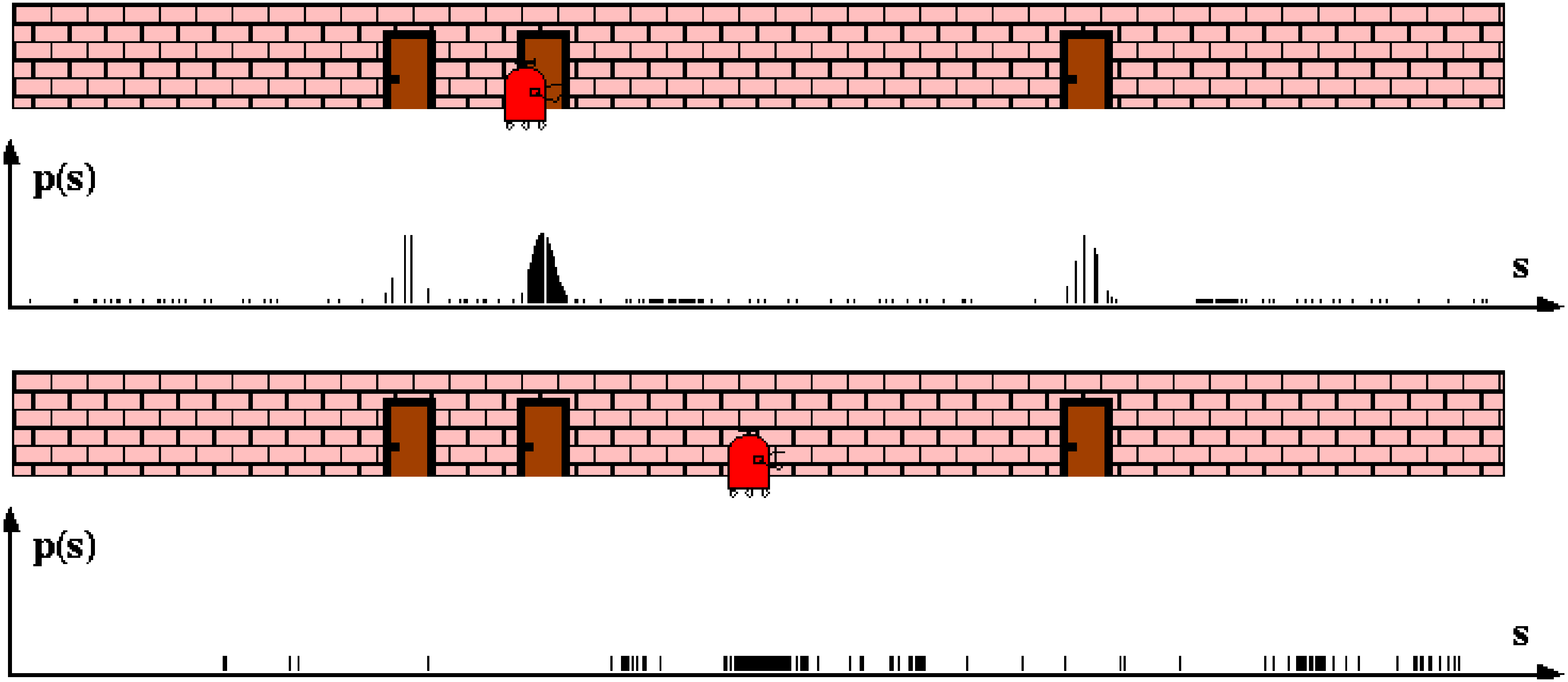
$$\boxed{\begin{array}{lcll} Bel(x) & \leftarrow & \alpha\ p(z \mid x)\, Bel^{-}(x) & \\ w & \leftarrow & \dfrac{\alpha\ p(z \mid x)\, Bel^{-}(x)}{Bel^{-}(x)} & = \quad \alpha\ p(z \mid x) \end{array}}$$

Particle Filter: Sensor Information



Particle Filter: Robot Motion

$$\boxed{Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x') \quad \mathrm{d}x'}$$



Particle Filter Algorithm

- Sample the next generation for particles using the proposal distribution

- Compute the importance weights :

  $$weight = target\ distribution\ /\ proposal\ distribution$$

- Resampling: “Replace unlikely samples by more likely ones”

- [Derivation of the MCL equations on the blackboard]

Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

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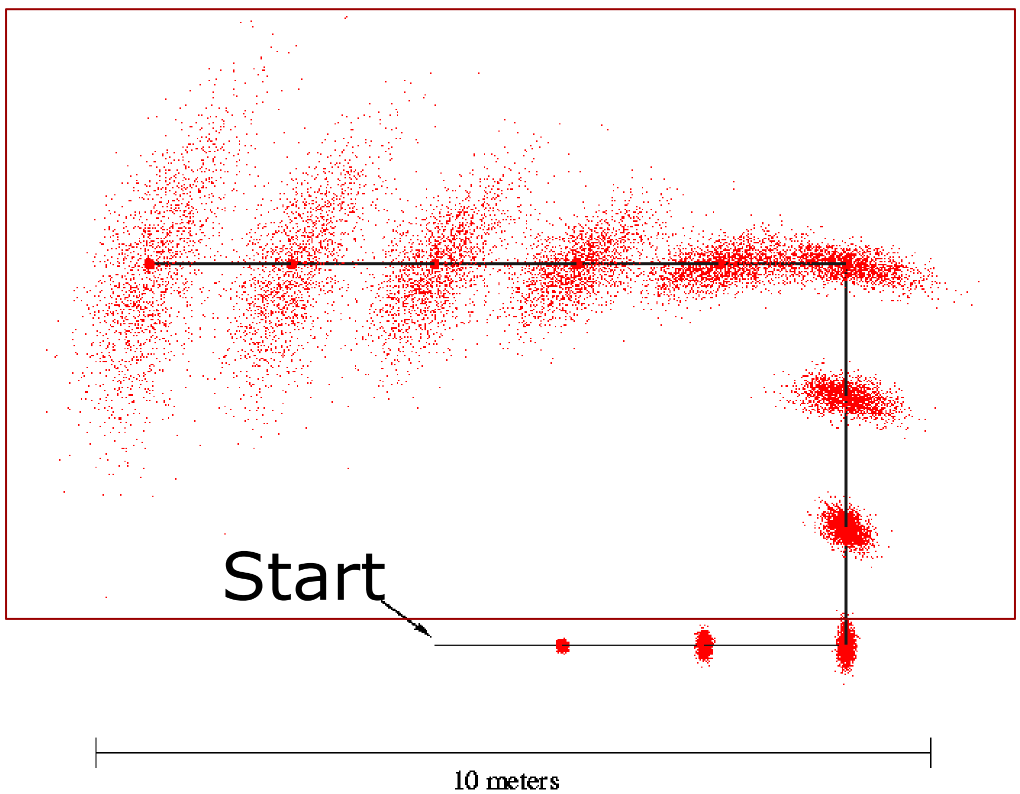
Monte Carlo Robot Localization

- Each particle is a potential pose of the robot

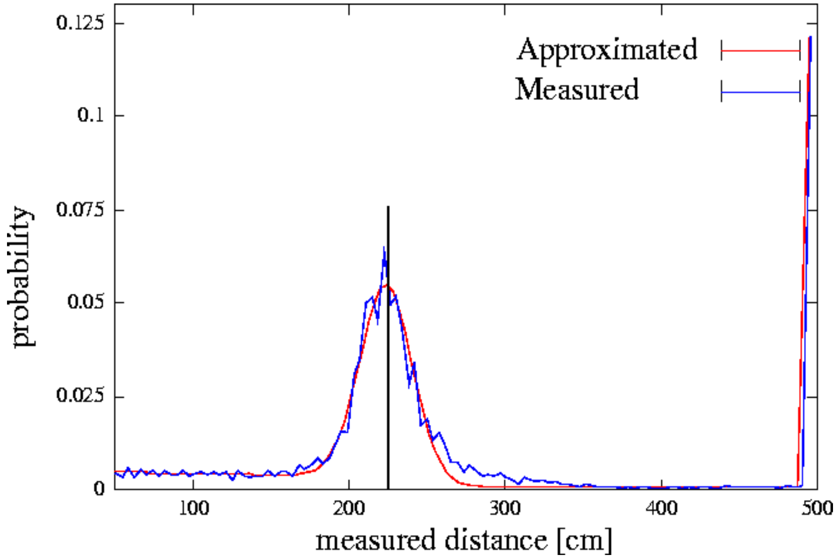
- Proposal distribution is the motion model of the robot (prediction step)

- The observation model is used to compute the importance weight (correction step)

