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School of Computer Science

# EVALUATION METHODS AND STATISTICS

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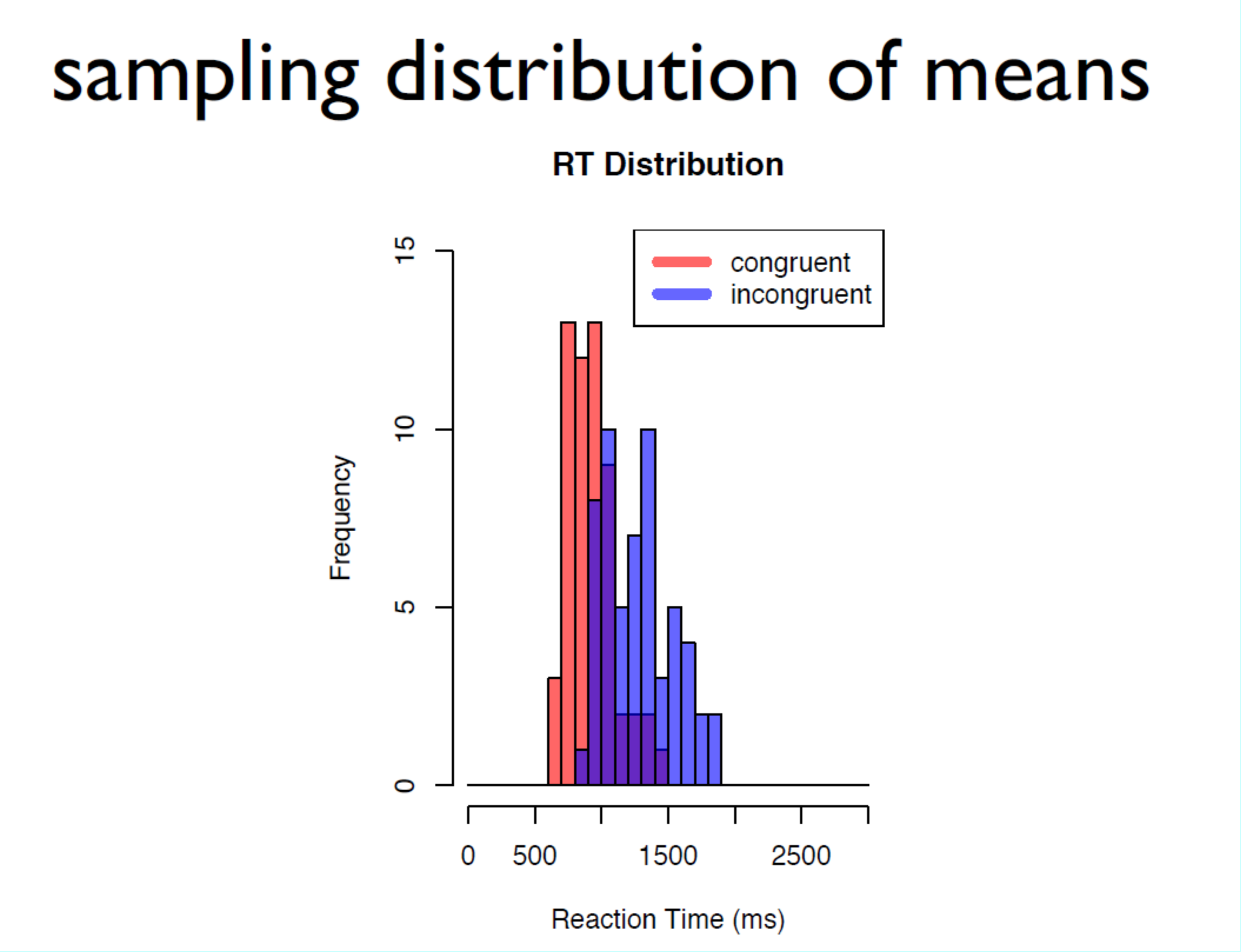
Chair of Pervasive and Ubiquitous Computing

