Advanced Statistics - Lab 04

1) Confirm by simulations the Thin-Shell Theorem for Gaussian vectors. More precisely, obtain by simulations the following probability

$$\Pr\left\{\left(||\boldsymbol{X}||_2 \leq (1-\delta)\sqrt{n}\right) \bigcup \left(||\boldsymbol{X}||_2 \geq (1+\delta)\sqrt{n}\right)\right\} \tag{1}$$

where $\boldsymbol{X} = [X_1, X_2, ..., X_n]$, where X_i, for $i = 1, 2, ..., n$, are i.i.d. zero-mean unit-variance Gaussian RVs, and plot it as a function of n for 3 different values of δ. Then, on the same figure, plot the theoretical bound of (1) for the same of values δ and as a function of n for a well chosen k. What is the best k that you can come up with such that the theoretical bound is tightest?

Upload the results on Moodle in a single PDF file or as the script itself that contains explanations, the code, and figures.

Important note: By failing to do the following, you will loose points:

- You must provide clear explanation of what your program is doing.
- Use a log-scale when piloting probabilities or tails.
- You must provide comments in your code in order for anyone to understand the code.
- You must not use in-bulid functions for obtaining the PDF, mean, variance, and probability.
- You must use different colors, lines, and markers in the plots, along with legends for each curve and suitable line-widths of the curves so that the figure is understandable.
- You muse clearly define what are the x and y axis in your figures.
- Finally, you must use caption that fully explains the figure.