Machine Learning

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Today's Objectives

- A quick recap of the last lecture
- Understanding image convolution and its applications
- Why do we need convolutional neural networks (CNNs)?
- Padding, Stride, Multiple Channels, and Multiple Filters
- Types of Layers in a CNN
- See a simple example of CNN step by step

Reflection

1. What are the features in machine learning and why are they important?
2. What is feature or representation learning, and why is it important?
3. How can artificial neural networks (ANNs) help us in feature learning?
4. What enables ANNs to learn hierarchical features?
5. What enables ANNs to learn nonlinear features?
6. What is backpropagation?

Recap (1)

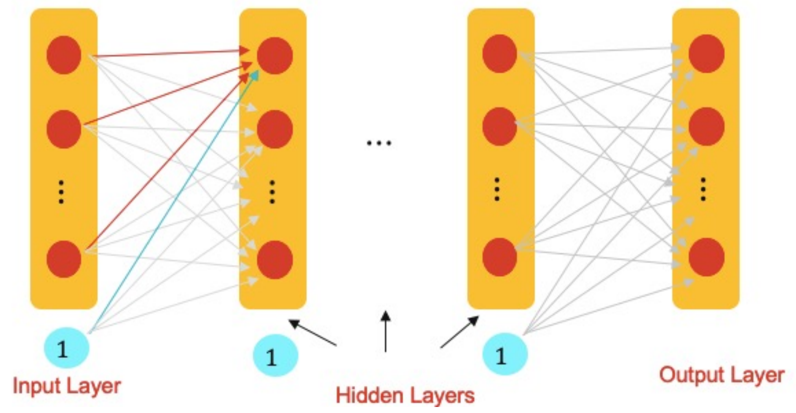
DNNs

1. **Feature learning** is better than using hand-crafted features
2. Feature learning should aim for learning **hierarchical features**
3. Higher level features **should not be simple linear combinations** of low-leve features

Deep Learning enables us to achieve these goals!

Feedforward Networks

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right)$$



Activation Functions

1. Sigmoid
2. Hyperbolic Tangent
3. Rectified Linear Unit
4. Leaky ReLU
5. Exponential ReLU

Recap (2)

Backpropogation

- At the heart of backpropagation is an expression for the partial derivative of the cost function C with respect to every weight w (or bias b) in the network.

Neural Learning

- Using Gradient Descent find the optimum weights and the biases for the DNN that minimze the error or the cost

$$\boxed{\delta_j^L = \frac{\partial C}{\partial a_j^L}\sigma'(z_j^L)}$$

$$\boxed{\frac{\partial C}{\partial b_j^l} = \delta_j^l}$$

$$\boxed{\delta_j^l = \sum_k w_{kj}^{l+1}\delta_k^{l+1}\sigma'(z_j^l).}$$

$$\boxed{\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1}\delta_j^l}$$

Recap (3)

Backpropogation Algorithm

1. **Input a set of training examples**

2. **For each training example x:** Set the corresponding input activation $a^{x,1}$, and perform the following steps:

   - **Feedforward:** For each $l = 2, 3, \ldots, L$ compute $z^{x,l} = w^l a^{x,l-1} + b^l$ and $a^{x,l} = \sigma(z^{x,l})$.

   - **Output error $\delta^{x,L}$:** Compute the vector $\delta^{x,L} = \nabla_a C_x \odot \sigma'(z^{x,L})$.

   - **Backpropagate the error:** For each $l = L-1, L-2, \ldots, 2$ compute $\delta^{x,l} = ((w^{l+1})^T \delta^{x,l+1}) \odot \sigma'(z^{x,l})$.

3. **Gradient descent:** For each $l = L, L-1, \ldots, 2$ update the weights according to the rule $w^l \rightarrow w^l - \frac{\eta}{m} \sum_x \delta^{x,l} (a^{x,l-1})^T$, and the biases according to the rule $b^l \rightarrow b^l - \frac{\eta}{m} \sum_x \delta^{x,l}$.

Recall: Features are Important!

- Also called "Representations"

- We want these features to be:
  - Informative, discriminative, and invariant to common variations, such as rotation, scaling, and noise
  - Generalizable – able to capture and represent the underlying patterns and structure of the data in a way that can be effectively used to make predictions on new, unseen data.
  - Hierarchical – hierarchical features are learned by progressively combining and abstracting lower-level features to form more complex and informative higher-level features.

Edges → Lines → Curves and Corners → Rectangles, Circles, Triangles, → Body and Tyres → Shape

- Therefore, in a deep learning model, such as a Convolutional Neural Network (CNN),
  - generalizable features are learned through the process of training the model on a large and diverse dataset,
  - Allows the model to learn to recognize and represent the relevant patterns and structures in the data.
  - The features learned by the model are typically hierarchical and increasingly abstract, capturing local and global relationships and dependencies in the data.

Image Convolution and Their Applications

Convolutions

- Sounds fancy but it is not!

- Every time we do image blurring, smoothing, sharpening, edge detection, etc. we are doing convolutions

- Convolutions are one of the most critical, fundamental building-blocks in computer vision and image processing.

What is a convolution?

"***In terms of deep learning, an (image) convolution is an element-wise multiplication of two matrices followed by a sum***"

1. Take two matrices (both have the same dimensions).
2. Multiply them, element-by-element (i.e., not the dot product, simple element-to-element multiplication).
3. Sum the elements of the resulting Matrix.

What is a convolution?

1	0	-1
1	0	-1
1	0	-1

element-to-element multiplication

3	0	1
1	5	8
2	7	2

=

3	0	-1
1	0	-8
2	0	-2

Sum all Elements

-5 =

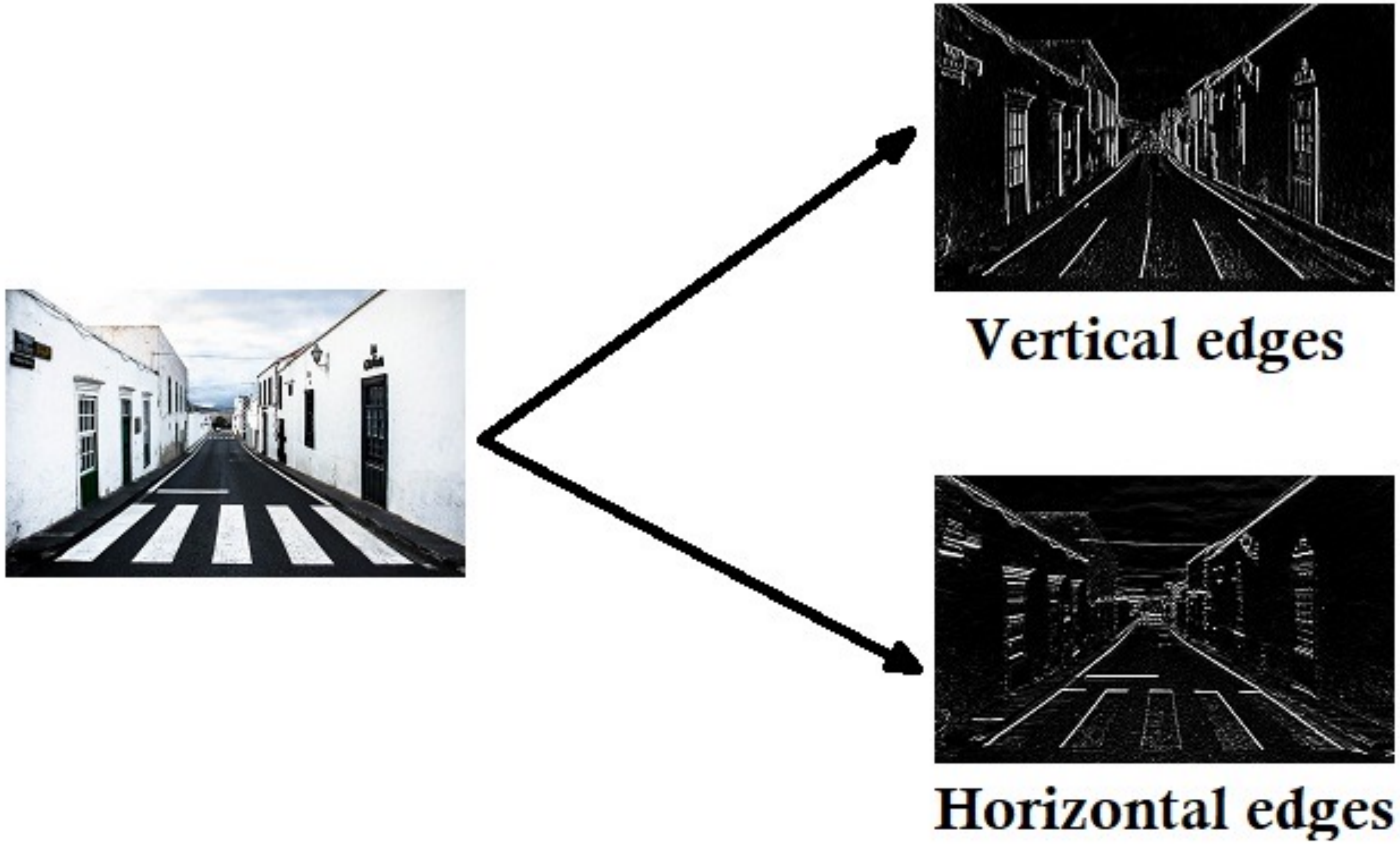
Applications of Convolutions in Images

- Edge Detection
- Image smoothing
- Image sharpening
- Image enhancement

Let’s look at edge detection in more detail.

Edge Detection

- Two types of edges
  - Horizontal Edges
  - Vertical Edges



So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

- Example of a simple image of resolution 6 x 6
- Each cell contains a grayscale value
- This is what an image looks like to a computer
- We want to detect vertical edges in this image

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

$*$

Convolution

3 x 3

Filter or Kernel

- To do that, we will use a small matrix called *Kernel*
- And convolve the Kernel with the image

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

- How do we select the size of the Kernel and the values in the kernel?

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

3 x 3

29 x 29

So how do we detect edges?

- Let's see why the result will be a 4 x 4 matrix.

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

4 x 4

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3^{1}	0^{0}	1^{-1}	2	7	4
1^{1}	5^{0}	8^{-1}	9	3	1
2^{1}	7^{0}	2^{-1}	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

4 x 4

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3^{1}	0^{0}	1^{-1}	2	7	4
1^{1}	5^{0}	8^{-1}	9	3	1
2^{1}	7^{0}	2^{-1}	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

-5			

4 x 4

Next we roll the window forward on step!

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0^{1}	1^{0}	2^{-1}	7	4
1	5^{1}	8^{0}	9^{-1}	3	1
2	7^{1}	2^{0}	5^{-1}	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

-5			

4 x 4

Calculate the result of convolution!

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0^{1}	1^{0}	2^{-1}	7	4
1	5^{1}	8^{0}	9^{-1}	3	1
2	7^{1}	2^{0}	5^{-1}	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

-5	-4		

4 x 4

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

-5	-4	0	8

4 x 4

We have computed the entire first row, what now?

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

-5	-4	0	8
-10			

4 x 4

We move the window down and start from left again

So how do we detect edges?

- Vertical Edge Detection in a Grey Scale Image

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6 x 6

*

Convolution

1	0	-1
1	0	-1
1	0	-1

3 x 3

Filter or Kernel

=

-5	-4	0	8
-10	-2	2	3
0	-2	-4	-7
-3	-2	-3	-16

4 x 4

- Dimensions of output depend on: (1) Size of Kernel, (2) Step size

Kernels for Vertical and Horizontal Edge Detection

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

Edges are *Features*

- Edges represent the boundary of an object in an image
- We can use them to identify the objects: face, car, street signs, etc.
- Thus, edges can be thought of as features, or predictors.

More Generally

- Kernels (or filters) and Convolution can help us detect features in a given input.