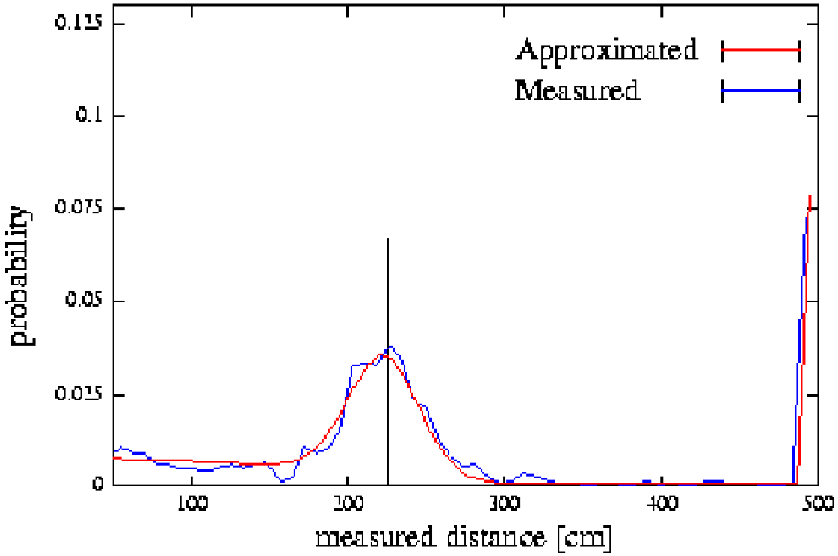
Motion Model



Proximity Sensor Model

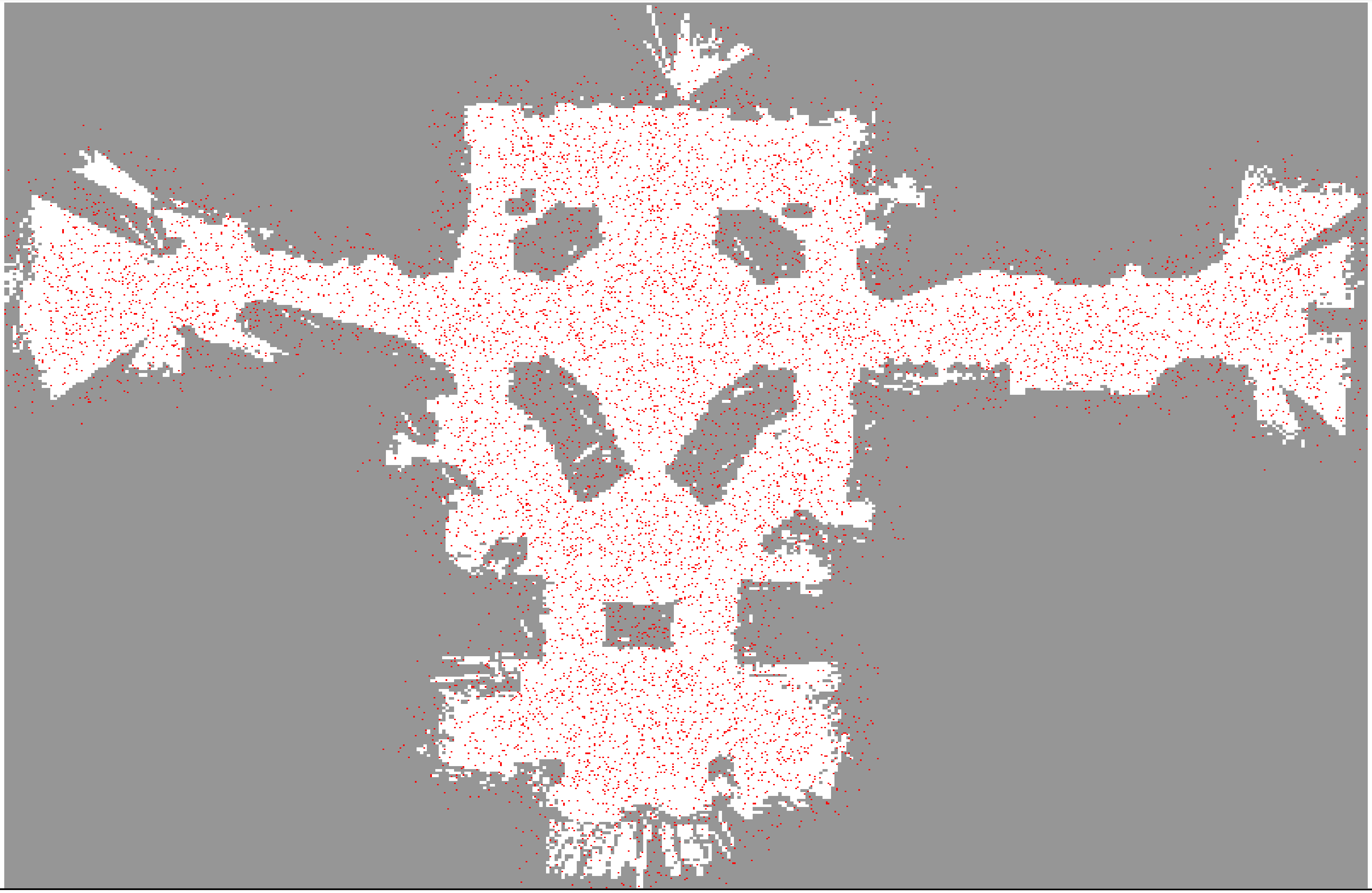


Laser sensor

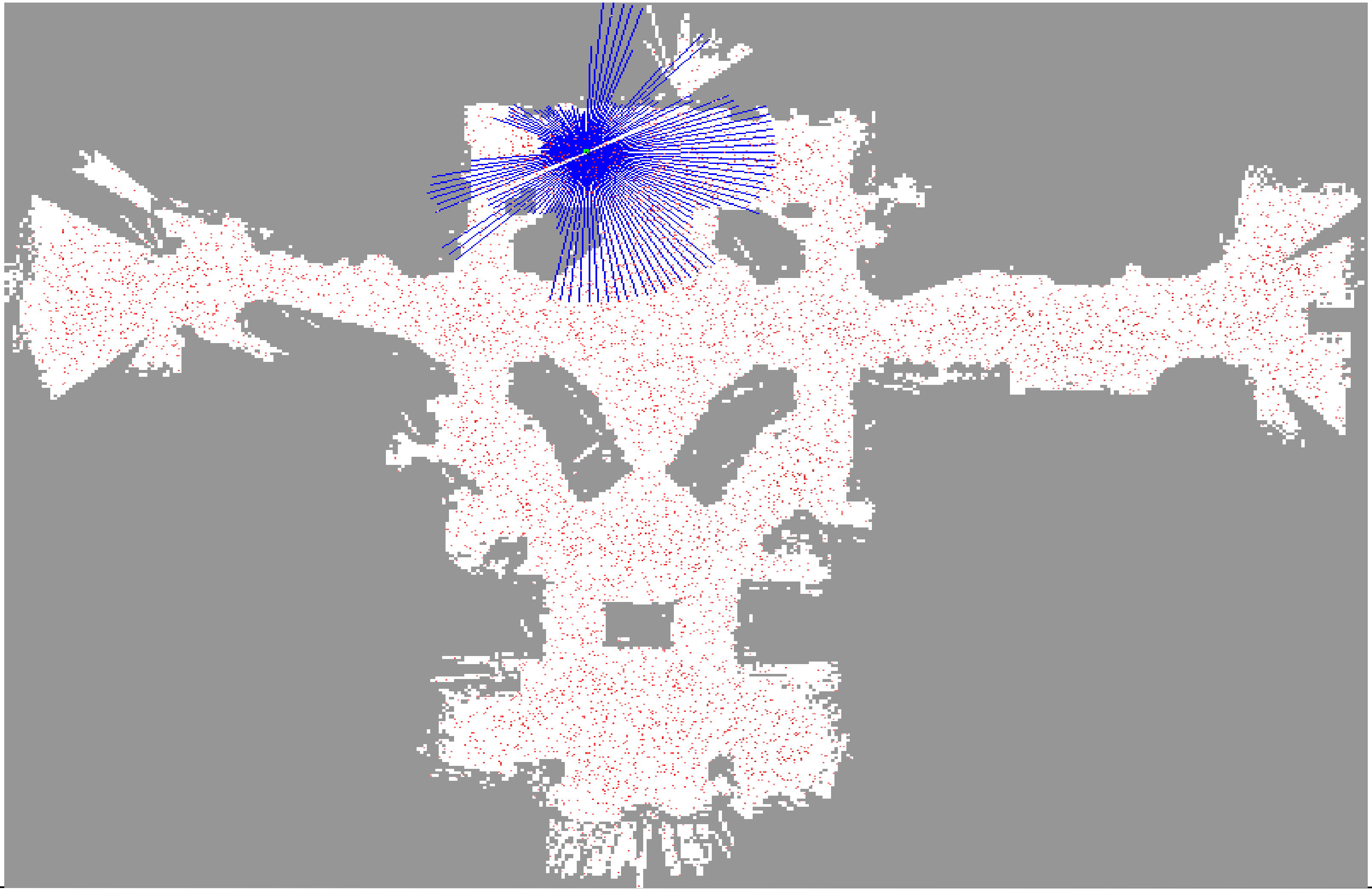


Sonar sensor

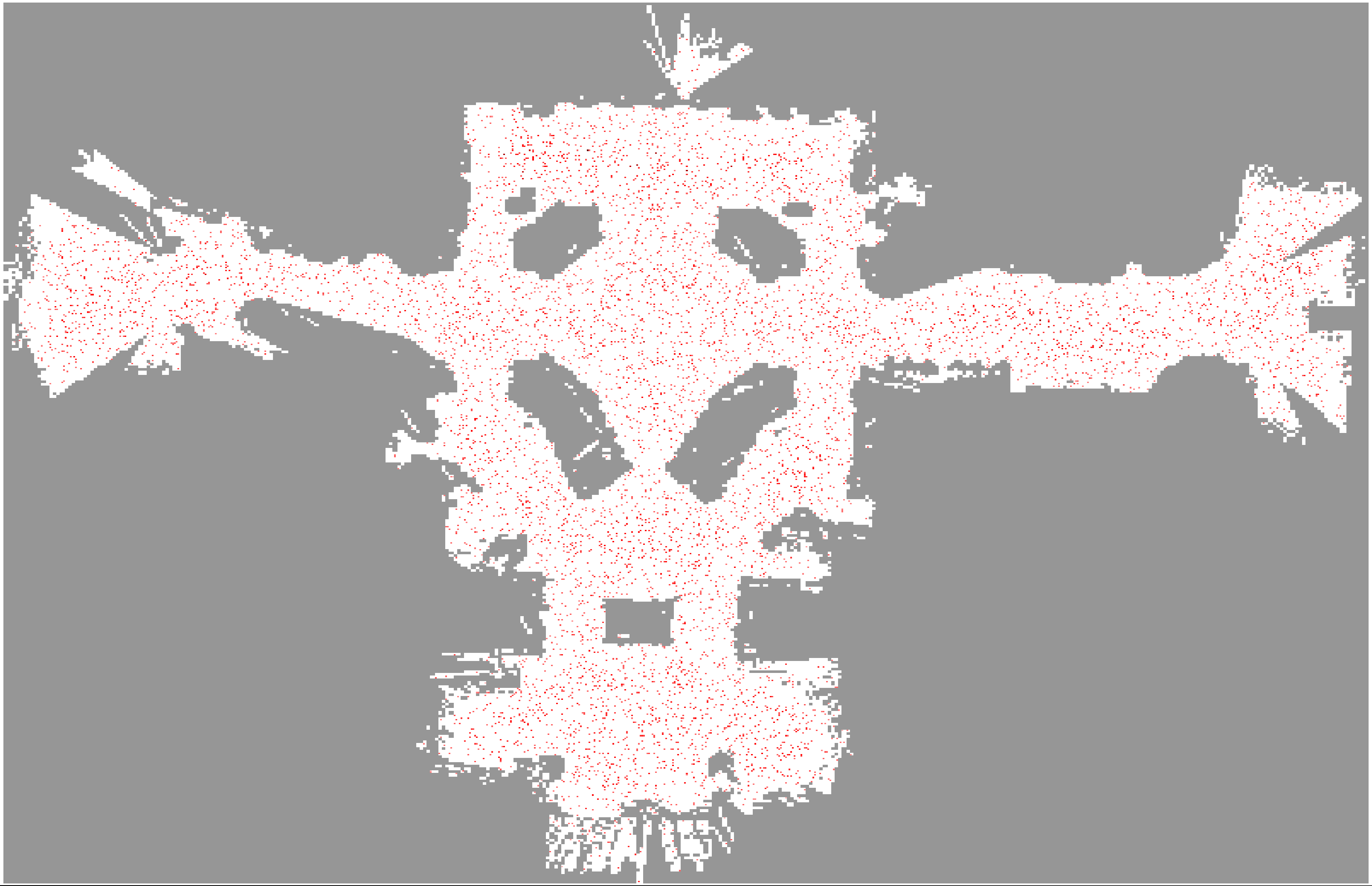
