CE7453: Photogrammetric Computer Vision

Lecture 3

Radiometry and Photometric stereo

Acknowledgements: Most of the slides in this lecture come from Ping Tan. part of the materials of the all the lecture notes are from Cyrill Stachniss, Marc Pollefey, Wolfgang Foerstner, Bernhard Wrobel, James Hays, A. Dermanis, Armin Gruen, Alper Yilmaz.

BRDF

BRDF is a four-parameter function that describes the reelecting property of the surface material.

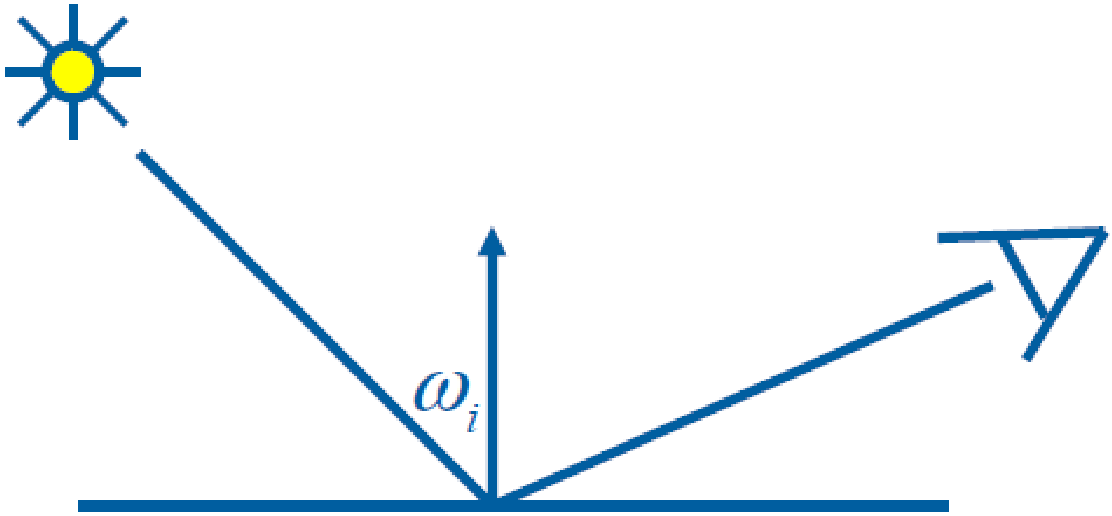
$$BRDF = \rho(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{L_o(\theta_o, \phi_o)}{E_i(\theta_i, \phi_i)} = \frac{L_o(\theta_o, \phi_o)}{L_i(\theta_i, \phi_i) cos\theta_i d\boldsymbol{\omega}}$$

Once knowing BRDF, the intensity of the pixel received from the viewing point can be computed:

$$L_o(\theta_o, \phi_o) = \rho(\theta_i, \phi_i, \theta_o, \phi_o) L_i(\theta_i, \phi_i) cos\theta_i d\boldsymbol{\omega}$$

Reflection Equation

Single light source

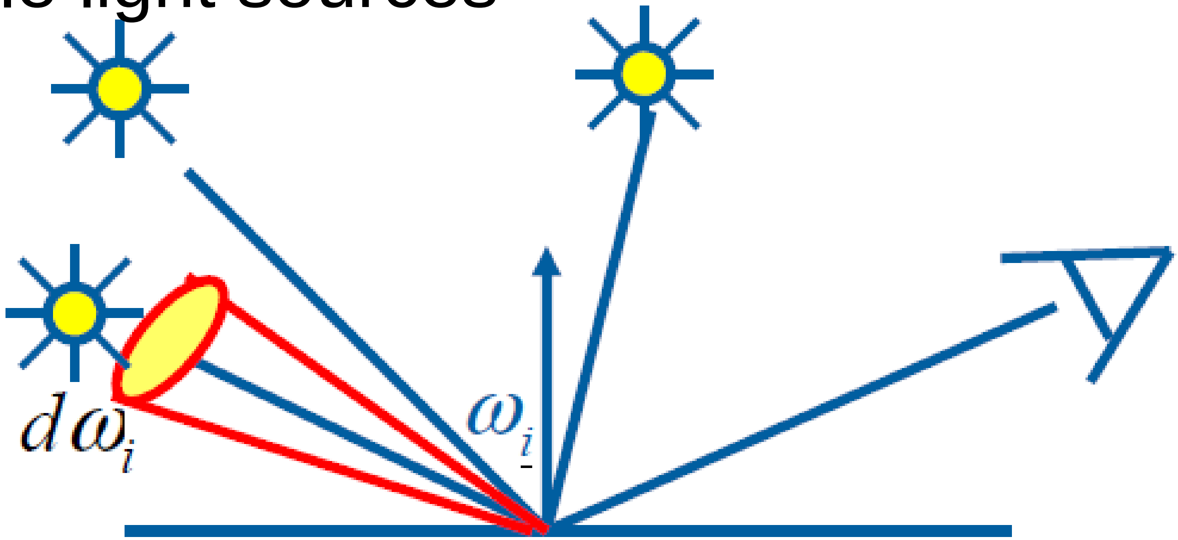


$$L_o(\theta_o,\varphi_o)=\rho_{bd}(\theta_o,\varphi_o,\theta_i,\varphi_i,)L_i(\theta_i,\varphi_i)\cos\theta_i$$

Reflected Radiance (Output Image)	BRDF	Incident radiance (from light source)	Cosine of Incident angle

Reflection Equation - Cont.

Multiple light sources



Replace sum with integral

$$L_o(\theta_o,\varphi_o)=\int_{\Omega}\rho_{bd}(\theta_o,\varphi_o,\theta_i,\varphi_i,)L_i(\theta_i,\varphi_i)\cos\theta_i d\omega_i$$

Reflected Radiance (Output Image)

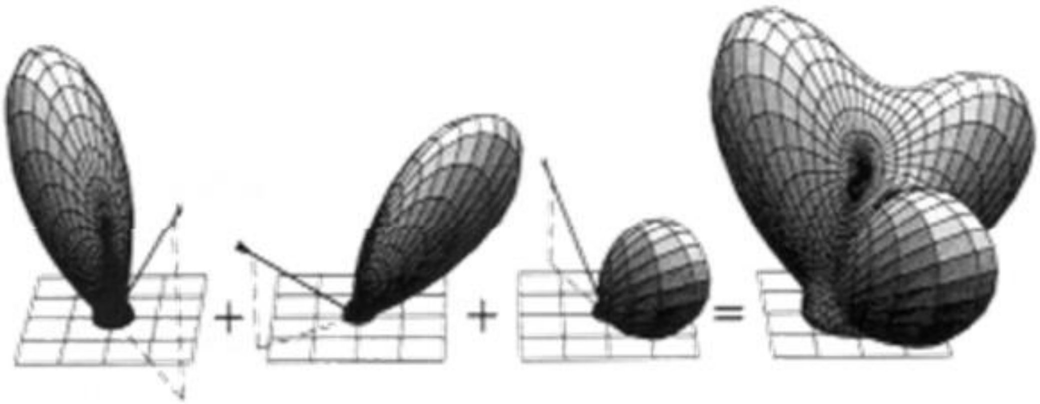
BRDF

Incident radiance (from light source)

Cosine of Incident angle

Properties of BRDF

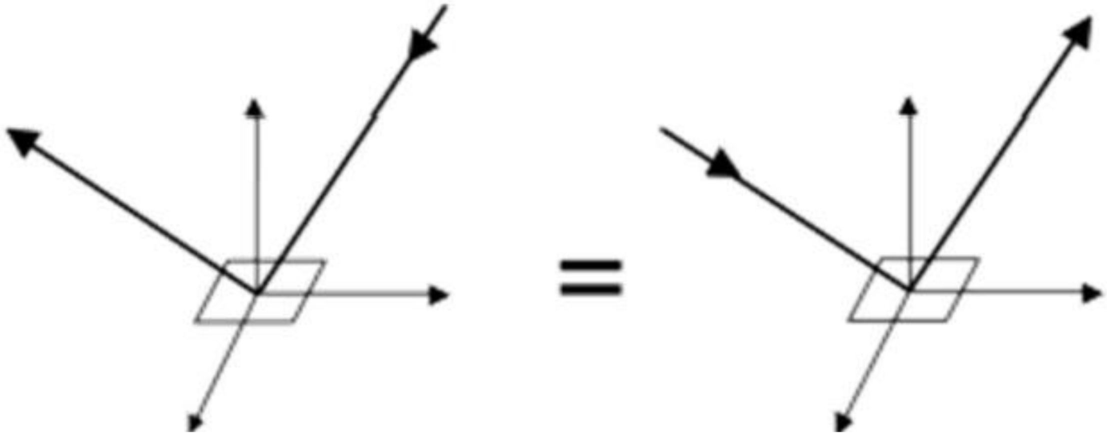
1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

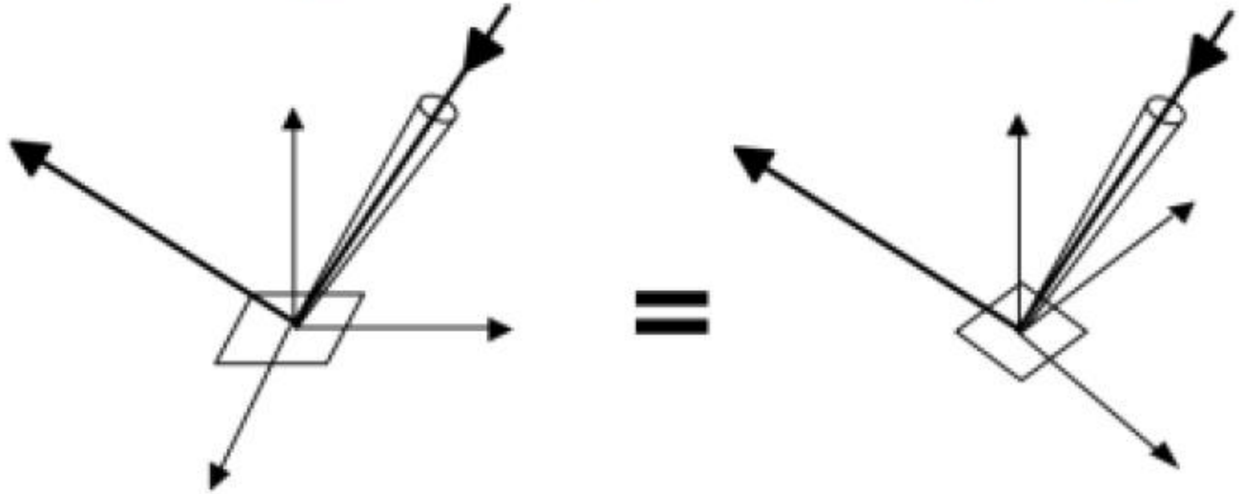
$$\rho(w_o \to w_i) = \rho(w_i \to w_o)$$



Properties of BRDF - Cont.

3. Isotropic vs. anisotropic

$$\rho(\theta_o,\varphi_o,\theta_i,\varphi_i)=\rho(\theta_o,\theta_i,\varphi_o-\varphi_i)$$



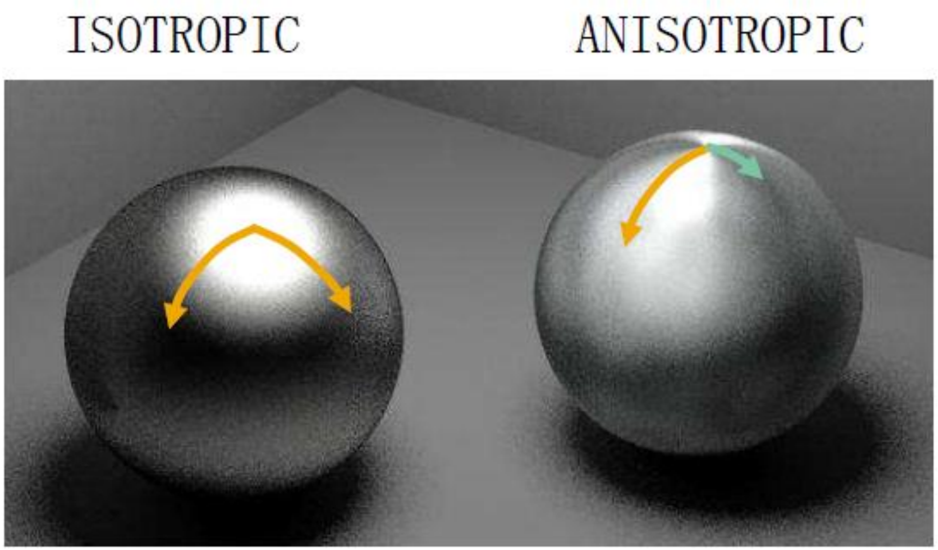
Reciprocity *and* isotropy

$$\rho(\theta_o,\theta_i,\varphi_o-\varphi_i)=\rho(\theta_o,\theta_i,\varphi_i-\varphi_o)=\rho(\theta_o,\theta_i,|\varphi_i-\varphi_o|)$$

4. Energy conservation

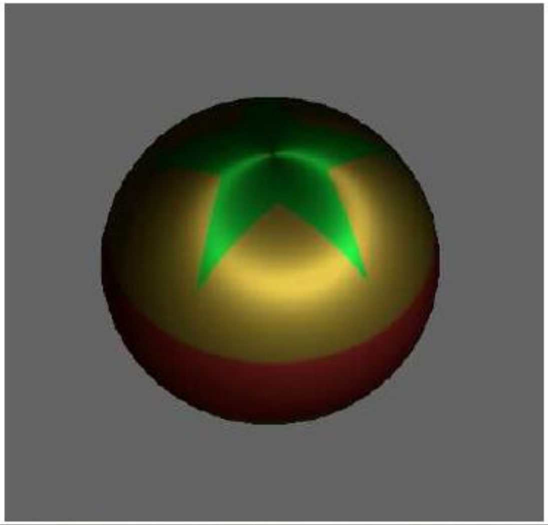
Isotropic vs Anisotropic

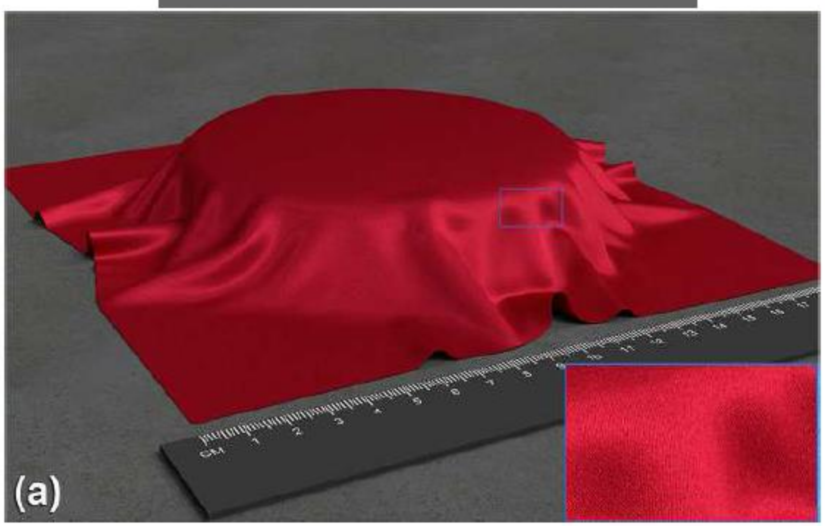
- Isotropic: Most materials (you can rotate light and view as a fixed pair about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



[Westin et al. 92]

Examples of anisotropic material



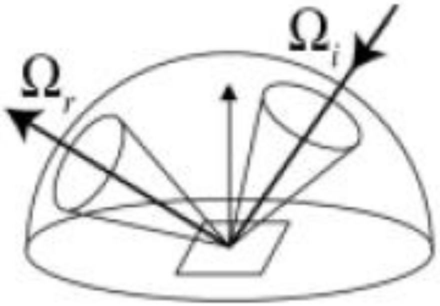


BRDF – Cont.