- Thus Kernels (or filters) can be thought of as ***feature detectors***

- Idea: *to get good features <u>train good feature detectors</u>*

Learning Kernels or Filters for Classification

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

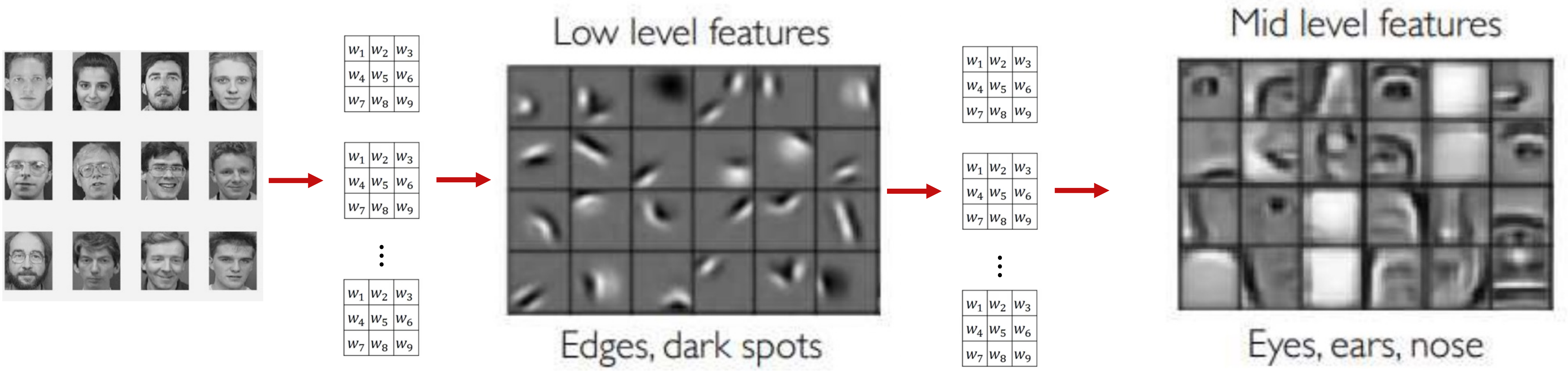
- Filter is represented by the parameters that we want to learn
- Learning happens under a loss function
- In other words, we learn those filters that helps us discover features that improve classification

Hierarchical Features

- Higher-level features can be built from lower-level features
- For example, edges can be combined to detect noses, eyes, and lips
- Nose, eyes, and lips can be combined to detect faces
- Etc.
- Idea: *we can learn kernels from the data in a hierarchical fashion*

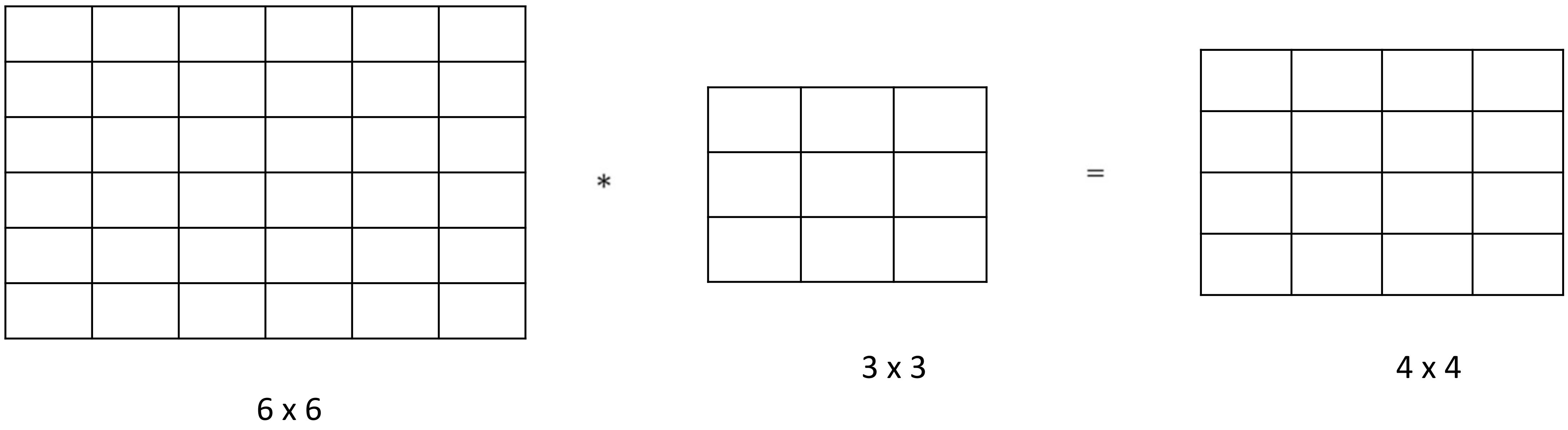
And this brings us to CNNs.

Hierarchical Feature Detectors

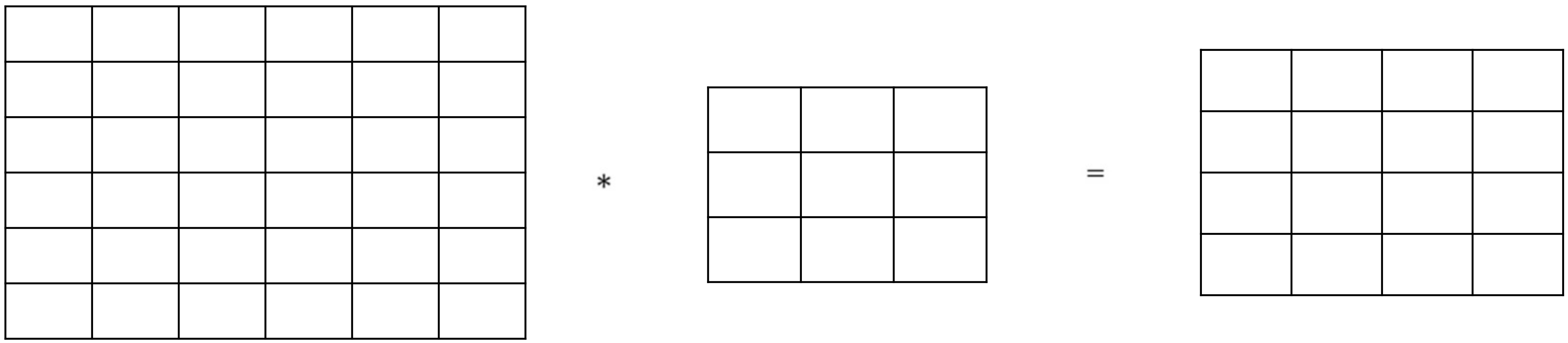


Padding, Stride, Multiple Channels, and Multiple Filters

Dimensions in Convolution



In General



6 x 6 3 x 3 4 x 4

w x h f x f (w – f + 1) x (h – f + 1)

f is usually odd!

Problem

1. Thus, the size decreases (shrinking output)

2. As well as, the pixels on the edges are used less than those in the center of the image

- And in order to fix these problems, we can do padding (pad the image with additional border of pixels)

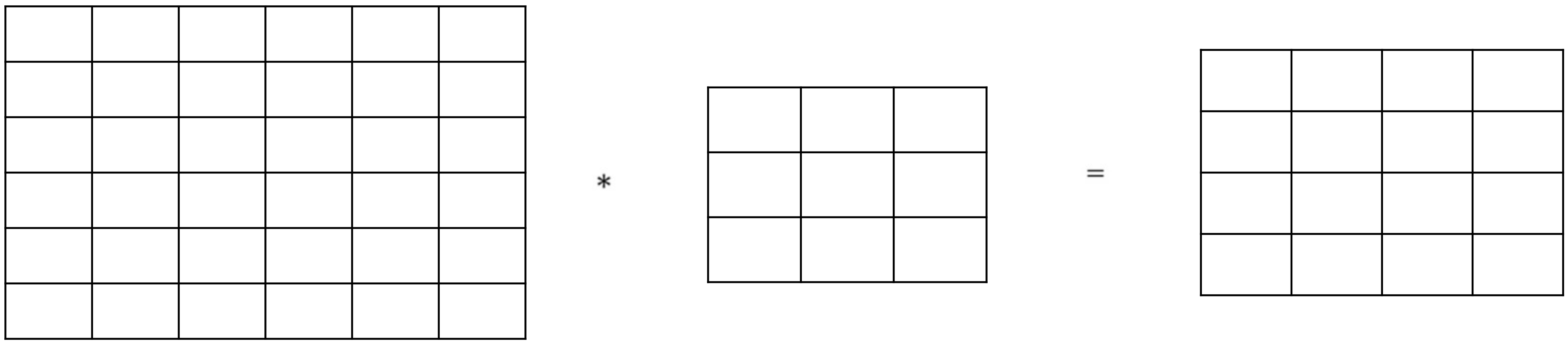
Padding

95	242	186	152	39
39	14	220	153	180
5	247	212	54	46
46	77	133	110	74
156	35	74	93	116

→

0	0	0	0	0	0	0
0	95	242	186	152	39	0
0	39	14	220	153	180	0
0	5	247	212	54	46	0
0	46	77	133	110	74	0
0	156	35	74	93	116	0
0	0	0	0	0	0	0

Original Image is 6 x 6, Output Image is 4 x 4



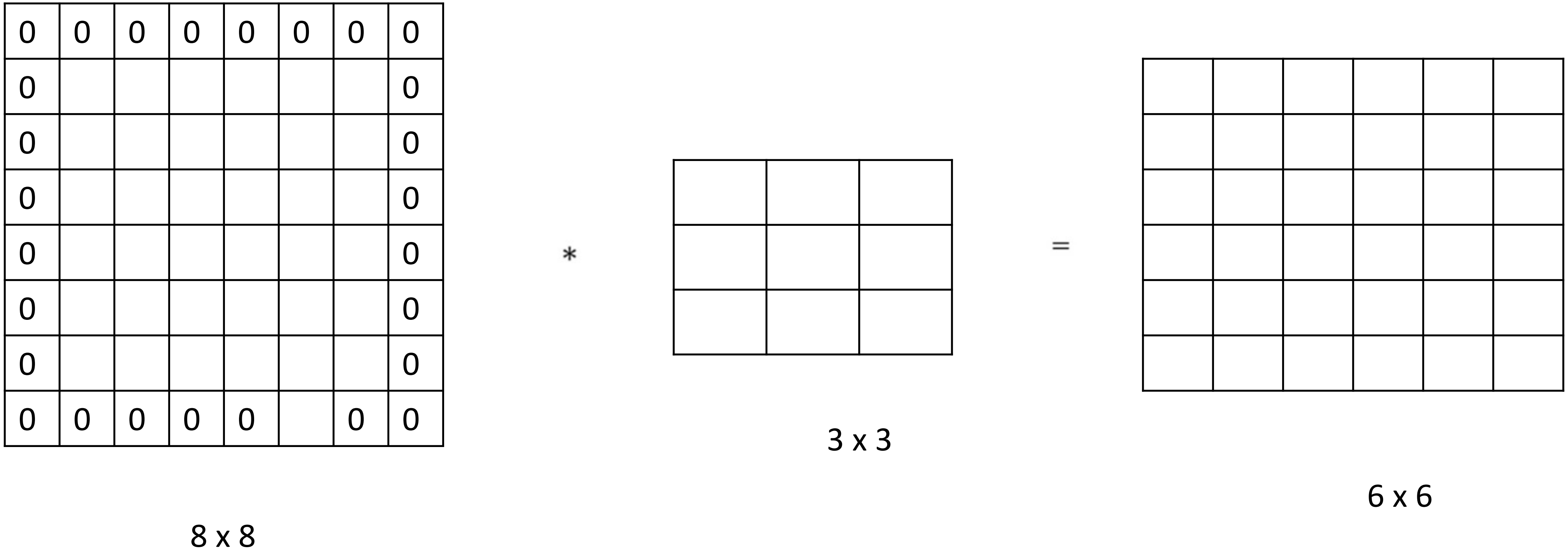
3 x 3

6 x 6

4 x 4

If we want the output image to be 6 x 6 then we have to do the padding.

New Dimensions After Padding



Padding (Dimensions)

- Input Size: w x h
- Filter Size: f x f
- Padding: p
- Output Size: $(w + 2p – f + 1)$ x $(h + 2p – f + 1)$

Valid and Same Convolutions

- Define whether convolution is with or without padding
- Valid: when convolution is done without padding.
- Same: when padding is done to keep the output size the same as input size

$$p = \frac{f - 1}{2}$$

Stride

95	242	186	152	39
39	14	220	153	180
5	247	212	54	46
46	77	133	110	74
156	35	74	93	116

*

0	1	0
1	-4	1
0	1	0

=

692	-315	-6
-680	-194	305
153	-59	-86

When moving filter across the input, we are stopping at each coordinate

What if we did a stride of 2?

Stride

95	242	186	152	39
39	14	220	153	180
5	247	212	54	46
46	77	133	110	74
156	35	74	93	116

*

0	1	0
1	-4	1
0	1	0

=

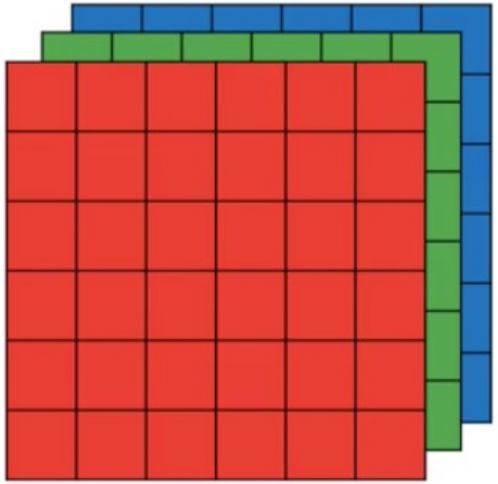
692	-6
153	-86

With a Stride of 2

Dimensions with Stride

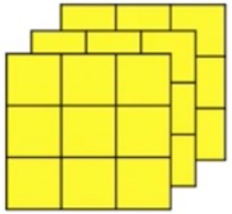
- Input Size: w x h
- Filter Size: f x f
- Padding: p
- Stride: S
- Output Size: $\left\lfloor \frac{w+2p-f}{s} + 1 \right\rfloor$ x $\left\lfloor \frac{h+2p-f}{s} + 1 \right\rfloor$

