Newton's Interpolating Polynomial

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Part I

The Polynomial

Let $y_0 = f(x_0), y_1 = f(x_1), y_2 = f(x_2)...y_n = f(x_n)$ be the n+1 values of a function $y = f(x)$ where no x appears more than once, the Newton's Interpolating Polynomial for this function described as follows:

$$Pn(x) = d_0 + d_1(x - x_0) + d_2(x - x_0)(x - x_1) + \dots d_n(x - x_0)(x - x_1)(...)(x - x_{n-1})$$

Where $d_i \mid 0 \leq i \leq n$, is the divided difference operator of i$^{\text{o}}$ order and x is the (future) value we want to find.

Part II

Divided Differences

The divided differences are defined as:

$$d_0 = f[x_0] = y_0$$

$$d_1 = f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

$$d_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$\vdots$$

$$d_n = \frac{f[x_1, x_1, x_2, \dots, x_n] - f[x_0, x_1, x_2, \dots, x_{n-1}]}{x_n - x_0}$$

By doing this with the known points, we can generate a table with all the divided differences.

Table 1: Divided Differences Table

x	f[x]	$f[x_i, x_j]$	$f[x_i, x_j, x_k]$	...
x_0	y_0			
x_1	y_1	$f[x_0, x_1] = \frac{f[x_1]-f[x_0]}{x_1-x_0}$		
x_2	y_2	$f[x_1, x_2] = \frac{f[x_2]-f[x_1]}{x_2-x_1}$	$f[x_0, x_1, x_2] = \frac{f[x_2,x_1]-f[x_1,x_0]}{x_2-x_0}$	
...	...			
x_n	y_n	$f[x_{n-1}, x_n] = \frac{f[x_n]-f[x_{n-1}]}{x_n-x_{n-1}}$	...	

And so on...

The element on the first line and column i of the table (without considering the column with x) is the divided difference operator of iº order.