Energy Conservation

- For any incident lighting $L_i(\omega)$

$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$



$$= \frac{\int_{\Omega_r}\int_{\Omega_i} f_r(\omega_i \to \omega_r) L_i(\omega_i)\cos\theta_i\, d\omega_i \cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$\leq 1$$

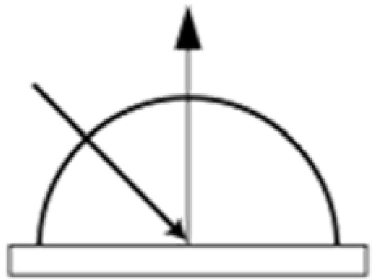
Typical types of Reflectance

Diffuse reflection:

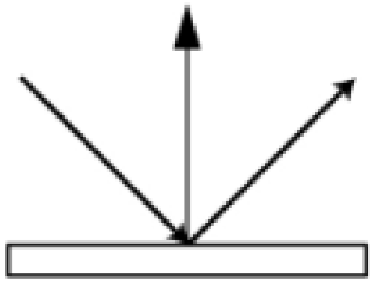
- The surface look the same from all directions (many vision algorithms depend on this!)
- Matte surfaces

Specular reflection:

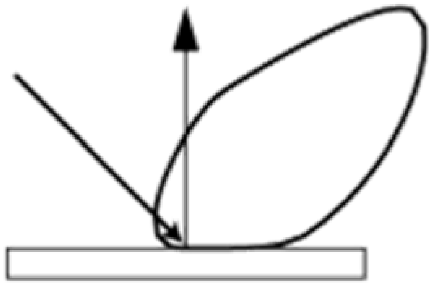
- The surface look different from different directions (causes troubles to many vision algorithms)
- Shiny surfaces



ideal diffuse reflectior (e.g. walls)



ideal specular reflection (e.g. mirror)



specular reflection (e.g. plastic, metal, porcelain)

Diffuse vs. Specular Reflection

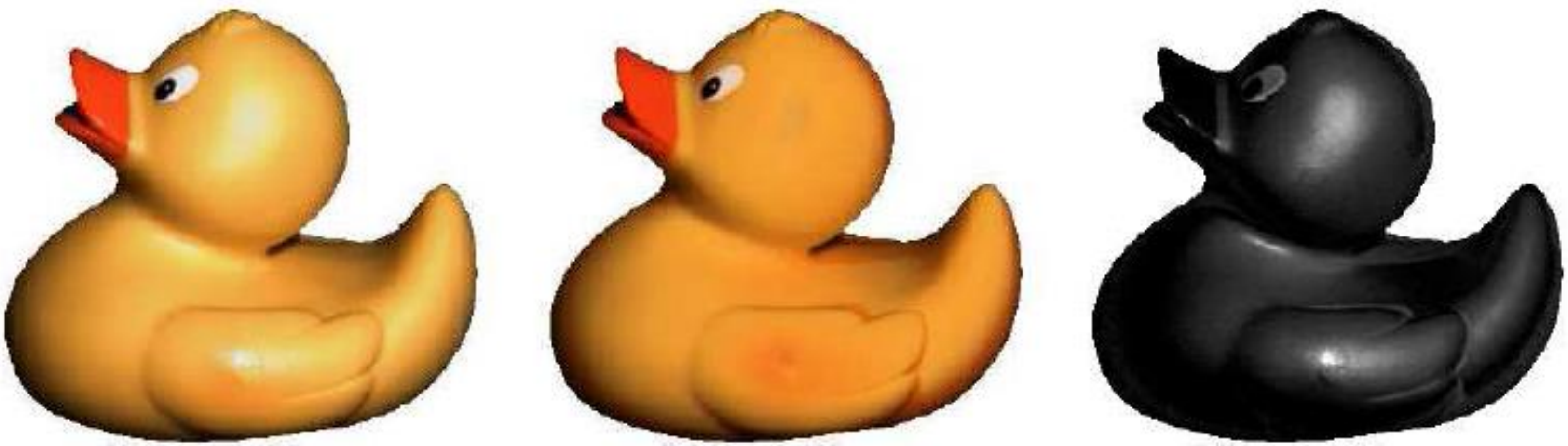
Most of the real surfaces have both components

diffuse component

diffuse + specular

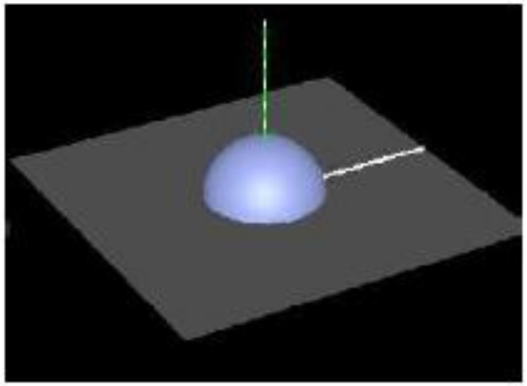
Diffuse vs. Specular Reflection – Cont.

- Diffuse reflection:
  - has the same color as the object surface
  - is unpolarized
- Specular reflection:
  - has the same color as the light source
  - has the same polarization as the light source

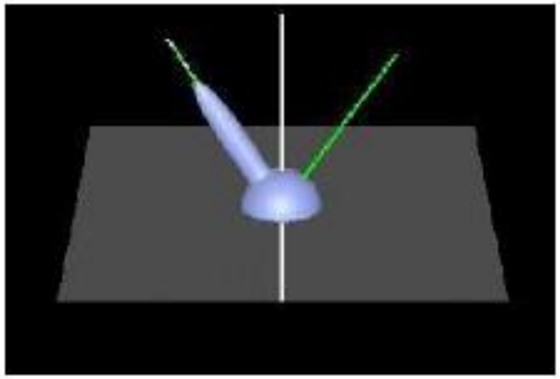


the original image | diffuse reflection | specular reflection

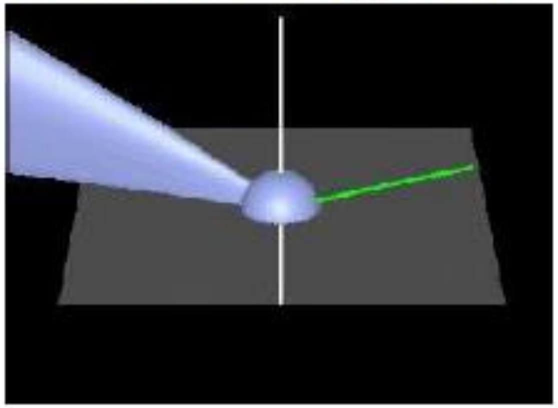
Common BRDF Models



Lambert's Model



Torrance-Sparrow



Lambertian Model

Empirical mathematic model for diffuse reflection

- Assume the BRDF is a constant

$$\rho(\theta_o, \varphi_o, \theta_i, \varphi_i) = \rho$$

- Observed Pixel intensity should be

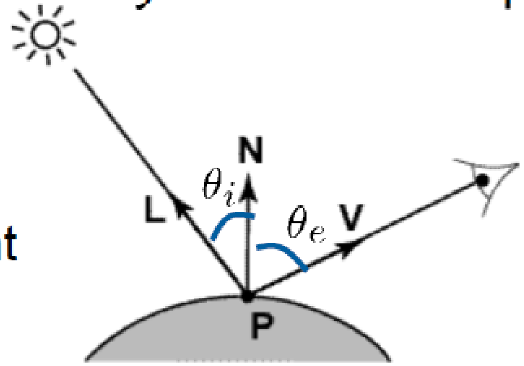
$$L_o = L_i \rho \cos\theta_i = L_i \rho (\mathbf{N} \cdot \mathbf{L})$$

Features of this model:

- Named after Johann H. Lambert
- A pixel's brightness does not depend on viewing direction
- Brightness *does* depend on direction of illumination
- This is the model most commonly used in computer vision

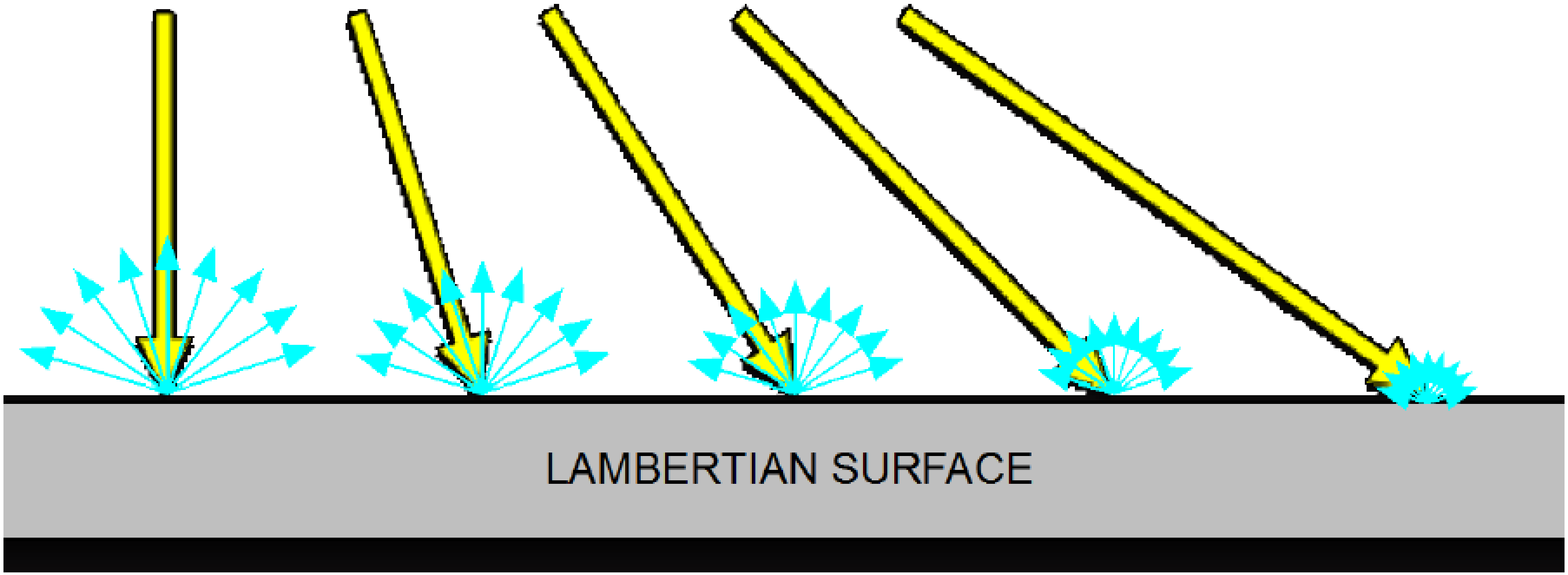
$\mathbf{N}, \mathbf{L}, \mathbf{V}$ are unit vectors

