OFDM signal generation with IFFT

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Abstract— **Regarding the rising importance of Orthogonal Frequency Division Multiplexing (OFDM) in recent communication systems, this work explains how an OFDM signal can be synthesized with the use of the Inverse Discrete Fourier Transform (IDFT) of which efficient implementation is the Inverse Fast Fourier Transform (IFFT).**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become a popular technique for transmission of signals over wireless channels. Due to the benefits of this modulation technique, OFDM is used in several applications ranging from cellular systems, wireless local area networks (WLANs) and even optical light modulation. Regarding that, this article intends to explain the reason for using this technique, especially over wireless channels, and how it works.

II. DEVELOPMENT

A. Wireless Channel Characteristics

In conventional communication systems where the channel frequency response is approximately flat for a certain frequency range, typically a narrower range, single carrier modulations are commonly used. In these modulations schemes a data sequence (modulated at the symbol rate of the source) is sending over the channel in a single carrier. However in situations where the symbol rate are very high and the transmission medium is the air, some problems may occur.

One of the most important problems it's the presence of multipath components. Since the propagation path between transmitter and receiver contains objects that reflect, refract or spread the transmitted signal, in receiver the effect of these components translates as the reception of copies of transmitted signal with different delay, attenuation and phase shifts. The sum of these multiples copies causes variations on the instantaneous power of the received signal, therefore producing fading.

As a consequence of the internet popularization, there is a rising demand of higher data rates and to achieve this, one of the solutions is to decrease the symbol time period. However, this solution increase the signal bandwidth and if channel coherent bandwidth is smaller than the signal bandwidth then the channel creates a frequency selective fading as a direct consequence, the spectral response of the signal will show dips due to the multipath. In addition, this type of fading is also dispersive, since the signal energy associated with each symbol is spread out in time. This causes transmitted symbols that are adjacent in time to interfere with each other and this phenomenon is called intersymbol interference (ISI).

B. Basic Principles of OFDM

Two important concepts in terms of OFDM are the orthogonality of waveforms and the relation of the discrete fourier transform with such waveforms. Considering the time-limited complex exponential signals $\{e^{j2\pi f_k t}\}_{k=0}^{N-1}$, which represents differents subcarries at $f_k = \frac{k}{T_S}$ in the OFDM signal, where $0 \leq t \leq T_{sym}$ and T_{sym} is the symbol period.

These signals are defined to be orthogonal if the integral of the product for their common (fundamental) period is zero, that is,

$$
\begin{aligned}
&\frac{1}{T_{sym}} \int_0^{T_{sym}} e^{j2\pi \frac{k}{T_{sym}} t} e^{-j2\pi \frac{i}{T_{sym}} t} dt = \\
&= \frac{1}{T_{sym}} \int_0^{T_{sym}} e^{j2\pi \frac{k-i}{T_{sym}} t} dt \\
&= \begin{cases} 1 & \text{if } \forall \text{ integer } k = i \\ 0 & \text{otherwise.} \end{cases}
\end{aligned} \tag{1}
$$

Taking the discrete samples with the sampling instances at $t = nT_S = nT_{sym}/N$, $n = 0, 1, 2 \ldots, N-1$, the previous equation 1 can be written in the discrete time domain as

$$
\begin{aligned}
&\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi \frac{k}{T_{sym}} \cdot nT_S} e^{-j2\pi \frac{i}{T_{sym}} \cdot nT_S} = \\
&\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi \frac{k-i}{T_{sym}} \cdot \frac{nT_{sym}}{N}} = \\
&= \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi \frac{k-i}{N} n} \\
&= \begin{cases} 1 & \text{if } k - i = mN, \quad m \in \mathbf{Z} \\ 0 & \text{otherwise.} \end{cases}
\end{aligned} \tag{2}
$$

Thus, considering this inherent orthogonality of OFDM and that there aren't frequency deviations, there will be no ISI between symbols transmitted on different carriers, therefore will be no ICI.

OFDM transmitter maps the message bits into a sequence of PSK and QAM symbols which will be converted into N parallel stream. Each one of the N symbols is carried out by different subcarrier.