Convolutions on RGB Image



6 x 6 x 3

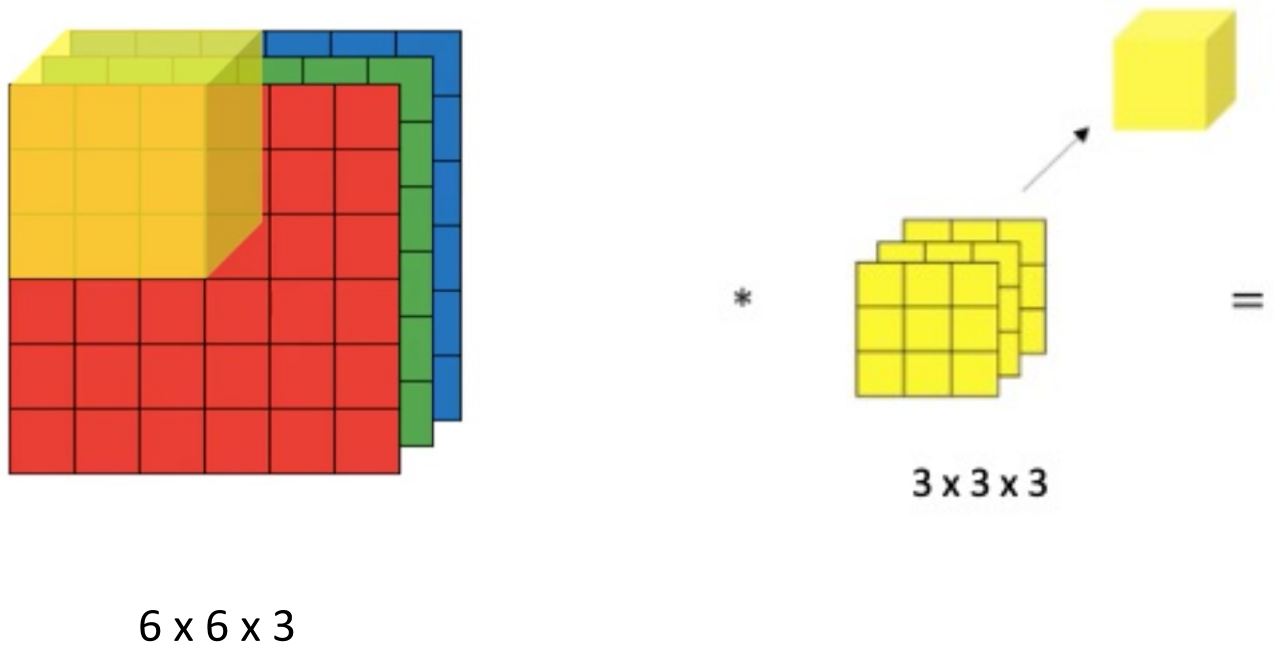
*



3 x 3 x 3

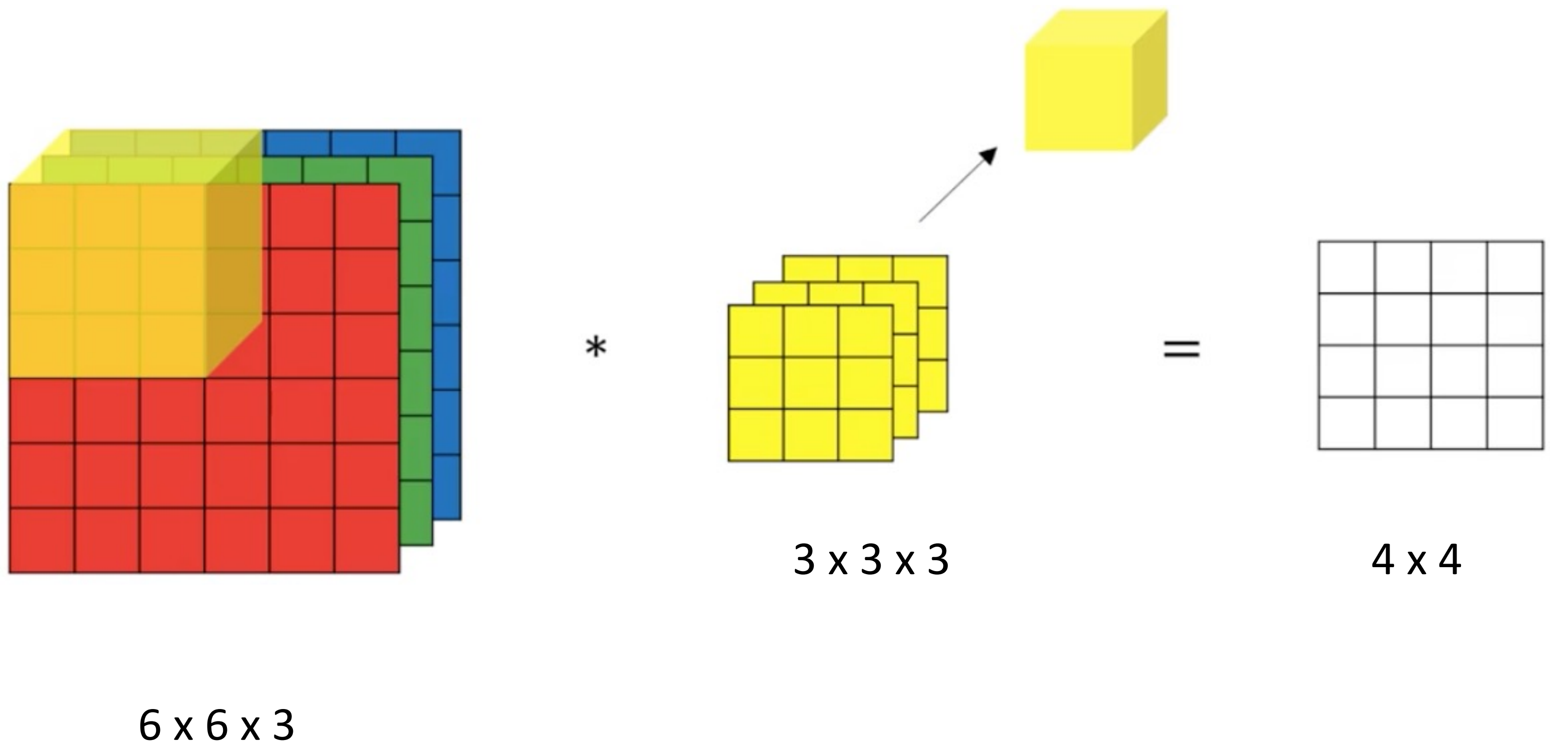
=

Convolutions on RGB Image

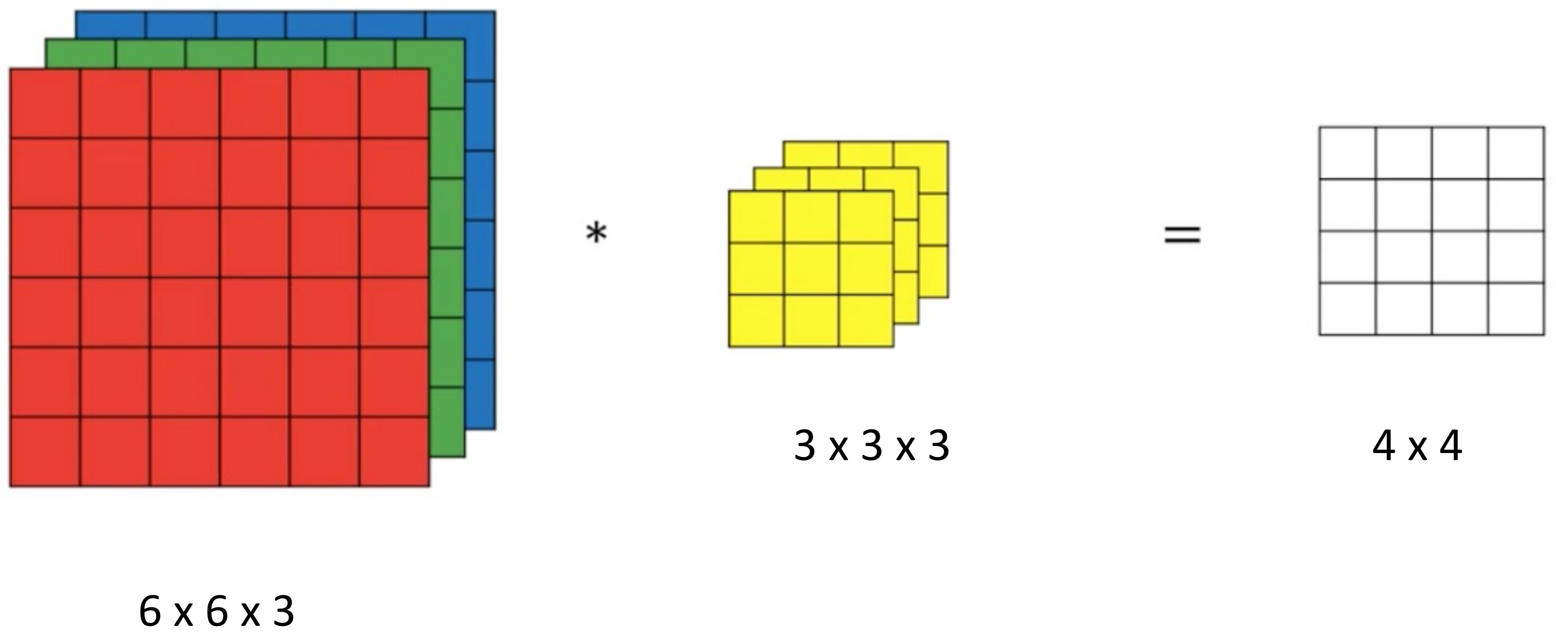


You do $f \times f \times ch$ multiplication, and then add the result of all multiplications

Convolutions on RGB Image



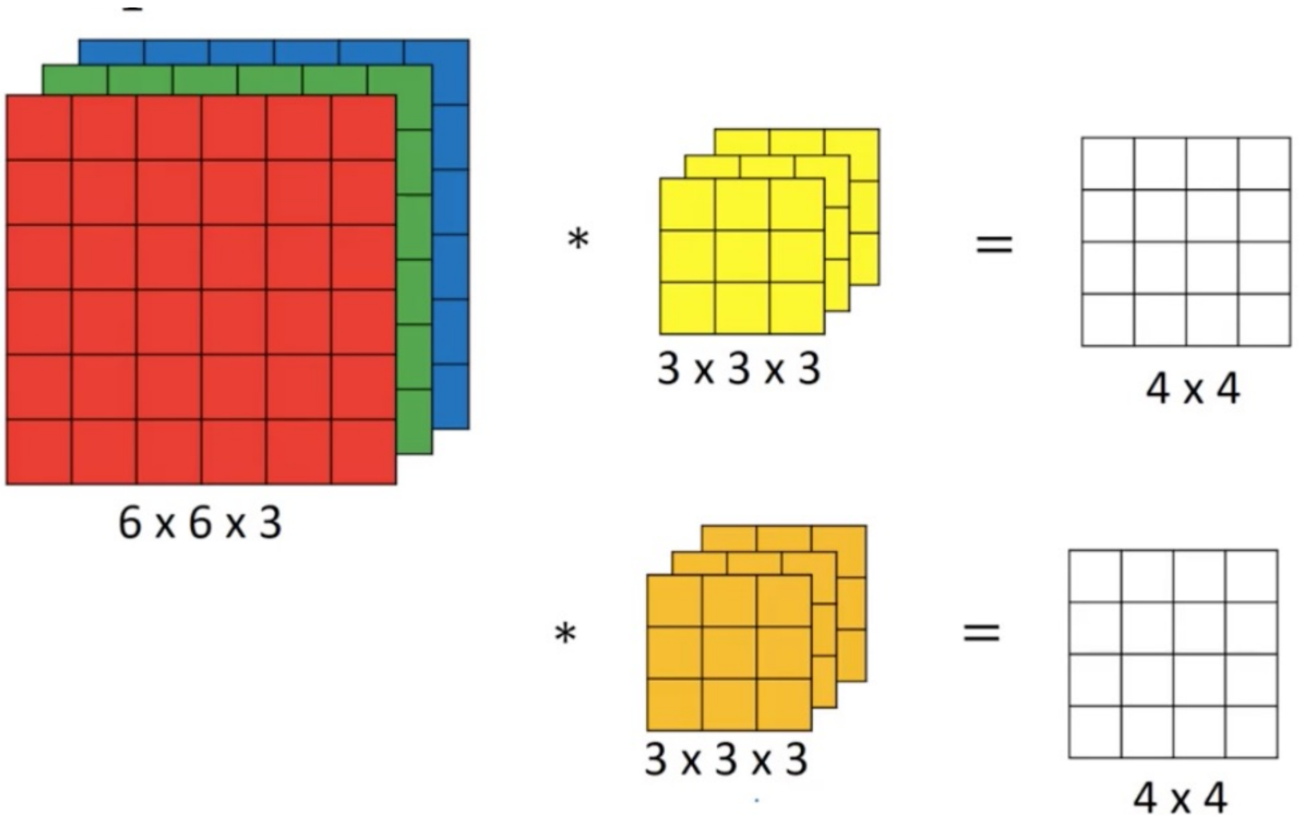
Convolutions on RGB Image



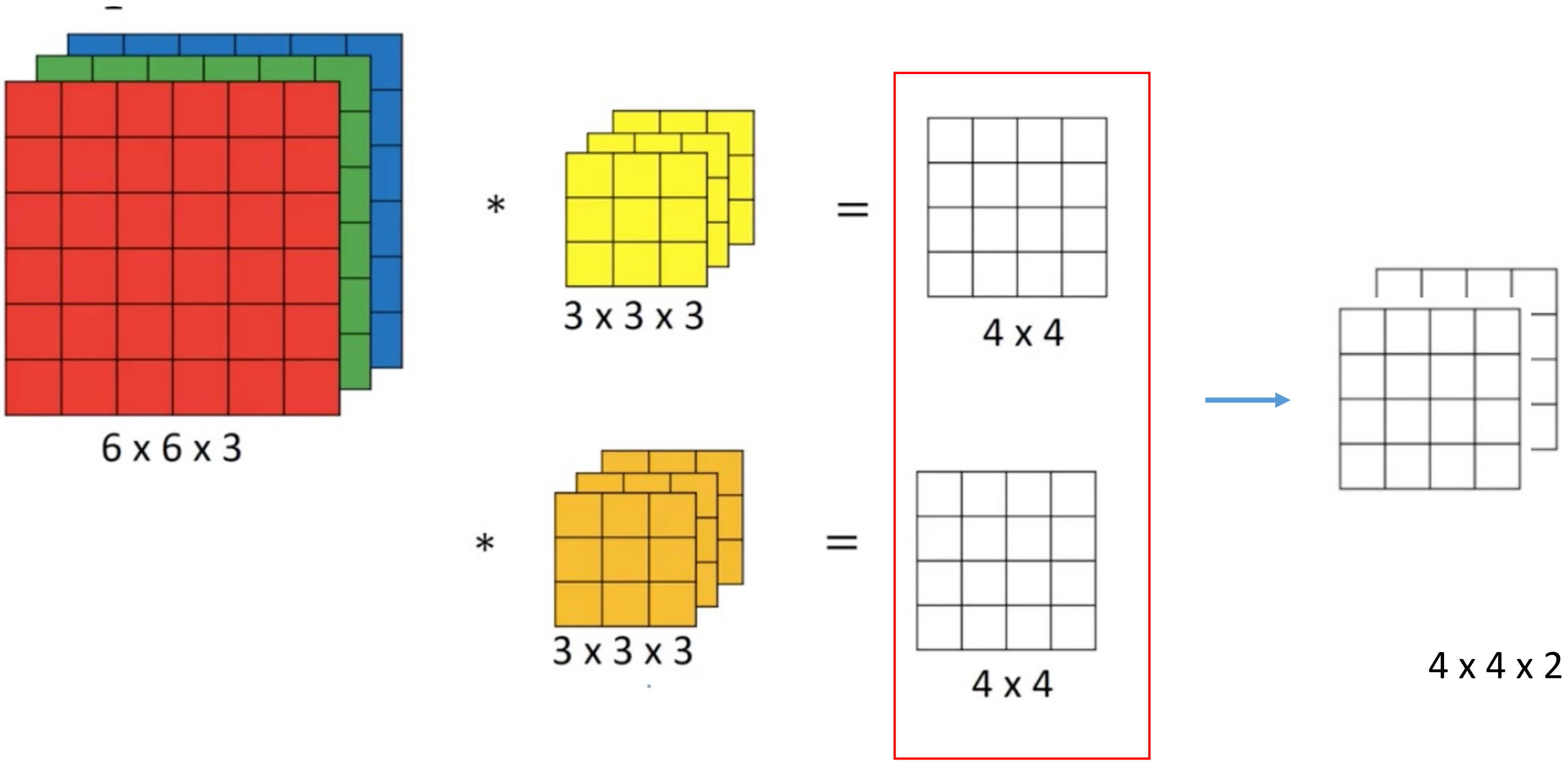
Three dimensional input, three dimensional kernel!