Example I-1



Example I-2

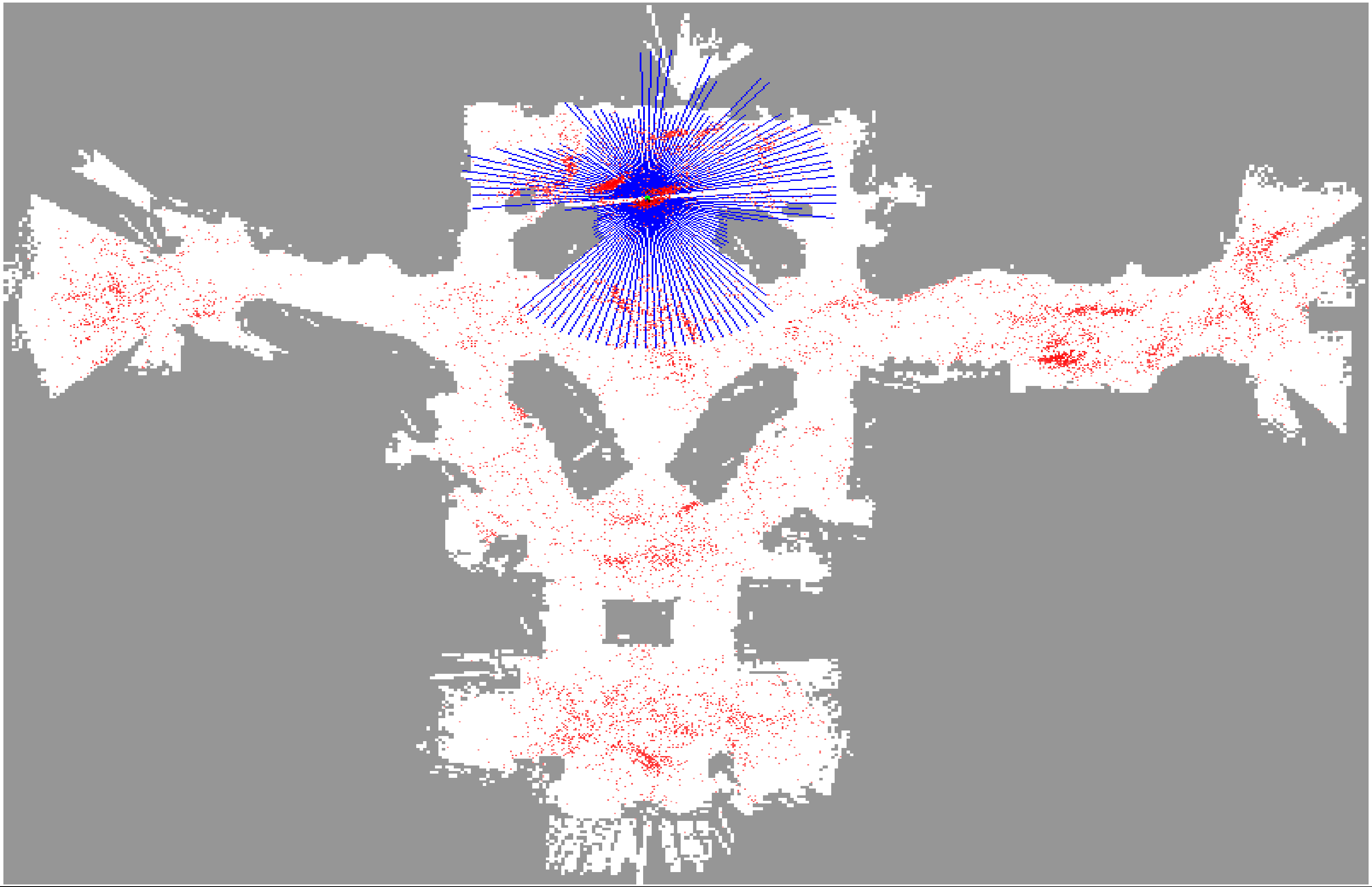


Example I-3



Example I-4

Example I-5

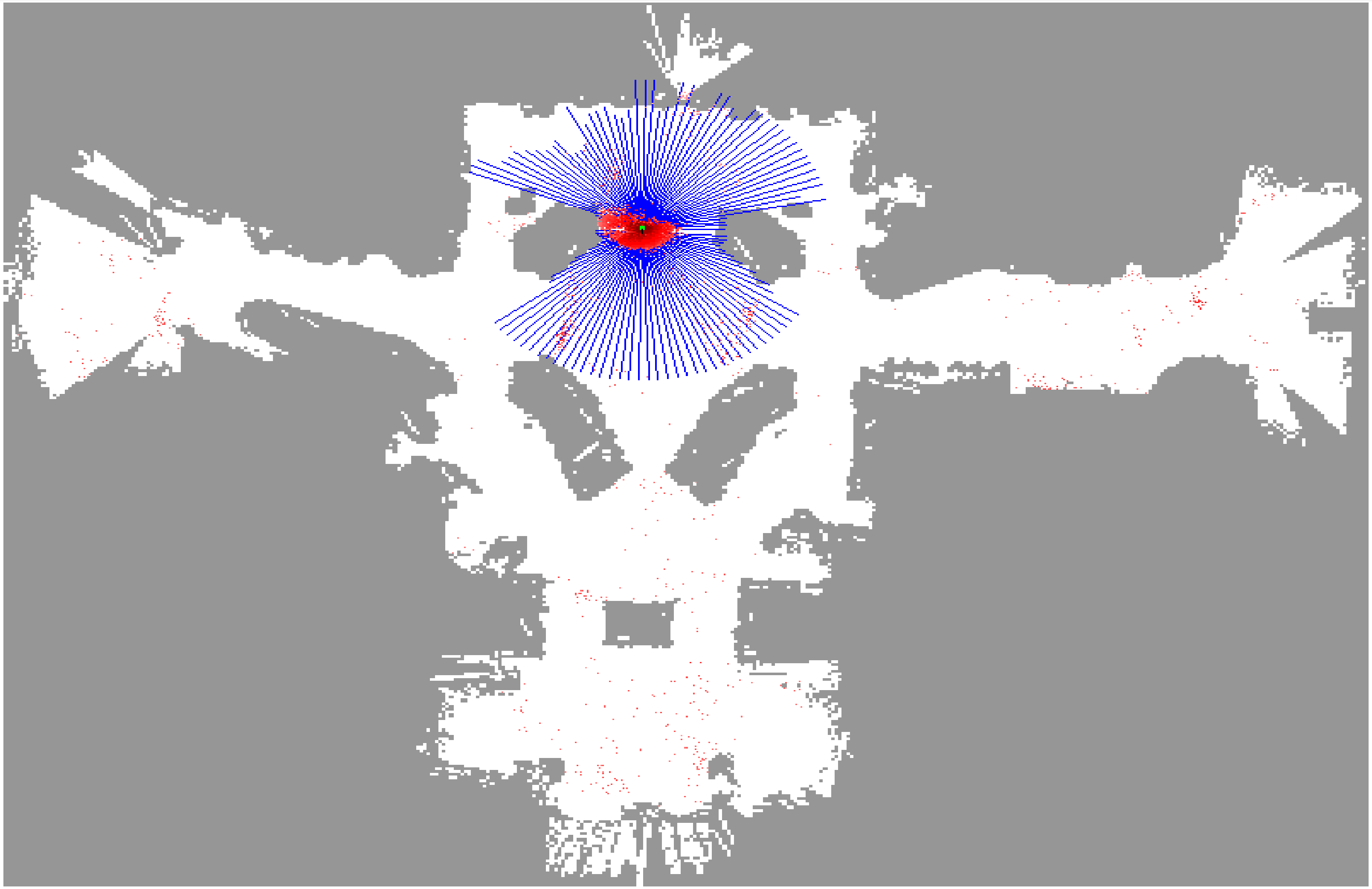


Example I-6

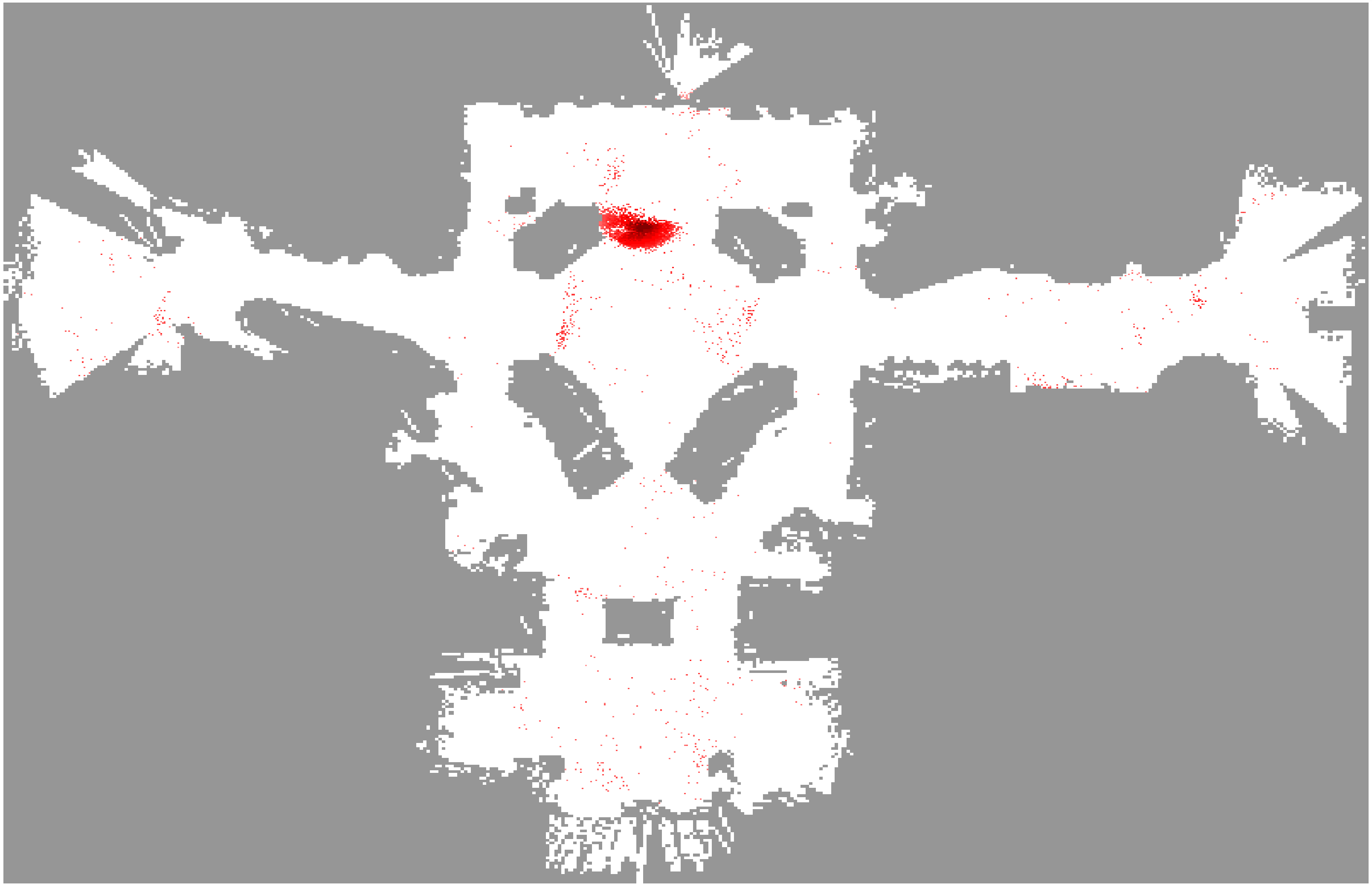
Example I-7

Example I-8