Let $X_l[k]$ denote the $l-th$ transmit symbol at the $k-th$ subcarrier, $l = 0, 1, 2, \ldots, \infty$, $k = 0, 1, 2, \ldots, N-1$. Due to

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the S/P conversion, the duration of transmission time for N symbols is extended to NT_S, which forms a single OFDM symbol with a length of T_{sym}. Let $\Psi_{l,k}(t)$ denote the $l-th$ OFDM signal at the $k-th$ subcarrier, which is given as

$$\Psi_{l,k}(t) = \begin{cases} e^{j2\pi f_k(t-lT_{sym})} & , 0 \leq t \leq T_{sym} \\ 0 & \text{elsewhere.} \end{cases} \quad (3)$$

Then the OFDM signal in the continuous-time domain can be expressed as:

$$x_l(t) = \sum_{l=0}^{\infty} \sum_{k=0}^{N-1} X_l[k] e^{j2\pi f_k(t-lT_{sym})} \quad (4)$$

The continuous-time OFDM signal in equantion 3 can be sampled at $t = lT_{sym} + nT_S$ with $T_S = T_{sym}/N$ and $f_k = k/T_{sym}$ to yield the corresponding discrete-time OFDM symbol as

$$x_l[k] = \sum_{k=0}^{N-1} X_l[k] e^{j2\pi kn/N} \text{ for } n = 0, 1, 2, \ldots, N-1 \quad (5)$$

The equation 5 is the N-point IDFT of PSK or QAM data symbols $\{X_l[k]\}_{k=0}^{N-1}$ and can be computed efficiently using the IFFT algorithm. In addition, considering this inherent orthogonality of OFDM and that there aren't frequency deviations, there will be no ISI between symbols transmitted on different carriers, therefore will be no ICI.

C. What is OFDM

The Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation strategy. In counterpoint with a single carrier modulation system of which the data modulate a carrier with single frequency, the OFDM strategy uses multiples carriers to modulate the data segmented in as many channels as subcarriers available.

The catch with the OFDM is that the carriers obey the principle of orthogonality, with each subarrier closely spaced, allowing each data channel bandwidth to superposed between themselves, but without loss of information, different that would happen in a Frequency Division Multiplex (FDM) channel. Therefore the major data stream is segmented into minor flows with lower rate and large symbol time.

D. OFDM transmitter with IFFT

For sake of simplicity an OFDM transmitter can be split in some sections, so each transformation in the original sequence of bits and its effects in the output signal becomes clearly. The steps followed to generate a OFDM signal is as follows:

- Convert original frame from series to parallel representation (S/P);
- Map blocks of bits in a M-QAM constellation;
- Ensure the conjugate symmetry property of DFT upon M-QAM sequence of constellation points;
- Modulation with IDFT (IFFT for higher efficiency);
- Further insertion of the cyclic prefix.

1) Serial-to-parallel conversion: The original serial bit stream that is normally modulated with a single carrier is split in a serial-to-parallel buffer with frames of size N_f bits. Soon after each frame is parsed into N_c blocks, so each block of bits can be modulated in a multi carrier fashion and each block representing a channel. Each individual block can be encoded as the designer wishes. The equation 6 translates the i-th block has b_i bits, and when summed all bits, the frame size is achieved:

$$\sum_{i=1}^{N_c} b_i = N_f \quad (6)$$

It is important to disclosure that this paper will work with sequences of positive non-null index (in the majority of time), so each sequence can be represented conveniently in a software simulation.

2) Constellation mapping: The N_c blocks must have its bits modulated with a M-QAM modulation, so each M-QAM symbol can be mapped into a constellation map. A constellation point is represented by a complex number with an in-phase value (real) and a quadrature value (imaginary). Therefore, each channel represented by a different M-QAM constellation with same symbol time of $1/T$ will employ $M = 2^{b_i}$ signal points. Each constellation point in the k-th channel can be represented by X_k, where $k = 1, .., N_c$.

3) Symmetry of DFT: The $\{X_k\}$ sequence is read as the DFT over an OFDM time signal. So applying the IDFT over the sequence $\{X_k\}$ it is expected to obtain the OFDM time signal to be transmitted. Sure the IDFT can be applied in the sequence $\{X_k\}$ as it is, but it will generate a complex time series that needs an additional modulation step via a phase and quadrature modulator.

The solution to avoid the use of such modulator is transforming the sequence $\{X_k\}$ in such way that its magnitude is an even function of frequency and its phase response is an odd function, therefore ensuring the conjugate symmetry property of the DFT.