But a 2 dimensional output!

Multiple Filters

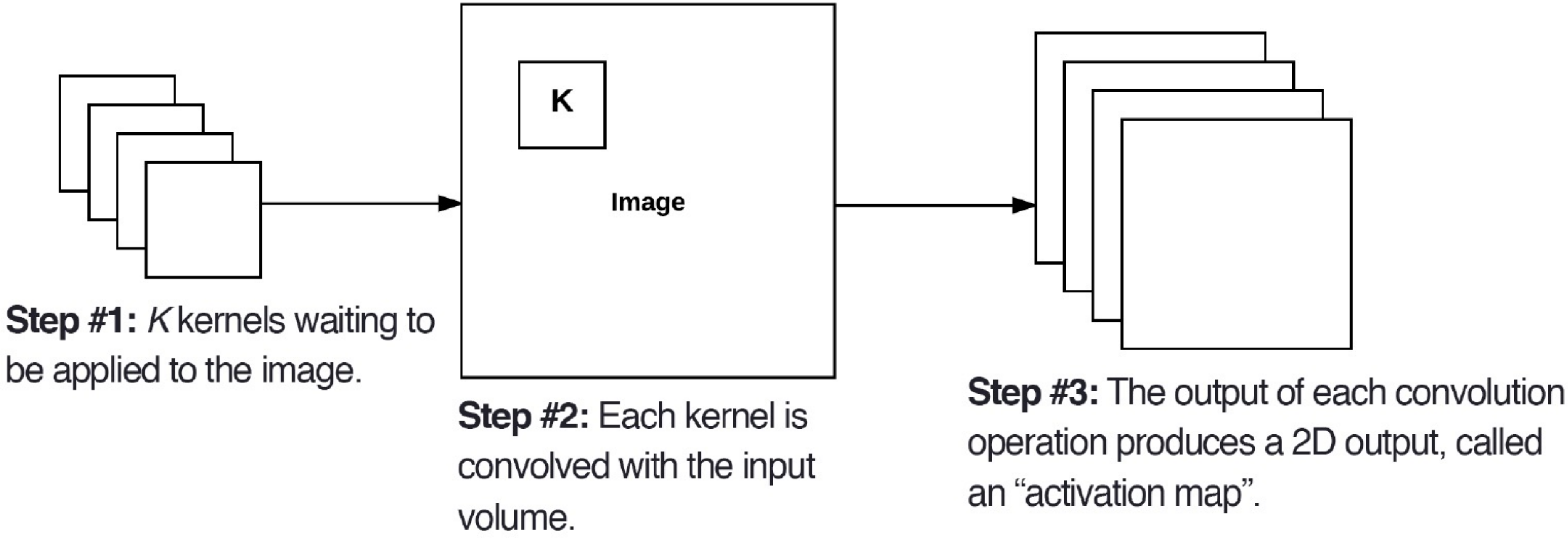


Multiple Filters



Activation Maps – combined as a volume

In General


K
Image
Step #1: *K* kernels waiting to be applied to the image.
Step #2: Each kernel is convolved with the input volume.
Step #3: The output of each convolution operation produces a 2D output, called an “activation map”.

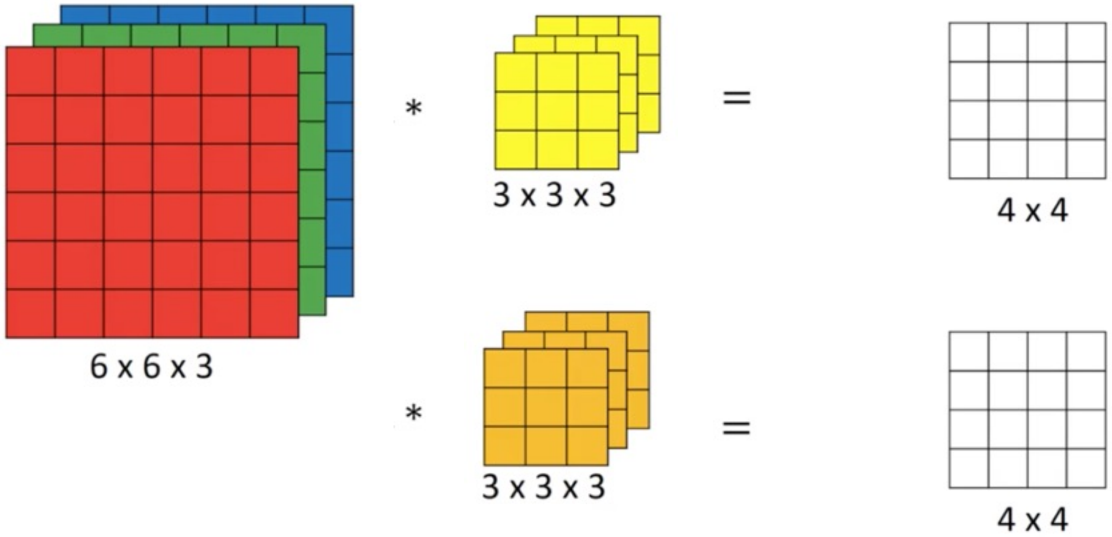
Different types of Layers in a CNN

Types of Layers in a Convolutional Net:

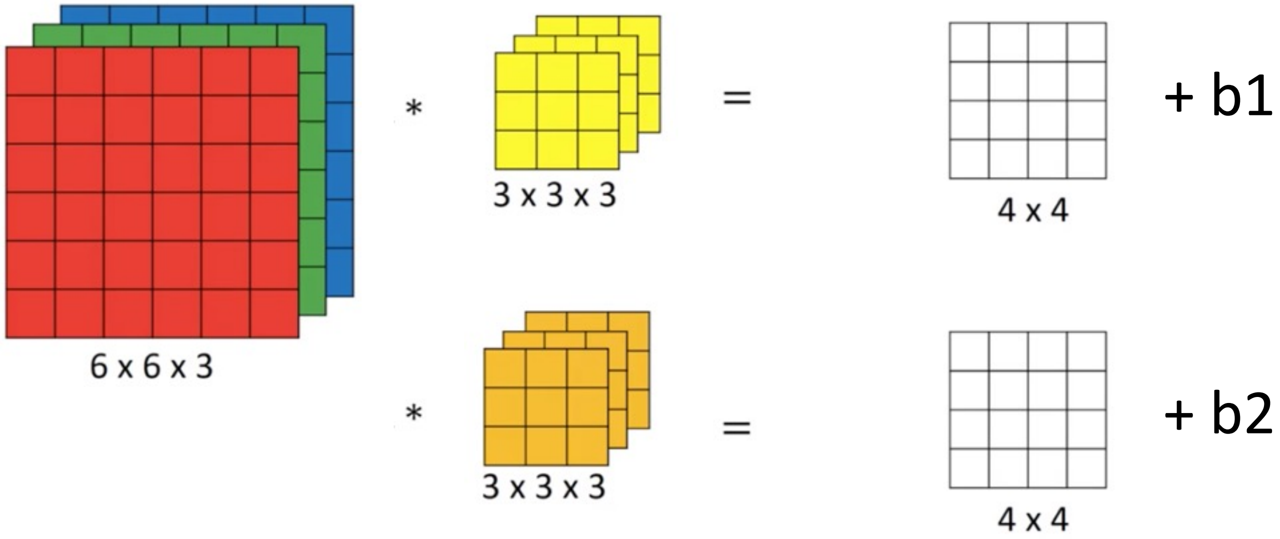
- Convolutional Layers (CONV)
- Pooling Layers (POOL)
- Fully connected (FC)

What Happens in a CONV layer?