L_i, L_o are intensity of incoming and outgoing light

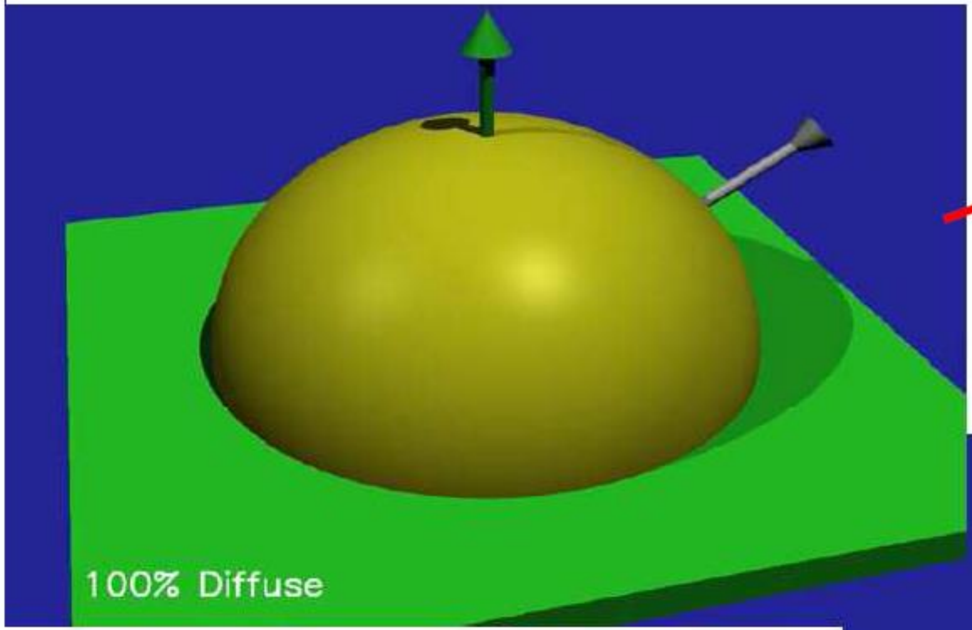


Lambertian Model

Lambert's Cosine Law



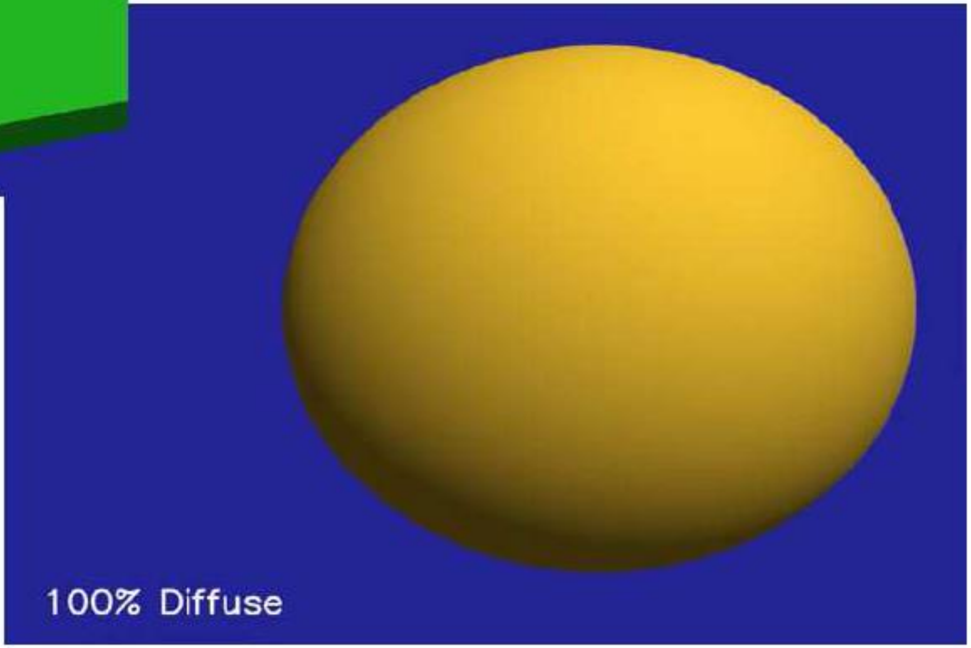
Lambertian Model



plot a BRDF as a function of (θ_o, φ_o) for a fixed (θ_i, φ_i)

3D plot of reflected intensity

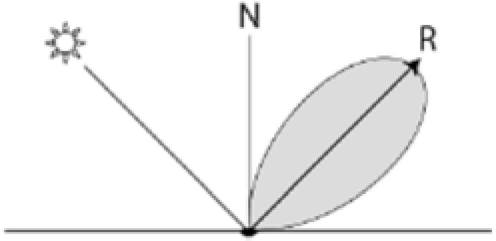
Appearance of a diffuse (dull) sphere



Phong Model

- Apparently we need models for shiny surfaces – specular reflection. Phong model gives the mathematical formulation of the specular reflection.

- Assuming light is concentrated on the "mirrored direction"
- Intensity of light falls off by cosine law
- Observed pixel intensity should be:



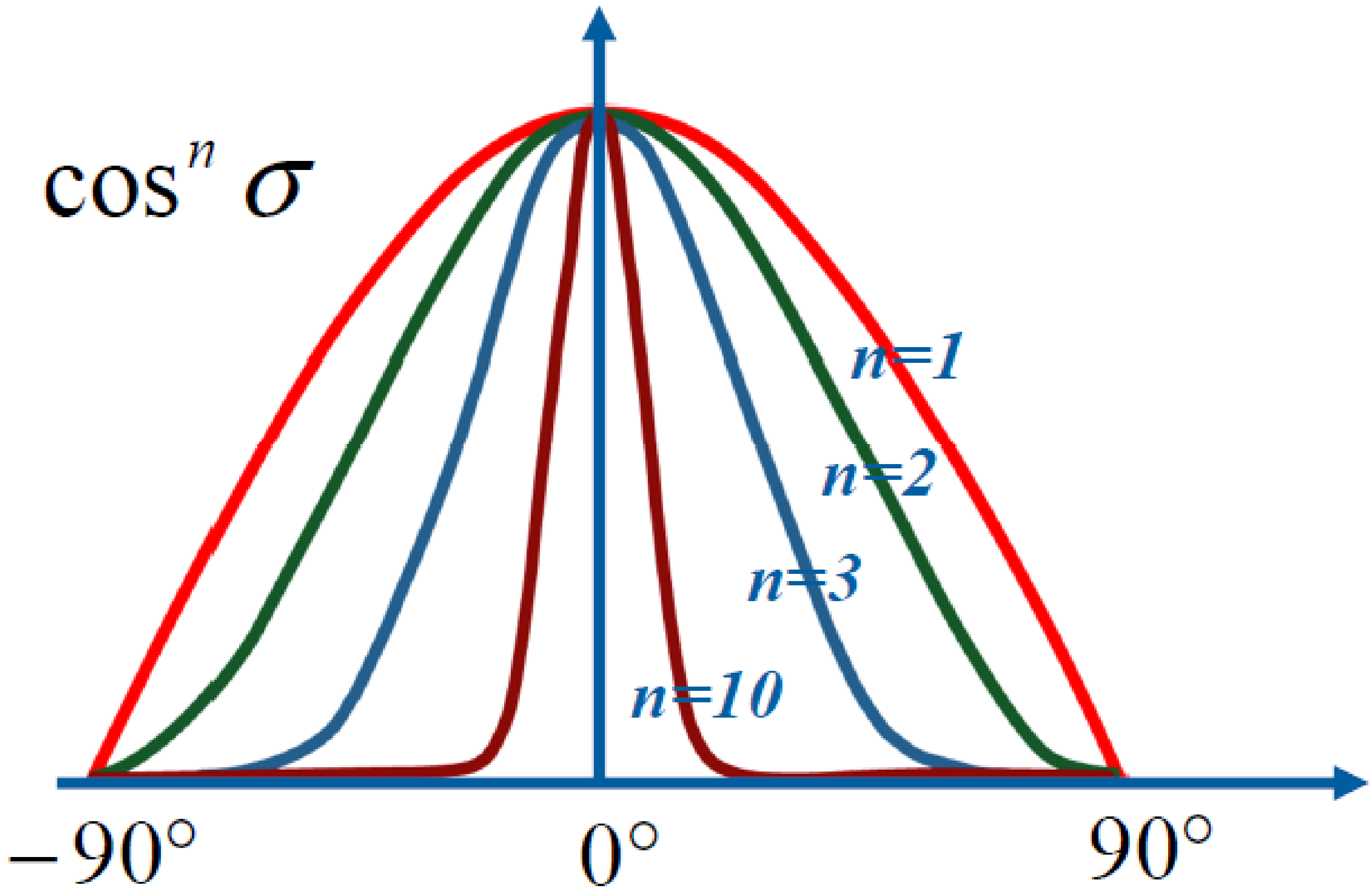
$$L_r = L_i(\mathbf{V} \cdot \mathbf{R})^n$$

$$\mathbf{R} = 2(\mathbf{N} \cdot \mathbf{L})\mathbf{N} - \mathbf{L}$$

Named after Bui Tuong Phong

Phong Model – Cont.

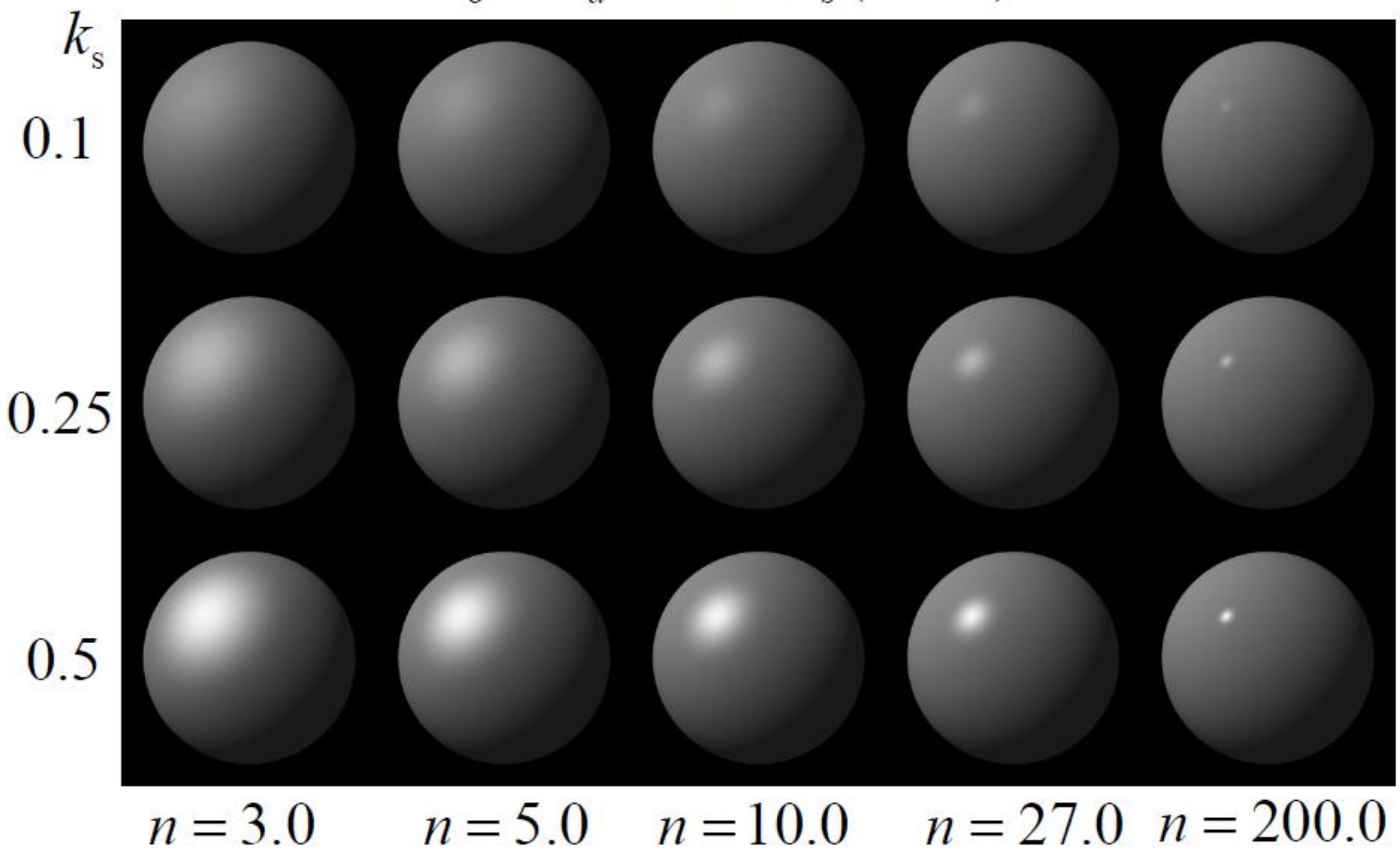
Shininess n controls the size of the highlight spot



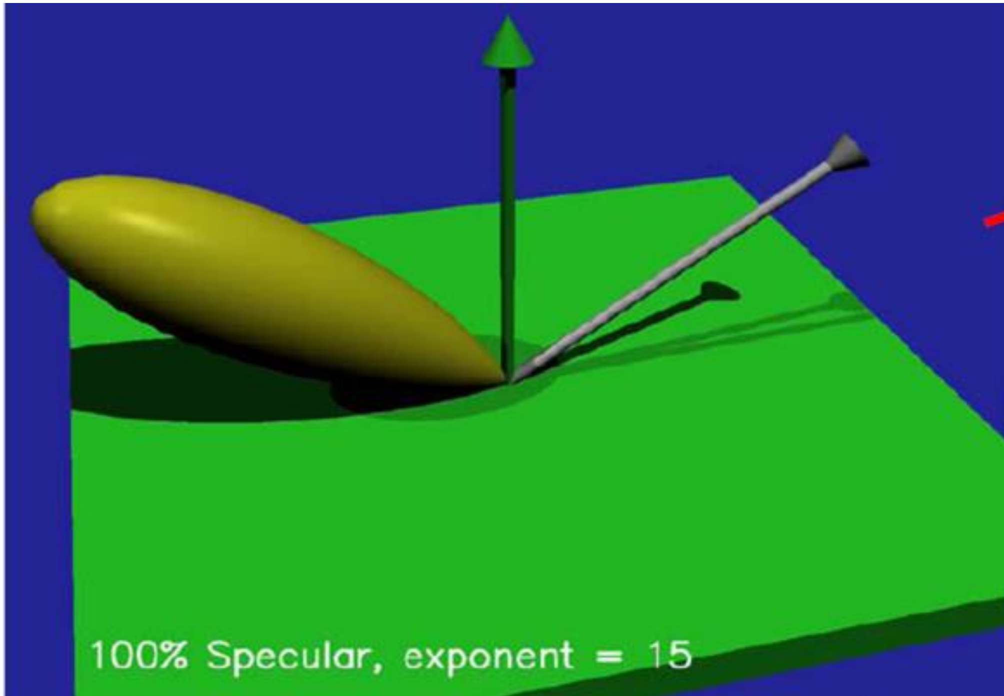
Phong Model – Cont.

Linear combination of Lambert's model and Phong Model

$$L_o = k_d \mathbf{N} \cdot \mathbf{L} + k_s (\mathbf{V} \cdot \mathbf{R})^n$$



Phong Model – Cont.

