Math 151a, Homework 3

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1. 2.2.9

   We know that the $\sin(x)$ function is continuous from $[0, 2\pi]$. Furthermore we know the value of the $\sin(x)$ function will lie within $[-1, 1], \forall x$. Therefore we know that $\pi - 0.5 \leq f(x) \leq \pi + 0.5, \forall x$ Therefore $f(x) \in [0, 2\pi], \forall x$ By theorem 2.3, this implies that $f(x)$ has at least one fixed point in $[0, 2\pi]$

   $$f'(x) = 0.25 \cos(\frac{x}{2})$$

   $f'(x)$ exists on $(0, 2\pi)$ and $|f'(x)| \leq 0.25, \forall x$. Therefore, by theorem 2.3, we can conclude that there is one unique fixed point of $f(x)$ on $(0, 2\pi)$.

```
function [x] = fixedpoint(a, b, tol, N0, p0)
F = @(x) pi + 0.5 * sin(0.5*x);
j = 1;
p = p0;
while j < N0
    p = F(p);
    if abs(p-p0)<tol
    % close enough to actual root, stop
        break;
    else
        p0=p;
        j = j + 1;
    end
end
fprintf('Iteration number = %d \n', j);
fprintf('p = %.4f \n',p);
fprintf('f(p) = %.4f \n', pi + 0.5 * sin(0.5*p));
```

   Using the code above, to approximate the fix point to within 10^{-2}, we get $x \approx 3.6270$ in 3 iterations if we start with a $p_0 = \pi$.

   By corollary 2.5, we know that $|p_n - p| \leq \frac{k^n}{1-k}|p_1 - p_0|, \forall n \geq 1$. This implies that $10^{-2} \leq \frac{k^n}{1-k}|p_1 - p_0| = \frac{0.25^n}{0.75}|3.6416 - \pi| = \frac{2}{3} * 0.25^n$ Doing some algebra and taking the log of both sides results in an n value of $n \geq 3.0294$. This is similar to the actual iteration in which a fixed point was located.

2. 2.2.13a

3. 2.2.13b

4. 2.2.20

5. 2.2.24

6. 2.3.19

7. 2.3.23

8. 2.3.25 2.3.28

   (a)