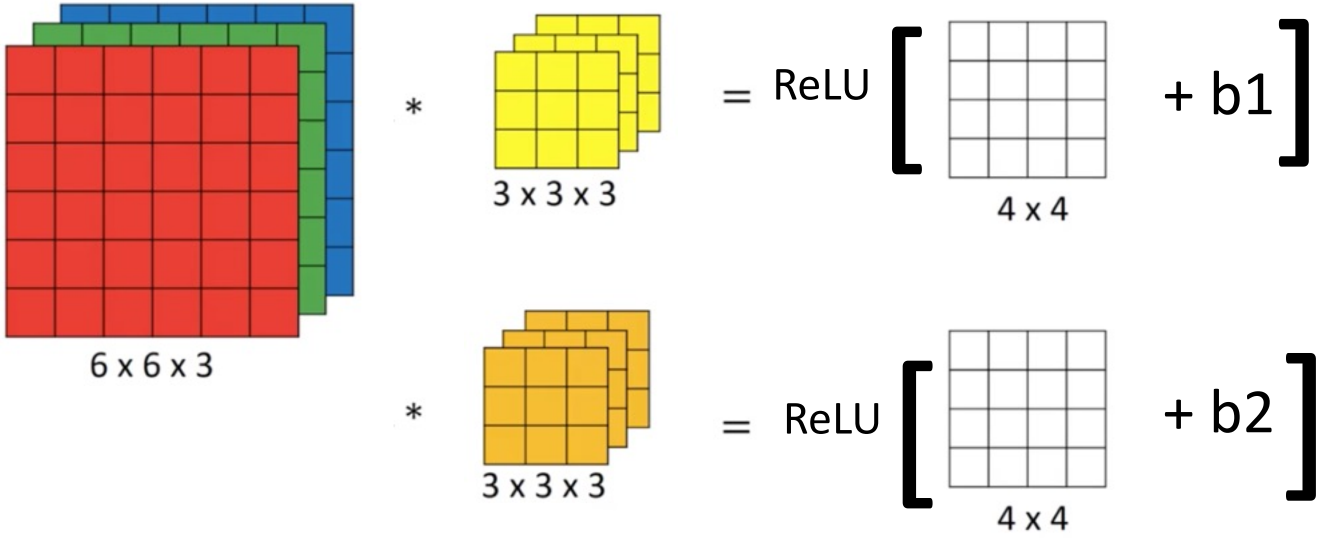
Conv Layer of a CNN



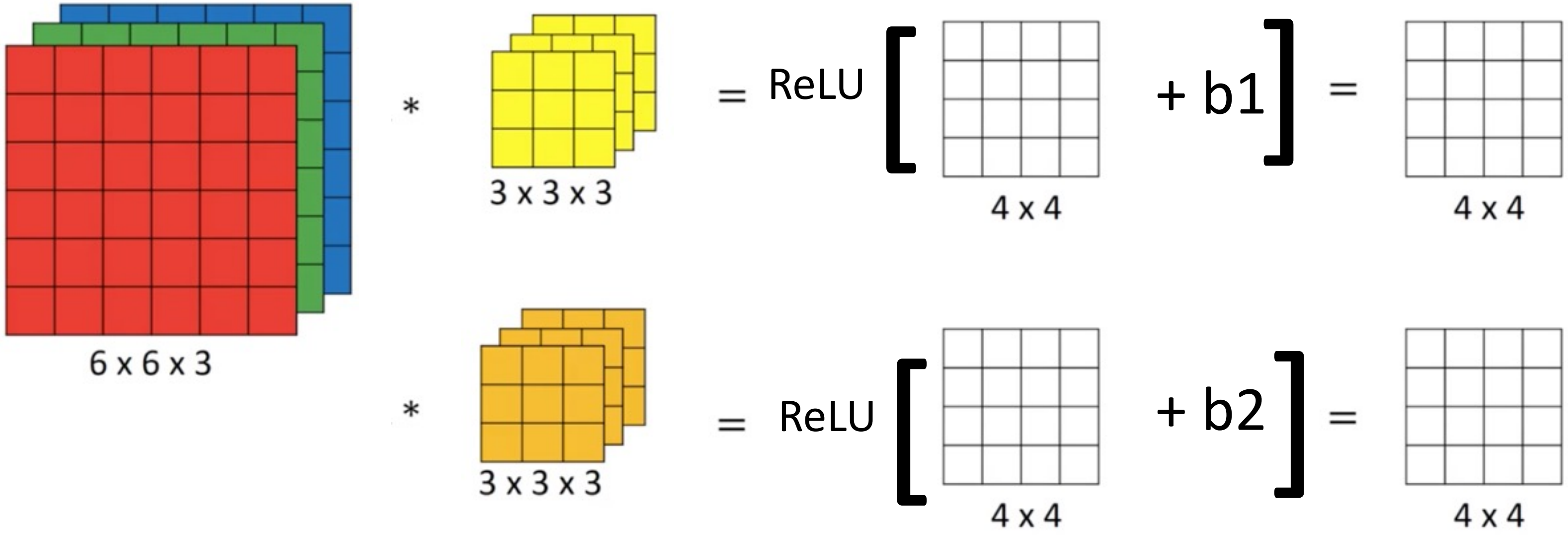
Conv Layer of a CNN



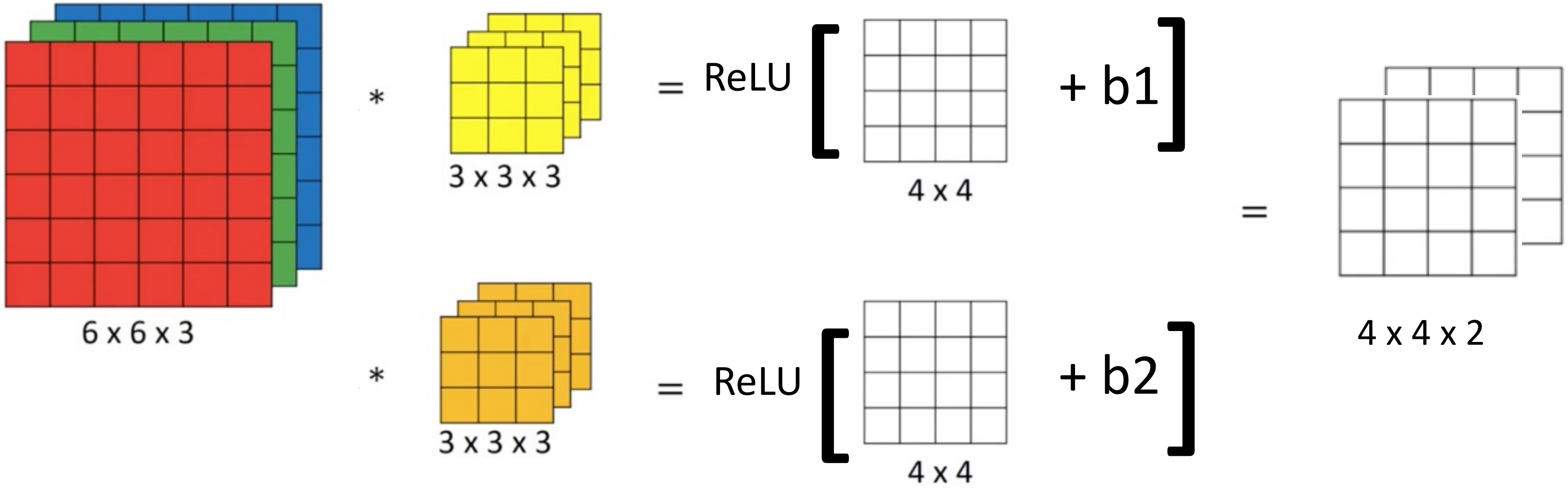
Conv Layer of a CNN



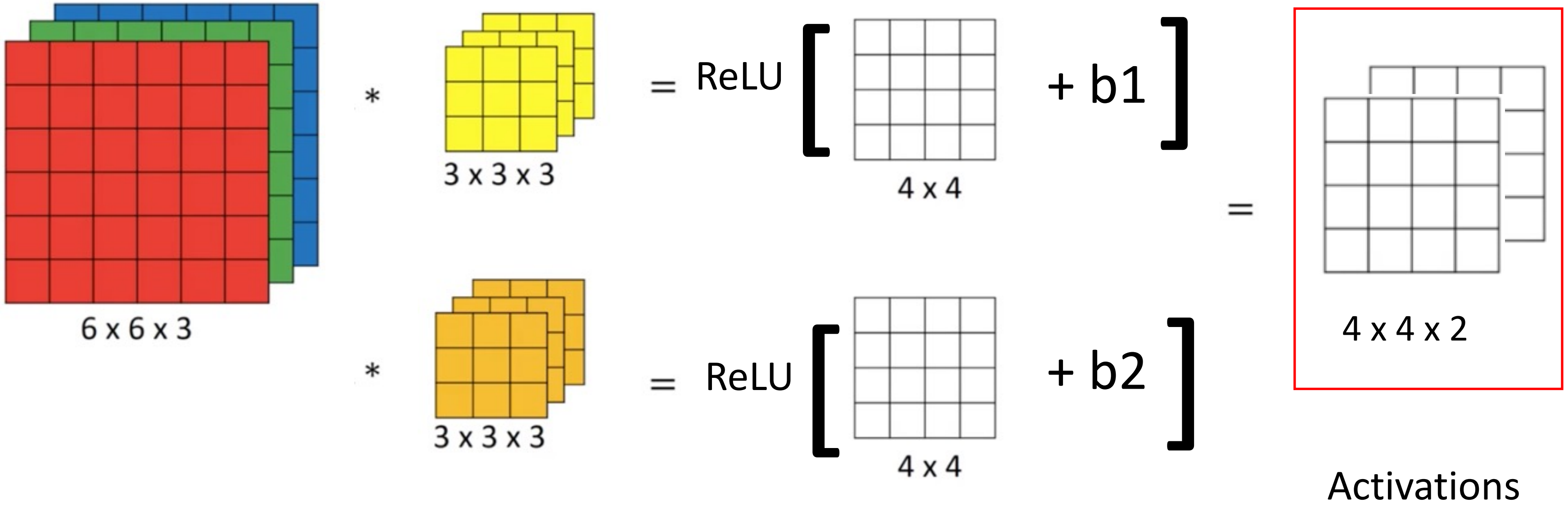
Conv Layer of a CNN



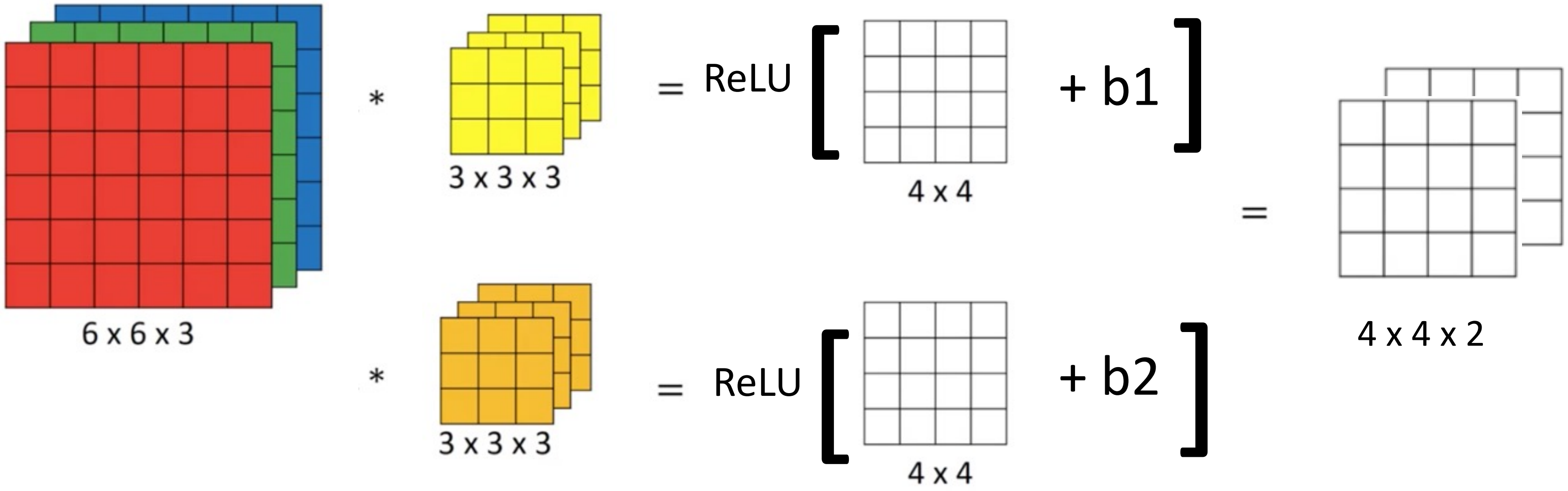
Conv Layer of a CNN



Conv Layer of a CNN



Conv Layer of a CNN



ReLU is just for example, you can apply any other activation function

Number of Parameters in a Conv Layer

- Let's say that in a layer:
  - You have 10 filters
  - Each filter is of size 3 x 3 x 3
  - How many parameters does this layer have?

Number of Parameters in a Conv Layer

- Let’s say that in a layer:

  - You have 10 filters
  - Each filter is of size 3 x 3 x 3
  - How many parameters does this layer have?

  - For each filter we have 27 + 1 parameters (1 for bias)
  - Total number of parameters for 10 filters = 280

What Happens in a POOLING layer?

Pooling

- The pooling layer summarises the features present in a region of the feature map generated by a convolution layer.

- So, further operations are performed on summarised features instead of precisely positioned features generated by the convolution layer.

- This makes the model more robust to variations in the position of the features in the input image.

Max Pooling Filter

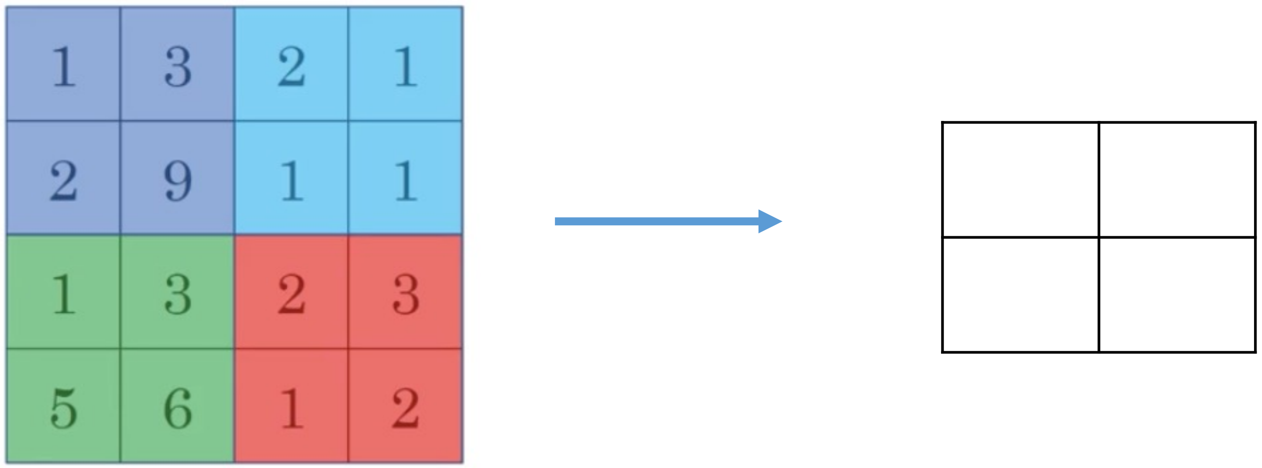
- Process a region of size f x f and reduce it to single value: maximum value in that region

Pooling Layer: Max Pooling

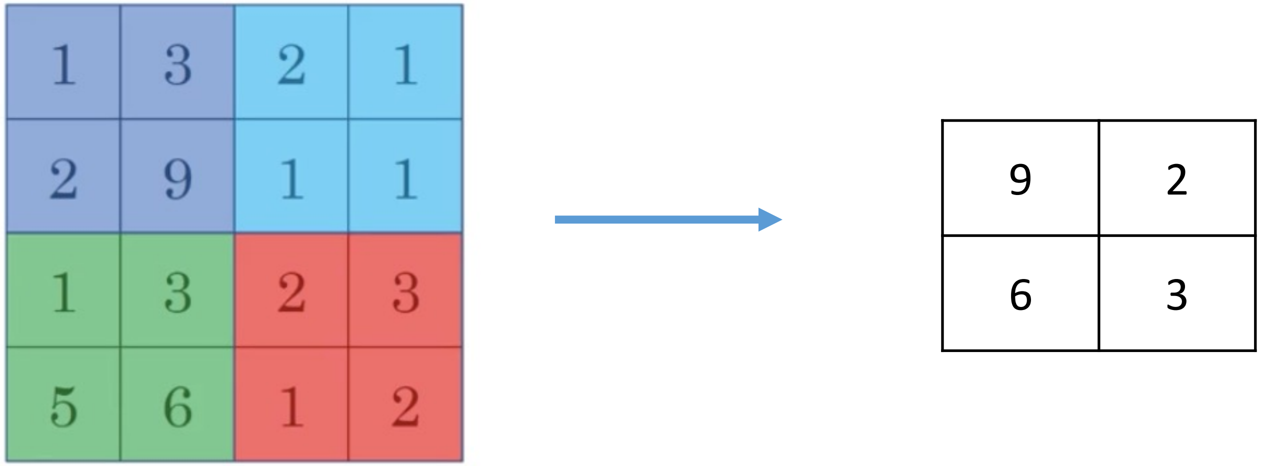
1	3	2	1
2	9	1	1
1	3	2	3
5	6	1	2

Let' say this is the input of 4 x 4, and we want to do max pooling with over a size of 2 x 2

Pooling Layer: Max Pooling



Pooling Layer: Max Pooling



- Filter size and stride length are the hyerparameters of the pooling layer
- There are no parameters to learn
- You do pooling on each channel in the input separately

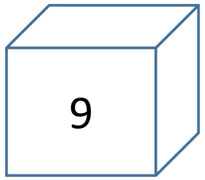
Finally, Let’s look at a Complete CNN

Example: *Digit Classification via CNN*

32 x 32 x 3

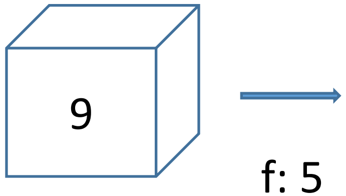
- Task: Classify images of digits (0-9) in RGB format
- CNN Model:
  - Convolutional layers to extract features
  - No padding
  - Pooling layers to reduce dimensionality
  - Fully connected layers for final classification
- Training:
  - Labeled dataset of RGB digit images
  - Goal: minimize classification error on validation/test set