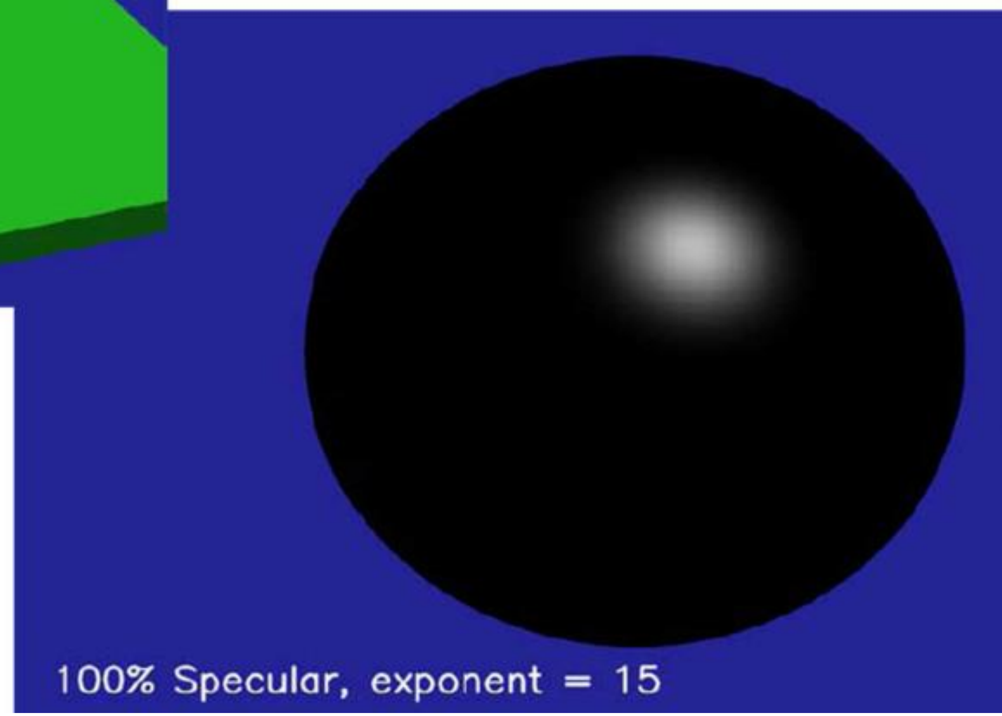


plot a BRDF as a function of (θ_o, φ_o) for a fixed (θ_i, φ_i)

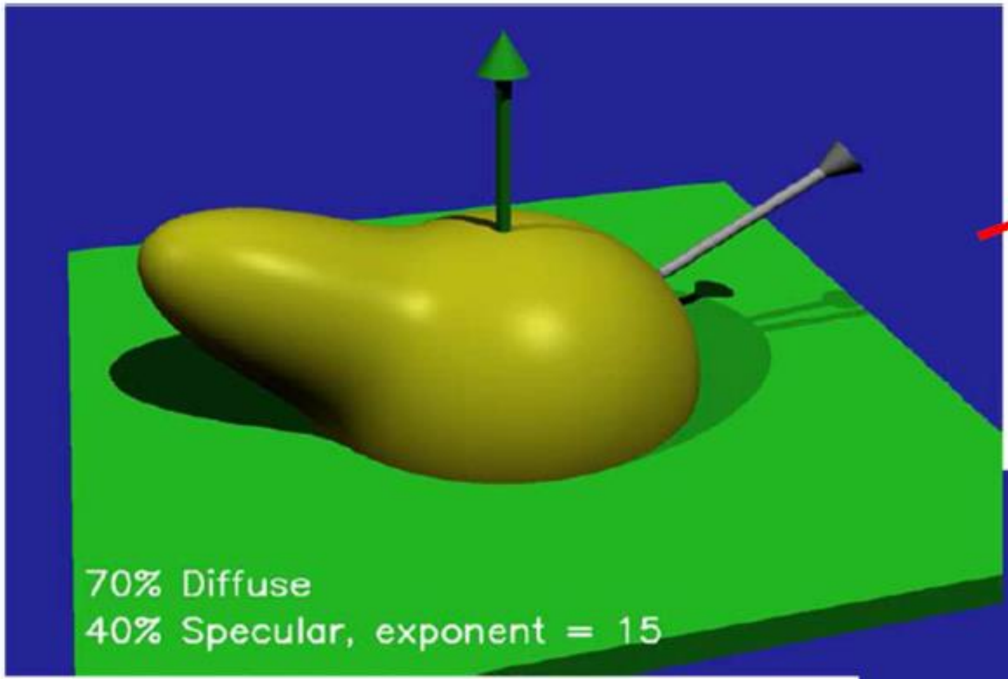
3D plot of reflected intensity

$n = 15$

Appearance of a specular sphere

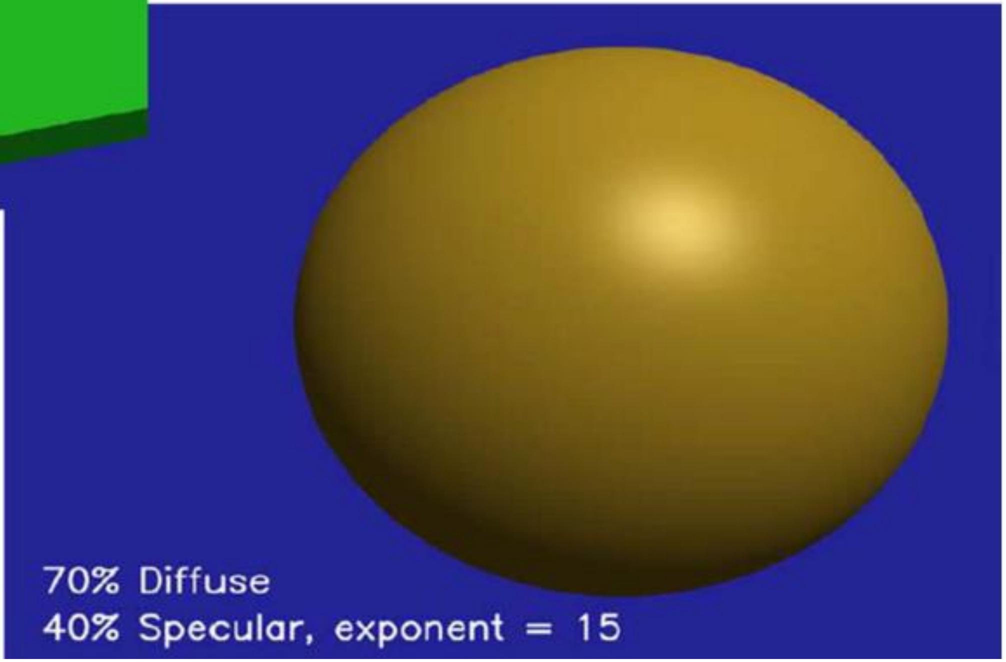


Phong Model – Cont.



plot a BRDF as a function of (θ_o,φ_o) for a fixed (θ_i,φ_i)

$k_d = 0.7$
$k_s = 0.4$
$n = 15$

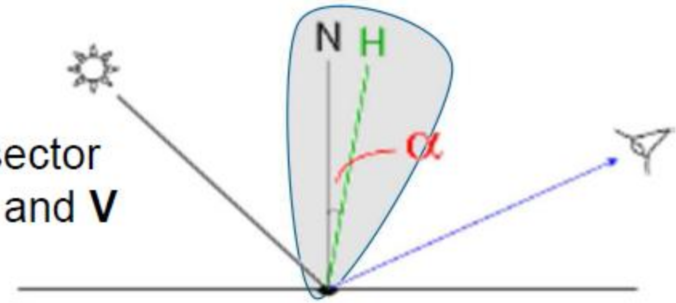


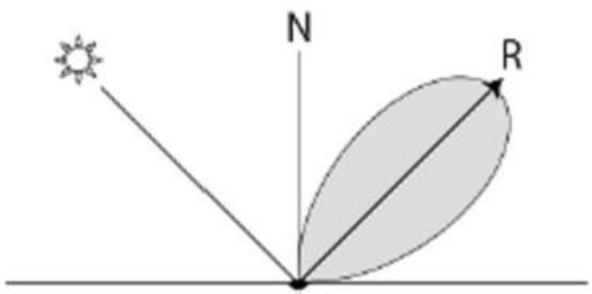
Blinn-Phong Model

Formulation similar to Phong model, observed intensity falls off by cosine law

$$L_r = L_i (\mathbf{N} \cdot \mathbf{H})^{n_s} \qquad \mathbf{H} = (\mathbf{L} + \mathbf{V})/2$$

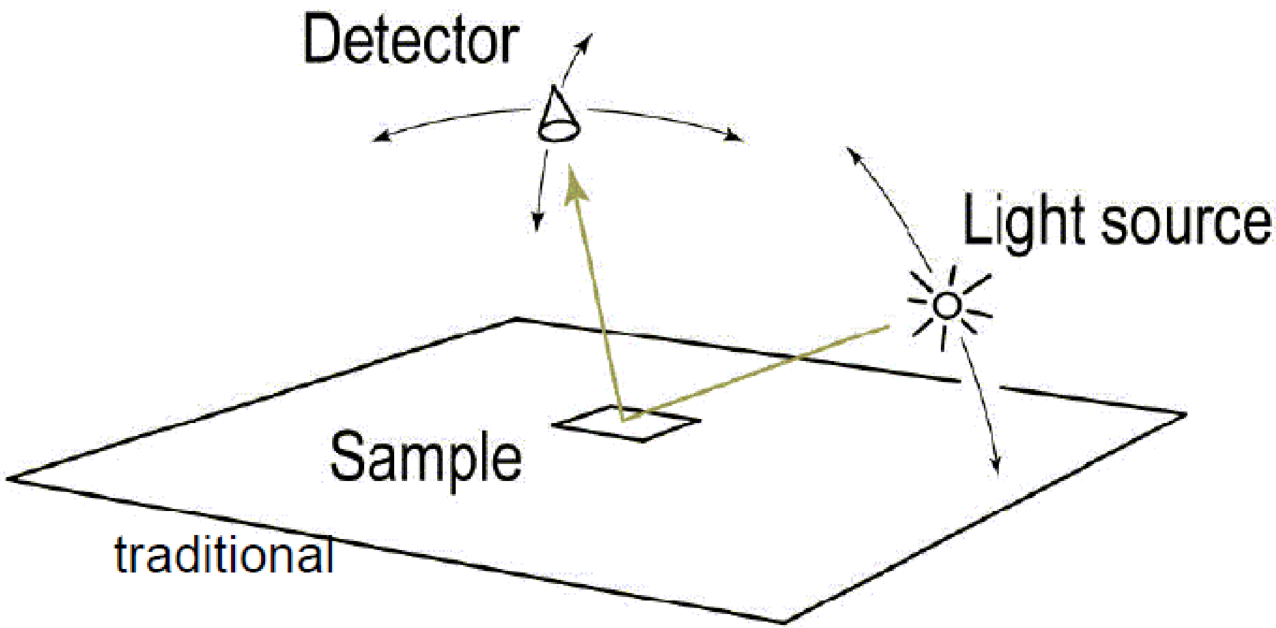
H is the bisector between **L** and **V**





The computation of H is faster than R

Measuring BRDF



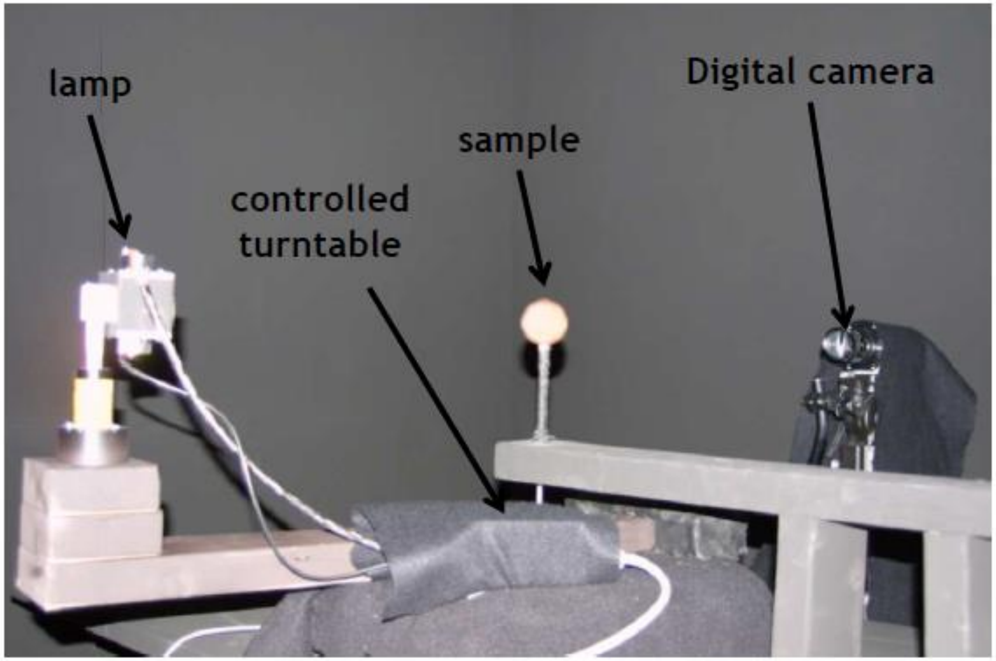
- Capture the BRDF by moving a camera + light source
- Need careful control of illumination, environment

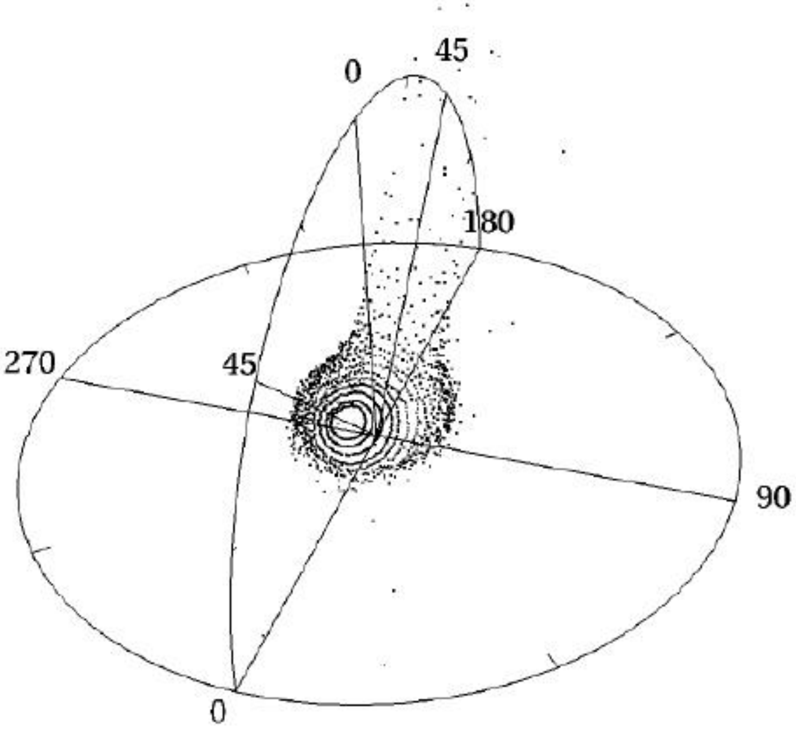
Representation of Measured Data

- Measure-then-fit analytic models
  - Fitting can reduce noise but also is limited by the model
  - Non-obvious error metric for fitting – often biased to specular which has large values
  - Difficult optimization – nonlinear; depends on initial guess
- Tabulated BRDF
  - 4D table
  - Not editable

