The Myth of Linearity in Bell's Classical Benchmark

A Thesis and Formal Proof

Submitted by

Iwl Robran

Category: Quantum Physics / Foundations of Physics

Date: May 29, 2025

This paper is submitted as an independent work

The Myth of Linearity in Bell's Classical Benchmark

A Thesis and Formal Proof By Iwl Robran

Author's Note

This work was conceived, developed, and structured independently by the author. AI tools were used specifically for assistance with formalizing a key mathematical step involving Fourier decomposition and field convolution. All conceptual insights, philosophical critique, and framing of Bell's assumptions originate solely with the author.

Abstract

Bell-type experiments frequently contrast quantum predictions—particularly the $\cos^2(\varphi)$ correlation curve—with a simplified "classical" benchmark that assumes linear decay in correlation with angular separation. This comparator, $P_{classical}(\varphi) = 1 - (2\varphi/\pi)$, predicts 0.75 correlation at 22.5°, diverging from the quantum prediction of ≈0.85, and is used to claim that quantum mechanics violates classical expectations.

This paper challenges the validity of that comparator.

We show that linear correlation decline cannot arise from smooth, localized classical fields. Using convolution and Fourier analysis on the circle, we prove that angular overlap between such fields must produce non-affine (curved) functions. The widely used linear benchmark, therefore, is not a physically realistic classical model—it is a construct of binary logic systems with stepwise transitions.

We further argue that $\cos^2(\varphi)$-like correlation curves are a natural outcome of classical field overlap, not uniquely quantum. Bell's framework may still rule out binary hidden-variable theories, but it unfairly excludes a vast class of plausible field-based models. The foundational contrast drawn by Bell is, therefore, philosophically incomplete.

Section 1: Entangled Photons and Angular Filters

In a typical Bell-type setup:

- A source emits entangled photon pairs.
- One photon travels to Detector A, the other to Detector B.
- Each detector includes a polarizing filter, independently set to some angle.

Quantum mechanics predicts the correlation between detection outcomes as:
$P\_quantum(\varphi) = \cos^2(\varphi)$

This function aligns closely with experimental data. Key values include:
- $\varphi = 0° \to P = 1.0$
- $\varphi = 22.5° \to P \approx 0.85$
- $\varphi = 45° \to P = 0.5$
- $\varphi = 90° \to P = 0.0$

The quantum explanation appeals to wavefunction collapse: when a photon encounters a filter, it collapses into an eigenstate aligned with that filter. The entangled partner's behavior is correlated based on the angular separation between the first filter's eigenstate and the second filter's orientation.

Section 2: The Classical Comparator

In contrast, the traditional "classical benchmark" assumes a linear drop in correlation as the angular difference increases. This is often stated as:
$P\_classical(\varphi) = 1 - (2\varphi / \pi)$

At $\varphi = 22.5°$, this model yields a correlation of 0.75, compared to the quantum prediction of ≈0.85. The gap is used to demonstrate violations of Bell's inequalities and argue for the non-classical nature of entanglement.

However, this linear comparator:

• Does not arise from physical models involving fields.

• Originates from logic-based hidden-variable models that treat outcomes as binary.

• Behaves like a step-function model with preset outcomes and no smooth overlap.

This linear assumption has become deeply embedded in pedagogical and even experimental comparisons, yet it reflects a mathematical convenience, not a physical reality.

Section 3: Fields, Not Bits

In physical models, especially classical ones involving polarization, we work with:

• Smooth fields (no sharp transitions),

• Localized distributions (fields with preferred orientation),

• And Fourier representations (decompositions into harmonics).

Suppose we model the interaction of a photon's polarization field with a rotated polarizer as a field overlap. The correlation between the two detectors becomes:

$C(\varphi) = \int f(\theta) \cdot g(\theta - \varphi)\, d\theta$

(i.e., the circular convolution of two angular fields)

Our core claim: If f and g are smooth and localized, the resulting overlap function $C(\varphi)$ must be non-affine—that is, curved.

