INNOPOLIS UNIVERSITY

Lab 03

Measurement scales

Applied statistics and experiments

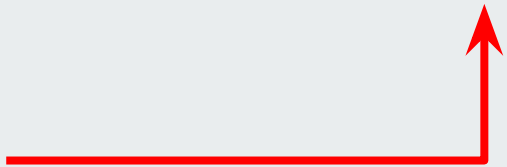
September, 2024

Agenda

1. Representational theory of measurement
2. Measurement scales
    a. Nominal
    b. Ordinal
    c. Interval
    d. Ratio

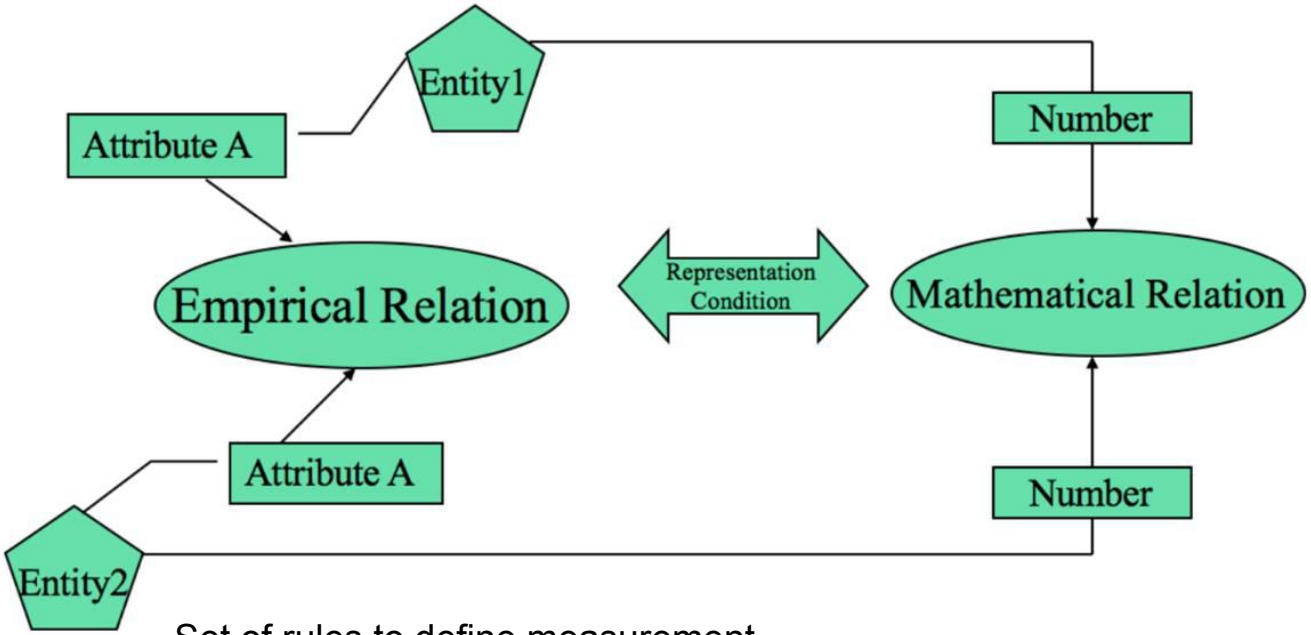
Lecture Recap

https://quizizz.com/join



Join and enter game code

Representational theory of measurement



Set of rules to define measurement

Representational theory of measurement

Frankie is taller than Wonderman.

Person F

Entity1

Attribute A

height

Empirical Relation

Attribute A

Entity2

Person W

A is taller than B if and only if M(A)>M(B)

Representation Condition

F is taller than W

84

Number

M(F)>M(W)