# Stroop Data (again): are the distributions different?

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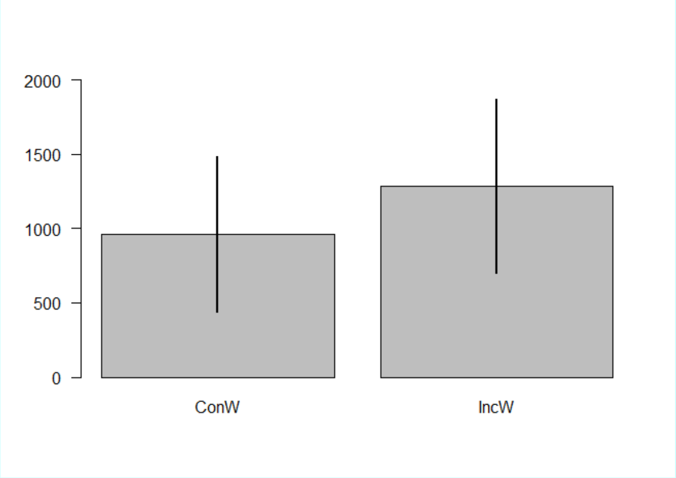
# Testing for Differences in Means

- If the sample mean difference is greater than we expect then
  - We have, by chance, collected samples that are not typical of the population

  Or
  - The two samples are from different populations




- We can compare the means to decide which explanation is most plausible

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# t-distribution

- Assume that samples are drawn from a normal distribution
- A t-distribution is symmetrical but with more scores in the tails (leptokurtic)
- Developed by W.S. Gosset in 1908, working at the Guinness brewery in Dublin
- ‘Hypothesis test statistic’
- Published under pseudonym ‘student’
- A t-distribution is defined as a measure of central tendency divided by a measure of spread.

$$t = \frac{m - \mu}{\sqrt{\frac{s^2}{N}}}$$

$\mu = 0$, $m$ = mean of D

- There are numerous t-distributions, so we define the form required for a specific analysis by the degrees of freedom of a distribution.
- The areas under the normal distribution curve are calculated in terms of t rather than z values, e.g., 95% confidence interval is z = 1.96 but t = 2.045

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# Types of t-test

- Dependent means / Paired Samples
  - Each participant completes all conditions

- Independent Samples
  - Different participants are allocated to each condition

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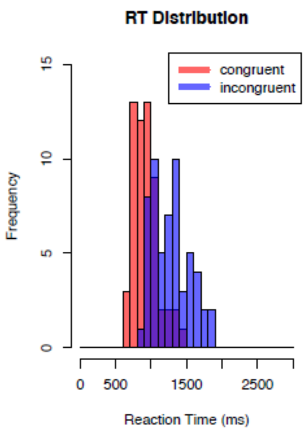
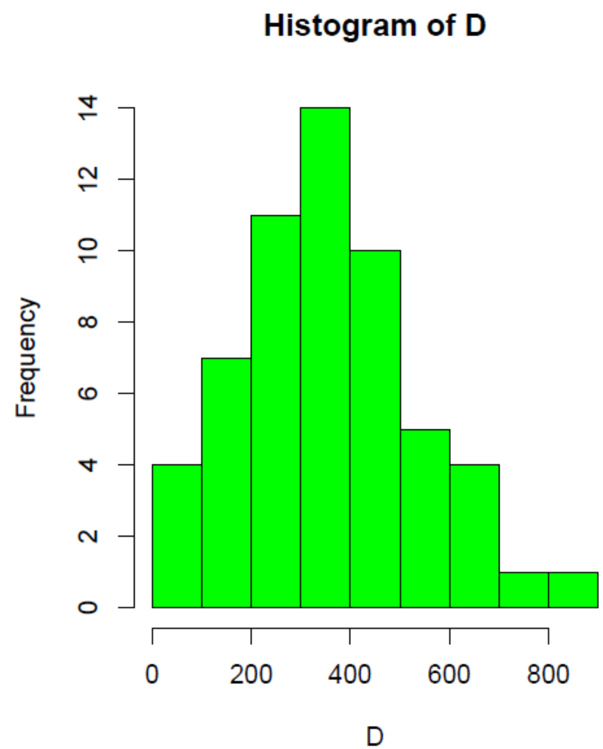
# What is the differences (between the reaction time for the congruent and incongruent conditions)?