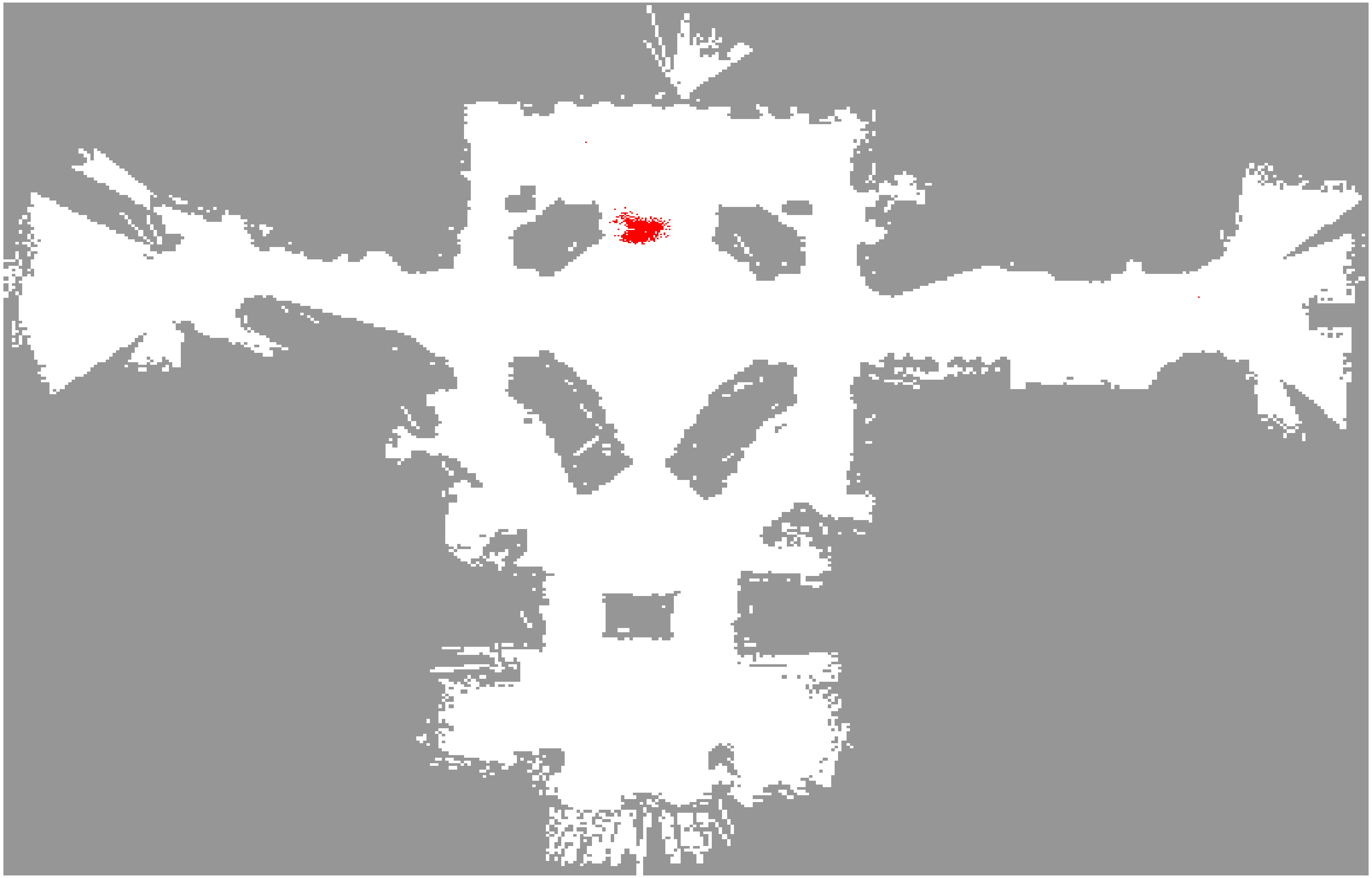
Example I-9



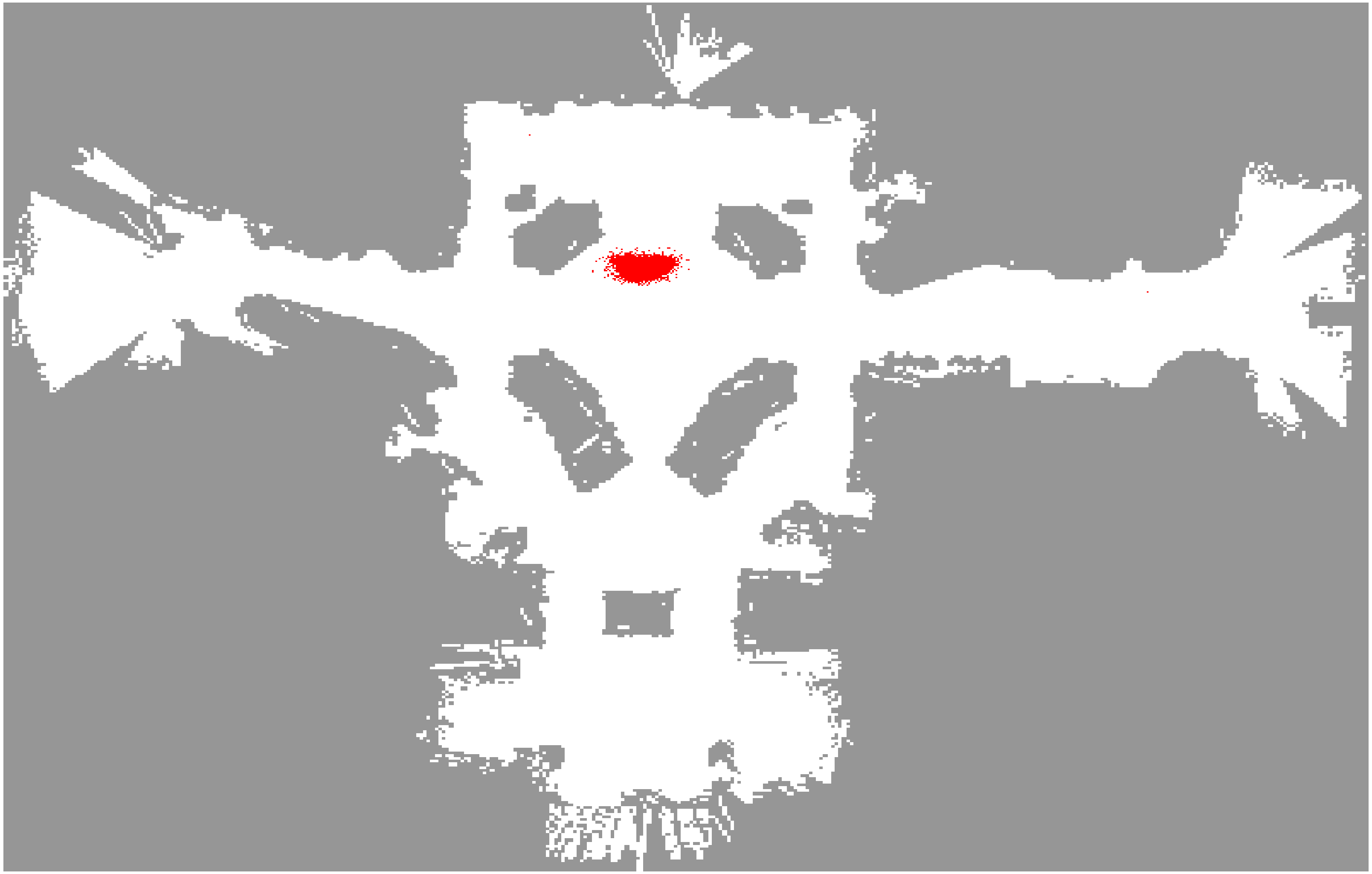
Example I-10



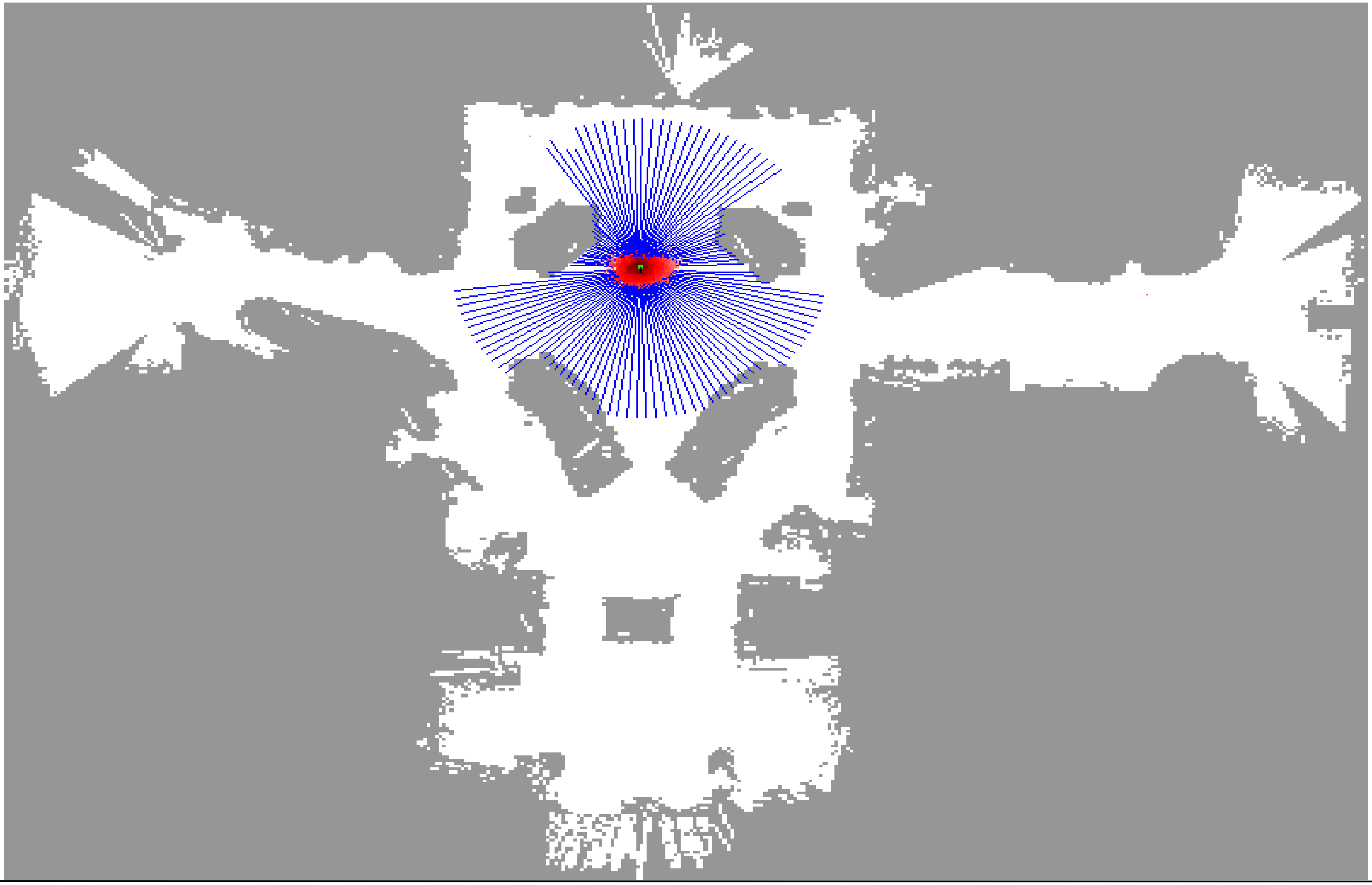
Example I-11



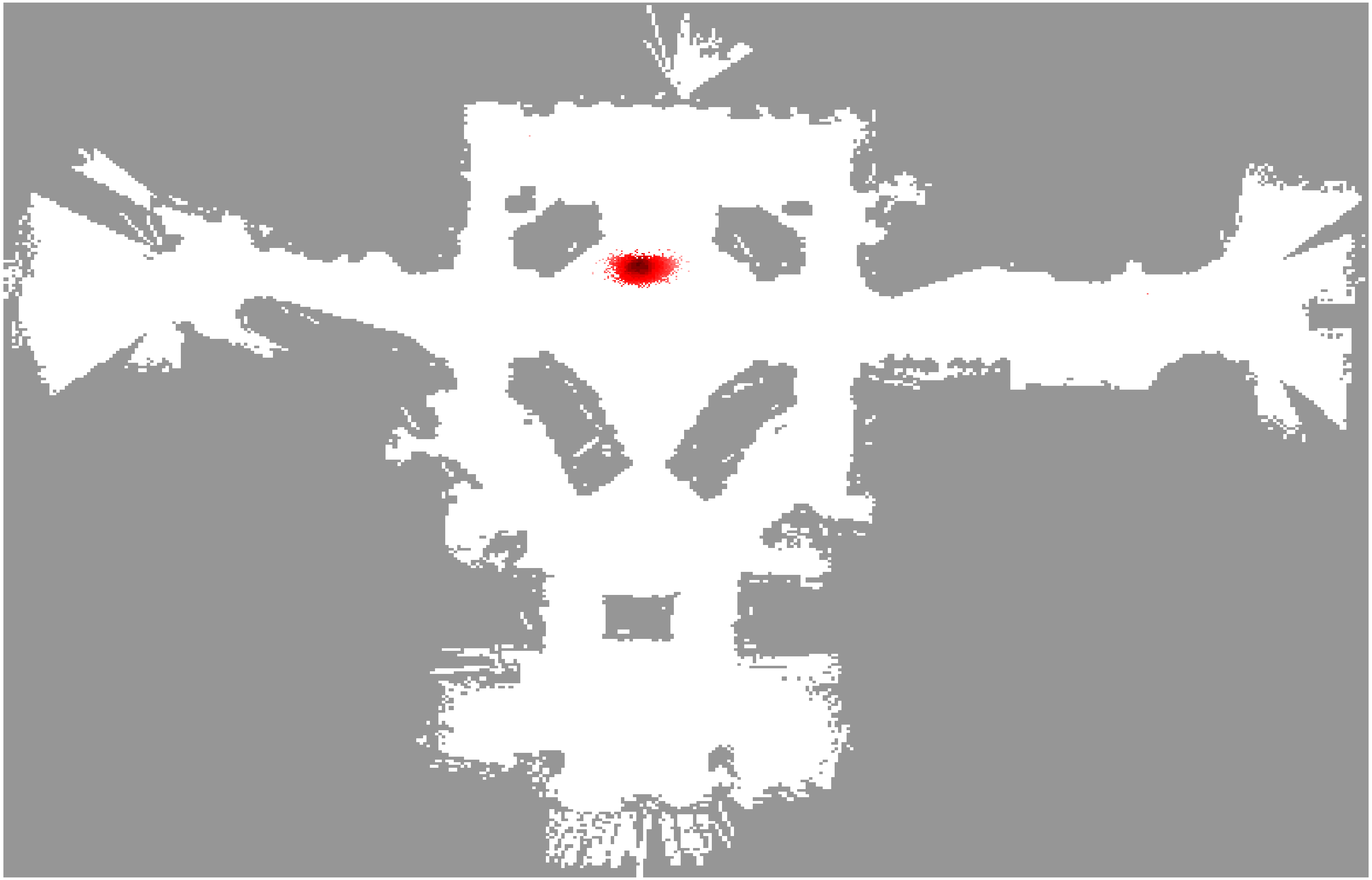
Example I-12