Mathematical Relation

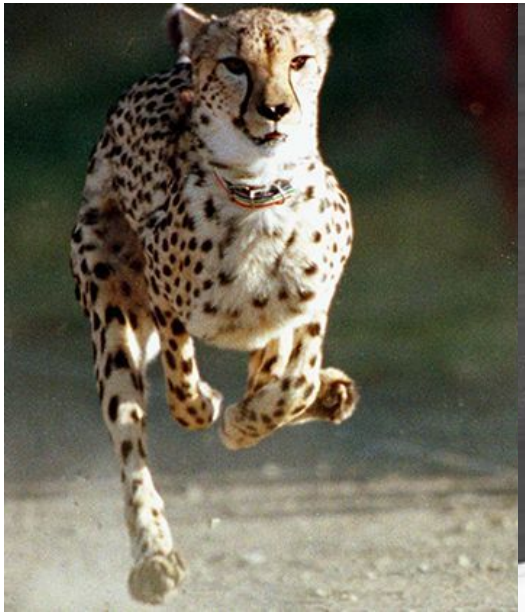
Number

72

Set of rules to define measurement

Acceleration

Representational theory of measurement



Is cheetah faster than F1 car (2016 Mercedes W07)?

Representational theory of measurement

A is faster than B
if and only if
M(A)>M(B)

M(Super car)>M(Cheetah)

F1 car (2016 Mercedes W07)

Cheetah

Entity1

Attribute A

acceleration

Empirical Relation

Representation Condition

Mathematical Relation

Number

60 mph in 3 s.

Attribute A

Super car is faster than Cheetah

Number

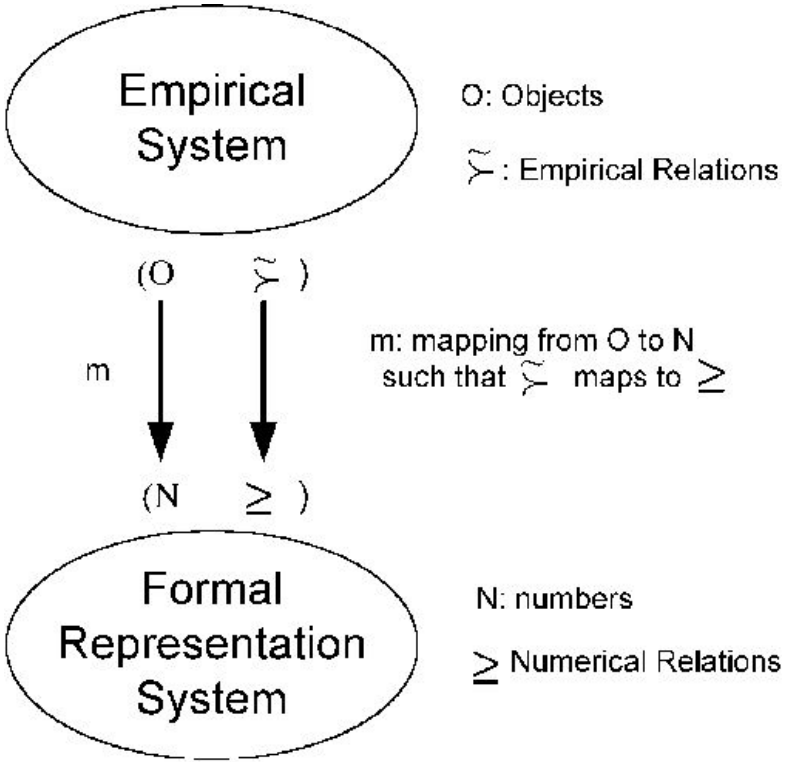
Entity2

Super car

62 mph in 2.4 s.

Set of rules to define measurement

Representational theory of measurement



Set of rules to define measurement

Empirical relations

- Unary
  - **tall**
  - Defined on individual entities.
- Binary
  - **taller than**
  - Defined on two entities
- Ternary
  - **higher than**
  - Defined on three entities
- etc…

Frankie is taller than Wonderman.

Frankie is tall.

Wonderman is tall.

Peter is not tall.

Frankie is not much taller than Wonderman.

Frankie is much taller than Peter.

Peter is higher than Frankie if sitting on Wonderman's shoulders.

