# Bayesian Cat I

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**Question 1: Single parameter poisson**

Consider the count of airline crashes per year over a 10-year period form 1976 to 1985:

```
y <- c(24,25,31,31,22,21,26,20,16,22)
n<- length(y)
n
bar_x<- mean(y)
bar_x
```

***a. Show that the Gamma distribution is the conjugate prior for a Poisson mean.***

Solution in the steps below;

1. write down the likelihood $p(X|\lambda)$ for $\lambda$.

The pdf and likelihood function of Poisson Distribution with parameter $\lambda$ is given by;

$$P(x|\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$L(\lambda; \mathbf{x}) = \frac{e^{-n\lambda}\lambda^{n\bar{x}}}{\prod_{i=1}^{n} x_i\,!} \propto e^{-n\lambda}\lambda^{n\bar{x}}$$

2. Write down the prior density for $\lambda$.

The prior density Gamma with parameter

$$n, \lambda$$

is given by;

$$P(x|\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\lambda^{(\alpha-1)}e^{(-\lambda\beta)}$$

Ignoring the constant term:

$$P(\lambda) = \lambda^{\alpha-1}e^{-\beta\lambda}$$

3. Multiply them together to obtain the posterior density, and notice that it has the same form as the gamma distribution.

Using Bayes Rule

$$\text{posterior} \propto \text{likelihood} \times \text{prior},$$

That is

$$P(\lambda|x) = P(x|\lambda) \times P(\lambda)$$

Therefore:

$$P(\lambda|x) = e^{-n\lambda}\lambda^{n\overline{x}} \times \lambda^{\alpha-1}e^{-\beta\lambda}$$

$$P(\lambda|x) = \lambda^{n\overline{x}+\alpha-1}e^{-\lambda(n+\beta)}$$

Thus we can notice that the posterior takes a Gamma form:

$$Post(\lambda|x) \sim Gamma(\alpha^*, \beta^*)$$

Where

$$\alpha^* = n\overline{x} + \alpha$$

$$\beta^* = n + \beta$$

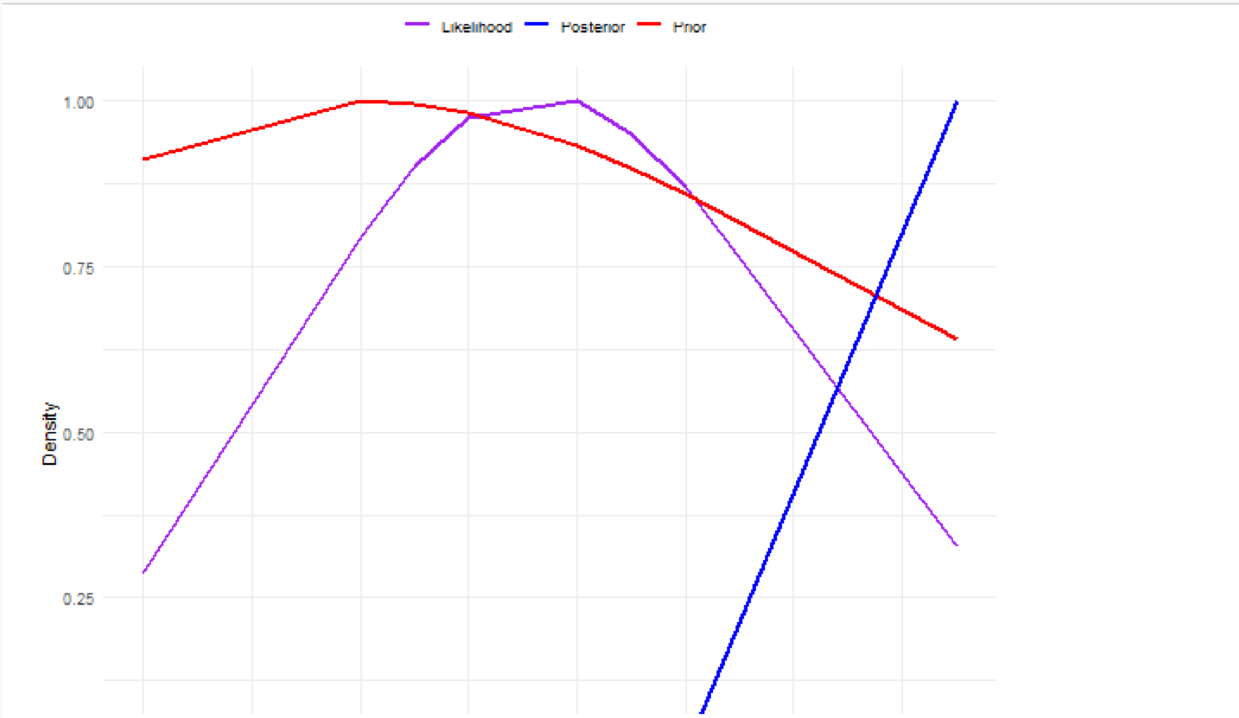
***b. Show graphics of the resulting posterior distribution for choices of various gamma priors***