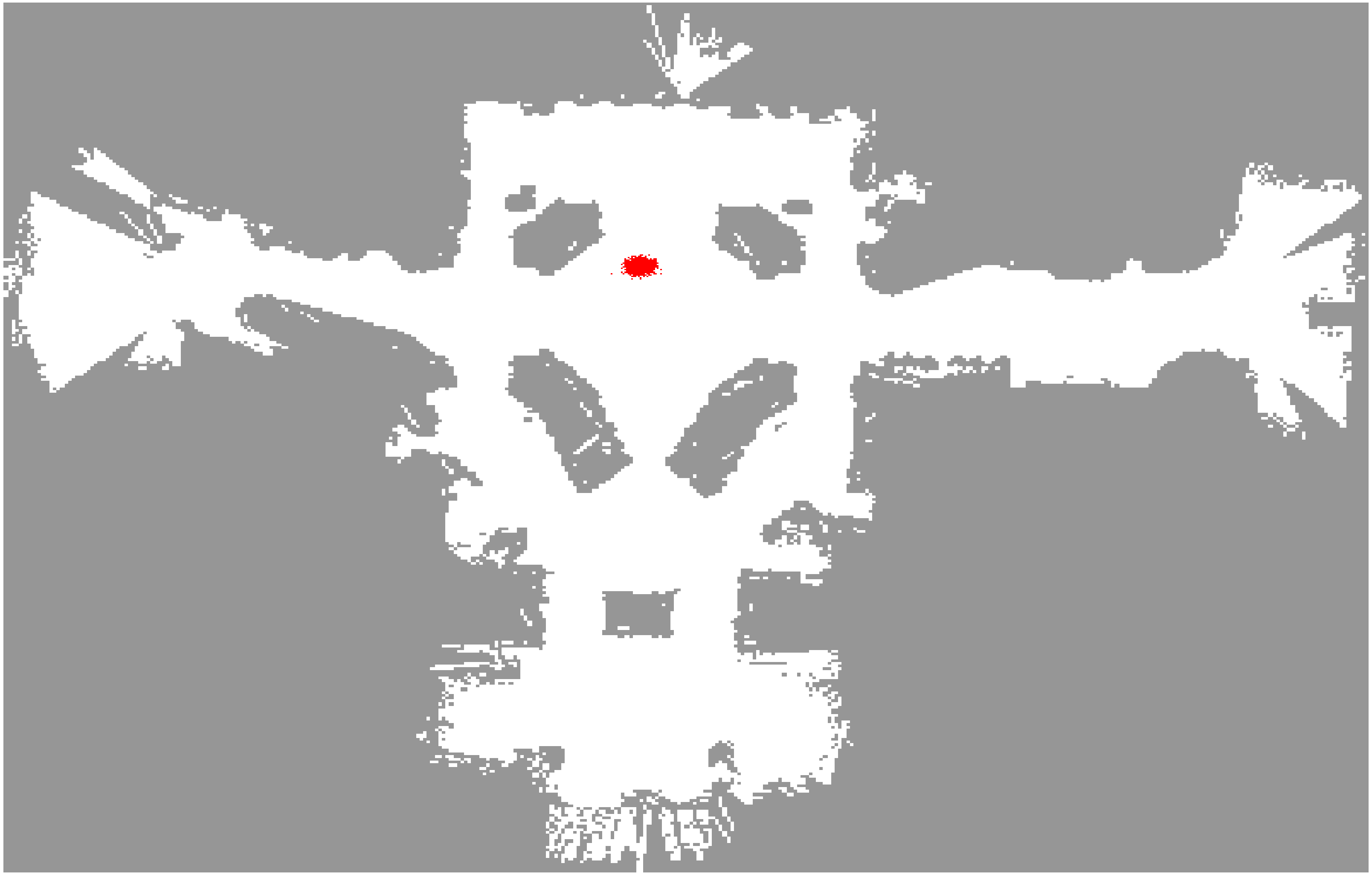
Example I-13



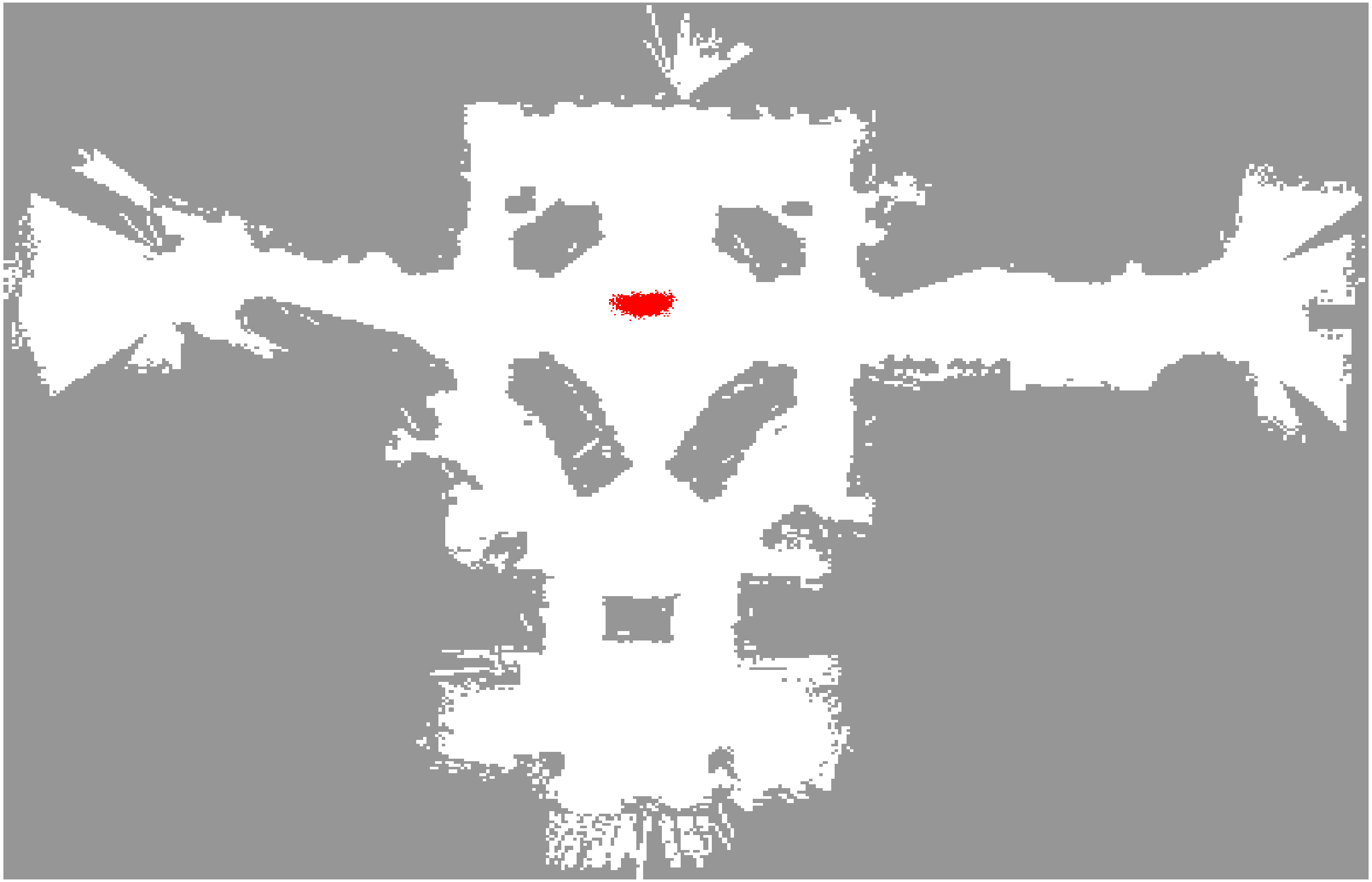
Example I-14



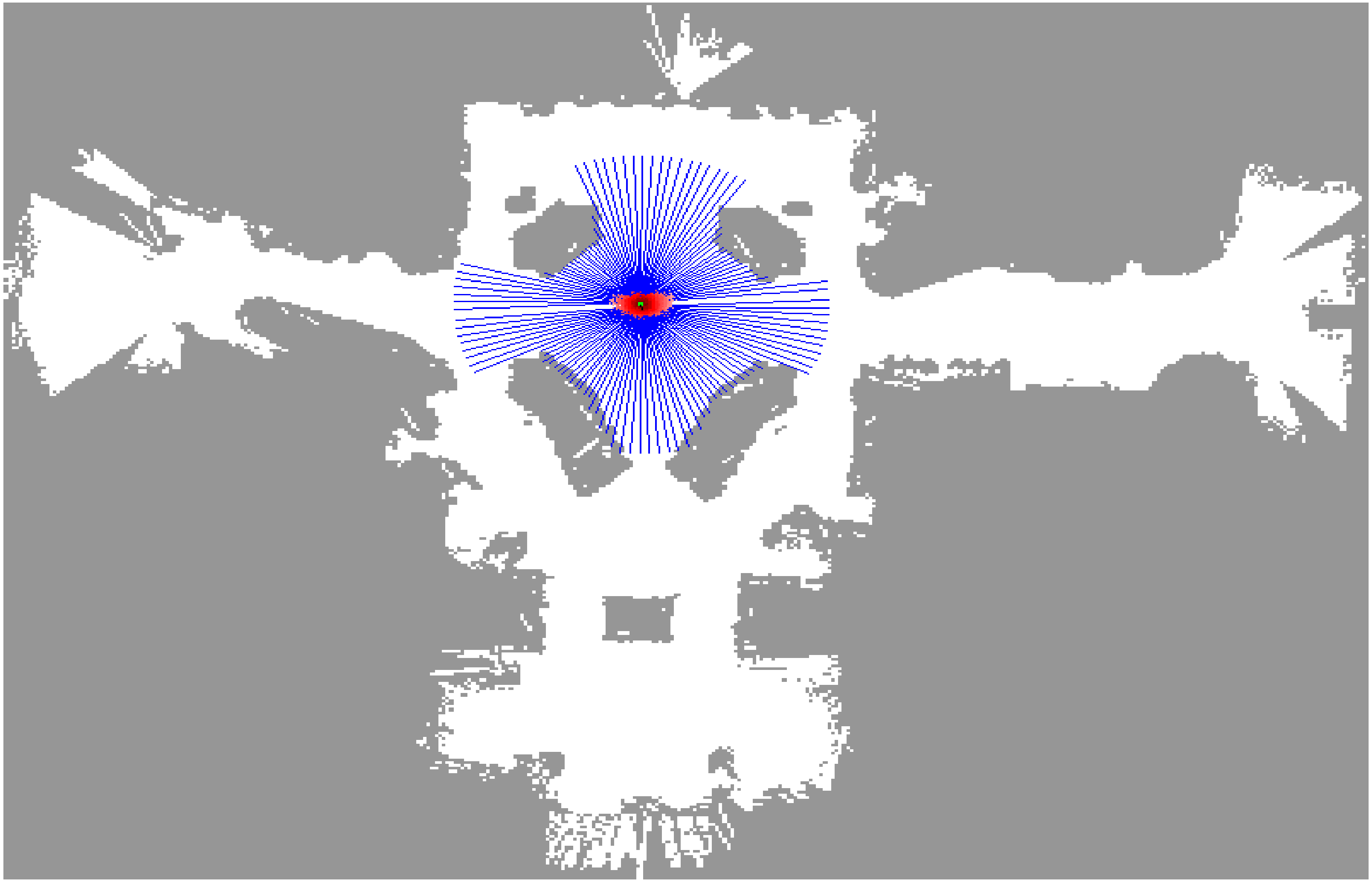
Example I-15



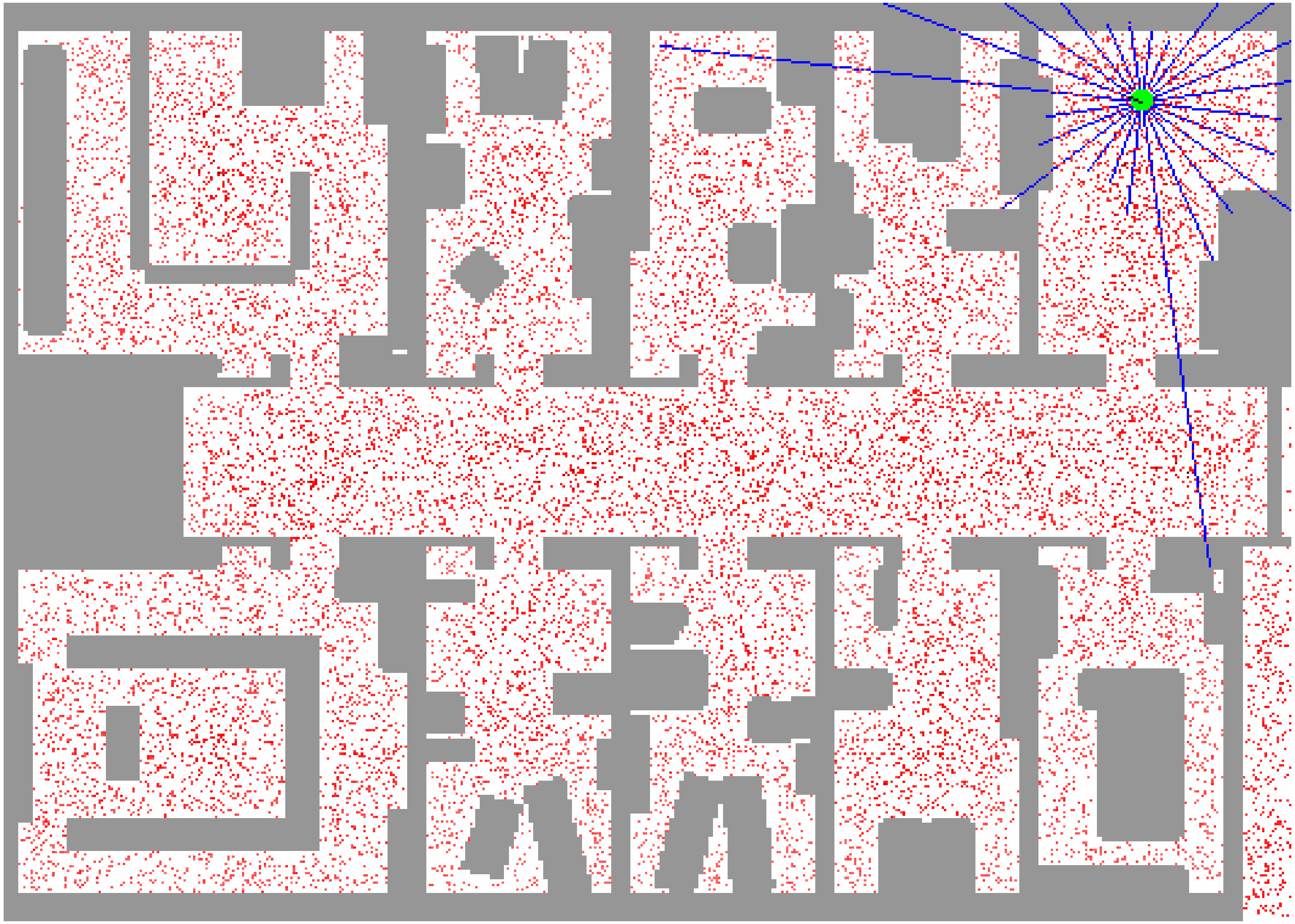
Example I-16



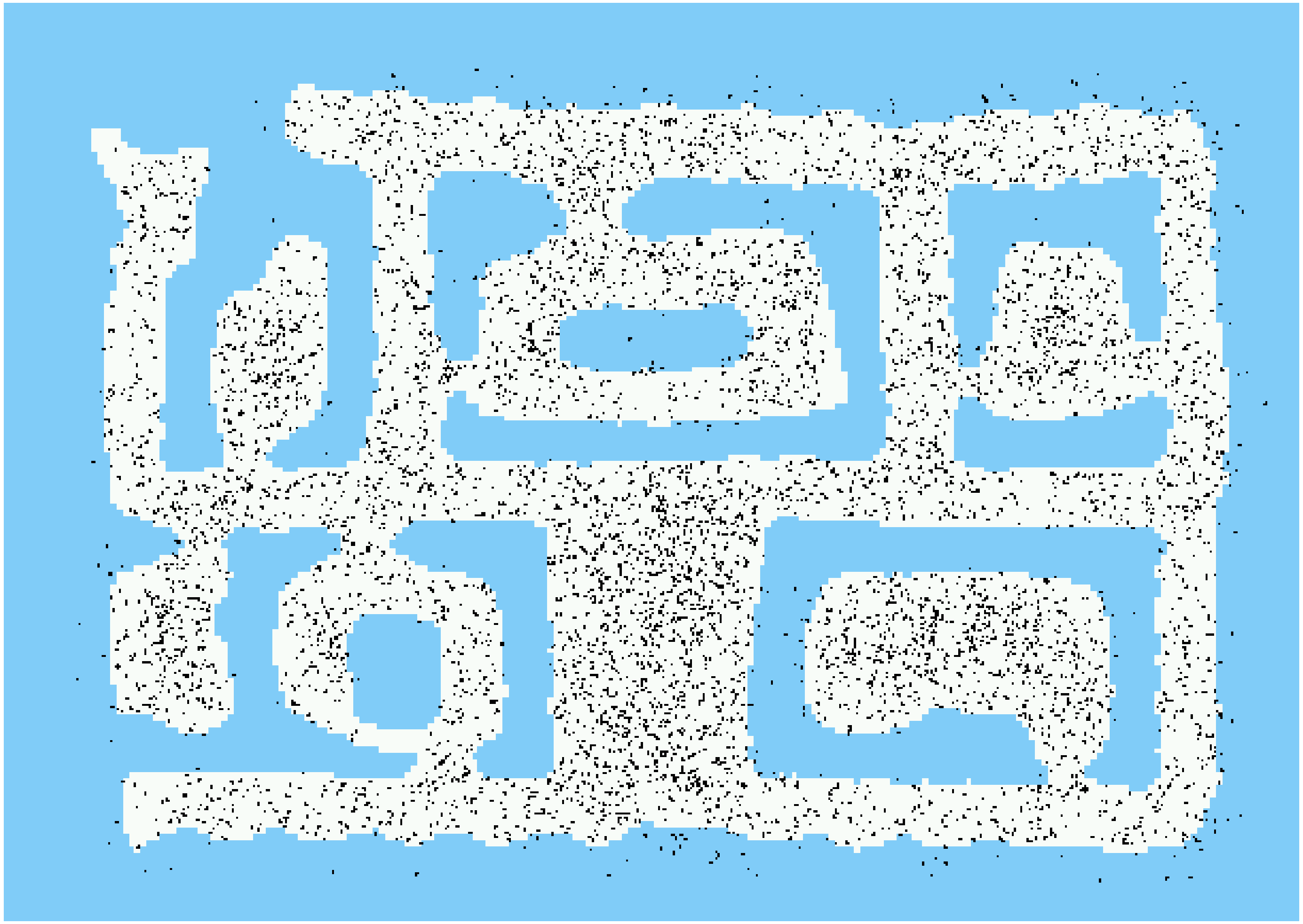