Empirical relations

Empirical relation	Numerical/Mathematical relation	Representation condition (For any measure M)
taller than	M(A)>M(B)	A is taller than B if and only if M(A)>M(B)
tall	M(A) > 70	A is tall if and only if M(A)>70
much taller	M(A) > M(B) + 15	A is much taller than B if and only if M(A)> M(B)+15
higher than	0.7 M(A) + 0.8 M(C) > M(B)	A is higher than B if sitting on C's shoulders if and only if 0.7 M(A) + 0.8 M(C) > M(B)

M(Frankie) = 84 (Frankie is mapped to 84)

M(Wonderman) = 72

M(Peter) = 42

Mapping (measure for Height)

Empirical relation	Numerical/Mathematical relation	Representation condition (For any measure M)
taller than (satisfied)	M(A)>M(B)	A is taller than B if and only if M(A)>M(B)
tall (satisfied)	M(A) > 70	A is tall if and only if M(A)>70
much taller (satisfied)	M(A) > M(B) + 15	A is much taller than B if and only if M(A)> M(B)+15
higher than (satisfied)	0.7 M(A) + 0.8 M(C) > M(B)	A higher than B if sitting on C's shoulders if and only if 0.7 M(A) + 0.8 M(C) > M(B)

M(Frankie) = 84 (Frankie is mapped to 84)