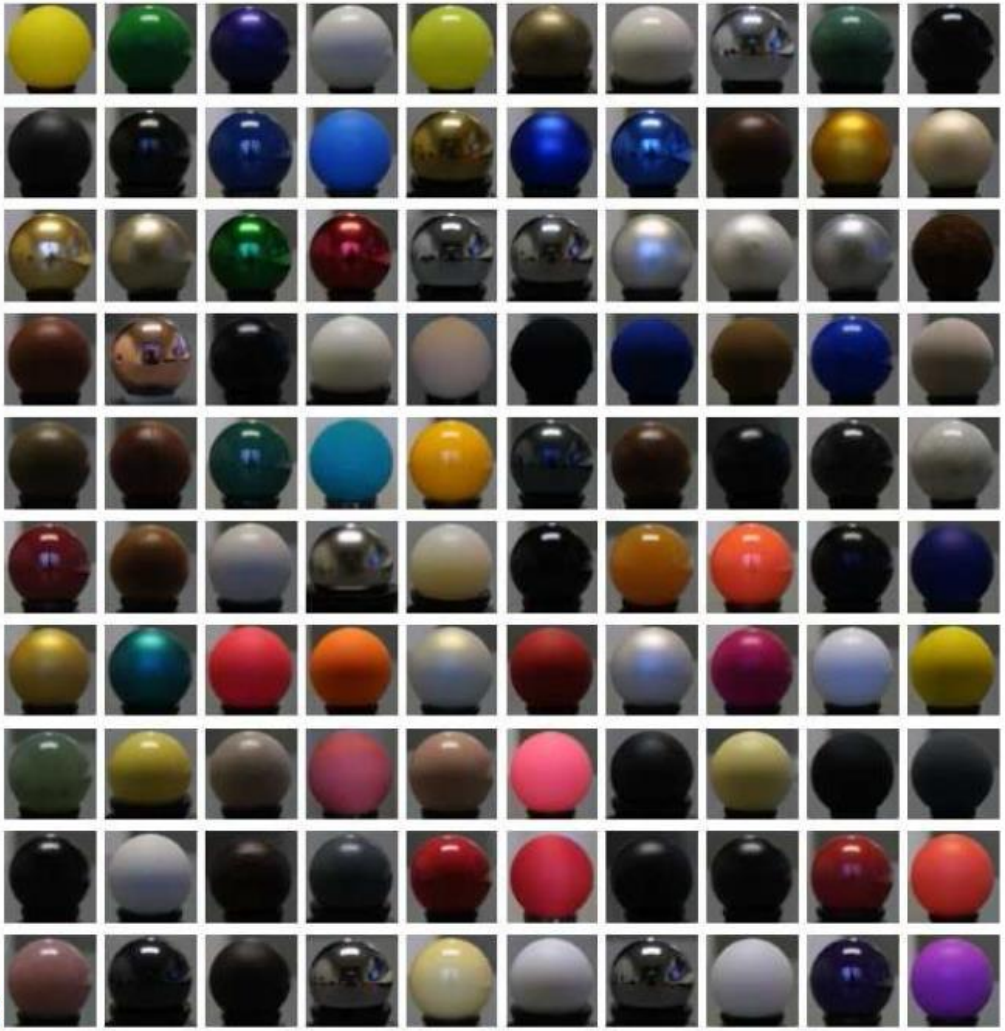
Acquisition

- A spherically homogeneous sample of the material can make the acquisition simpler





Acquisition – Cont.

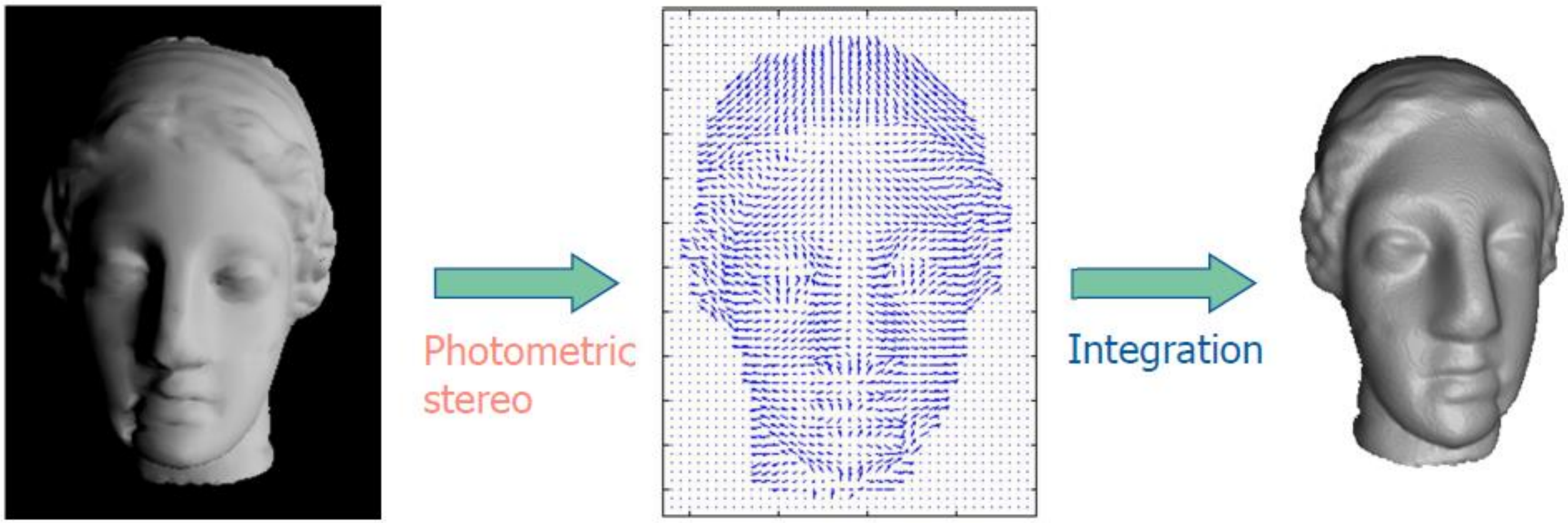


130 materials were scanned; 100 of them shown here

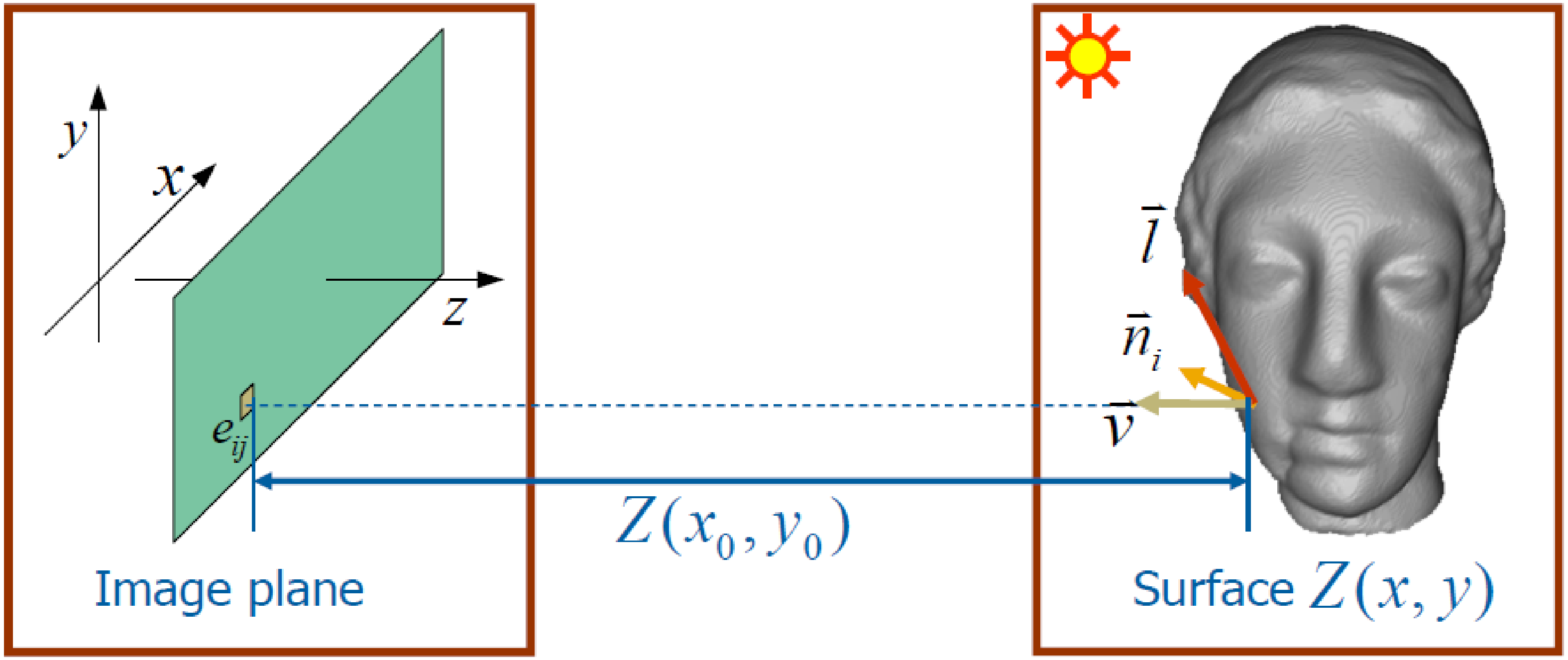
MERL BRDF database, freely available online

Photometric stereo

- Input: Images with
  - Fixed view & changing lighting
- Goal:
  - Recover surface (normal map)

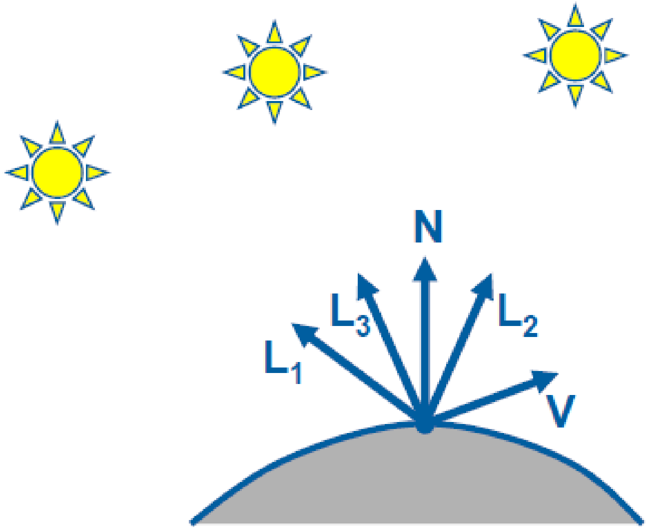


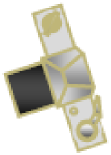
Assumptions



- Lambert's reflectance model
- Camera centered coordinate system
- Orthographic camera ($\vec{v}$ is the same for all pixels)
- Directional illumination ($\vec{l}$ is the same for all pixels)

Mathematic Formulation





$$I_1 = k_d \mathbf{N} \cdot \mathbf{L_1}$$
$$I_2 = k_d \mathbf{N} \cdot \mathbf{L_2}$$
$$I_3 = k_d \mathbf{N} \cdot \mathbf{L_3}$$

Can write this as a matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = k_d \begin{bmatrix} \mathbf{L_1}^T \\ \mathbf{L_2}^T \\ \mathbf{L_3}^T \end{bmatrix} \mathbf{N}$$

$$\mathbf{G} = \mathbf{L}^{-1}\mathbf{I} \qquad k_d = \|\mathbf{G}\| \qquad \mathbf{N} = \frac{1}{k_d}\mathbf{G}$$

Record the lighting directions

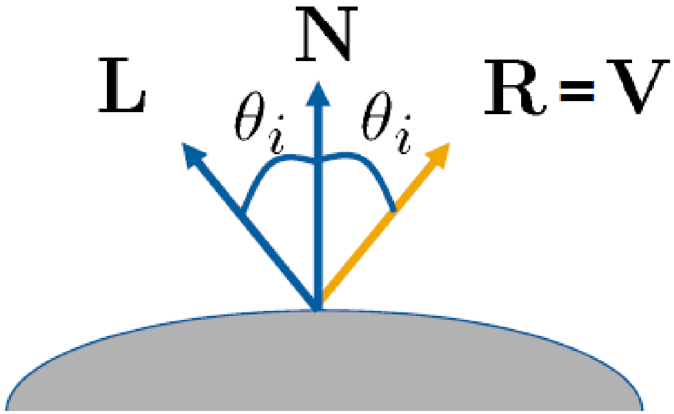
- Trick: place a shiny sphere in the scene

  - the location of the highlight tells what light direction is

Recall Specular reflection

- For a perfect mirror, light is reflected about N

$$L_r = \begin{cases} L_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$



- Highlight can be seen at a pixel where V = R
  - then L can be computed with normal **N** at that pixel

$$\mathbf{L} = 2(\mathbf{N} \cdot \mathbf{R})\mathbf{N} - \mathbf{R}$$

- How to get normal direction **N?**
  - normal of each point on a sphere can be determined

Dealing with Shadows

- This formulation does not consider shadows

$$I = k\mathbf{N} \cdot \mathbf{L}$$

- Pixel in shadows have zero intensity $I = 0$
- A precise formulation is nonlinear:

$$I = \max\left(0, k\mathbf{N} \cdot \mathbf{L}\right)$$

- A simple trick is:

$$I^2 = I\mathbf{N} \cdot \mathbf{L} \qquad \begin{bmatrix} I_1{}^2 \\ \vdots \\ I_n{}^2 \end{bmatrix} = \begin{bmatrix} I_1\mathbf{L}_1 \\ \vdots \\ I_n\mathbf{L}_n \end{bmatrix} k_d\mathbf{N}$$

