Quiz Week 2

Let $X_i \sim \mathcal{N}(0,1)$, for $i = 1, 2, ..., n$, be n i.i.d. zero-mean unit-variance Gaussian random variables (RVs). Let Z be defined as

$$Z = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i. \tag{1}$$

Find an upper bound on its right tail

$$\Pr\{Z \geq z\} \tag{2}$$

for $z > 0$ using the moment generating function (MGF) method. To this end, you may need the following integral

$$\int_{-\infty}^{\infty} \exp(-ax^2 + bx)dx = \frac{\sqrt{\pi}}{\sqrt{a}} \exp\left(\frac{b^2}{4a}\right), \tag{3}$$

which holds for any $a > 0$ and $b > 0$.

On Moodle, upload the numerical value of your upper bound for $z = 1$ up to and including the first decimal, i.e., upload this numerical value in the form $A.B$, where B is the first decimal after the dot.