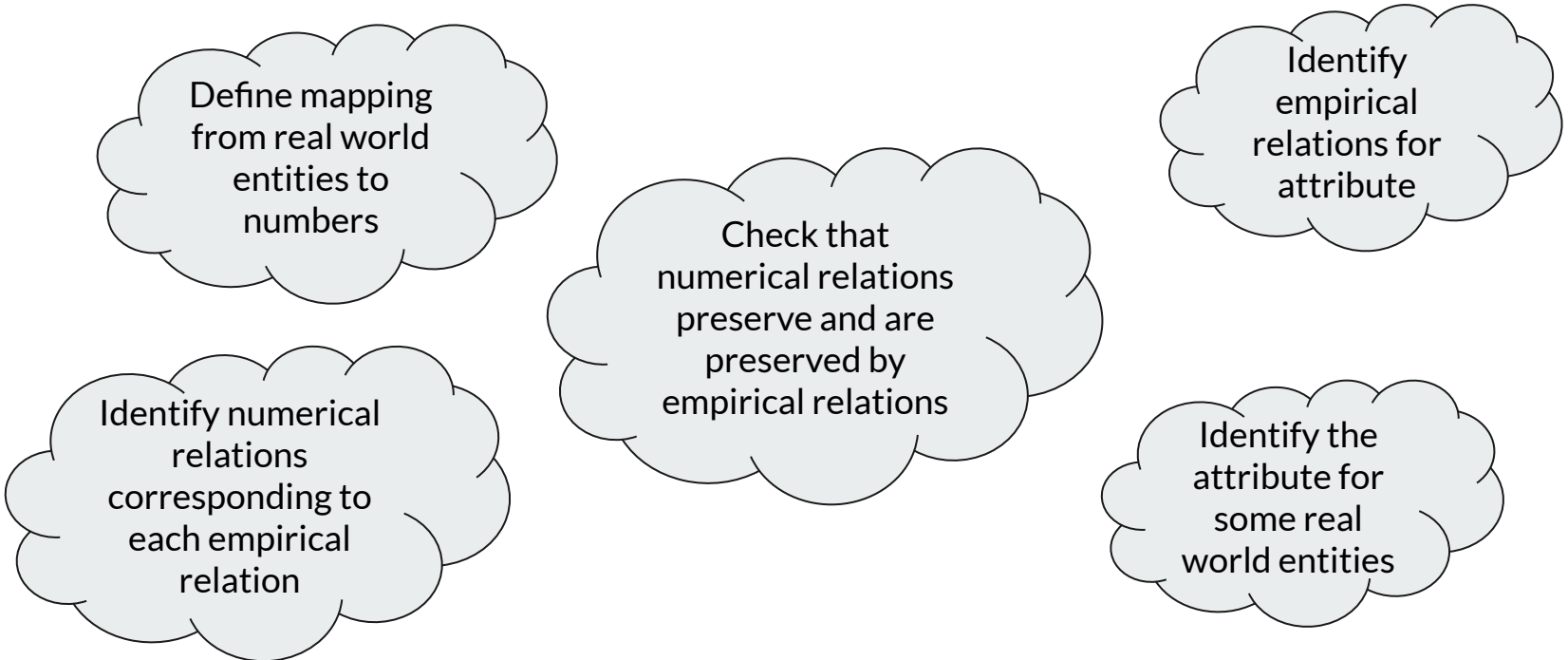
M(Wonderman) = 72

M(Peter) = **70**

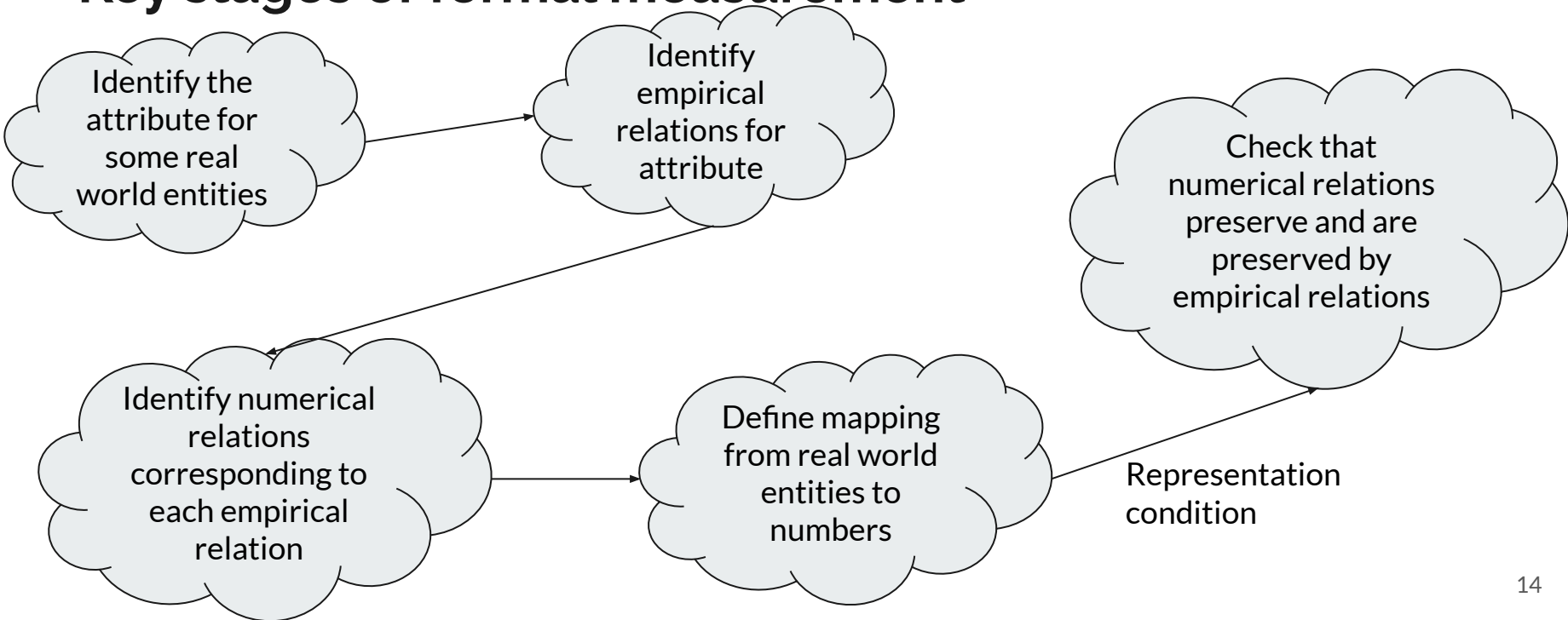
Mapping (not a valid measure for Height)

Empirical relation	Numerical/Mathematical relation	Representation condition (For any measure M)
taller than (satisfied)	M(A)>M(B)	A is taller than B if and only if M(A)>M(B)
tall (satisfied)	M(A) > 70	A is tall if and only if M(A)>70
much taller (not satisfied)	M(A) > M(B) + 15	A is much taller than B if and only if M(A)> M(B)+15
higher than (satisfied)	0.7 M(A) + 0.8 M(C) > M(B)	A higher than B if sitting on C's shoulders if and only if 0.7 M(A) + 0.8 M(C) > M(B)

