ASSIGNMENT 1

Course: Probability and Statistics

Student Name:

Date of Birth:

Question 1: Three bits are transmitted over a digital communications channel. Each bit is either distorted or received without distortion. Let A_i denote the event that the ith bit is distorted, $i = 1, 2, 3$.

(a) Describe the sample space for this experiment.

(b) Describe the outcomes in each of the following events:

$$A_1, \quad A_2, \quad A_2', \quad A_2 \cap A_3.$$

(c) Are the A_i's mutually exclusive?

Question 2: If the last digit of a weight measurement is equally likely to be any of the digits 0 through 9.

(a) What is the probability that the last digit is 0?

(b) What is the probability that the last digit is greater than or equal to 4?

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Question 3: If $P(A) = 0.4$, $P(B) = 0.7$, and $P(A \cap B) = 0.2$, determine the following probabilities:

$$P(A \cup B), \qquad P(A' \cap B), \qquad P(A \cup B'), \qquad P(B \mid A), \qquad P(A \mid B')$$

Question 4: A batch of 300 containers for frozen orange juice contains 10 that are defective. Two are selected, at random, without replacement from the batch.

(a) What is the probability that the second one selected is defective given that the first one was defective?

(b) What is the probability that both are acceptable?

Question 5: The edge roughness of slit paper products increases as knife blades wear. Only 2% of products slit with new blades have rough edges, 5% of products slit with blades of average sharpness exhibit roughness, and 7% of products slit with worn blades exhibit roughness. Assume that 45% of the blades in manufacturing are new, 30% are of average sharpness, and 25% are worn.

(a) What is the probability that a randomly selected product exhibits edge roughness?

(b) If a randomly selected product was found to exhibit edge roughness, what is the probability that this product was slit with worn blades?

Question 6: A batch of 150 containers for frozen orange juice contains 3 that are defective. Two are selected, at random, without replacement, from the batch. Let A and B denote the events that the first and second container selected is defective, respectively.

(a) Are A and B independent events?

(b) If the sampling were done with replacement, would A and B be independent?

Question 7: An inspector working for a manufacturing company has a 98% chance of correctly identifying defective items and a 0.6% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 1.2% of nonconforming items.

(a) What is the probability that an item selected for inspection is classified as defective?

(b) If an item selected at random is classified as defective, what is the probability that it is indeed good?

Question 8: The random variable X has the probability distribution.

X	-1	2	3	6	10
$f(x)$	0.2	0.1	0.25	0.3	0.15

(a) Determine $P(X \leq 3), \quad P(-2 < X < 10)$

(b) Determine the mean and the standard deviation of X.

Question 9: A multiple choice test contains 20 questions, each with five answers. Assume a student just guesses on each question.

(a) What is the probability that the student answers more than two questions correctly?

(b) What is the student's expected number of correct answers

Question 10: The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 20 calls per hour.

(a) What is the probability that there are exactly 10 calls in one hour?

(b) What is the probability that there are 2 or less calls in one hour?

(c) What is the probability that there are exactly 6 calls in 30 minutes?