Geometry

I. Saltini

1 Topological space

Definition. A **neighbourhood topology** is a function $\nu : T \to \mathscr{P}(T)$ mapping **points** (elements of T, from here on denoted by x or y) to **neighbourhoods** (subsets of T, from here on denoted by n or m). It must satisfy the following axioms.

TS.1 A neighbourhood of a point must contain the point itself.

$$\forall x \forall n : n \in \nu(x) \to x \in n$$

TS.2 All supersets of a neighbourhood of a point are neighbourhoods of that point.

$$\forall x \forall n \forall m : (n \in \nu(x) \land n \subseteq m) \to m \in \nu(x)$$

TS.3 The intersection of two neighbourhoods of the same point is a neighbourhood of that point.

$$\forall x \forall n \forall m : (n \in \nu(x) \land m \in \nu(x)) \to n \cap m \in \nu(x)$$

TS.4 Any neighbourhood n of a point must contain another neighbourhood m of the same point, such that n is a neighbourhood of all points in m.

$$\forall x \forall n \exists m \forall y : (n \in \nu(x) \land m \subseteq n \land m \in \nu(x) \land y \in m) \to n \in \nu(y)$$

Definition. A **topological space** (T, ν) is a set T equipped with a neighbourhood topology ν.

Definition. $u \subseteq T$ is **open** if it is a neighbourhood of all the points it contains.

$$\forall x : x \in u \to u \in \nu(x)$$

Definition. $u \subseteq T$ is **closed** its complement $\complement(u)$ is open.

Definition. $u \subseteq T$ is **clopen** if it is both open and closed.