Key stages of formal measurement



Key stages of formal measurement



Examples on measures in SE

Entity	Attribute	Measure
Program code	Reliability	Mean time to failure in CPU hours
Program code	Reliability	Rate of occurrence of failures in CPU hours
Test set	Efficiency	Number of faults found per number number of test cases
Completed project	Duration	Days from start to finish

Representational theory of measurement

Empirical Relations: The data we obtain as measures should represent attributes of the entities we observe, and manipulation of the data should preserve relationships that we observe among the entities

Measurement: The mapping from the empirical world to the formal, relational world

Measure: The number or symbol assigned to an entity by this mapping in order to characterize an attribute

The real world is the domain of the mapping, and the mathematical world is the range. This mapping, along with the empirical and numerical relations is known as a **measurement scale.**

Measurement scales

What is a measurement?

- The process by which numbers are assigned to attributes of entities in the real world in such a way as to characterize them according to clearly defined rules.

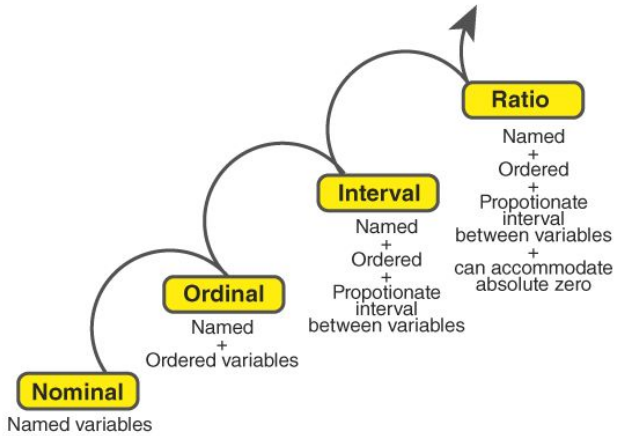
What is a measurement scale?

- a triple <E,N,M> where
  - E is the set of empirical (observed) phenomenon
  - N is the set of numerical (mathematical) objects
  - M is the mapping between them using a valid and repeatable mechanism

Scale types

Each represents differing degrees of empirical knowledge about entities and

their attributes. The following are shown in order of increasing richness of empirical knowledge.



Nominal scale

What is a *nominal scale?* A scale that categorizes items

Partitions the set of empirical entities into categories (equivalence classes) with respect to a certain attribute

Operations which apply to the nominal scale:

- Count
  - Mode
- Equality (=)

Nominal scale

- How would you describe your behavioral pattern?
  - E - Extroverted
  - I - Introverted
  - A - Ambivert
- What is your gender?
  - M - Male
  - F - Female

Ordinal scale

What is an *ordinal scale?*

A scale that categorizes and rank orders items

Used to augment the nominal scale type with information about the ordering of elements in a category

Higher number indicates more of a characteristic being measured

Operations which apply to the Ordinal Scale:

- All nominal scale statistics
- Comparsion (>, <)
- Median
- Range

Ordinal scale

“How satisfied are you with our products?”

1. Totally Satisfied
2. Satisfied
3. Neutral
4. Dissatisfied
5. Totally Dissatisfied

“How happy are you with the customer service?”

1. Very Unhappy
2. Unhappy
3. Neutral
4. Happy
5. Very happy

What is an *interval scale?*

A scale that categorizes and rank orders items, and has equal intervals

Interval scale

Captures information about the size of the intervals that separate the classes

Requires a well-defined unit of measurement

Operations which apply to the Interval Scale:

- All nominal and ordinal scale statistics
- addition and subtraction (+, -)
- Mean
- Variance
  - Standard deviation

Interval scale

Examples:

- Celsius Temperature.
- Fahrenheit Temperature.
- IQ (intelligence scale).
- SAT scores.
- Time on a clock with hands.

Ratio scale

What is a *ratio scale?*

A scale that categorizes and rank orders items, has equal intervals, and a zero that means the absence or none of the thing being measured

Has an absolute zero.

Operations which apply to the Ratio Scale:

- All nominal, ordinal and interval scale statistics
- Multiplication and division (*, /)
- Measurement mapping must start at zero and increase at equal intervals known as units

Ratio scale

Examples:

- Weight.
- Height.
- Sales Figures.
- Ruler measurements.
- Income earned in a week.
- Years of education.
- Number of children.
- Age. (It could be argued that age isn't on the ratio scale, as age 0 is culturally determined. For example, Chinese people also have a nominal age, which is tricky to calculate.)