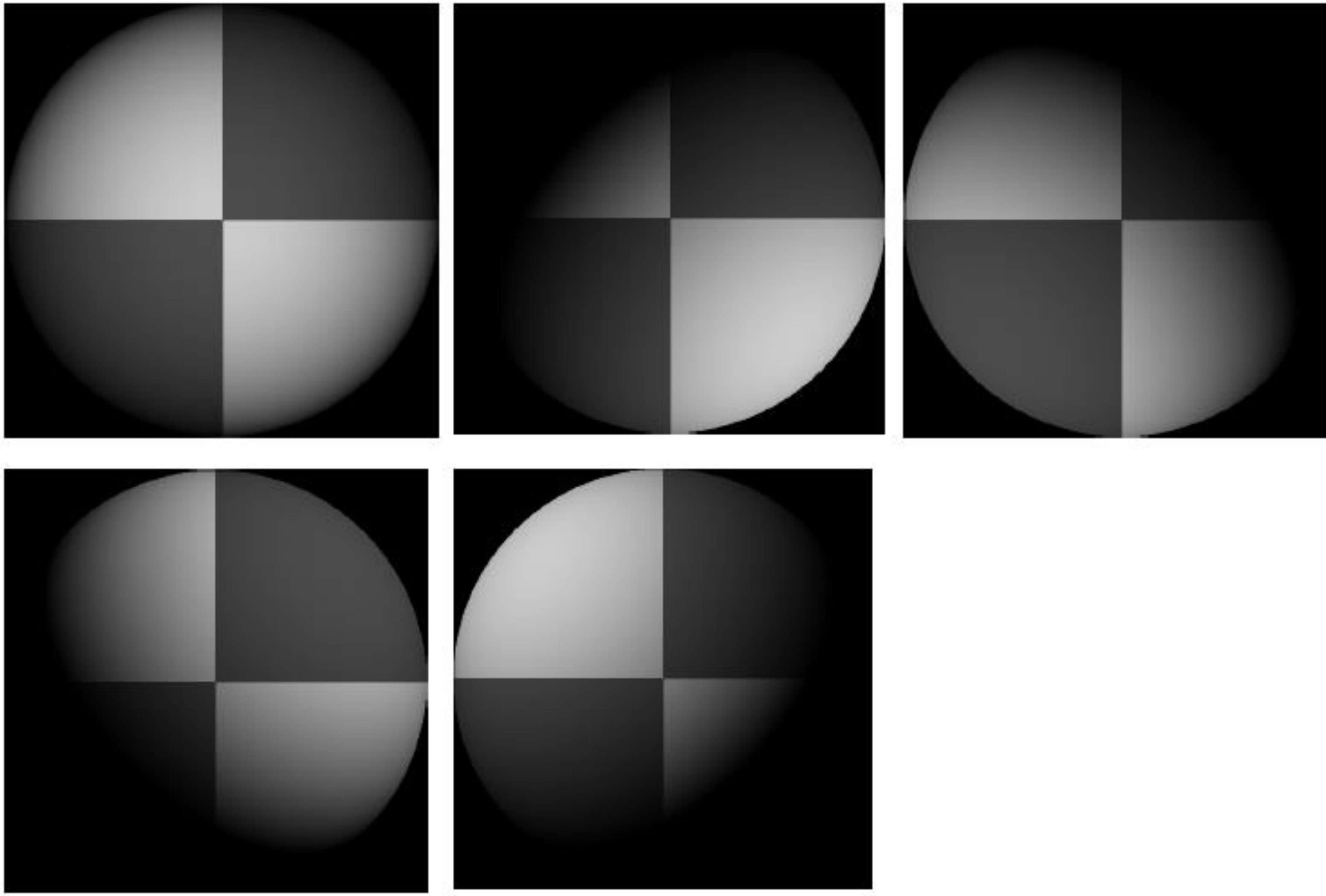
  - Essentially, each equation is given a weight
  - Larger weights are given to brighter pixels
  - Shadowed pixels are given weight of 0

Dealing with Shadows – Cont.

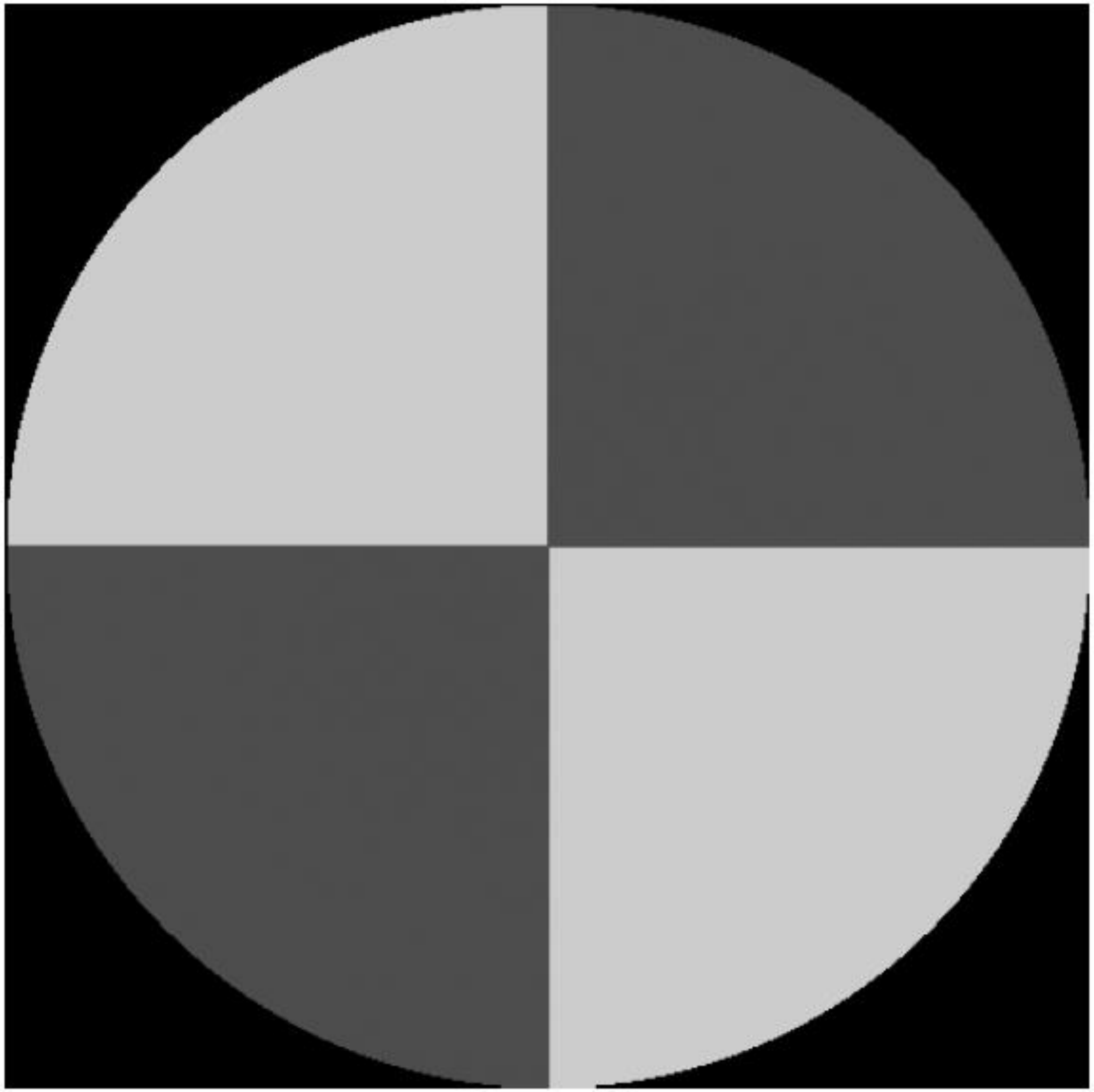
- Take lots of pictures and discard pixels that are too dark (10% of the darkest pixels)

- Similarly we can discard pixels that are too bright (specular reflections)

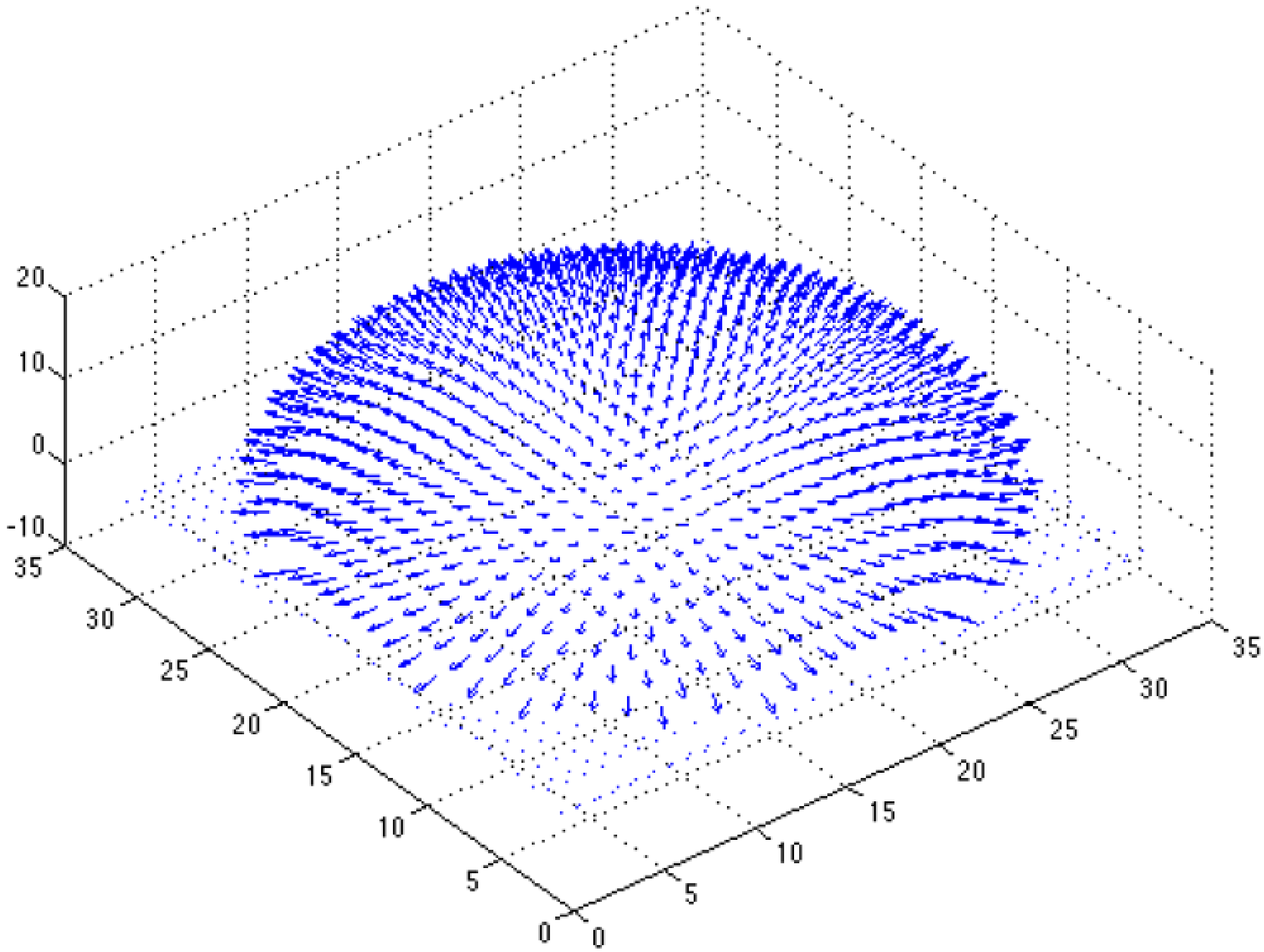
Example Figures



Recovered reflectance (K_d)



Recovered normal field



Depth from Normals (Method I)

- Recall the surface is written as

$$(x, y, f(x, y))$$

- This means the normal has the form:

$$N(x,y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}}\right)\begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}$$

- If we write the known normal **n** as

$$\mathbf{n}(x,y) = \begin{pmatrix} n_1(x,y) \\ n_2(x,y) \\ n_3(x,y) \end{pmatrix}$$

- Then we obtain values for the partial derivatives of the surface:

$$f_x(x,y) = n_1(x,y) / n_3(x,y)$$

$$f_y(x,y) = n_2(x,y) / n_3(x,y)$$

Depth from Normals I – Cont.

- We can now recover the surface height at any point by integration along some path, e.g.

