Thesis 3 — Classical Probability as Smooth Field Modulation Under Rotation

Recovering $\cos^2(\varphi)$ as a Statistical Attractor of Local Bounded Systems

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Author’s Note

The argument was conceived and developed independently by the author. Computational tools were used solely for editing, formatting, and symbolic refinement. All insights, errors, and conclusions are the responsibility of the author.

Thesis Statement

The curved correlation outcomes observed in rotationally aligned systems—such as the $\cos^2(\varphi)$ distribution seen in Bell-type experiments—do not require quantum superposition or metaphysical indeterminacy. They arise naturally from the convolution of:

1. Projection geometry ($\cos(\varphi)$, $\cos^2(\varphi)$) arising from angular alignment,
2. Physical indeterminacy—real but unresolved variables such as thermal drift, phase jitter, wavelength cycles, or detector threshold slop,
3. Bounded ensemble variation, which behaves as a smooth modulation field $\varepsilon(\theta)$, distributing these effects across angular space.

When angular alignment governs a system's behavior, and bounded modulation acts upon it, the resulting statistical surface inevitably converges toward $\cos^2(\varphi)$. This outcome is not a quantum artifact—it is a classical inevitability: the surface that emerges when deterministic geometry is modulated by real-world variability.

Foundational Framing

$\text{Cos}^2(\varphi)$ is not merely a prediction of quantum theory. It is a geometric invariant: the square of the projection between two vectors as one is rotated with respect to the other. Any system in which detection, coincidence, or interaction strength depends on angular alignment will naturally tend toward a $\cos^2(\varphi)$ correlation.

This includes:
- Fields under rotation
- Signal overlap
- Polarization systems
- Filter alignment and projection-based interactions

Thus, Thesis 2 (Angular Modulation in Classical Systems) supplies the core principle: $\cos^2(\varphi)$ is the expected curvature wherever rotation and alignment interact. Thesis 3 now extends this to the real world, showing how classical systems—modulated by local, bounded variation—reproduce this curve in practice.

Foundational Lemma — Classical Systems Cannot Obey Binary or Factorable Angular Logic

Lemma:
In any physically grounded classical system where detection depends on the angular alignment between system orientation and measurement apparatus, the detection

probability must vary continuously and smoothly—typically as a function of geometric projection (e.g., $\cos^2(\varphi)$).

Therefore, such systems cannot:
- Produce binary (±1) outcomes at arbitrary angles
- Exhibit stepwise or linear detection transitions
- Obey factorable joint probability constraints such as: $P(A, B \mid a, b, \lambda) = P(A \mid a, \lambda) \cdot P(B \mid b, \lambda)$

Justification:
- Geometric projection is inherently continuous: $v_1 \cdot v_2 = |v_1||v_2|\cos(\varphi)$
- Detection probabilities that depend on energy, amplitude, or overlap scale with the square of this projection: $P(\varphi) \propto \cos^2(\varphi)$
- Any binary, stepwise, or linear correlation assumption contradicts this physical structure and cannot arise from convolution of real, smooth, local fields.

Core Lemma — Bounded Modulation Across Angular Divergence Yields $\cos^2(\phi)$

Let a classical system exhibit angularly dependent interaction strength modeled by:

$$f(\theta) = \cos^2(\theta)$$

Let $\varepsilon(\theta)$ be a smooth, bounded, symmetric kernel (e.g., Gaussian) representing real-world variation:
- Thermal drift
- Wavelength phase cycles
- Detector jitter
- Alignment slop

Let φ be the nominal angular offset between two components. Then the ensemble-averaged outcome over many trials is given by:

$$C(\varphi) = \int_{-\pi}^{\pi} \cos^2(\theta) \cdot \varepsilon(\theta - \varphi)\, d\theta$$

If:
- $\varepsilon(\theta)$ is bounded within $[-\delta, +\delta]$,
- $\varepsilon(\theta)$ is normalized: $\int \varepsilon = 1$,
- $\varepsilon(\theta)$ is symmetric around 0,

Then:

$$C(\varphi) \approx \cos^2(\varphi)$$

And in the narrow-bound limit:

$$\lim_{\delta \to 0} C(\varphi) = \cos^2(\varphi)$$

Interpretation and Summary

Even when a system is subject to variation, uncertainty, or small inconsistencies, so long as these are bounded and centered, the overall statistical behavior remains faithful to geometric expectations. Rather than distorting or randomizing the projection curve, real-world modulation smooths and reinforces it.

Key Insight:
Classical probability is structured modulation—not metaphysical indeterminacy. In real systems, error and noise are not signs of randomness beyond understanding—they are manifestations of bounded physical variation. These variations, when treated as an ensemble field—smooth, finite, and locally distributed—produce structured statistical behavior.

Closing Principle

$\cos^2(\varphi)$ is not fragile.
It is not destroyed by indeterminacy; it is revealed by it.
In classical systems with bounded modulation under rotation, it is the default statistical surface.

Falsification Condition – Statistical Emergence of $\cos^2(\phi)$

1. Ensemble Convergence Expectation
The model predicts that, over sufficiently large datasets (e.g., $N \geq 500$–1,000), the observed correlations between angularly correlated particles passed through polarizing filters at varying relative angles will statistically converge toward a $\cos^2(\varphi)$ distribution. This is a signature of ensemble-based geometric emergence—not pairwise determinism.

2. Low-N Divergence Check
If $\cos^2(\varphi)$ correlations appear consistently at low-N (e.g., $N \leq 10$), this suggests non-statistical behavior or premature convergence—falsifying the ensemble emergence claim.

3. Comparative Divergence at $\varphi = 0°$
Quantum mechanics predicts near-perfect correlation (≥90%) at 0°/0° filter settings, even at low-N.
- If the classical model also produces 9 out of 10 or better correlation at $\varphi = 0°$ without modulation or smoothing, the statistical nature of convergence is undermined and the model is falsified.
- If instead, the classical model requires large-N to achieve high correlation at 0°, it supports the claim of emergent convergence.

4. Caveat – False Falsification Safeguard
Because even classical systems can occasionally generate high correlation in small trials by chance:
- Single outlier trials do not falsify the model.
- Valid falsification requires repeatable, statistically significant low-N convergence inconsistent with ensemble behavior.

5. Benchmark Integrity Clause
Quantum mechanical results used for comparison must be:
- Pre–post-selection,
- Free of coincidence window trimming,
- Uncorrected for visibility,
- Unaffected by correlation expectation filters.
Only raw or minimally processed data may be used to evaluate convergence behavior and thresholds.

References

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