Example: *Digit Classification via CNN*



32 x 32 x 3

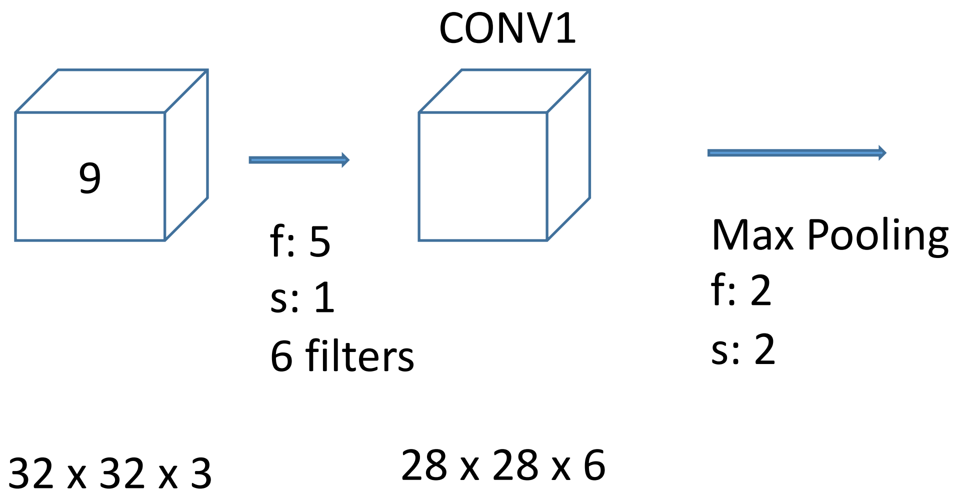
Example: *Digit Classification via CNN*



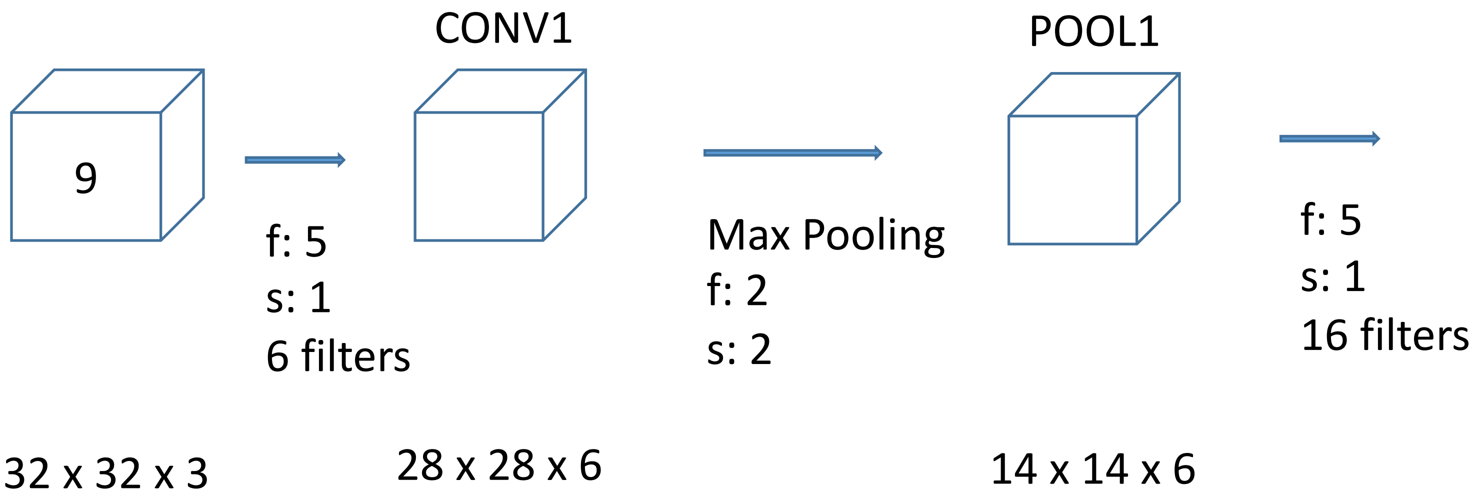
f: 5
s: 1
6 filters

32 x 32 x 3

Example

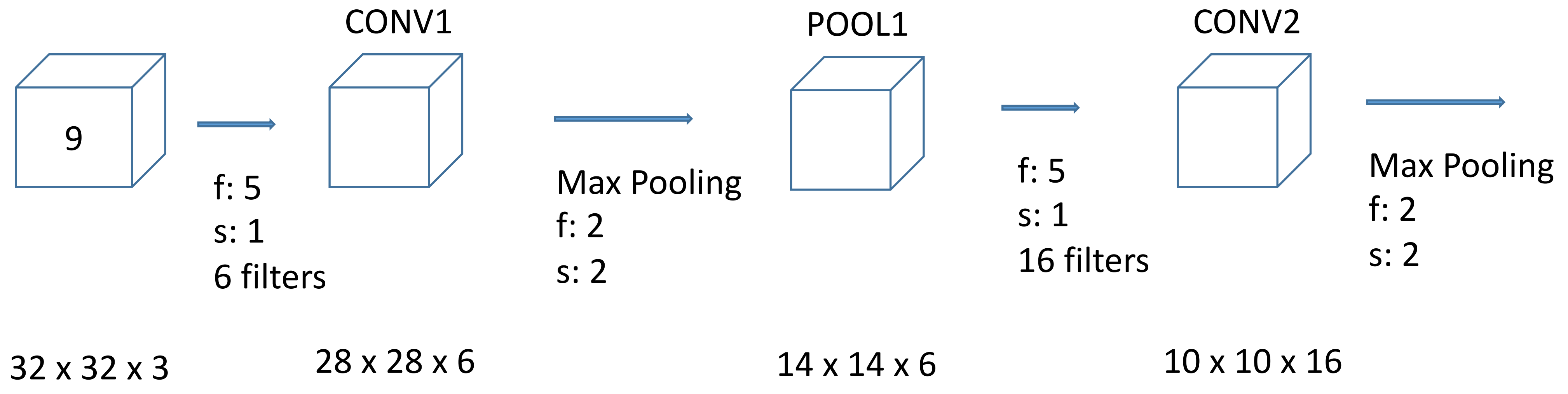


Example



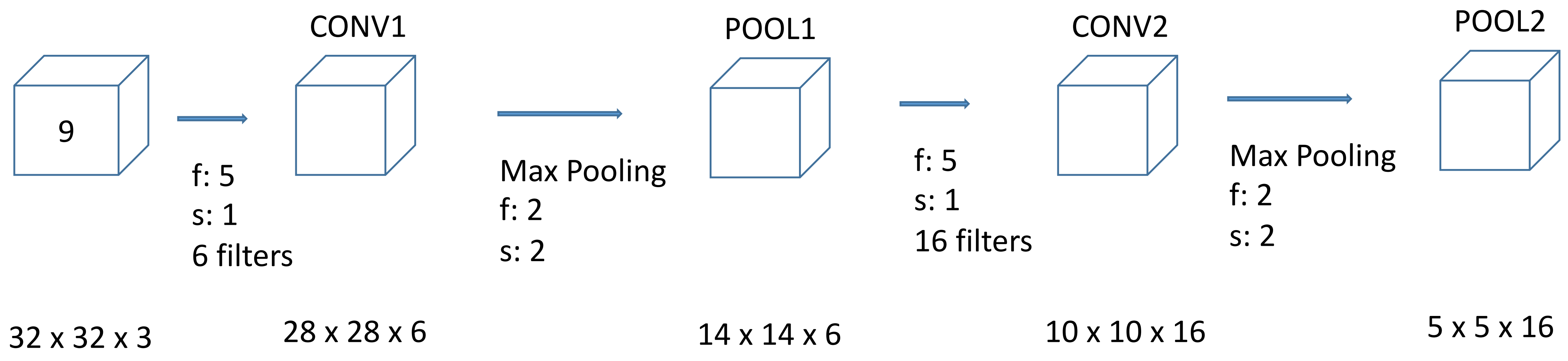
CONV1
POOL1
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6

Example



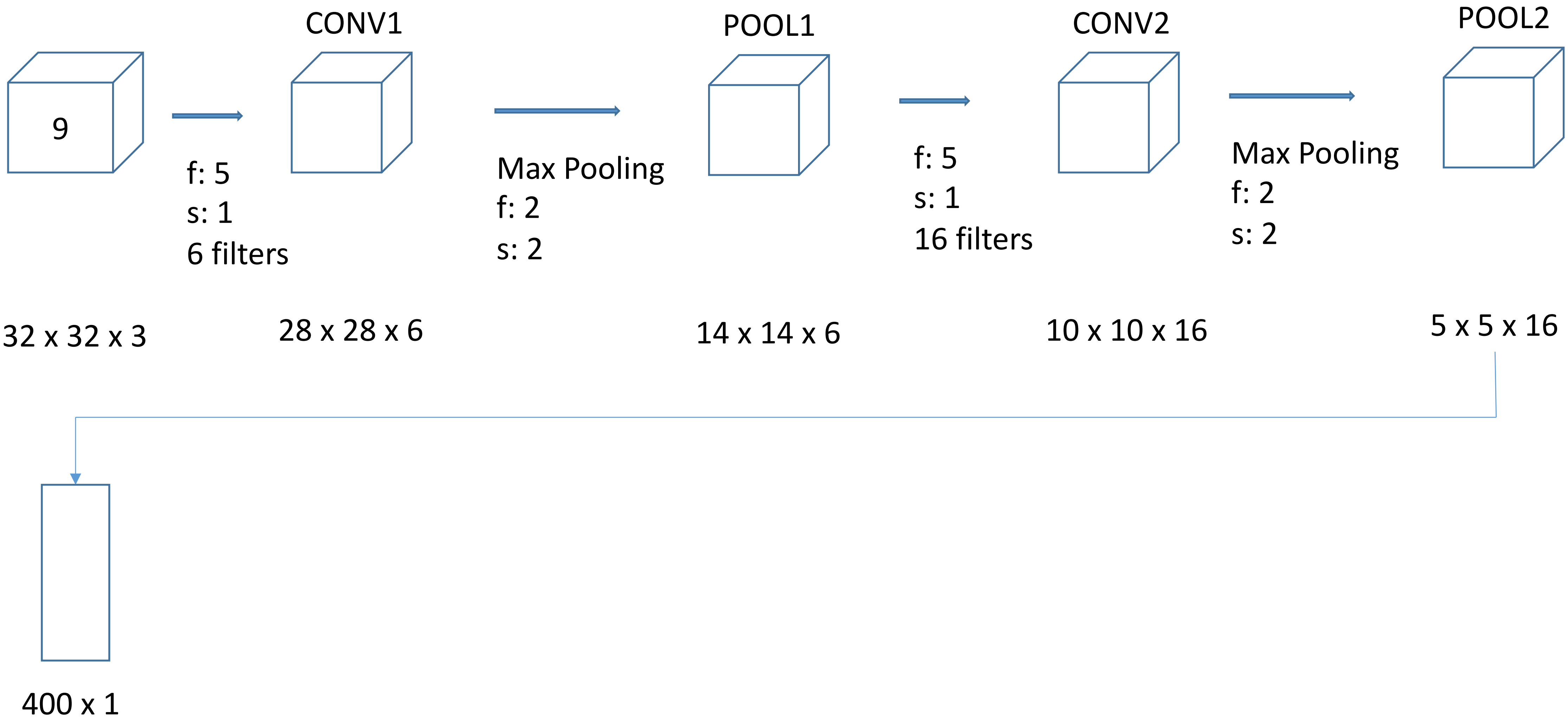
CONV1
POOL1
CONV2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16

Example



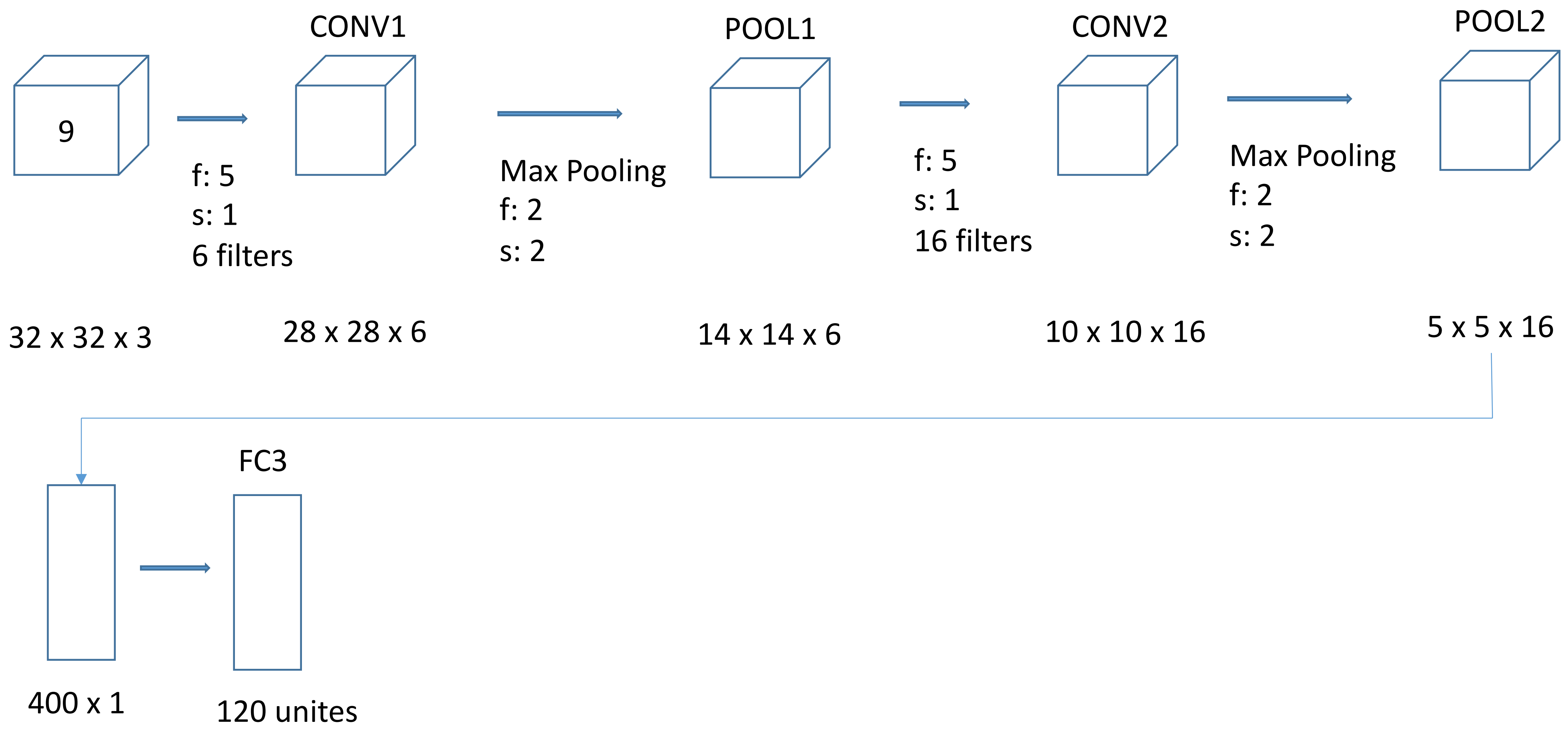
CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16