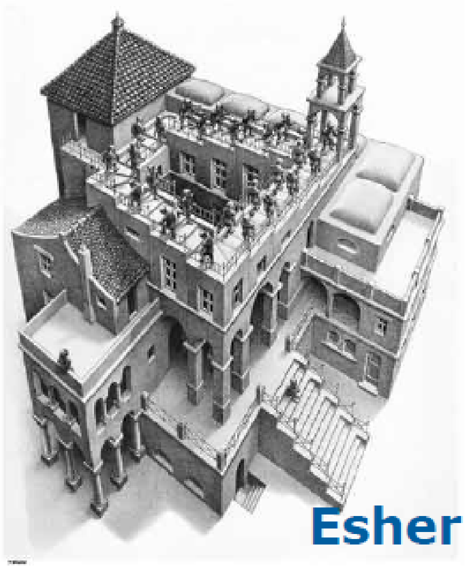
$$f(x,y) = \int_0^x f_x(s,y)ds + \int_0^y f_y(x,t)dt + c$$

- Recall that mixed second partials are equal --- this gives us a **check.** We must have:

$$\frac{\partial f_x(s,y)}{\partial y} = \frac{\partial f_y(s,y)}{\partial x}$$

(or they should be similar, at least)

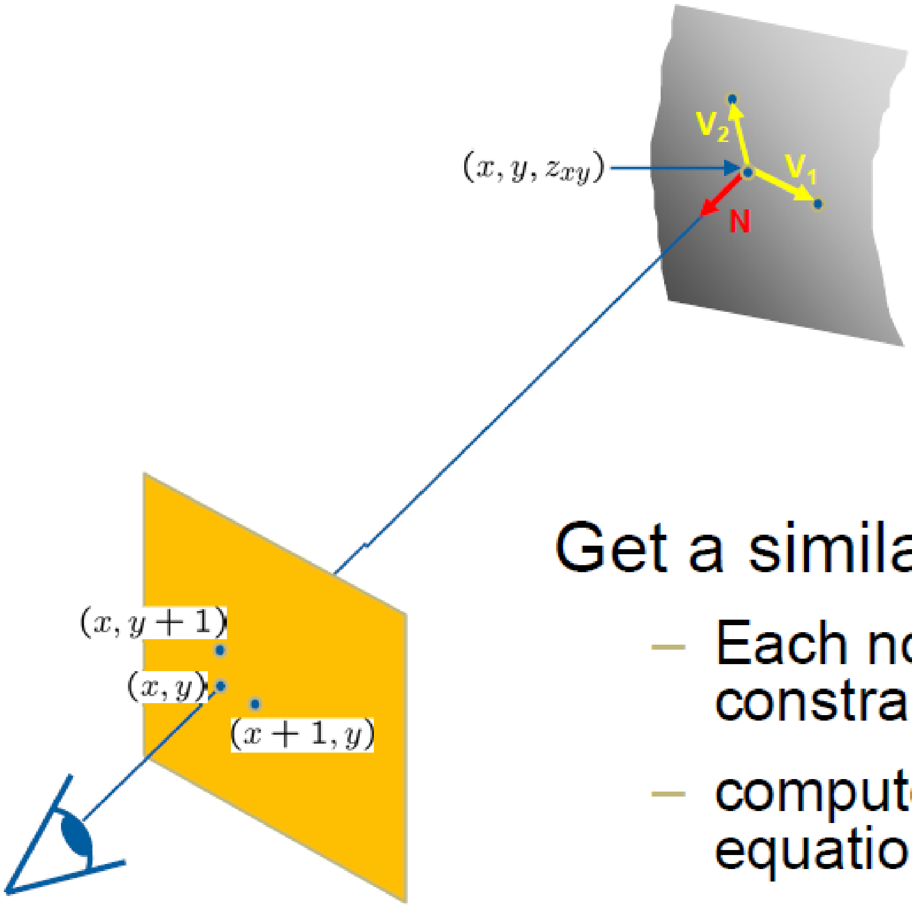
- Due to imaging and estimation noise, this almost never happens.
- This method never works on real data



Esher

Depth from Normals (Method II)

The tangent vector $\mathbf{V}_1$ is perpendicular to $\mathbf{N}$



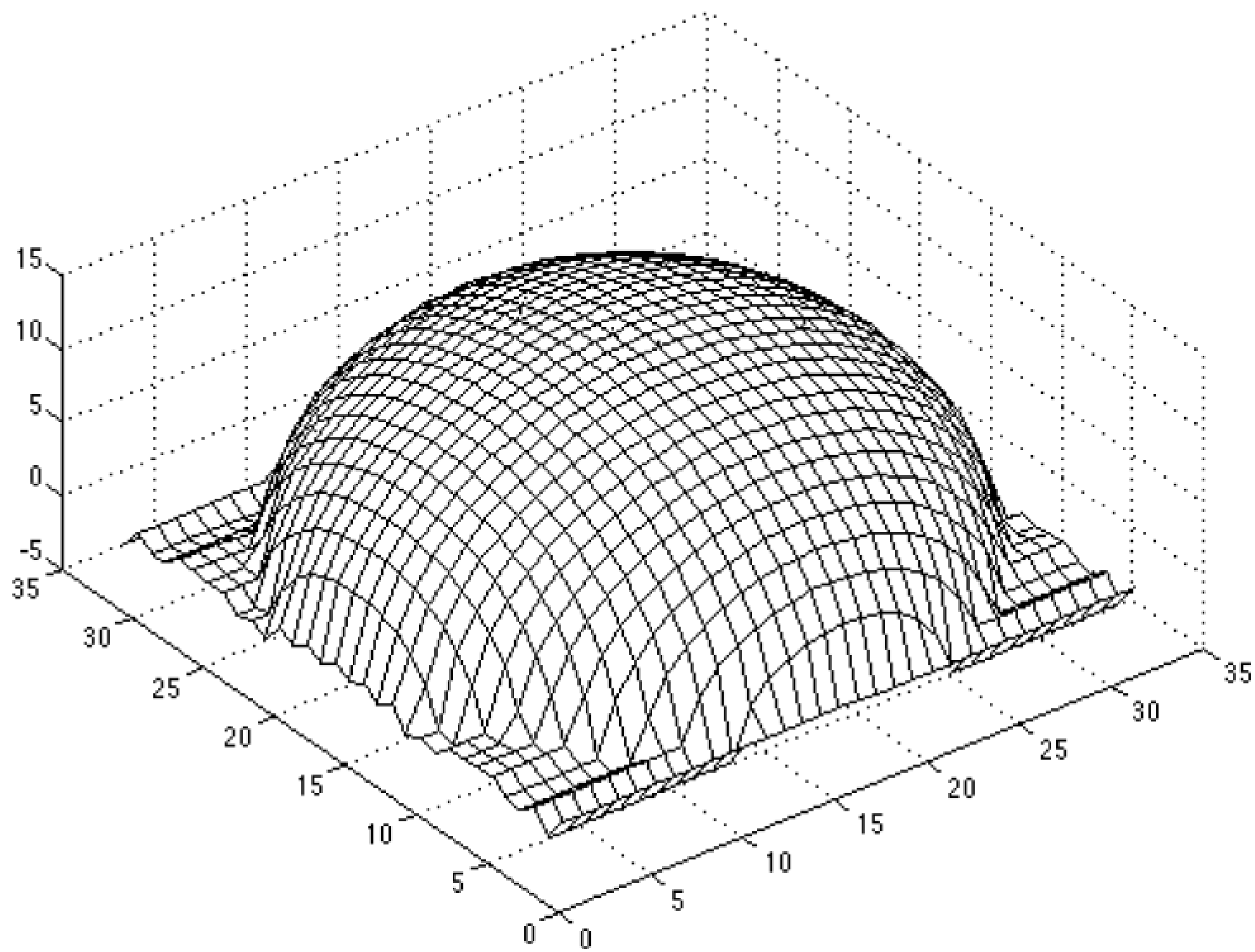
$$
\begin{aligned}
V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\
&= (1, 0, z_{x+1,y} - z_{xy})
\end{aligned}
$$

$$
\begin{aligned}
0 &= N \cdot V_1 \\
&= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\
&= n_x + n_z(z_{x+1,y} - z_{xy})
\end{aligned}
$$

Get a similar equation for $\mathbf{V}_2$

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Surface Recovered



Limitations for Lambertian Photometric Stereo

- Cannot handle shiny, semi-translucent objects.
- Shadows, multiple reflections
- Camera and lights have to be distant
- Light Calibration requirements

- measure light source detections, intensities

- Camera response function.

Example-based Photometric Stereo

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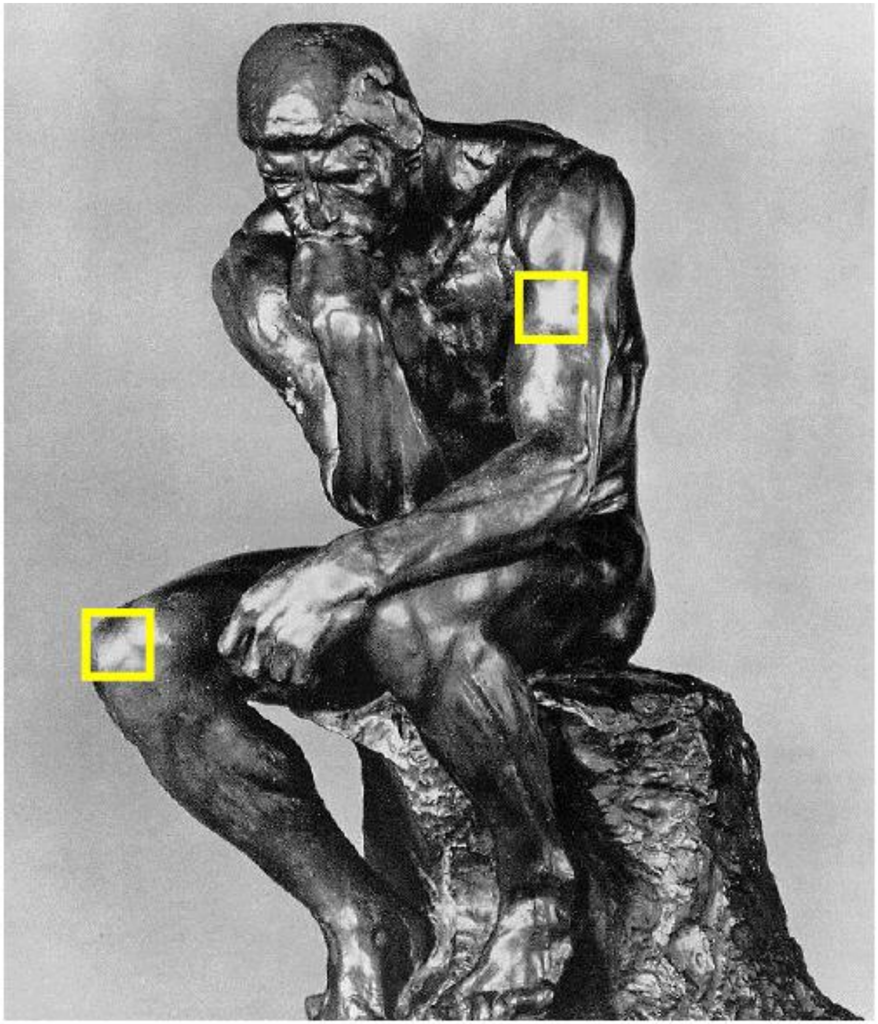
University of Washington

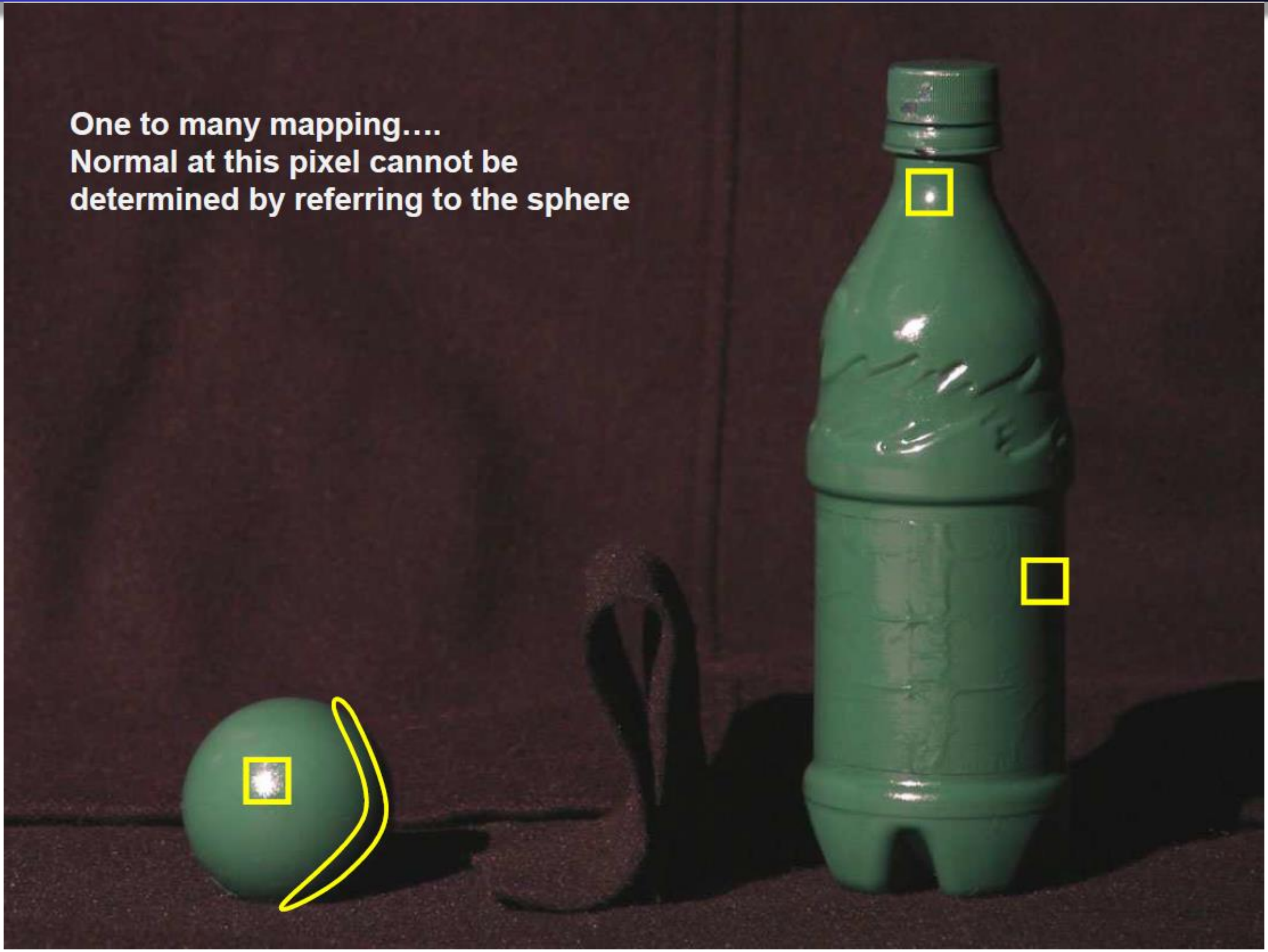
Example-based Photometric Stereo – Cont.

Shinny Areas



"Orientation consistency": points of similar orientation have similar intensity

Example-based Photometric Stereo – Cont.



Example-based Photometric Stereo – Cont.

Example-based Photometric Stereo – Cont.