This can be done extending the sequence $\{X_k\}$ of size N_c into a sequence of size $2N_c$ and inserting into this upper half of the sequence the complex conjugate of the original sequence (lower half). The new points of the sequence can be populated as:

$$X_{2N_c-k} = X_k^*, \quad k = 2, ..., 2N_c \quad (7)$$

The first point of the original $\{X_k\}$ is divided in two real points and distributed in the beginning of the sequence ($X_1 = \Re(X_1)$) and at the middle of the sequence ($X_{N_c} = \Im(X_1)$).

4) Multicarrier modulation with IFFT: Once the new sequence $\{X_k\}$ with length $2N_c$ is obtained and respects the conjugate symmetry properties, each point can be modulated by a different subcarrier over a symbol period. This is made with the help of the IDFT, but to achieve high efficiency, the IFFT algorithm is largely used.

As we can see in the equation 8, the variable x_n represent a sample of an OFDM symbol and the full sequence $\{x_n, 0 \leq$

$n \leq 2N_c - 1\}$ nonetheless represents an OFDM symbol. It is important to see that each X_k point is modulated by a complex exponential whose frequency is dependent of k, ensuring that each individual channel is modulated by a different carrier. Once n varies over $n = 0, 1, ..., 2N_c - 1$, we can see that the sequence $\{X_k\}$ is maintained until, at least, one period of each subcarrier is achieved, concluding the synthesis of the sequence $\{x_n\}$ of length $2N_c$ that represents an OFDM symbol of which contains N_c different M-QAM symbols.

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=1}^{2N_c} X_k e^{j2\pi n(k-1)/(2N_c)}, \quad n = 0, 1, ..., 2N_c - 1 \tag{8}$$

5) Cyclic prefix: Considering a dispersive channel with impulse response $c(t)$, and $n(t)$ some additive noise, the continuous OFDM signal in the medium will suffer the following distortion:

$$r(t) = x(t) * c(t) + n(t) \tag{9}$$

If the designer chooses the frequency bandwidth of each subchannel to be Δf as small as possible, the symbol duration $T = 1/\Delta f$ will be large if we compare with the channel dispersion with ν samples. A solution to avoid ISI is to add a guard band of duration $\nu T/(2N_c)$ so the effect of the channel in one symbol is isolated from another.

To observe the effect of the channel dispersion, we have simulated a transmission of a NRZ polar signal in two different hypothetical channel scenarios. The results are shown in the figure 1. Both channel are modulated by a first order transfer function for the sake of simplicity. The first channel has a settling time of 4 seconds $(1/(s+1))$ and the second has a settling time of 0.4 seconds $(10/(s+10))$. The symbol duration of the NRZ signal was choose to be 1 second, so as we can see in the figure 1, the dynamics of the channel 1 exerts high influence in all the symbol time and does not have time to settle its response before the next symbol, so it distort the receive signal in the point to harm the quality of the signal. Therefore the channel 2 that has a settling time lower than the symbol time, having a minor influence in the symbols and restricting its influence inside a symbol period.

An alternative way to avoid ISI in the OFDM signal is to append a cyclic prefix in each block of $2N_c$ samples $\{X_0, ..., X_{2N_c-1}\}$. The cyclic prefix is a sequence extract from the previous block of samples that consists in the ν last samples of the block $\{X_{2N_c-\nu}, ..., X_{2N_c-1}\}$ and then added to the beginning of the sequence.

Once ν is the number of samples that the channel dispersion lasts the final sequence have a length of $2N_c+\nu$ samples and the cyclic prefix will work as a time guard band against ISI. Further in reception these samples are discarded, once only the original sequence matters to extract information.

There is also another way to overcome the channel response, which its estimation and compensation with equalizers. But for this, there is a need to measure the channel response of which can be accomplished by transmitting either a known modulated sequence on each of the subcarriers or transmitting the unmodulated subcarriers.

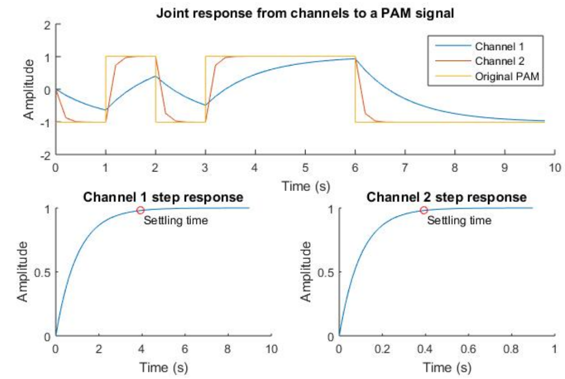

Fig. 1. Different channel influence in a transmission. Source: own.

