No calculator allowed in this examination

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Nature Inspired Search and Optimisation

Main Summer Examinations 2019

Time allowed: 1:30

[Answer all questions]

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1

The travelling salesman problem (TSP) can be described as follows: Given N cities, $1, 2, \cdots, N$ and the distances $d_{i,j}$ between each pair of them (here i and j are one of the $1, 2, \cdots, N$ cities), find a permutation $(x_1, x_2, \cdots, x_N)$ of $(1, 2, \cdots, N)$ such that the sum of distances $D = d(x_1, x_2) + d(x_1, x_2) + \cdots + d(x_{N-1}, x_N) + d(x_N, x_1)$ is minimum.

(a) What are two immediate neighbourhood solutions of a TSP? Draw one immediate neighbourhood solution of the solution to the 6-city TSP in Figure 1. Based on your observation, write in pseudocode the 2-opt algorithm which searches for immediate neighbourhood solutions for TSP.

[6 marks]

(b) Comment on the suitability of exhaustive search, local search and stochastic local search for solving large-scale TSP with more than 10000 cities. Discuss their advantages and disadvantages.

[6 marks]

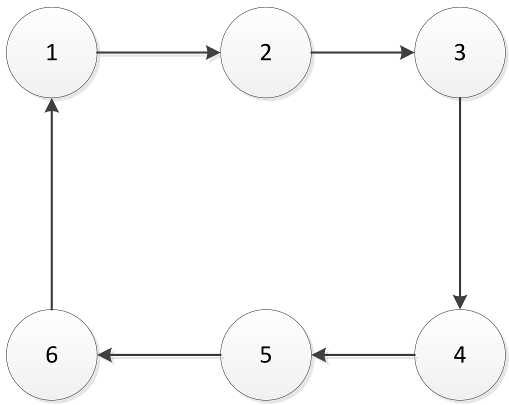

Figure 1: A solution (route) for a 6-city TSP: $1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 6$

Question 1 continued over the page.

(c) The simulated annealing algorithm is given in Table 3. Explain the purpose of the step marked "Should we move to it?", and write a suitable function $P(e, e_{new}, T)$.

[8 marks]

$x := x_0$; $e := f(x)$ // x_0: initial solution; $f(x)$: objective function
$x_{best} := x$; // Initial "best" solution
$k := 0$
while $(k < k_{max})$
    $T := temperature(t_0)$ // Temperature calculation.
    $x_{new} := neighbour(x)$ // Pick some neighbour.
    $e_{new} := f(x_{new})$ // Compute its objective function value.
    if $P(e, e_{new}, T) > R(0, 1)$ then // Should we move to it?
        $x := x_{new}$; $e := e_{new}$ // Yes, change state.
    if $e_{new} < e_{best}$ then // Is this a new best?
        $x_{best} := x_{new}$; $e_{best} := e_{new}$ // Save as 'best found'.
    $k := k + 1$
Output x_{best}

Table 1: Pseudocode of Simulated Annealing algorithm for minimisation. The annealing schedule $temperature()$ defines how to decrease the temperature from an initial temperature t_0.

Question 2

A company plans to manufacture a product with a £8000 budget. To manufacture this product, the company needs to pay the costs of labour and material. The unit cost of labour and material are £8.2 pounds and £18.5 pounds, respectively. For d_1 units of labour and d_2 units of material, the company will produce $d1 \times d2$ units of the product. The company asks you to maximises the quantity of the products that the company can manufacture.

(a) Formulate the task as a constraint optimisation problem.

[4 marks]

(b) Design an evolutionary algorithm for solving the constraint optimisation problem, justifying all your design decisions.

(i) Describe a suitable chromosome representation of an individual and what evolutionary operators you would use.

[3 marks]

(ii) Describe the constraint handling technique you would use. Discuss the advantages and the disadvantages of the method.

[3 marks]

Question 2 continued over the page.

(c) Consider two bitstrings x and z, each of of length n, as shown in the figure below. I.e., the bit-strings differ in the first $a+b$ bit-positions, and they are equal in the last $c+d$ bit-positions. In the first a bit-positions, bitstring x takes the value 0, while bitstring y takes the value 1. In the following b bit-positions, bitstring x takes the value 1 while bitstring 0. In the next c bit-positions, both bitstrings take the value 0, and in the final d bit-positions, both bitstrings take the value 1.

Assume that m bitstrings $y^{(1)}, \dots, y^{(m)}$ are obtained by applying the mutation-operator with mutation rate p_{mut} on the result of applying uniform crossover to bitstring x and bitstring z, i.e., for all $i \in \{1, \dots, m\}$,

$$y^{(i)} := \texttt{mutation}(\texttt{crossover}(x, z))$$

What is the *expected number* of 1-bits in bitstring $y^{(1)}$?

You observe that the *average number* of 1-bits in the m bitstrings $y^{(1)}, \dots, y^{(m)}$ is

$$\frac{a+b}{2} + d + 1 - \frac{d}{c}.$$

What can you infer about the mutation rate p_{mut}?

	a	b	c	d
x	00...000	11...111	00...000	11...111
z	11...111	00...000	00...000	11...111

[10 marks]

Question 3

(a) What distinguishes co-evolutionary algorithms from traditional evolutionary algorithms? Explain the difference between competitive co-evolution, and cooperative co-evolution. Provide one example application for each of these two types of co-evolution.

[8 marks]

(b) What is fitness sharing? What problem is fitness sharing designed to alleviate?

[4 marks]

(c) Suppose you have a fitness function $f : \{0,1\}^n \rightarrow [0,\infty)$ to be maximised and a population of λ individuals $y_1, \ldots, y_\lambda \in \{0,1\}^n$. A function g is defined as

$$g(x) := \frac{f(x)}{\sum_{j=1}^{\lambda} s(x, y_j)}$$

where

$$s(x,y) := \begin{cases} 1 - \left(\frac{d(x,y)}{\sigma_{\text{share}}}\right)^{\alpha} & \text{if } d(x,y) \leq \sigma_{\text{share}}, \text{ and} \\ 0 & \text{otherwise,} \end{cases}$$

and $d(x,y)$ is the Hamming distance between bitstrings x and y.

Suggest how to implement fitness sharing in an evolutionary algorithm using the function g. Explain how the setting of parameters α and σ_{share} impact the fitness sharing mechanism. What are reasonable values for the parameters α and σ_{share}?

[8 marks]

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.