This immediately rules out the linear comparator as a legitimate outcome of field-based models.

Section 4: The Core Proof Sketch

(Should this section be abbreviated and lead into section 5, so there aren't two complete final proofs?)

Let:

$f(\theta)$ and $g(\theta)$ be smooth, square-integrable, real-valued functions defined on the circle $\theta \in [0, 2\pi)$.

These represent classical angular fields—e.g., polarization amplitudes.

Assume both are:

- Smooth (infinitely differentiable, $C\infty$),
- Localized (e.g., narrow angular support or fast Fourier decay).

We define the angular correlation function as:

$C(\varphi) = \int_0^{2\pi} f(\theta) \cdot g(\theta - \varphi)\, d\theta = (f * g)(\varphi)$

Fourier Representation:

Let F_n and G_n be the nth Fourier coefficients of f and g, respectively. Then, by the convolution theorem:

$C(\varphi) = \sum_n F_n G_{-n} e^{i n \varphi}$

This sum is smooth and periodic, with fast-decaying harmonics due to the smoothness of f and g.

Contradiction Argument:

Assume, for contradiction, that:

$C(\varphi) = a\varphi + b$, for real constants a and b.

Then:

- $C(\varphi)$ is affine (i.e., linear),
- But this function is not periodic unless $a = 0$,
- And linear functions are not expressible as finite or convergent trigonometric series.

Therefore:

A non-constant linear function cannot arise from a convolution of smooth, periodic fields.

Hence:

A linear comparator function is incompatible with the convolution of any two smooth, localized classical fields.

Section 5: Philosophical Implications

What does this mean?

Bell's inequality correctly rules out binary logic models that assign definite yes/no outcomes to all possible filter angles.
But the linear comparator used as a classical stand-in is itself non-physical—a strawman.
Classical field models—those grounded in smooth, localized physics—do not predict linear correlations.
In fact, overlap-based models naturally produce curved correlation functions, such as $\cos(\varphi)$ or $\cos^2(\varphi)$.

Thus:
The contrast between "quantum curved vs. classical linear" is false.
A realistic classical comparator might resemble the quantum prediction—not differ from it.

Section 6: Final Thought

The myth of the linear comparator persists because it’s easy to understand and mathematically tidy.

But the physical world is not built on binary switches.

If classical systems are modeled as smooth fields, they must yield non-linear angular correlations. The “classical benchmark” used in Bell's analysis is not derived from any physical system—it is an idealized logical construction.

Quantum theory may still be the most complete model we have. But its superiority should be judged against realistic classical contenders, not abstract caricatures.

Appendix: Formal Proof (Full Text)

Claim:

A linear correlation function cannot arise from the circular convolution of two smooth, localized classical fields.

1. Setup: Classical Angular Fields

Let:

• $f(\theta), g(\theta) \in L^2([0, 2\pi))$ be real-valued, smooth, and localized.

Define:

$C(\varphi) = \int_0^{2\pi} f(\theta) \cdot g(\theta - \varphi)\, d\theta = (f * g)(\varphi)$

2. Fourier Representation

Let F_n and G_n be the nth Fourier coefficients of f and g:

$C(\varphi) = \sum_n F_n G_{-n} e^{i n \varphi}$

Since f and g are smooth, their coefficients decay faster than any polynomial. The result is a rapidly convergent sum.

3. Contradiction Argument

Assume:

$C(\varphi) = a\varphi + b$, an affine function.

But such functions:

• Are not periodic unless $a = 0$ (i.e., constant),

• Cannot be represented by a convergent Fourier series on the circle,

• And if forced to be periodic (e.g., sawtooth), introduce discontinuities—violating the smoothness of f and g.

4. Spectral Consequences

The spectrum of an affine function:

• Decays slowly (or not at all),

• Requires global (non-localized) support in angular space,

• Violates the physical assumptions of compact, smooth classical fields.

Therefore:

No combination of smooth, localized classical fields can produce a linear comparator.