Confirming the code for different values of alpha and beta:

```
library(shiny)
library(tidyverse)

## -- Attaching packages --------------------------------------- tidyverse 1.3.0 --

## v ggplot2 3.3.3     v purrr   0.3.4
## v tibble  3.0.5     v dplyr   1.0.3
## v tidyr   1.1.2     v stringr 1.4.0
## v readr   1.4.0     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

# ui codes ----------------------------------------------------------------

ui <- fluidPage(
  sidebarLayout(
    sidebarPanel(
    sliderInput("n", "Play to increase sample size", min = 1,
                max = 10, step = 1,value = 10, animate = T),
    tags$hr(),
    numericInput("beta", "Beta value", value = 5),
    tags$hr(),
```

```
    numericInput("alpha", "Alpha Value", value = 5), width = 2
    ),

    mainPanel(
      h3("The likelihood and posterior distributions converge as the sample
size n increases"),
      br(),
plotOutput("my_plot", height = "500px")
    )

  )
)

# server code ------------------------------------------------------------

server <- function(input, output, session){



# define posterior function ----------------------------------------------
y <- c(24,25,31,31,22,21,26,20,16,22)
Lambda <- mean(y)
post <- function(n = 10,alpha = 5, beta = 1/Lambda){

lambda<-Lambda  # 1data initialization

LL <- dpois(y,lambda)  # quartiles of a binomial

# prior
Prior <- dgamma(y,shape = alpha,scale = beta)  #beta prior distribution

# posterior
alpha1 <- n*lambda + alpha
beta1 <- n+ beta

Postr <- dgamma(y,shape=alpha1,scale = beta1)

ggplot(data = NULL, aes(y, LL/max(LL), col = "Likelihood")) + geom_line(size
= 1.0)+
  geom_line(aes(y, Prior/max(Prior), col = "Prior"), size = 1.0) +
  geom_line(aes(y, Postr/max(Postr), col = "Posterior"), size = 1.0) +
  labs(y = "Density", x= expression(y)) + theme_minimal() +
  scale_colour_manual("", values = c("Likelihood"="purple", "Prior"="red",
"Posterior"="blue")) +
  theme(legend.position = "top")

}
```

```
output$my_plot <- renderPlot({
  post(n=input$n,alpha = input$alpha, beta = input$beta)
})


}

shinyApp(ui, server)
```




***c. Show a table of the prior, MLE and posterior estimates of the poison mean under different choices of the gamma priors in (a) above.***

1.When $\alpha = 3$ and $\beta = 6$

```
Table_Estimates <- function(alpha,beta){

y <- c(24,25,31,31,22,21,26,20,16,22)
lambda <- mean(y)
MLE <- dpois(y,lambda)
Prior01 <- dgamma(y,shape = alpha,scale = beta)
Postr01 <- MLE*Prior01

Estimates <- rbind(MLE,Prior01,Postr01)

return(Estimates)
```

```
}
df <- data.frame(Table_Estimates(3,6))
df

##                   X1          X2           X3           X4          X5
## MLE     0.081083527 0.077191517 0.0264639843 0.0264639843 0.079016501
## Prior01 0.024420852 0.022430344 0.0126877560 0.0126877560 0.028638384
## Postr01 0.001980129 0.001731432 0.0003357686 0.0003357686 0.002262905
##                   X6          X7          X8           X9         X10
## MLE     0.073040463 0.070659927 0.064447467 0.0233562465 0.079016501
## Prior01 0.030826496 0.020536205 0.033031475 0.0411753785 0.028638384
## Postr01 0.002251582 0.001451087 0.002128795 0.0009617023 0.002262905
```