Example



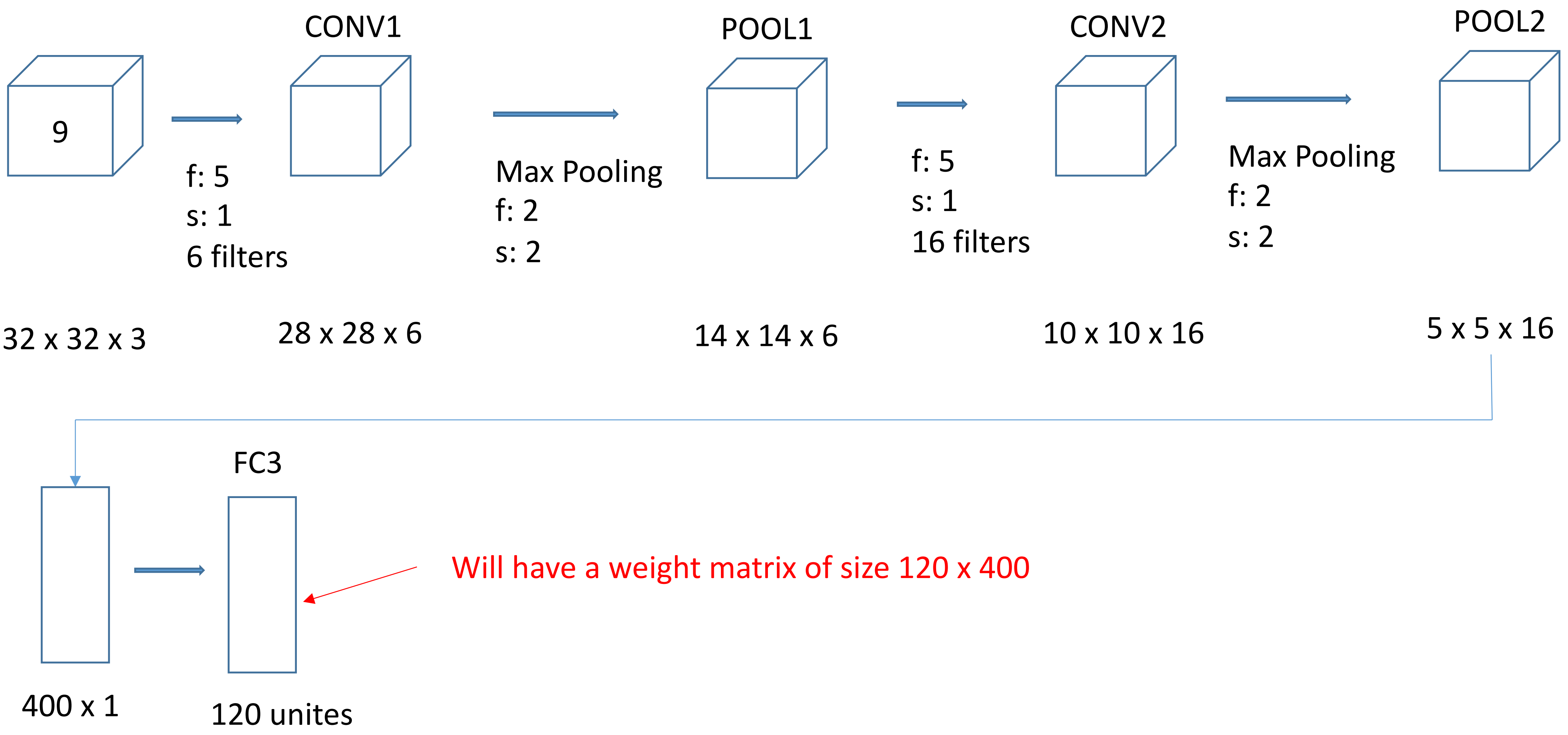
CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
400 x 1

Example



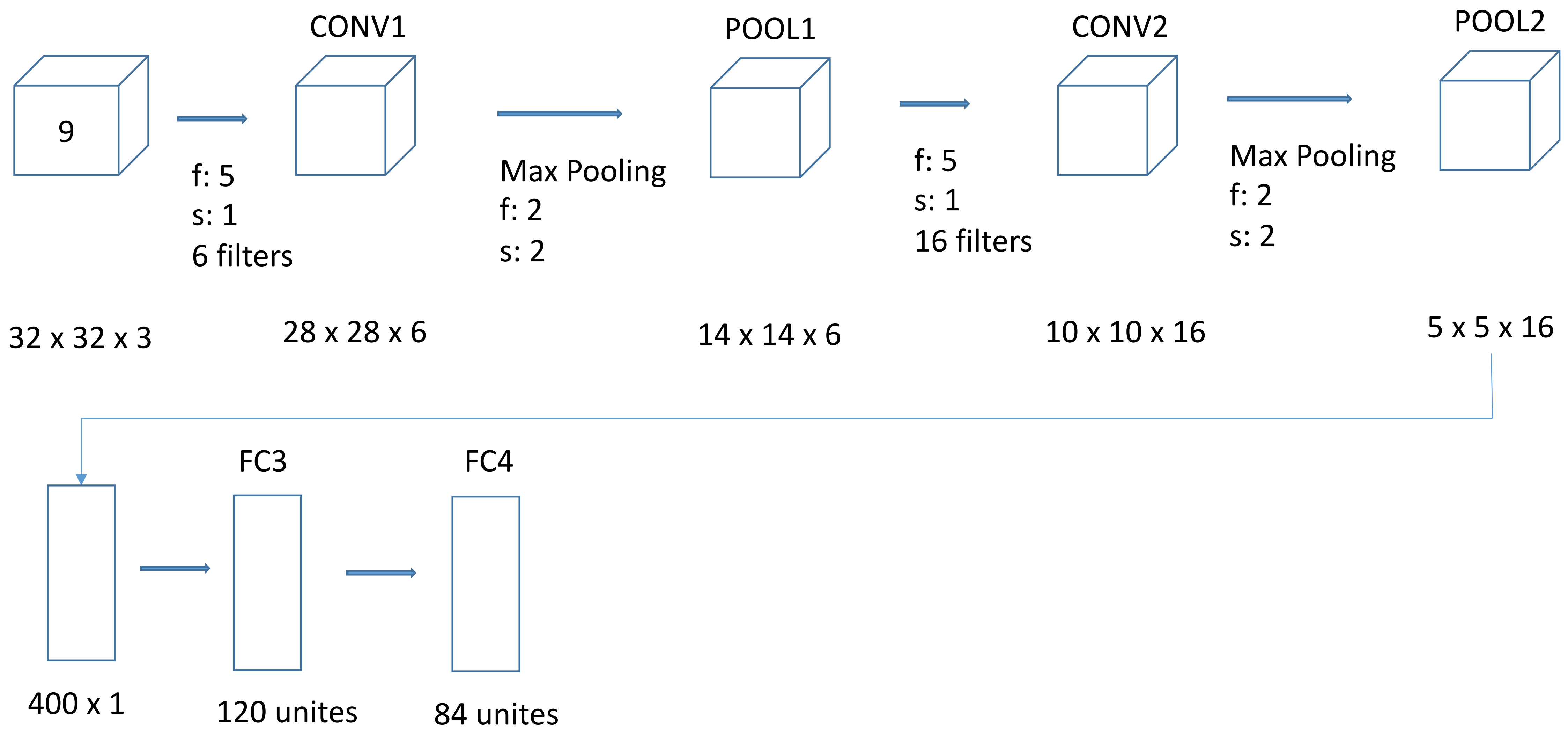
9
CONV1
POOL1
CONV2
POOL2
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
FC3
400 x 1
120 unites

Example

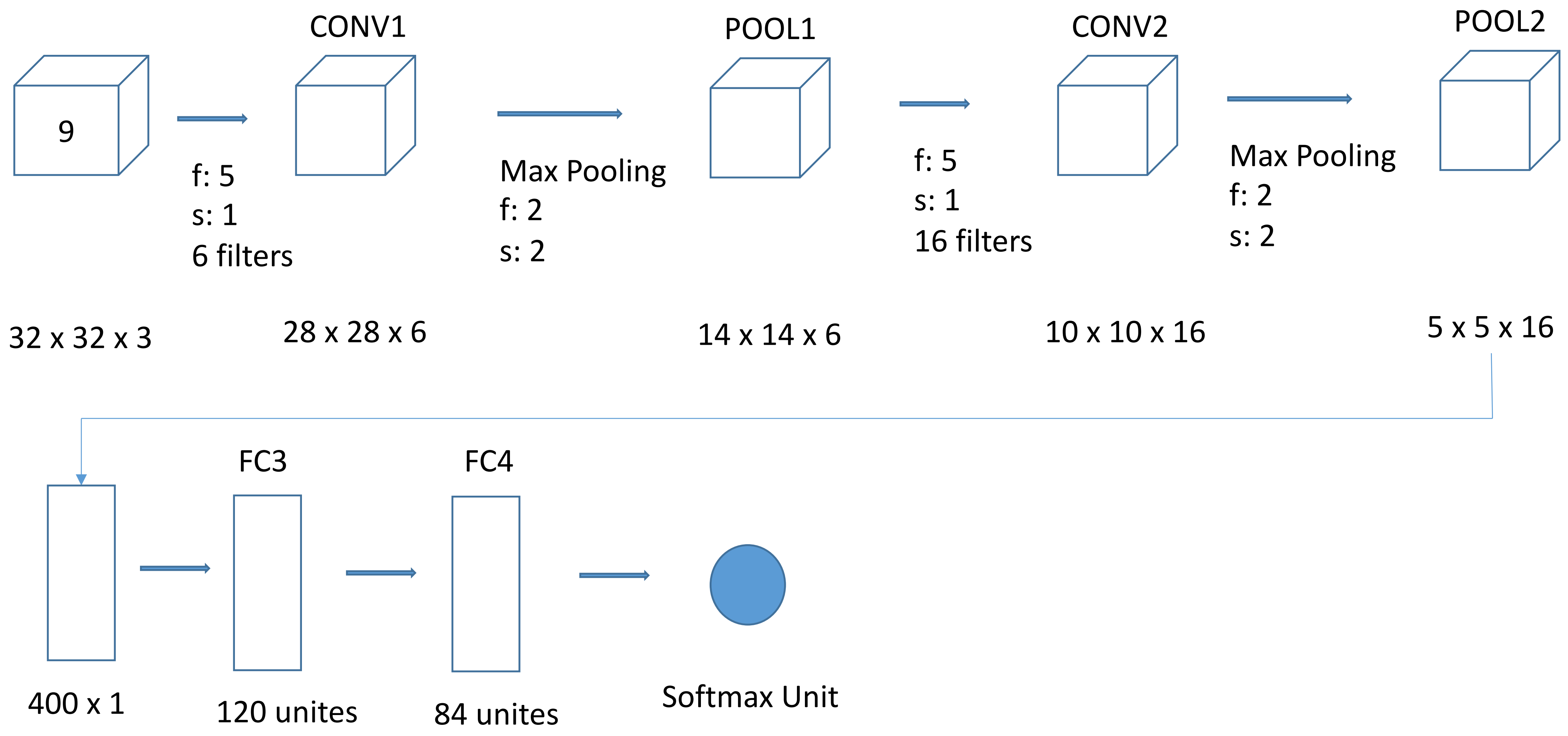


CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
FC3
Will have a weight matrix of size 120 x 400
400 x 1
120 unites