| | ConW | IncW | D |
|---|---|---|---|
| 1 | 806 | 1087 | 281 |
| 2 | 709 | 1099 | 390 |
| 3 | 935 | 1086 | 151 |
| 4 | 1004 | 1548 | 544 |
| 5 | 1225 | 1616 | 391 |
| 6 | 1391 | 1855 | 464 |
| 7 | 1051 | 1667 | 617 |
| 8 | 917 | 1231 | 314 |
| 9 | 1216 | 1306 | 90 |

...

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# Central Tendency and Distribution

- We can assume that the histogram shows that difference, D, follows a normal distribution
- So, we can calculate central tendency as the mean
- If the null hypothesis is true, the mean = 0
- We don't (as yet) know the spread, i.e., variance, of the population.
- But, we assume that the population variance is the same as the sample variance.

# t-statistic (paired samples)

$$t = \frac{\left(\sum D\right)/N}{\sqrt{\dfrac{\sum D^2 - \left(\dfrac{\left(\sum D\right)^2}{N}\right)}{(N-1)(N)}}}$$

D = difference between trials;
N = number of samples

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# Data Set (adapted from the source data to be small enough to work by hand)

| IncW | ConW |
|---|---|
| 1087 | 806 |
| 1099 | 709 |
| 1086 | 935 |
| 1548 | 1004 |
| 1616 | 1225 |
| 1855 | 1391 |
| 1667 | 1051 |
| 1231 | 917 |
| 1306 | 1216 |
| 896 | 1548 |
| 1472 | 840 |
| 1008 | 640 |
| 1023 | 752 |
| 1056 | 815 |
| 1040 | 800 |
| 1548 | 736 |

In this experiment, participants completed both conditions.
Each row represents one person’s mean reaction time for Incongruent words (different colour ink and word) and Congruent words (same colour ink and word)

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Calculating a Paired Samples t-statistic by hand…

# 1. Calculate difference in time for each condition

| | IncW | ConW | Diff |
|---|---|---|---|
| | 1087 | 806 | |
| | 1099 | 709 | |
| | 1086 | 935 | |
| | 1548 | 1004 | |
| | 1616 | 1225 | |
| | 1855 | 1391 | |
| | 1667 | 1051 | |
| | 1231 | 917 | |
| | 1306 | 1216 | |
| | 896 | 1548 | |
| | 1472 | 840 | |
| | 1008 | 640 | |
| | 1023 | 752 | |
| | 1056 | 815 | |
| | 1040 | 800 | |
| | 1548 | 736 | |
| Sum(Σ) | | | |

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Calculating a Paired Samples t-statistic by hand…

# 1. Calculate difference in time for each condition

| | IncW | ConW | Diff |
|---|---|---|---|
| | 1087 | 806 | 281 |
| | 1099 | 709 | 390 |
| | 1086 | 935 | 151 |
| | 1548 | 1004 | 544 |
| | 1616 | 1225 | 391 |
| | 1855 | 1391 | 464 |
| | 1667 | 1051 | 616 |
| | 1231 | 917 | 314 |
| | 1306 | 1216 | 90 |
| | 896 | 1548 | -652 |
| | 1472 | 840 | 632 |
| | 1008 | 640 | 368 |
| | 1023 | 752 | 271 |
| | 1056 | 815 | 241 |
| | 1040 | 800 | 240 |
| | 1548 | 736 | 812 |
| Sum(Σ) | | | |