**QUESTION TWO**

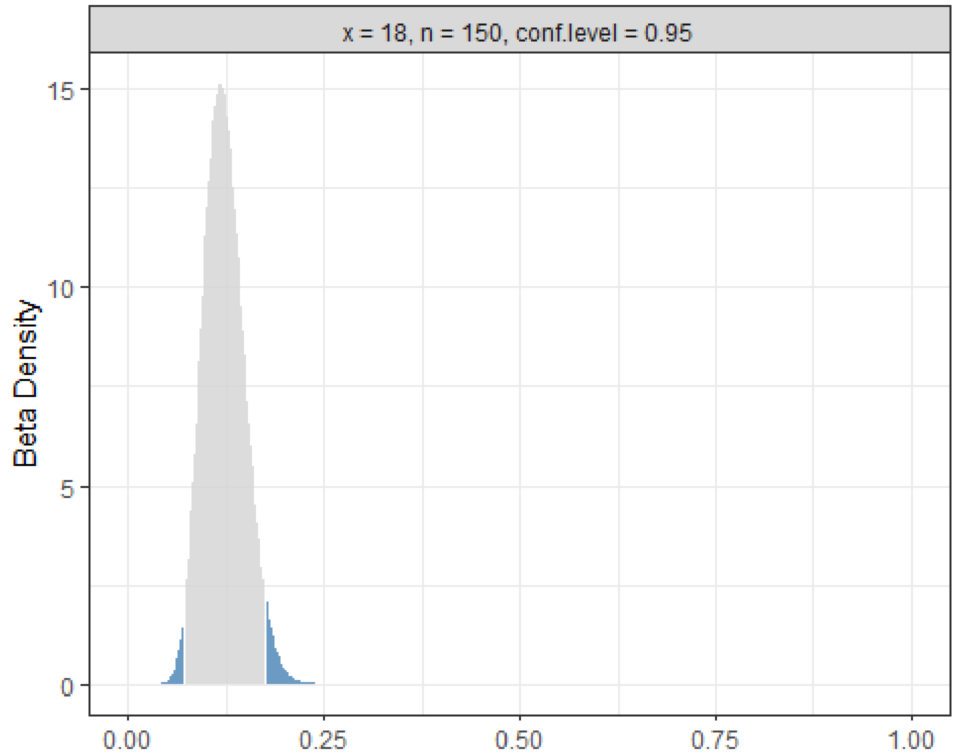
***a.Differentiate between Credible Intervals and the Highest Posterior Density (HPD) in Bayesian analysis.***

A credible interval is an interval within which an unobserved parameter value falls with a particular probability. It is an interval in the domain of a posterior probability distribution or a predictive distribution.On the other hand, a highest posterior density (interval) is basically the shortest interval on a posterior density for some given confidence level.

***b.Graphically demonstrate this for a disease prevalence given that out of 150 individuals 18 had the disease. Hint: assume Binomial distribution***

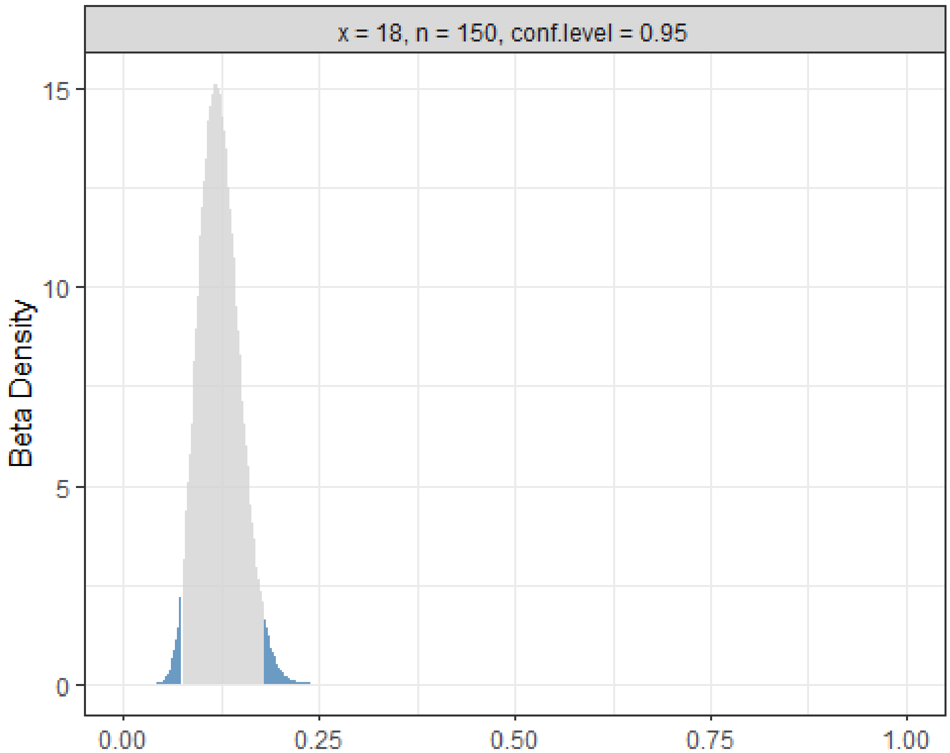
Highest Posterior Density (HPD)

```
library(binom)
HPD <- binom.bayes(x=18,n=150,type = "highest",conf.level = 0.95,tol = 1e-9)
print(HPD)

##   method  x   n shape1 shape2      mean      lower     upper  sig
## 1  bayes 18 150   18.5  132.5 0.1225166 0.07246161 0.1754186 0.05

binom.bayes.densityplot(HPD)
```




Credible Interval

```
library(binom)
Central <- binom.bayes(x=18,n=150,type = "central",conf.level = 0.95,tol =
1e-9)
print(Central)

##   method  x   n shape1 shape2      mean      lower    upper  sig
## 1  bayes 18 150   18.5  132.5 0.1225166 0.07534776 0.179135 0.05

binom.bayes.densityplot(Central)
```



x = 18, n = 150, conf.level = 0.95
Beta Density
15
10
5
0
0.00
0.25
0.50
0.75
1.00