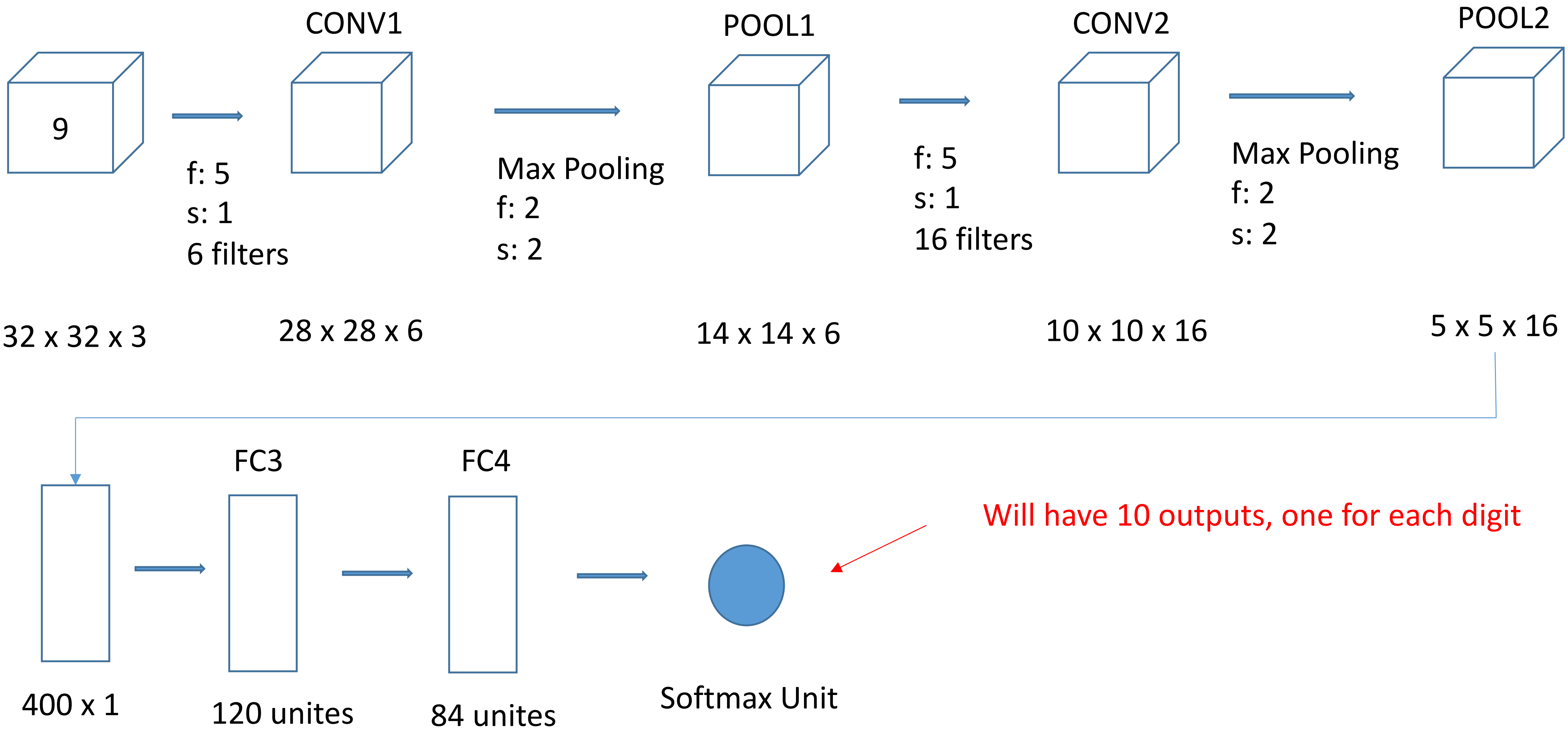
Example


CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
FC3
FC4
400 x 1
120 unites
84 unites

Example

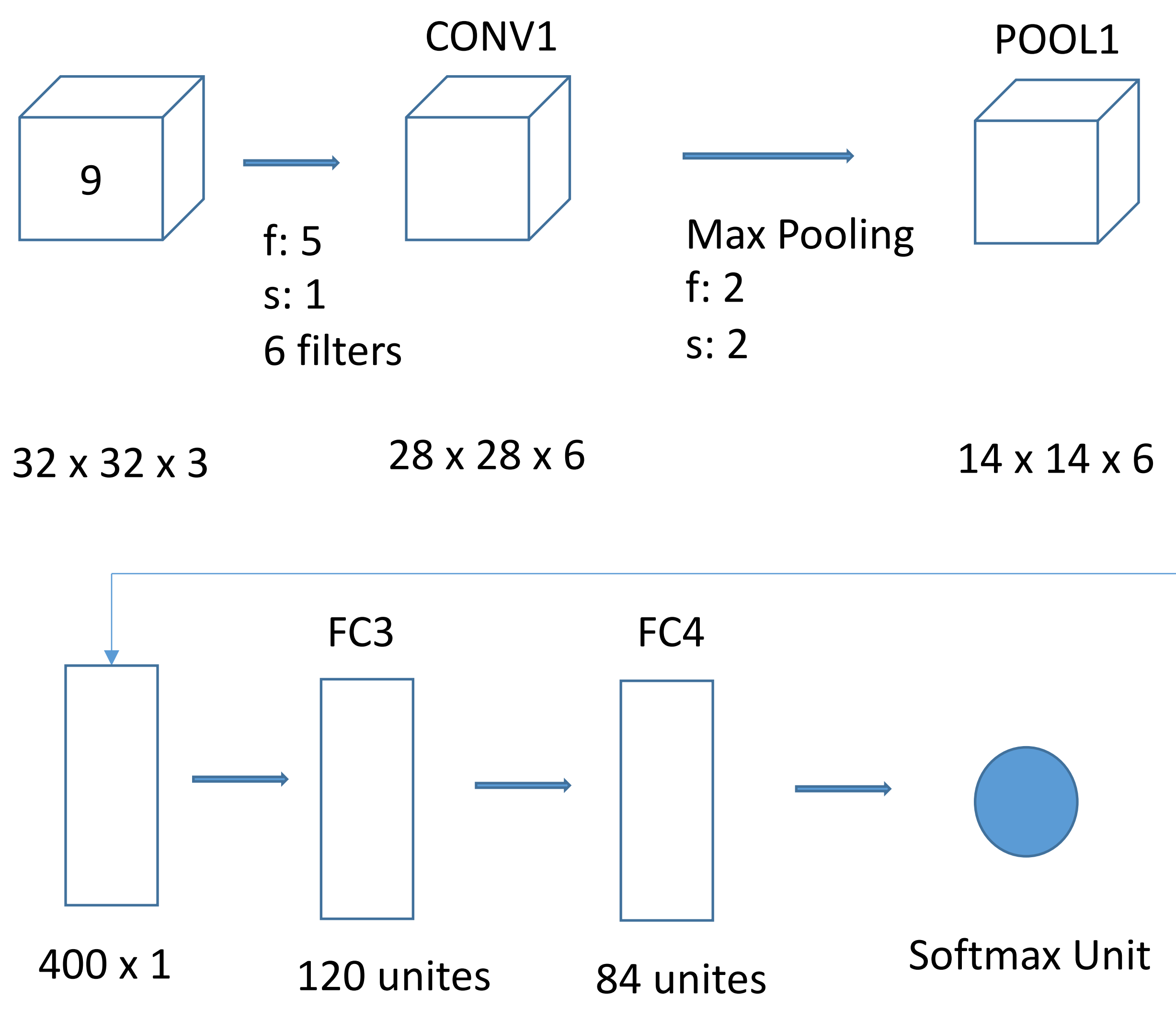

CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
FC3
FC4
400 x 1
120 unites
84 unites
Softmax Unit

Example



CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
FC3
FC4
Will have 10 outputs, one for each digit
400 x 1
120 unites
84 unites
Softmax Unit

Example



What happens in a forward pass?

What happens in a backward pass?

Backpropogation in CNNs

1. **Input a set of training examples**

2. **For each training example** x**:** Set the corresponding input activation $a^{x,1}$, and perform the following steps:

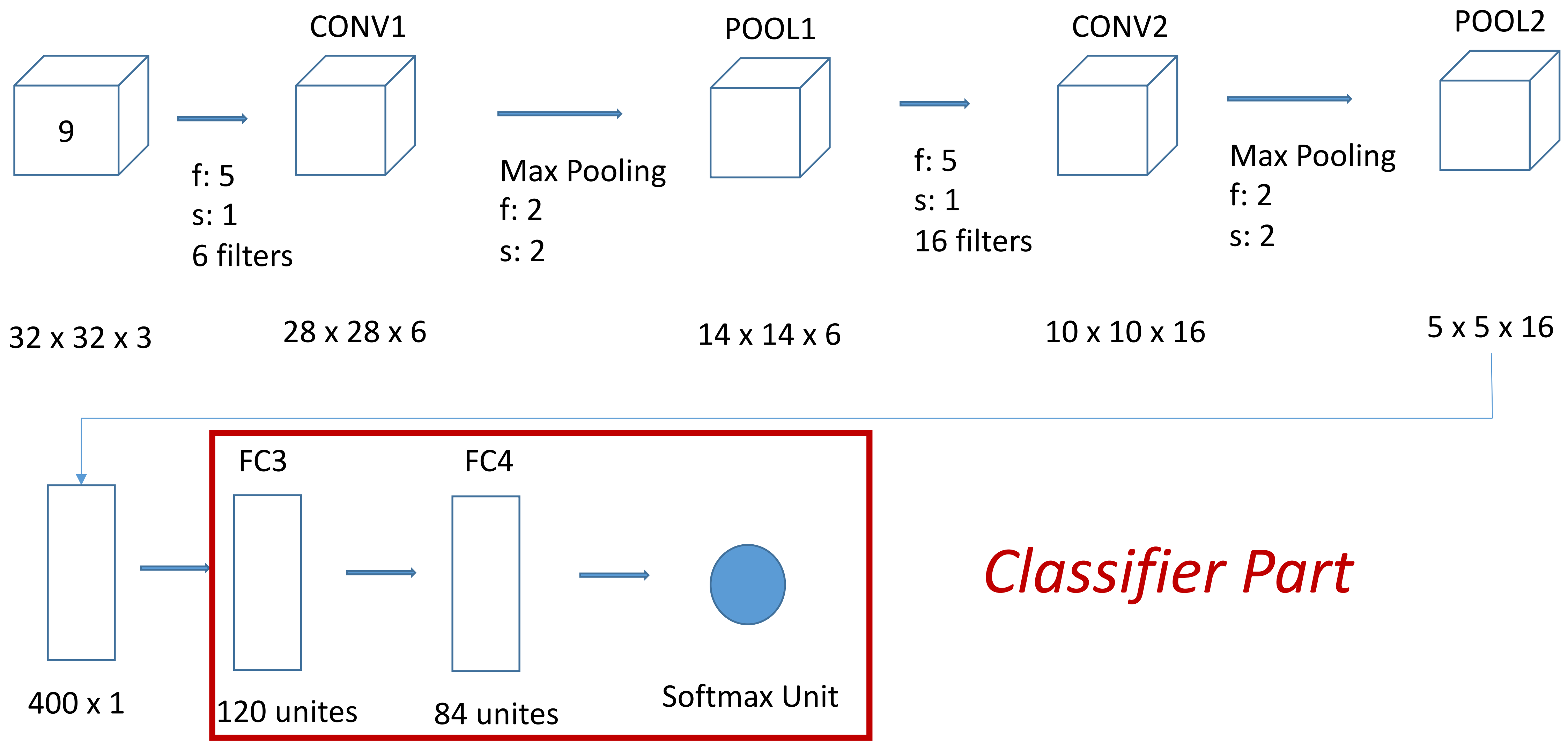
   - **Feedforward:** For each $l = 2, 3, \ldots, L$ compute $\boxed{z^{x,l} = w^l a^{x,l-1} + b^l}$ and $a^{x,l} = \sigma(z^{x,l})$.

   - **Output error** $\delta^{x,L}$**:** Compute the vector $\delta^{x,L} = \nabla_a C_x \odot \sigma'(z^{x,L})$.

   - **Backpropagate the error:** For each $l = L-1, L-2, \ldots, 2$ compute $\delta^{x,l} = ((w^{l+1})^T \delta^{x,l+1}) \odot \sigma'(z^{x,l})$.

3. **Gradient descent:** For each $l = L, L-1, \ldots, 2$ update the weights according to the rule $\boxed{w^l \rightarrow w^l - \frac{\eta}{m} \sum_x \delta^{x,l} (a^{x,l-1})^T}$, and the biases according to the rule $b^l \rightarrow b^l - \frac{\eta}{m} \sum_x \delta^{x,l}$.

Example



CONV1
POOL1
CONV2
POOL2
9
f: 5
s: 1
6 filters
Max Pooling
f: 2
s: 2
f: 5
s: 1
16 filters
Max Pooling
f: 2
s: 2
32 x 32 x 3
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
FC3
FC4
400 x 1
120 unites
84 unites
Softmax Unit
Classifier Part

CNNs as Feature Extractor

- A trained CNN (for classification) can also be used as a feature extractor for other classifiers.

- When treating networks as a feature extractor, we essentially "chop off" the classifier part

- But it can be at any other arbitrary point depending on the dataset

CNNs

- As you saw in the example, there could be a *large number of hyperparameters*

- A common practice is to look for their values in literature, and use what has worked for others

Reading

- Victor Powell. *Image Kernels Explained Visually*. http : / / setosa . io / ev / image - kernels/. 2015.
- Andrej Karpathy. *Convolutional Networks*. http://cs231n.github.io/convolutional- networks/
- Jost Tobias Springenberg et al. "Striving for Simplicity: The All Convolutional Net". In: *CoRR* abs/1412.6806 (2014). URL: http://arxiv.org/abs/1412.6806
- Sergey Ioffe and Christian Szegedy. "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift". In: *CoRR* abs/1502.03167 (2015). URL: http://arxiv.org/abs/1502.03167
- Andrew Ng's lecture on CNN

Summary

- Understanding image convolution and its applications
- Why do we need convolutional neural networks (CNNs)?
- Padding, Stride, Multiple Channels, and Multiple Filters
- Types of Layers in a CNN
- See a simple example of CNN step by step

Reflection

1. What is convolution operation? Why is it particularly important in image processing tasks, such as edge detection?
2. How do convolutional kernels, when trained, act as feature detectors? How does this tie into the concept of feature learning?
3. What are padding, stride and max pooling?
4. What is the function and significance of a convolutional layer, pooling layer and fully connected layers in a CNN?