Meaningfulness in measurement

Measurement scale must be considered to avoid making meaningless statements

Which of the following statements are meaningful:

a. **Fred is twice as tall as Jane**
b. **Joe weighs less than the elephant in the zoo**
c. **Customer satisfaction is twice than the customer satisfaction last year**
d. **Fred's student id is higher than Joe's student id**
e. **Failure x is twice as critical as failure y**

Scales of Measurement

	Nominal	Ordinal	Interval	Ratio
Properties	Identity	Identity Magnitude	Identity Magnitude Equality of intervals	Identity Magnitude Equality of intervals True zero
Mathematical Operations	Count	Count Order	Count Order Addition Subtraction	Count Order Addition Subtraction Multiplication Division

	Nominal	Ordinal	Interval	Ratio
Descriptive Statistics	Mode	Mode	Mode	Mode
		Median	Median	Median
		Range Statistics	Mean	Mean
			Range Statistics	Range Statistics
			Variance	Variance
			Standard Deviation	Standard Deviation
Inferential Statistics	chi-square test	Friedman test	t-test	t-test
	binominal test	Spearman Correlation	ANOVA	ANOVA
		Mann-Whitney U	Pearson Correlation	Pearson Correlation
		Kruskal-Wallis H	Regression	Regression

Example 01

As part of a test preparation course, students are asked to take a practice version of the Graduate Record Examination (GRE). This is a standardized test. Scores can range from 200 to 800 with a population mean of 500 and a population standard deviation of 100.

Interval scale (GRE scores have the properties of identity, magnitude, and equal intervals between scale points)

Example 01

Nominal	NO	No groupings or classifications were described.
Ordinal	NO	GRE scores have the property of identity and magnitude, so they do tell us lower or higher probability of success in graduate school. They also have the property of equal intervals between scale points.
Interval	YES	GRE scores have the properties of identity, magnitude and equal intervals between scale points.
Ratio	NO	GRE scores have the properties of identity, magnitude and equal intervals between scale points. They do not have a zero score; they range from 200 to 800.

Example 02

Children in elementary school are evaluated and classified as non-readers (0), beginning readers (1), grade level readers (2), or advanced readers (3). The classification is done in order to place them in reading groups.

Ordinal scale (The numbers identify and order the level of reading skill)

Example 02

Nominal	NO	Reading levels are identified but they are also ordered by skill level.
Ordinal	YES	The numbers identify and order the level of reading skill.
Interval	NO	The reading level scores have the property of identity and magnitude/order but there is not equal distance between scale points.
Ratio	NO	Even though the zero score may represent no reading skill (and thus is a “true zero”), the ratio scale of measurement requires both equal intervals and a true zero point.

Example 03

Innopolis University wants to know which dormitories the students prefer. The administration counts the number of applications for each dorm. Administrators, assign a rank to each dorm based on the number of applications received.

Ordinal scale (This ranking has the properties of identity and magnitude)

Example 03

Nominal	NO	The dorms are identified in this number system but they are also ordered.
Ordinal	YES	This ranking system has the properties of identity and magnitude.
Interval	NO	This ranking system identifies and orders students preferences but the interval between each rank is not equal. It does not tell us how much each dorm is preferred (or not) over the others.
Ratio	NO	This ranking system identifies and orders students preferences but the interval between each rank is not equal. It does not tell us how much each dorm is preferred (or not) over the others. There is also no true zero point on this scale.

Example 04

During a clinical interview, survivors of a tornado are asked to state “no” or “yes” to whether they have experienced specific symptoms of Post-Traumatic Stress Disorder (PTSD) in the past week. The interviewer adds up the total number of “yes” responses to create a Total PTSD Symptoms scale.