Example I-17



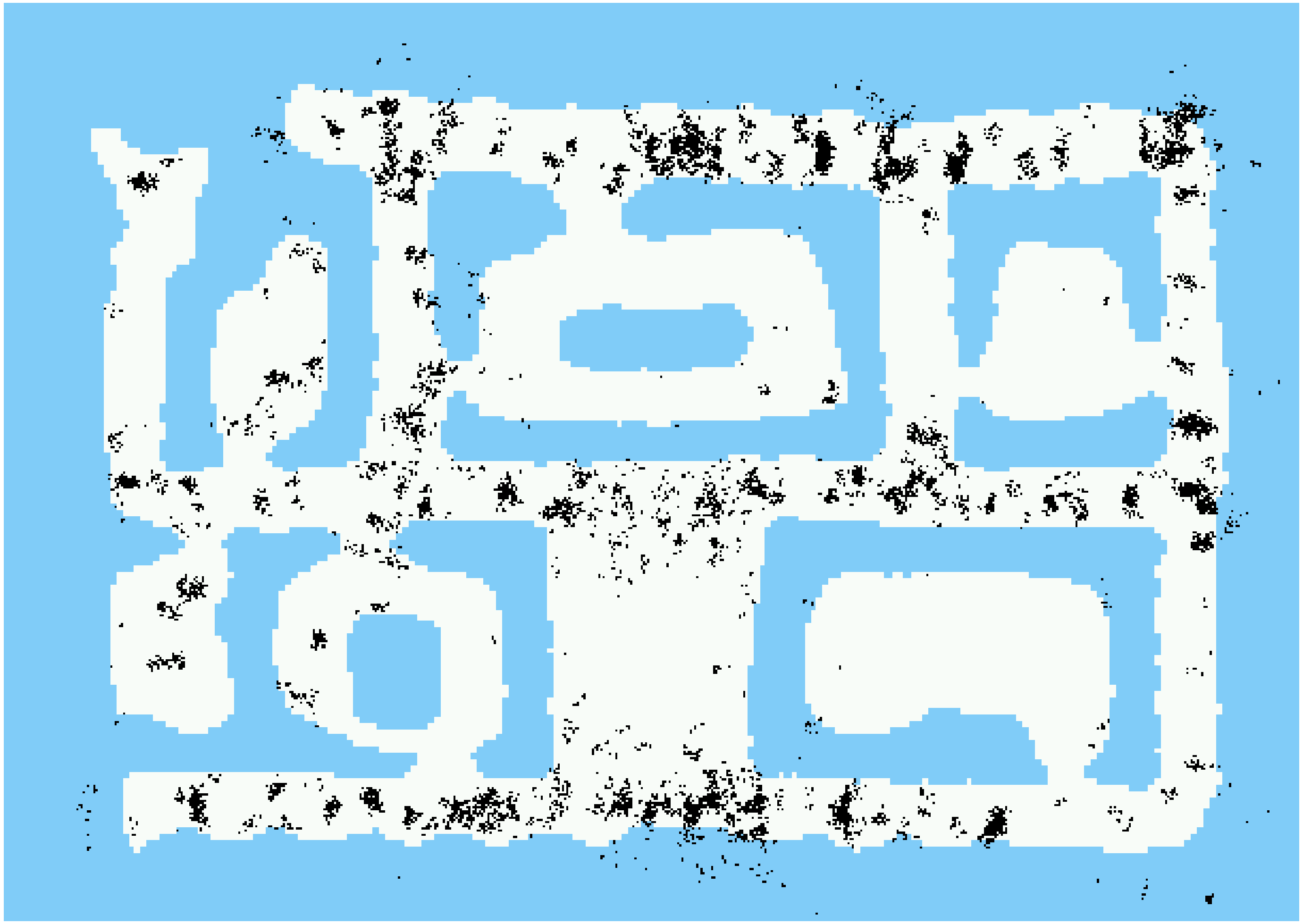
Example II (Sonar)



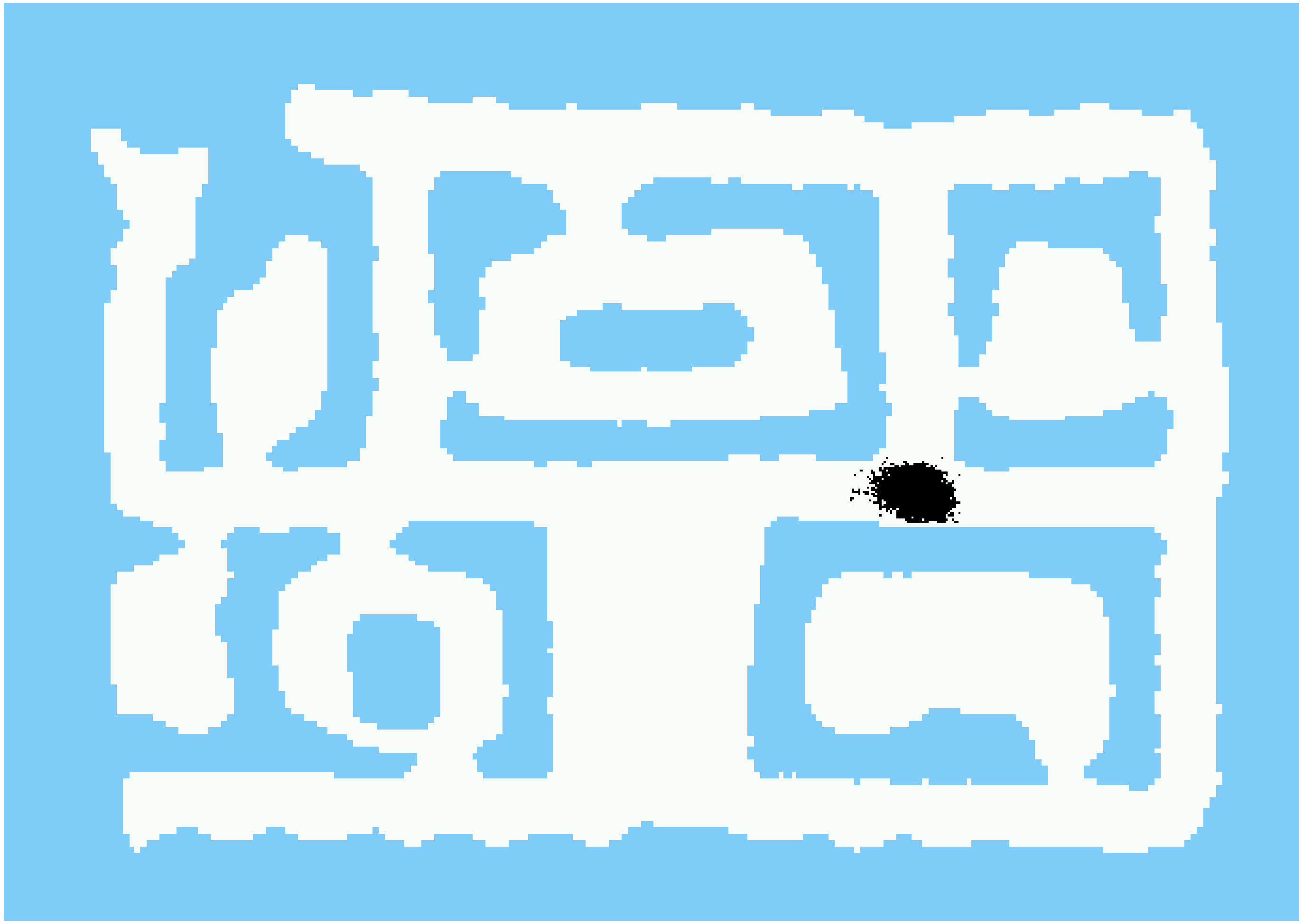
Example III (Initialization)



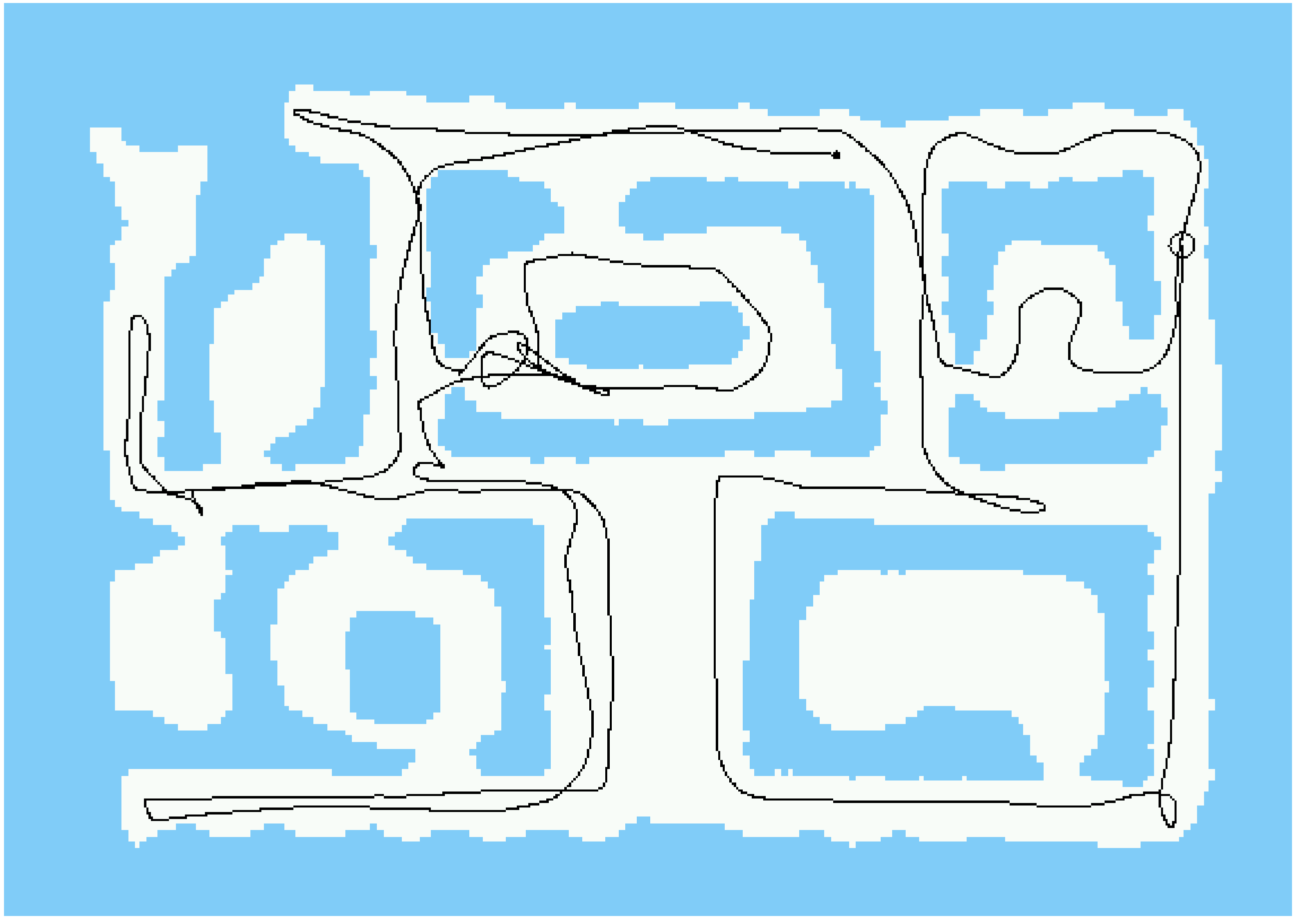
Example III (10 Ultrasound Scans)