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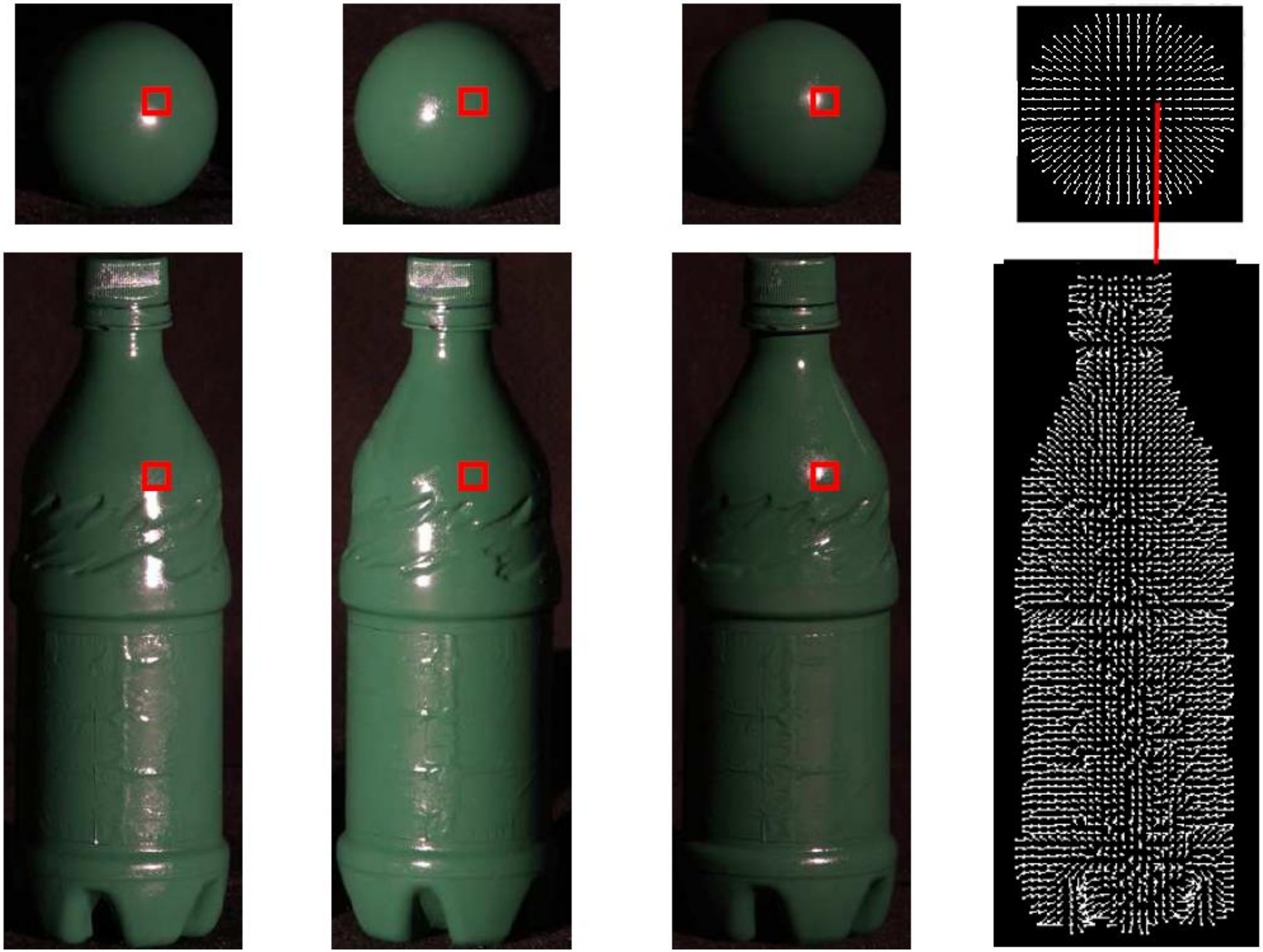
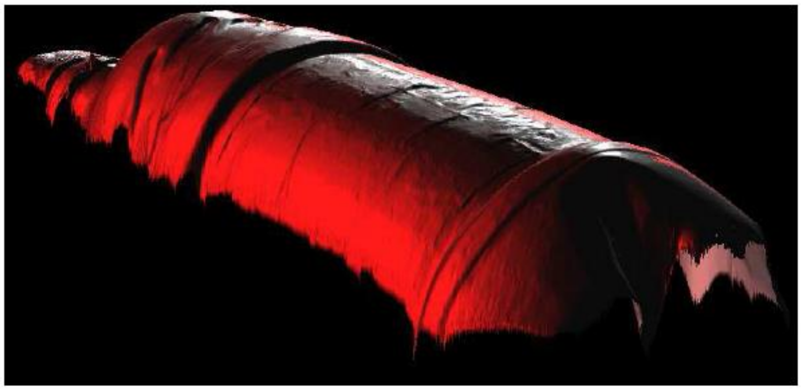
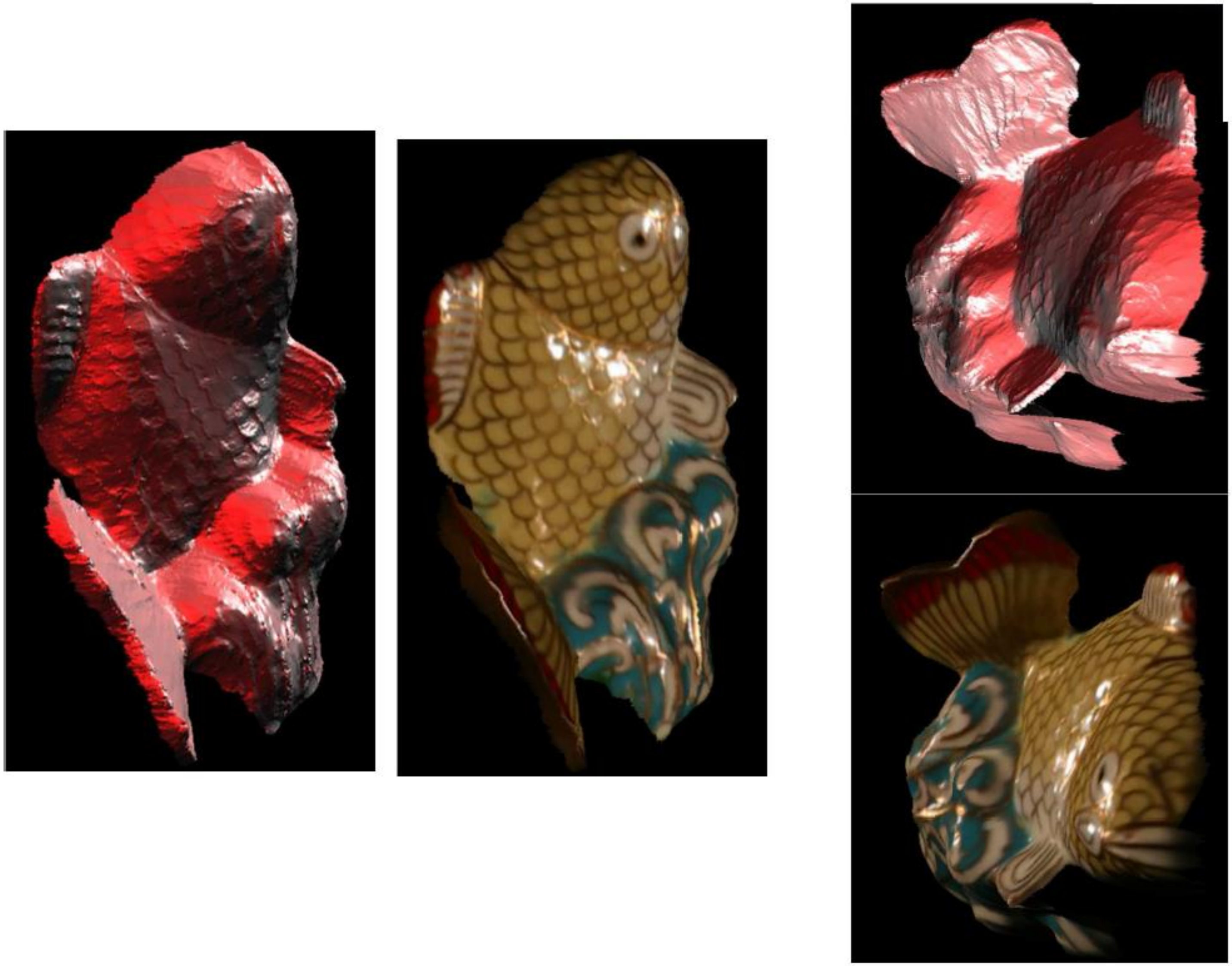
Example-based Photometric Stereo – Cont.



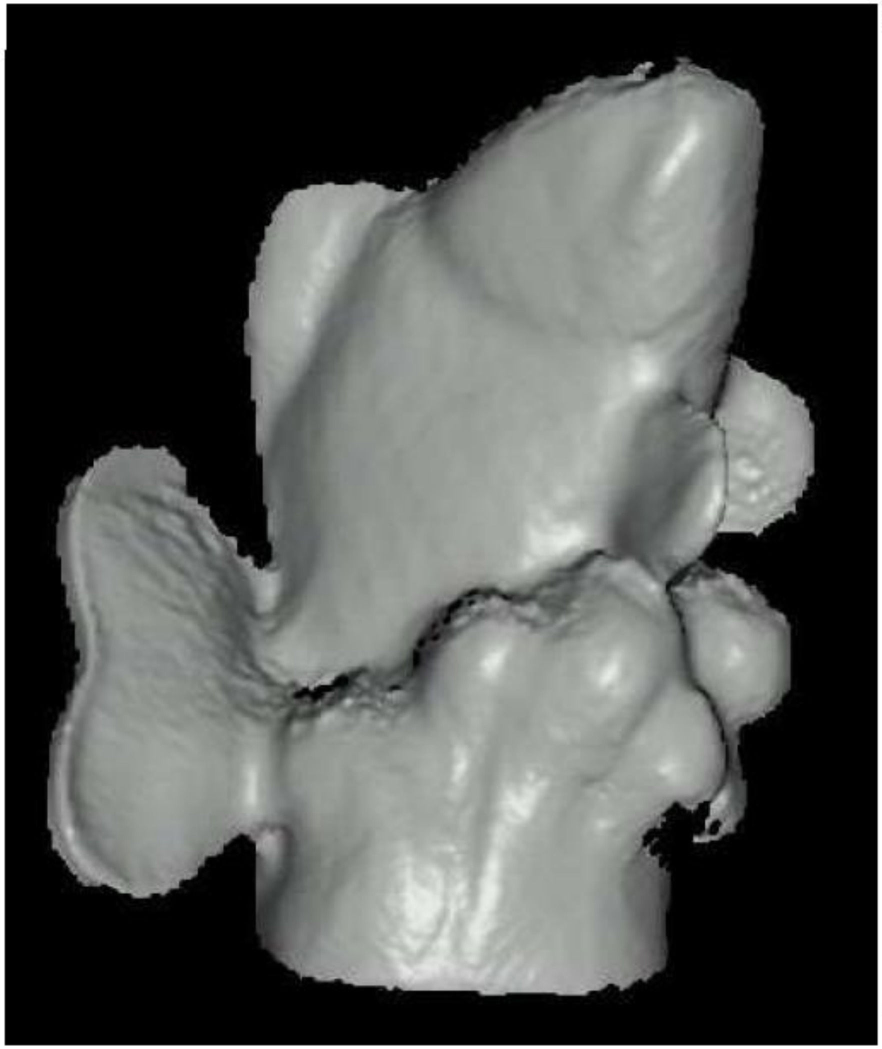
Example-based Photometric Stereo – Cont.



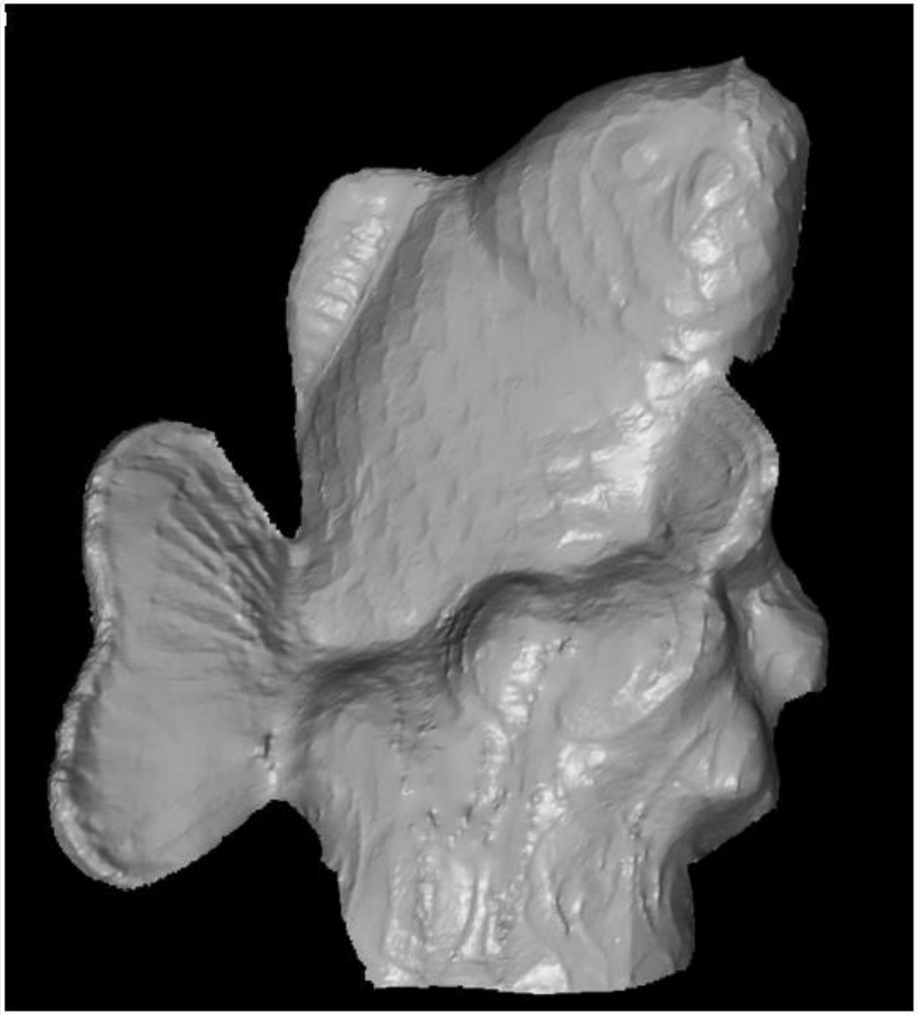
Example-based Photometric Stereo – Cont.



Example-based Photometric Stereo – Cont.



laser scan



photometric stereo

Next Class - Features

Questions?