Ratio scale (The "0" score is a true zero indicating that the person interviewed reported experiencing no symptoms of PTSD in the past week)

Example 04

Nominal	NO	The yes/no responses to each symptom are measured on a nominal scale, but the total score counts of the number of yes responses.
Ordinal	NO	The yes/no responses to each symptom are measured on a nominal scale, but the total score counts of the number of yes responses.
Interval	NO	Counting the number of “yes” responses creates a scale with equal intervals between scale points, but it also has a true zero. A score of “0” means that the person interviewed stated that they did not experience any of the specific symptoms of PTSD in the past week.
Ratio	YES	Counting the number of “yes” responses creates a scale with the properties of identity, magnitude, equal interval, and true zero. The “0” score is a true zero indicating that the person interviewed reported experiencing no symptoms of PTSD in the past week.

Example 05

Interviewers ask research subjects to describe in some detail their relationships with important people in their lives. Using established criteria, raters review these personal descriptions and assign a dominant attachment category (Secure - 1, Preoccupied - 2, Fearful - 3, Dismissing - 4).

Nominal scale (The numbers on this rating scale name the dominant attachment style. This scale only has the numerical property of identity.)

Example 05

Nominal	YES	The numbers on this rating scale name the dominant attachment style. This scale only has the numerical property of identity.
Ordinal	NO	The numbers in this rating system only identify the attachment style. There is no inherent order in the styles.
Interval	NO	This scale is simply classifying the dominant attachment style. Scale points are not ordered and we do not have a fixed interval between the styles.
Ratio	NO	There is no “0” score on this scale.

Example 06

All new clients in a mental health clinic are given a symptoms checklist that asks them to report how often they experienced each symptom in the past week. They assign a frequency rating to each symptom (0 – not at all, 1 – sometimes, 2 – often, 3 – almost all the time).

Ordinal scale

Example 06

Nominal	NO	Each number identifies a frequency level but the levels are ordered.
Ordinal	YES	This rating scale has the property of identity and order. Each number identifies a frequency level on an ordered scale.
Interval	NO	Each number identifies a frequency level on an ordered scale, but the interval between scale points is not equal.
Ratio	NO	Even though “0” represents the absence of a symptom in the past week, the ratio scale of measurement requires both equal intervals and a true zero point. The distance between points on this scale is not equal.

Example 07

In a medical study, the researcher measures pain levels in patients. The measurements ranges from "no pain" to "severe pain" and helps healthcare professionals assess and compare pain levels among individuals.

Ordinal scale

Example 07

Nominal	NO	Each number identifies a pain level but the levels are ordered.
Ordinal	YES	This pain level scale has the property of identity and order. Each number identifies a pain level on an ordered scale.
Interval	NO	Each number identifies a pain level on an ordered scale, but the interval between scale points is not equal.
Ratio	NO	Even though “0” represents the absence of the pain in the past week, the ratio scale of measurement requires both equal intervals and a true zero point. The distance between points on this scale is not equal.

Example 08

The survey asks participants to select their preferred mode of transportation from a list of options.

Nominal scale

Example 08

Nominal	YES	The responses are categorized based on the transportation modes, creating distinct categories without any specific order or numerical value assigned to each option.
Ordinal	NO	The preferences have the property of identity but they are not ordered since there is no numerical value assigned to each option.
Interval	NO	The interval between scale points are not equal.
Ratio	NO	There is no “0” score on this scale.

Example 09 (HW)

In a physics experiment, scientists measure the time it takes for an object to fall from a certain height in seconds. The measurements allow for precise calculations and comparisons, including determining ratios of falling times between different objects.

Example 10 (HW)

In a restaurant review platform, users rate their dining experiences on a scale from "poor" to "excellent." The ratings provide a ranking of the restaurants based on customer satisfaction but do not quantify the exact difference in satisfaction levels between each rating.

Upcoming class

- Goodness of measurement
- Validity
- Reliability

References

- https://www.amazon.com/Software-Metrics-Innovations-Engineering-Development/dp/1439838224
- https://psychology.emory.edu/clinical/bliwise/Tutorials/SOM/smmod/scalemea/PracticeExercises.htm
- https://www.questionpro.com/blog/nominal-ordinal-interval-ratio/
- https://statisticsbyjim.com/basics/nominal-ordinal-interval-ratio-scales/
- https://stattrek.com/statistics/measurement-scales
- https://link.springer.com/referenceworkentry/10.1007/978-3-642-04898-2_67

Attendance

https://baam.duckdns.org

Questions?