Solution to Quiz 2

Let $\lambda \geq$ be a constant. Then, the desired tail can be bounded as

$$\Pr\{Z \geq z\} = \Pr\left\{\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i \geq z\right\} = \Pr\left\{\lambda\sum_{i=1}^{n} X_i \geq \lambda\sqrt{n}z\right\} = \Pr\left\{\exp\left(\lambda\sum_{i=1}^{n} X_i\right) \geq e^{\lambda\sqrt{n}z}\right\}$$
$$\leq \frac{E\left\{\exp\left(\lambda\sum_{i=1}^{n} X_i\right)\right\}}{e^{\lambda\sqrt{n}z}} = \frac{E^n\left\{\exp\left(\lambda X\right)\right\}}{e^{\lambda\sqrt{n}z}} = \frac{e^{\frac{n\lambda^2}{2}}}{e^{\lambda\sqrt{n}z}} = e^{\frac{n\lambda^2}{2}-\lambda\sqrt{n}z} \tag{1}$$

Minimizing the above expression wrt $\lambda \geq 0$, we obtain the optimal λ as $\lambda = \frac{z}{\sqrt{n}}$. Inserting $\lambda = \frac{z}{\sqrt{n}}$ into (1), we obtain the right-tail bound as

$$\Pr\{Z \geq z\} \leq e^{-\frac{z^2}{2}}. \tag{2}$$

Plugging $z = 1$ into (2), we obtain the desired numerical value of the bound as 0.6.