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Calculating a Paired Samples t-statistic by hand…

# 2. Sum differences and Square the product

| | IncW | ConW | Diff |
|---|---|---|---|
| | 1087 | 806 | 281 |
| | 1099 | 709 | 390 |
| | 1086 | 935 | 151 |
| | 1548 | 1004 | 544 |
| | 1616 | 1225 | 391 |
| | 1855 | 1391 | 464 |
| | 1667 | 1051 | 616 |
| | 1231 | 917 | 314 |
| | 1306 | 1216 | 90 |
| | 896 | 1548 | -652 |
| | 1472 | 840 | 632 |
| | 1008 | 640 | 368 |
| | 1023 | 752 | 271 |
| | 1056 | 815 | 241 |
| | 1040 | 800 | 240 |
| | 1548 | 736 | 812 |
| Sum(Σ) | | | 5153 |
| $(\Sigma D)^2$ | | | 26553409 |

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Calculating a Paired Samples t-statistic by hand…

# 3. Square the differences

| | IncW | ConW | Diff | Diff^2 |
|---|---|---|---|---|
| | 1087 | 806 | 281 | |
| | 1099 | 709 | 390 | |
| | 1086 | 935 | 151 | |
| | 1548 | 1004 | 544 | |
| | 1616 | 1225 | 391 | |
| | 1855 | 1391 | 464 | |
| | 1667 | 1051 | 616 | |
| | 1231 | 917 | 314 | |
| | 1306 | 1216 | 90 | |
| | 896 | 1548 | -652 | |
| | 1472 | 840 | 632 | |
| | 1008 | 640 | 368 | |
| | 1023 | 752 | 271 | |
| | 1056 | 815 | 241 | |
| | 1040 | 800 | 240 | |
| | 1548 | 736 | 812 | |
| Sum(Σ) | | | 5153 | |
| $(\Sigma D)^2$ | | | 26553409 | |

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Calculating a Paired Samples t-statistic by hand…

# 3. Square the differences

| | IncW | ConW | Diff | Diff^2 |
|---|---|---|---|---|
| | 1087 | 806 | 281 | 78961 |
| | 1099 | 709 | 390 | 152100 |
| | 1086 | 935 | 151 | 22801 |
| | 1548 | 1004 | 544 | 295936 |
| | 1616 | 1225 | 391 | 152881 |
| | 1855 | 1391 | 464 | 215296 |
| | 1667 | 1051 | 616 | 379456 |
| | 1231 | 917 | 314 | 98596 |
| | 1306 | 1216 | 90 | 8100 |
| | 896 | 1548 | -652 | 425104 |
| | 1472 | 840 | 632 | 399424 |
| | 1008 | 640 | 368 | 135424 |
| | 1023 | 752 | 271 | 73441 |
| | 1056 | 815 | 241 | 58081 |
| | 1040 | 800 | 240 | 57600 |
| | 1548 | 736 | 812 | 659344 |
| Sum(Σ) | | | 5153 | |
| $(\Sigma D)^2$ | | | 26553409 | |

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Calculating a Paired Samples t-statistic by hand…

# 4. Sum the Squares

| | IncW | ConW | Diff | Diff^2 |
|---|---|---|---|---|
| | 1087 | 806 | 281 | 78961 |
| | 1099 | 709 | 390 | 152100 |
| | 1086 | 935 | 151 | 22801 |
| | 1548 | 1004 | 544 | 295936 |
| | 1616 | 1225 | 391 | 152881 |
| | 1855 | 1391 | 464 | 215296 |
| | 1667 | 1051 | 616 | 379456 |
| | 1231 | 917 | 314 | 98596 |
| | 1306 | 1216 | 90 | 8100 |
| | 896 | 1548 | -652 | 425104 |
| | 1472 | 840 | 632 | 399424 |
| | 1008 | 640 | 368 | 135424 |
| | 1023 | 752 | 271 | 73441 |
| | 1056 | 815 | 241 | 58081 |
| | 1040 | 800 | 240 | 57600 |
| | 1548 | 736 | 812 | 659344 |
| Sum($\Sigma$) | | | 5153 | |
| $(\Sigma D)^2$ | | | 26553409 | |