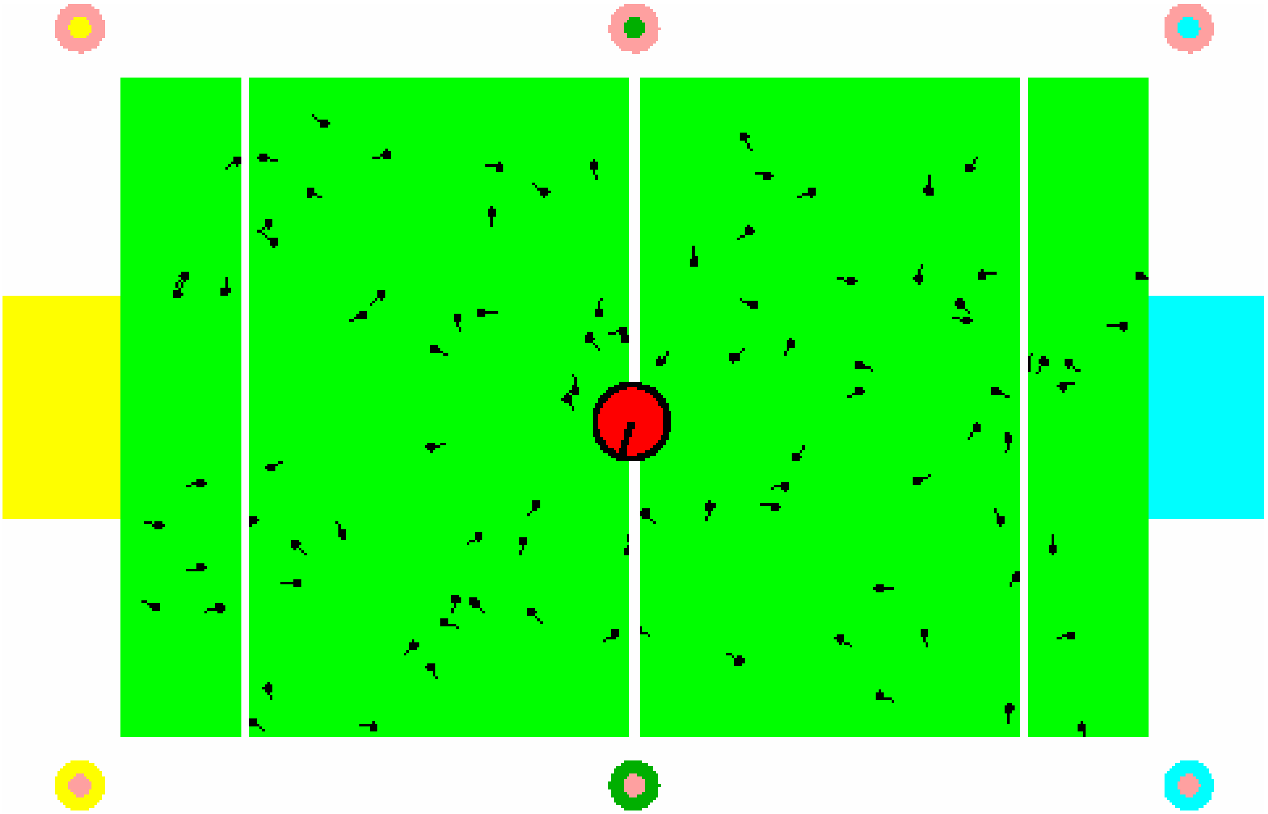
Example III (65 Ultrasound Scans)



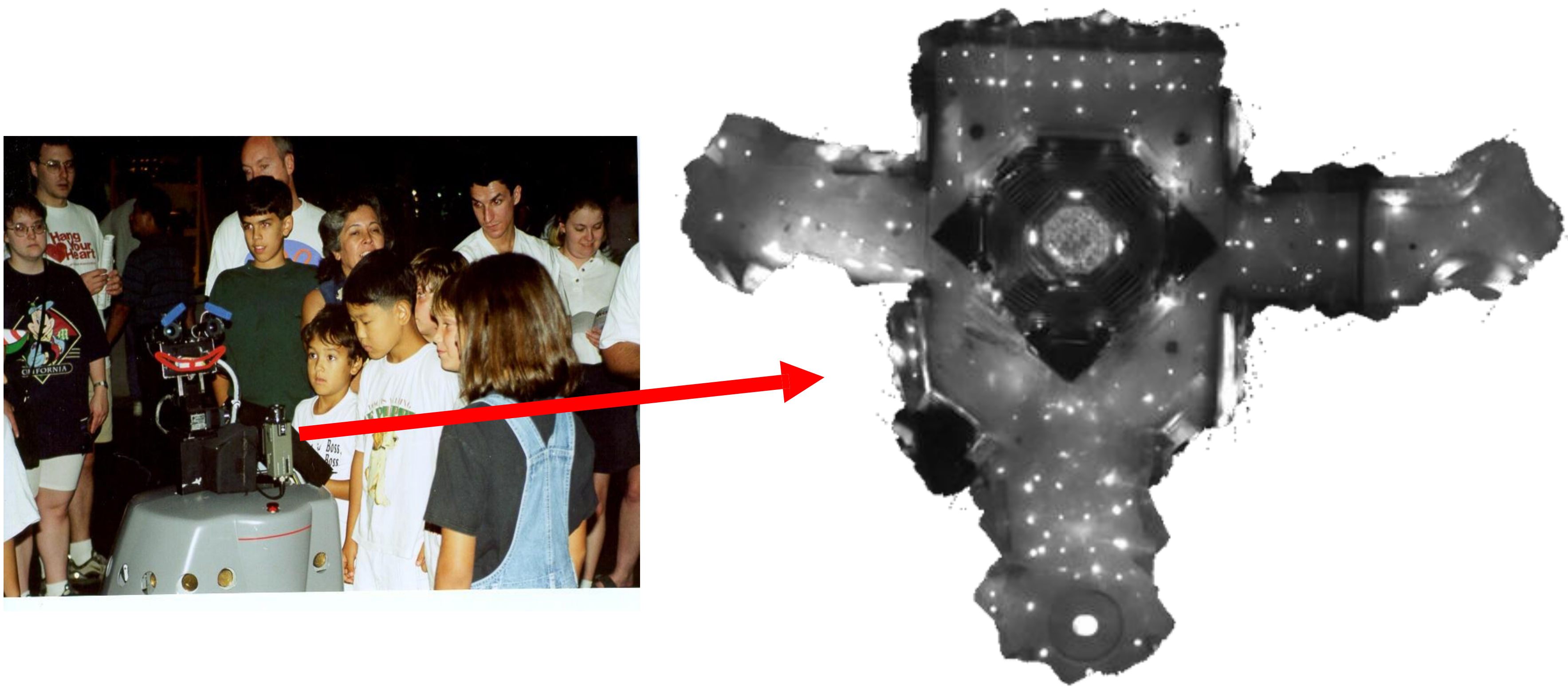
Example III (Path Estimation)



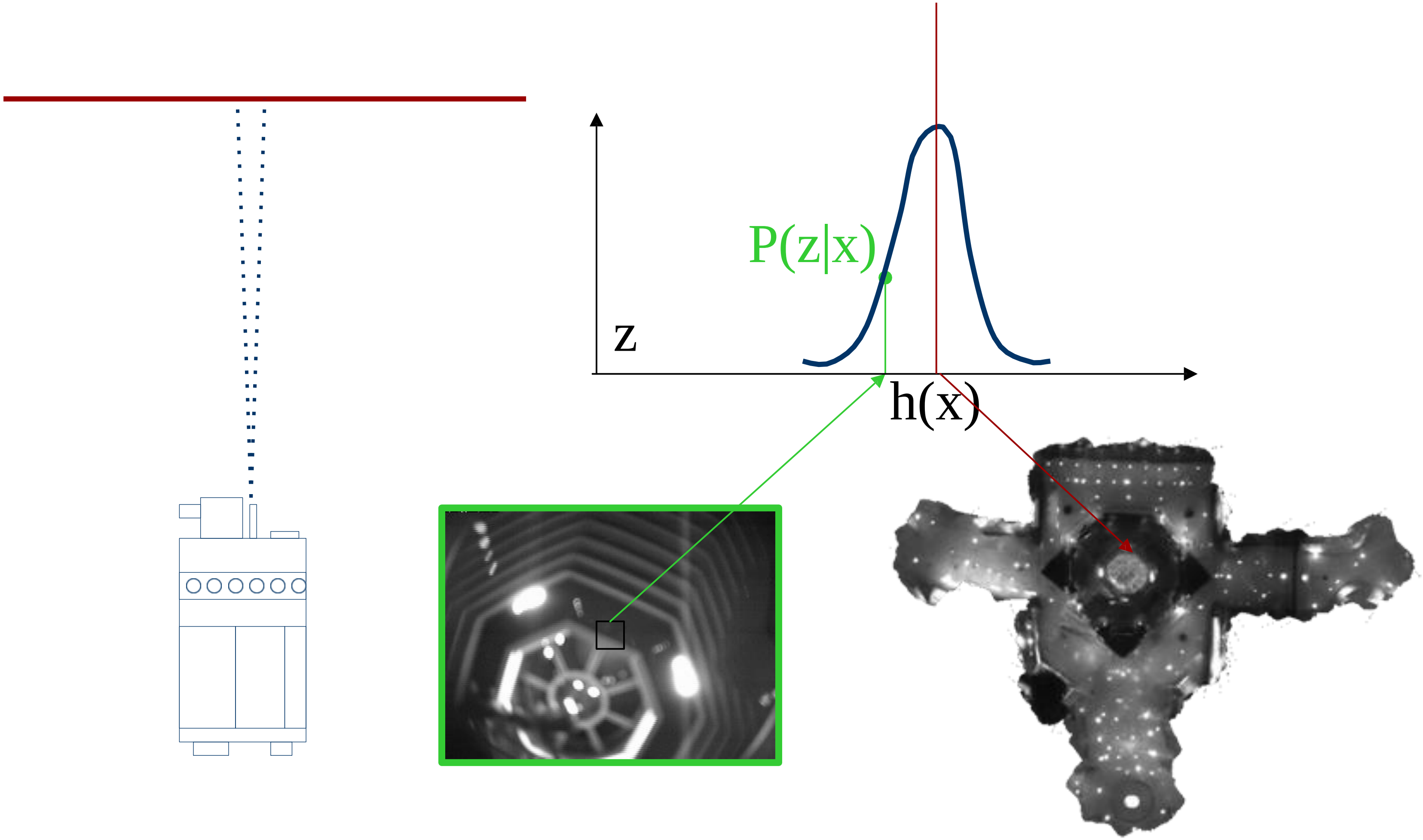
Example IV (Landmark-based)



Example V (Vision-based)

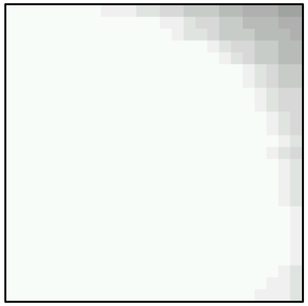


Example V (Vision-based)

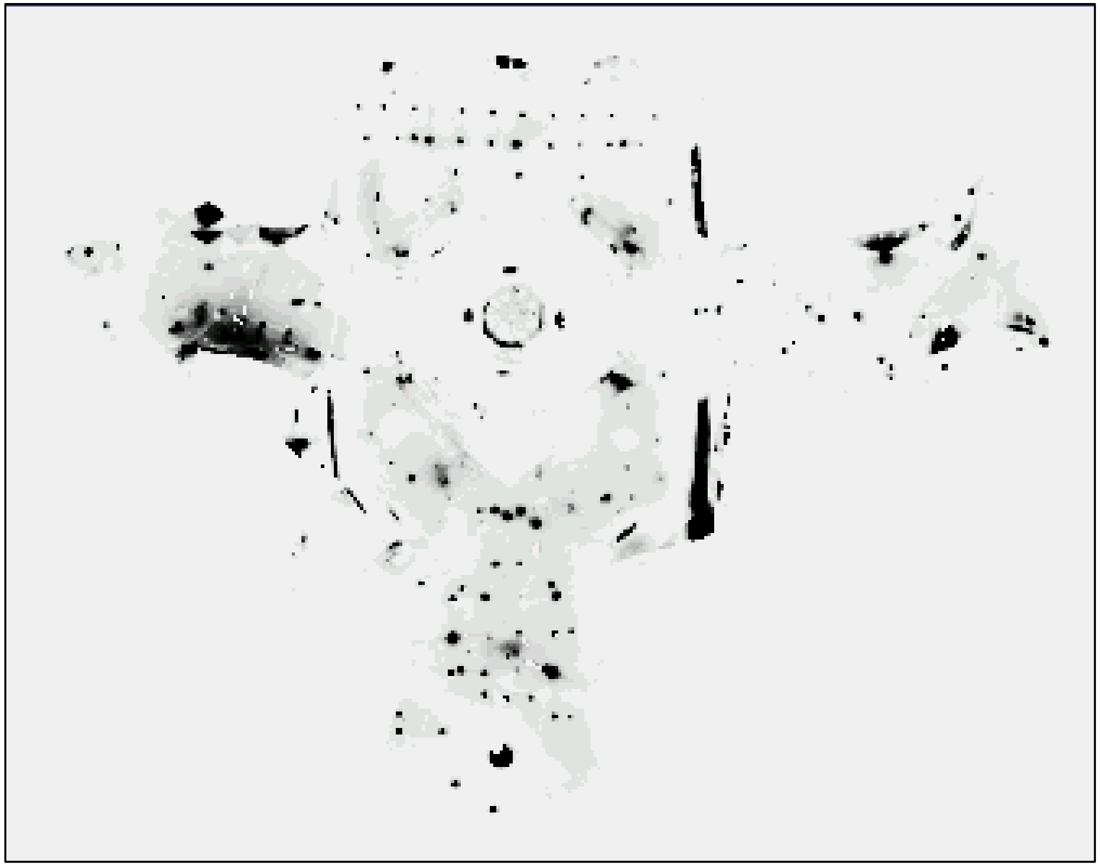


Example V (Under Light)

Measurement z:

