# Lecture 4: Measurement scales and functions that can be applied to scales



Measurement scales and functions that can be applied to scales



## Mapping

- Attribute values > numbers or symbols
- Attribute value domain >range
- $\bullet$  Empirical relation -> mathematical relation





• A measurement scale is a class of mapping that links empirical and number relations with specific properties



- Best possible numerical relation system?
- Representation of an empirical relation in a numerical system?
- Choosing a unique (and best) number system?



- Qualitative Scales
  - Nominal (gender)
  - Ordinal (arrival order)
- Numeric/Continuous Scales
  - Interval (temperatures in F)
  - Ratio (height)
  - Absolute (the actual count)



- Language(Program) = 1, if Program is written in Pascal
- Language(Program) = 2, if Program is written in C
- Language(Program) = 3, if Program is written in Fortran

Few mathematical operations are applicable (mode, histograms,  $\dots$ )



- Difficult(Program) = 1, if Program is easy to read
- Difficult(Program) = 2, if Program is not hard to read
- Difficult(Program) = 3, if Program is hard to read

We can have the median here...



- Nominal measure label variables without any quantitative value.
  Ex., Eye color.
- Ordinal measure categorize data in natural order. Size of steps between items is unknown. Ex., Customer satisfaction
- <u>Interval</u> measures preserve differences but not ratios. Ex., The absolute time when an event occurred.
- <u>Ratio</u> measures preserve also the ratio between entities. Ex., LOC in a program. *All math operations are applicable*.
- <u>Absolute</u> measures are counts. Ex., the number of if statements in a program.



Table 2.8: Summary of measurement scales and statistics relevant to each (Siegel and Castellan, 1988

Scale type	Defining relations	Examples of appropriate	Appropriate statistical tests
		Nominal	
	Frequency		
Ordinal	Equivalence	Median	Non-parametric
	Greater than	Percentile	-
		Spearman r	
		Kendall τ	
		Kendall W	
Interval	Equivalence	Mean	Non-parametric
	Greater than	Standard deviation	•
	Known ratio of any intervals	Pearson product-moment correlation	
		Multiple product-moment correlation	
Ratio	Equivalence	Geometric mean	Non-parametric
	Greater than	Coefficient of variation	and parametric
	Known ratio of any intervals		
	Known ratio of any		
	two scale values		



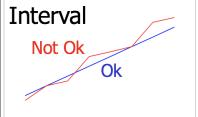
## Acceptable Mappings

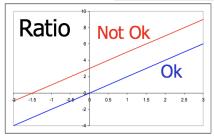
- For nominal, any 1:1 mapping is OK
- For ordinal, the mapping needs to be strictly increasing
- For interval, the mapping must have the form Y = aX + b, with a > 0
- For ratio, the mapping must have the form Y = aX, with a > 0
- For absolute, the only acceptable mapping is Y = X



## Examples of Mappings









## Meaningful Measures

- Measures are said to be meaningful if their truth value does not change when the measure is subject to transformation
- That is, they are defined on the appropriate scale. Mapping is used to verify the appropriateness of the scale.



## Examples

Meaningful	Not meaningful	
• The number of atoms in solid A is double the number of atoms in solid B	• The color of solid A is twice as black as the color of solid B	
• The number of people who agreed was double the number of people who disagreed	People agreed twice as much as they disagreed	

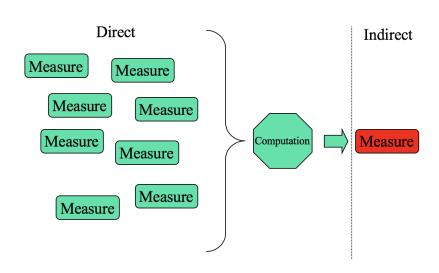


#### Kinds of metrics

- A metric is objective if it can be taken by an automated device; it is subjective otherwise
  - LOC are objective metrics, Function Points are subjective
- A metric is direct if it can be directly detected, indirect if it is the result of mathematical elaboration on other metrics
  - LOC, number of errors, and FP are direct
  - Number of errors per LOC (Error density) is indirect



#### Direct and Indirect Measurement





#### Direct or Indirect

- Immediately definable on one single calculation. Example: LOC, number of people in classroom, number of customer complaints
- Derived from a varied set of values. Example: ROI, number of tennis balls by weight, customer satisfaction



## Measurements, Statistics and Scales

- Measurement scales limit the type of operations on measure e.g., central tendency
- Objective or subjective measurement may limit the type of operations on measures
- Indirect measure depend on other measures' scales and thus are limited in meaningfulness and operations



#### Exercise: Measure of Mass



- What are the relations between their masses?
- Which of these are valid mappings?

$$M_1(A) = 1, M_1(P) = 130, M_1(E) = 1400$$

$$M_2(A) = 3, M_2(P) = 4, M_2(E) = 5$$

$$M_3(A) = 24, M_3(P) = 51, M_3(E) = 49$$

• Can we tell how intelligent they are from these mappings?