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Calculating a Paired Samples t-statistic by hand…

# 4. Sum the Squares

| | IncW | ConW | Diff | Diff^2 |
|---|---|---|---|---|
| | 1087 | 806 | 281 | 78961 |
| | 1099 | 709 | 390 | 152100 |
| | 1086 | 935 | 151 | 22801 |
| | 1548 | 1004 | 544 | 295936 |
| | 1616 | 1225 | 391 | 152881 |
| | 1855 | 1391 | 464 | 215296 |
| | 1667 | 1051 | 616 | 379456 |
| | 1231 | 917 | 314 | 98596 |
| | 1306 | 1216 | 90 | 8100 |
| | 896 | 1548 | -652 | 425104 |
| | 1472 | 840 | 632 | 399424 |
| | 1008 | 640 | 368 | 135424 |
| | 1023 | 752 | 271 | 73441 |
| | 1056 | 815 | 241 | 58081 |
| | 1040 | 800 | 240 | 57600 |
| | 1548 | 736 | 812 | 659344 |
| Sum(Σ) | | | 5153 | 3212545 |
| $(\Sigma D)^2$ | | | 26553409 | |

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Calculating a Paired Samples t-statistic by hand…

# 4. Calculate t-statistic (paired samples)

$$t = \frac{\left(\sum D\right)/N}{\sqrt{\dfrac{\sum D^2 - \left(\dfrac{\left(\sum D\right)^2}{N}\right)}{(N-1)(N)}}}$$

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# 4. Calculate t-statistic

$$t = \frac{(\sum D)/N}{\sqrt{\frac{\sum D^2 - \left(\frac{(\sum D)^2}{N}\right)}{(N-1)(N)}}}$$

$$t = \frac{5153/16}{\sqrt{\frac{3212545 - \left(\frac{26553409}{16}\right)}{(16-1)(16)}}}$$

$t = 4.00741909$

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# 5. Use t-table to check significance

- For 95% confidence interval, $\alpha = 0.05$
- D.F. (degrees of freedom) = N-1 = 15
- From the t-table, t should be >2.131
- The calculated t = 4.007, is larger than the value in the table
- Therefore, we reject the null hypothesis and state that the difference between the mean reaction times in these data is less than 5% likely to occur by chance.
- We write the result as **$t(15) = 4.007$, $p<0.05$**

## Two Tails T Distribution Table

| DF | A = 0.2 | 0.10 | 0.05 | 0.02 | 0.01 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|
| ∞ | $t_a$ = 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.091 | 3.291 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.656 | 318.289 | 636.578 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.328 | 31.600 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.214 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.894 | 6.869 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |

# Is this is a ‘good’ result?

- We consider the generalisability of the result by its Effect Size
- Effect size indicates the size of the difference between samples independent of sample size
  - This could relate to proportion of explained variance, e.g., using $R^2$ or (partial) eta squared
  - Or could relate to difference in averages, e.g., using Cohen’s d

| | **Small** | **Medium** | **Large** | **Very large** |
|---|---|---|---|---|
| $R^2$ | 1% | 9% | 25% | |
| Partial eta$^2$ | 0.02 | 0.13 | 0.26 | |
| d | 0.2 | 0.5 | 0.8 | >1 |

Here is a nice paper about Effect Size:
https://www.leeds.ac.uk/educol/documents/00002182.htm

Here is an online calculator for Effect size:
https://www.ai-therapy.com/psychology-statistics/effect-size-calculator

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# Power and Effect Size

- Power is the probability that the study produces significant difference between groups when a difference actually exits. Power is 1-$\beta$, and relates to Type II error.
- Power relies on sample size and effect size.
- Effect size is the magnitude of difference between groups.