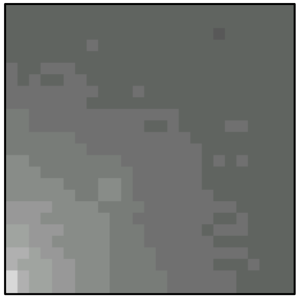


***P(z|x)*:**

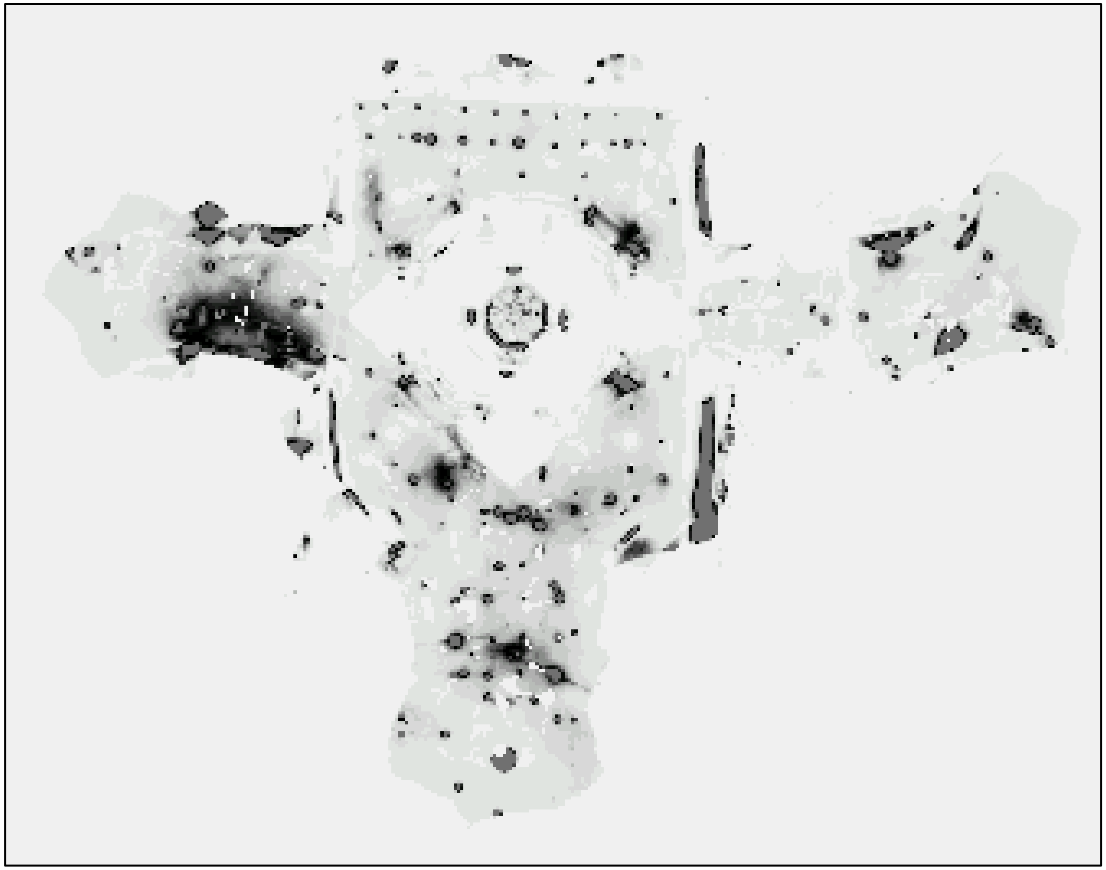


Example V (Next to Light)

Measurement z:



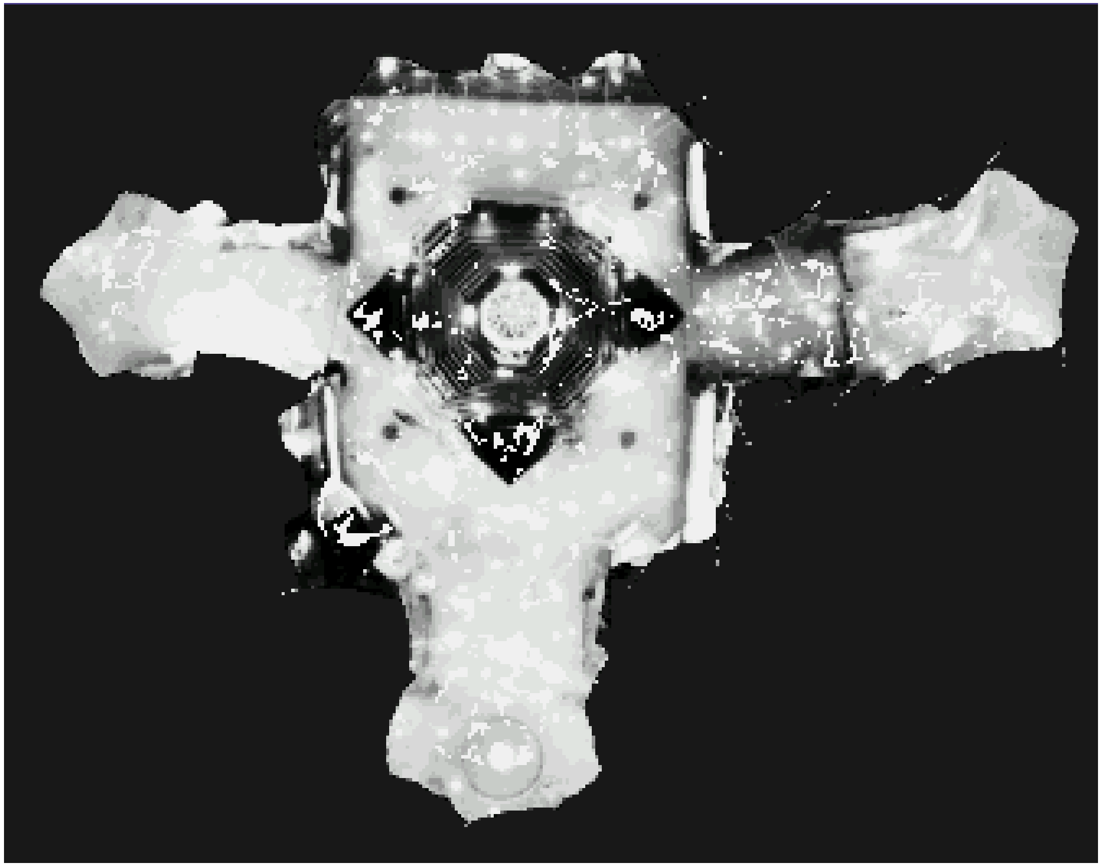
***P(z|x)*:**



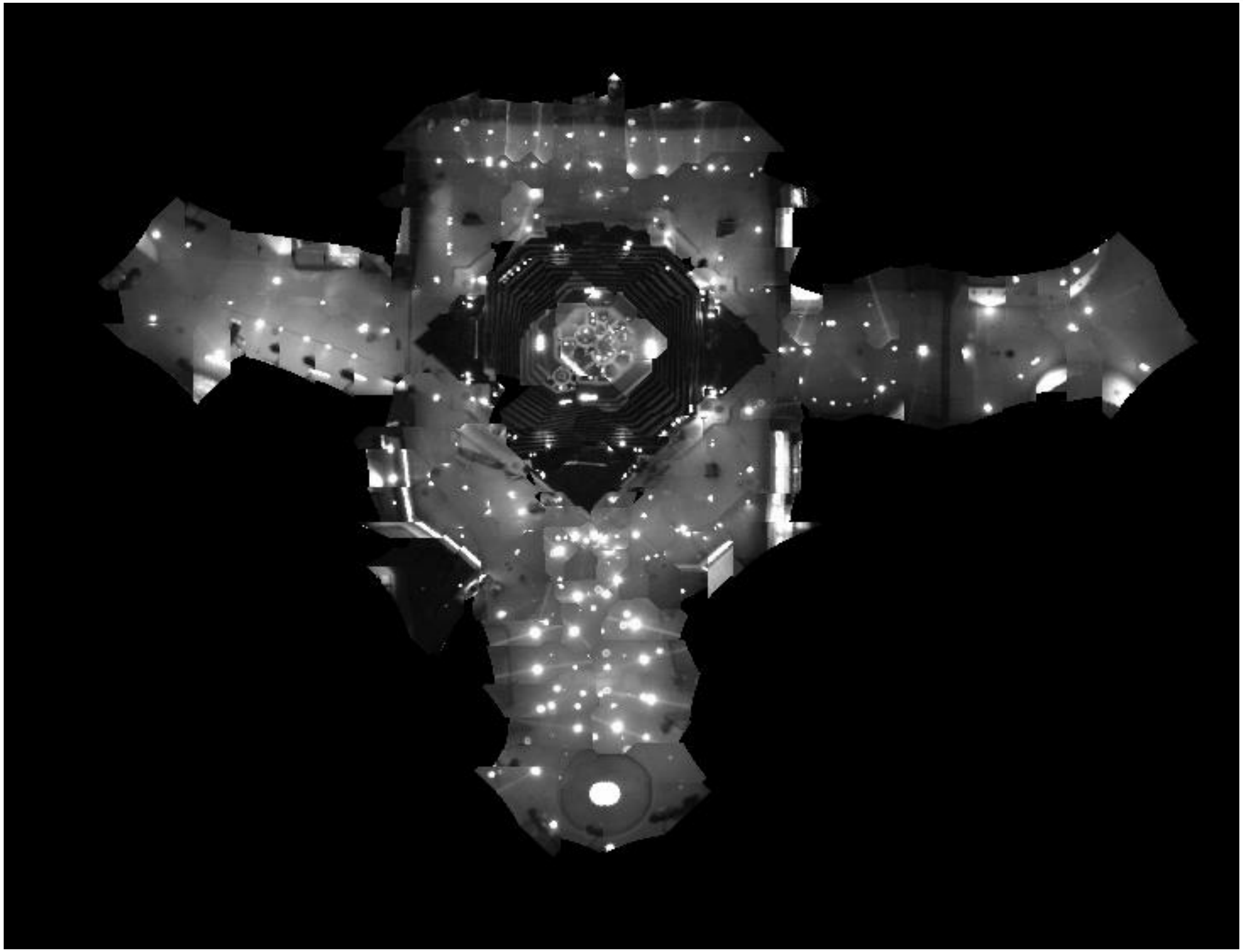
Example V (Elsewhere)

Measurement z:

***P(z|x)*:**



Example V (Global Localization)



Summary I

- Particle filters are an implementation of recursive Bayesian filtering

- They represent the posterior by a set of weighted samples

- They can model non-Gaussian distributions, Proposal to draw new samples

- Weight to account for the differences between the proposal and the target

- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter

Summary II

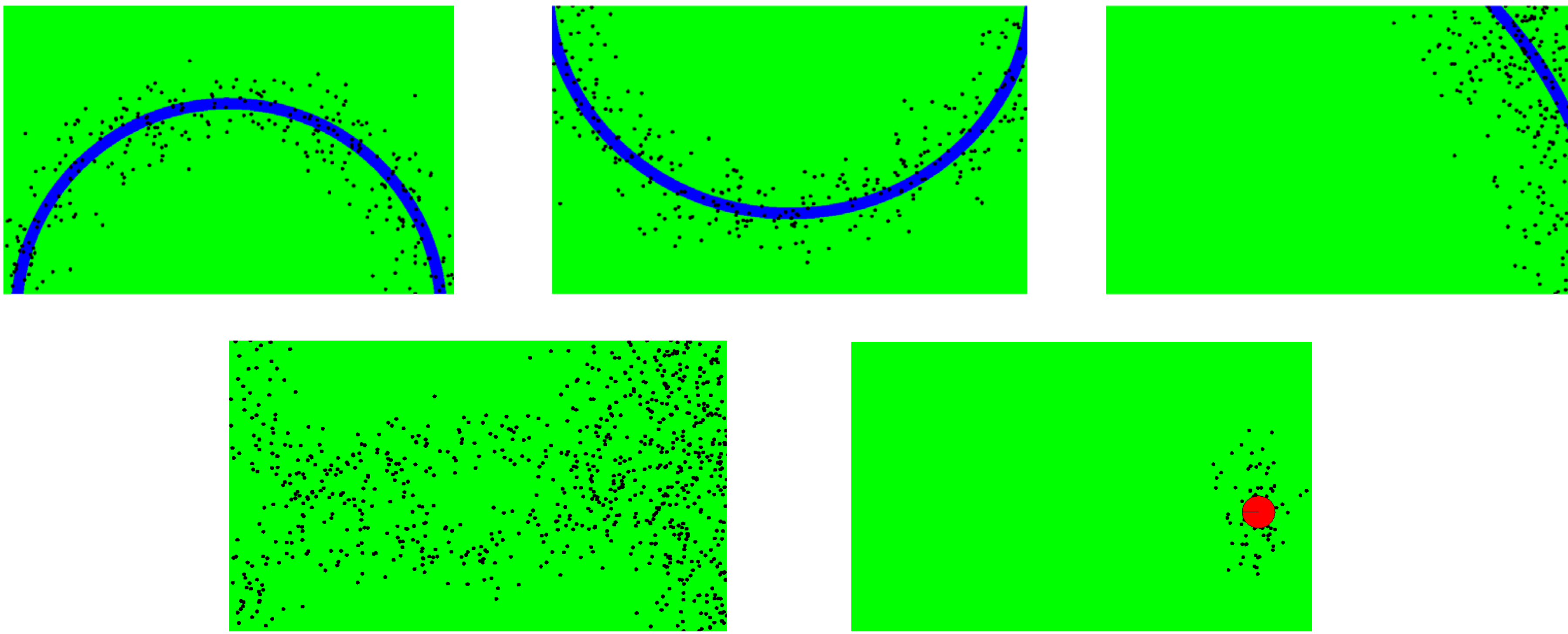
- In the context of localization, the particles are propagated according to the motion model.

- They are then weighted according to the likelihood of the observations.

- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.

Homework

Problem 1: Given the following observation models, please use importance sampling and resampling techniques to estimate the robot location.



Homework

Problem 2: Given a map and the ultrasound sensor model, please use importance sampling and resampling techniques to estimate the robot location and path.