III. RESULTS

After modulation we are able to check the resultant wave forms as shown in the current bibliography. The first set of wave forms we want to show are the disassembled wave forms in time and frequency with each subcarrier influence splited to allow an easy overview. Witte [7] show the figure 2 representing all the orthogonal subcarriers that be modulated by the symbols M-QAM, while figure 3 represent the specter of the OFDM signal with each subchannel superimposed over each other, but respecting orthogonality.

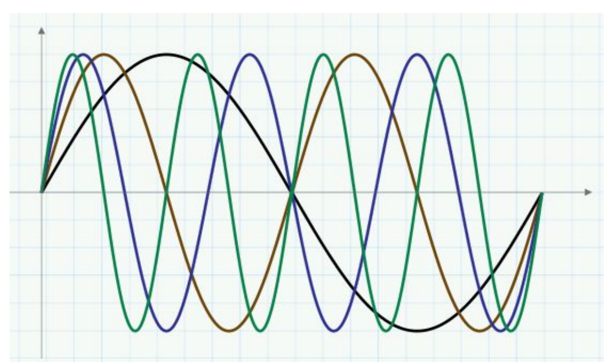

Fig. 2. Composition of time domain of 4 subcarriers for OFDM system. Source: Witte [7] .

Cheng [6] combine the previous wave forms in a simulation showing the overall result of the simulation. The figure 4 show the results of the simulation. We can see that the time signal is a composition of multiples sinusoidal subcarriers combined, while the spectrum denotes the results of the sum of multiples sinc spaced in frequency, accentuating the peak of each sinc function, once the orthogonality allows that each sinc peak are found in the subcarrier central frequency and the nulls of others sincs.

In the context of FDM, the OFDM has a more compact bandwidth since it uses orthogonal carriers, therefore a high

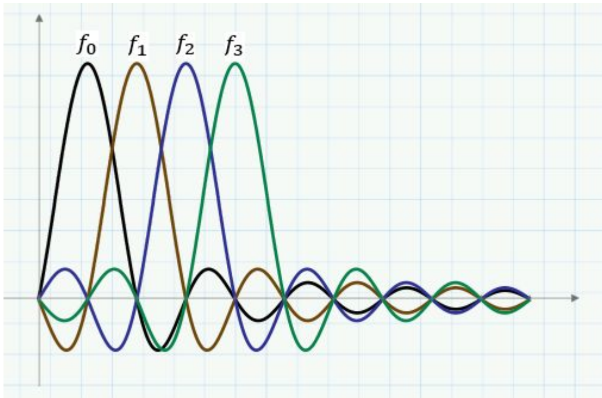

Fig. 3. Composition of frequency domain of 4 subcarriers for OFDM system. Source: Witte [7] .

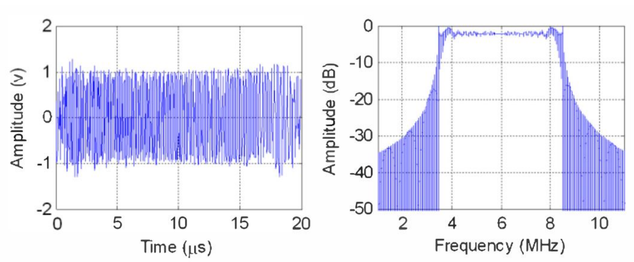

Fig. 4. In the left is a OFDM signal in time domain, while in the right is the OFDM signal spectrum. Source: Cheng [6].

spectral efficiency. Once FDM uses guard band between carriers, it presents a larger bandwidth. With the addition of the cyclic prefix in OFDM and its natural larger symbol period, the effects of ISI are reduced (even the effects of inter frame interference, IFI).

The downgrades of OFDM is that it has a high peak to average power ratio (PAPR), therefore presenting a signal and noise amplitude variation and high dynamic range that can impact RF amplifiers efficiency, since its linear range must accommodate the large amplitude variations. Other problem is, considering multiples subcarriers, the OFDM system is more likely to suffer from carrier offset and drift.

IV. CONCLUSIONS

Despite its downgrades, the OFDM system is very promising in future communications systems and show very good results in actual applications in the wireless market. The benefits of a high data capacity, high spectral efficiency and resilience to interference as a result of multipath effects provide some of the pillars that every communications scheme need today.

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