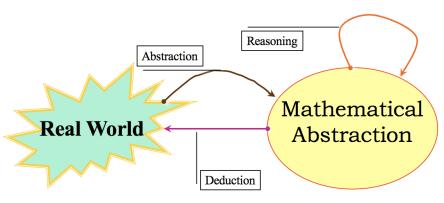
## Questions

- Is it wrong to assert that "lines of code" is a bad software measure?
- What scale is used in "lines of code" measurement?
- Discuss the notion of "distance" in a vector space and its meaningfulness as a measure
- What kind of measure would you use for "program quality?"



## Building Models out of Metrics

• A baby should double its weight at the age of month 6.





#### Model

- Mathematical abstraction
  - Indirect measurement
  - Control measurement
  - Prediction measurement
- **Prediction system** couples a model with procedures that allow forecasting



## Risks while building models



Figure 2.8: Using a suspect definition

from Fenton pp. 38



## Case study

Metrics to assess personal productivity

From: https://www.analyticsinhr.com/blog/employee-performance-metrics/



#### The work

- We now analyse how people are evaluating quantitatively the personal productivity.
- The full document is available at the website above.
- We can review it using our approach to metrics.
- We adopt a simplified GQM.



#### Goal

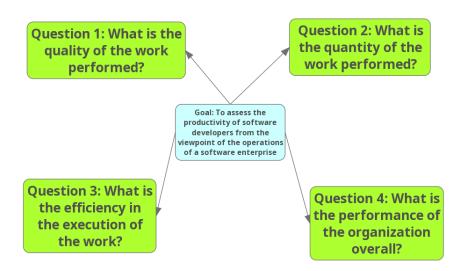
Goal: To assess the productivity of software developers from the viewpoint of the operations of a software enterprise

The text of this and the following slides comes from:

https://www.analyticsinhr.com/blog/employee-performance-metrics/



## Questions





## Proposed exercise (1/2)

- Create a mix team of 3 people of which at least one DS and one SE
- Complete the GQM with the metrics
- For every metrics determine:
  - if it direct or indirect
  - if it is subjective or objective
  - its measurement scale
- Provide a significant subset of the model that you would use to evaluate yourself
- Write the results on http://tiny.cc/IU\_EM\_F20\_L3, tab: EmployeesProductivity
  - Organize every cell as follows:
    - Referred Goal and Question as number, e.g., G1Q2
    - Metrics, newline
    - Direct or indirect, newline
    - Subjective or objective, newline
    - Measurement scale



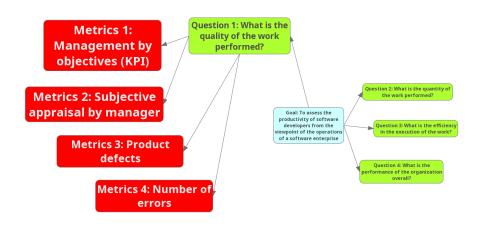
## Proposed exercise (2/2)

http://tiny.cc/IU\_EM\_F20\_L3, tab: EmployeesProductivity



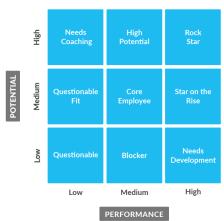


## Q1 Metrics (1)





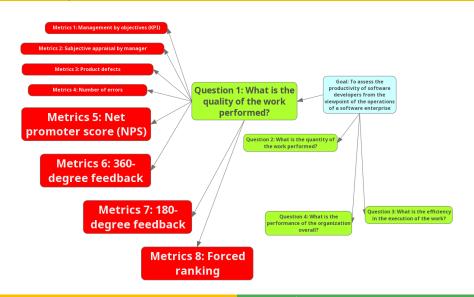
### Focus on Q1 M2







## Q1 Metrics (2)





## Focus on Q1 M6

#### The Cox-Box

I'm not very impressed with these employee satisfaction survey results.



#### by Gary P. Cox

I'll conduct a 360-degree feedback survey to assess my leadership skills...



Six Sigma Guy, have all my staff complete this anonymous 360-degree feedback survey.

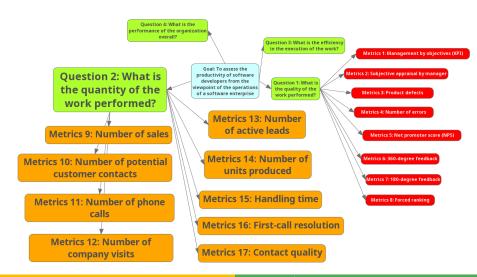
That's a great demonstration of your maturity as a leader. Boss.



Send comments and stories to Cox-Box@iSixSigma.com

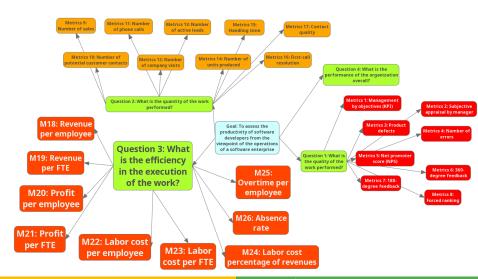


## Q2 Metrics





## Q3 Metrics





Questions?



## End of Lecture 4