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# Paired t-test effect size

- There are several formulae, I prefer Cohen's *d*…
  - For a paired t-test…

$$d = \frac{|m_1 - m_2|}{\sqrt{{s_1}^2 + {s_2}^2 - (2rs_1s_2)}}$$

where r is the correlation coefficient between groups

For this dataset, r = 0.3277 (we'll calculate this in a later lecture)

So, the Effect size for this result is 1.001

(meaning that the difference is larger than 1 standard deviation)

Here is an online calculator for Effect size:
https://www.ai-therapy.com/psychology-statistics/effect-size-calculator

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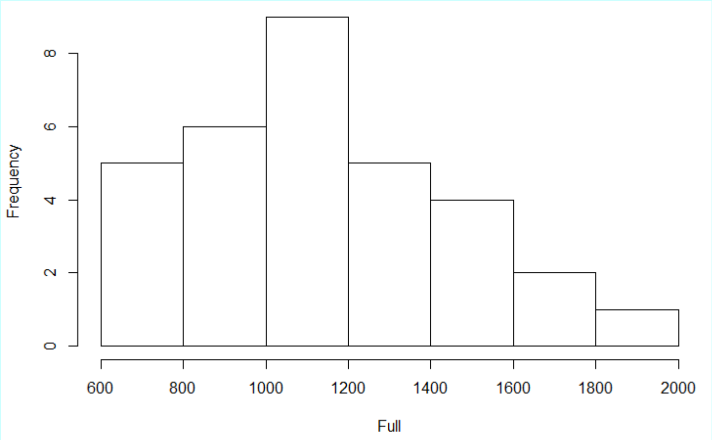
# Explore the data set in R

- Copy the data on slide 8 into a spreadsheet, such as Excel and save as .txt or .tsv
- In R user interface: File – Import dataset – From text…

```
>Attach (stroop)
>stroop <-data.frame(IncW, ConW)
>hist(IncW)
>hist(ConW)
> t.test(Incw, ConW, paired=TRUE, conf.level=0.95)
```




Paired t-test data: IncW and ConW (assuming null hypothesis, $\mu = 0$)

t = 4.0037, df = 15, p-value = 0.001151

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

150.6079 493.5171

sample estimates:

mean of the differences

322.0625

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# Calculating Power in R

- Note…Different tests calculate Power in different ways…t-tests use

  Pwr.t.test (n =  , d =  , sig.level =  , power =  , type = c(“two.sample”, “one.sample”, “paired”))

- For Paired t-test, we take the mean for each group…

  > mean(IncW)

  [1] 1293.625

  > mean(ConW)

  [1] 961.562

- …and we need to hypothesise a difference to detect (let’s say 200) and the power that we wish to use (let’s say, .8)

```
> library(pwr)
> pwr.t.test(d=(1283.625-961.5625)/200, power=0.8,sig.level=0.05, type ="paired",alternative="two.sided")
```

```
Paired t test power calculation
n = 5.242336
d = 1.610313
sig.level = 0.05
power = 0.8
alternative = two.sided
NOTE: n is number of *pairs*
```

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```
pwr.t.test(d=(1283.625-961.5625)/200, power=0.8,sig.lieve=0.05,type="paired", alternative="two.sided")
```

# We can explore this further…

- When we set the difference to 500, Cohen's d = 0.64, n = 20.9 (or 21)
- If we see beta to .95 (and keep difference at 200), d = 1.61, n = 7.2 (7)
- And beta .95, d = 500, d= 0.64, n = 33.3 (33)

- Where n = participants in paired (repeated measures) design

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# Independent t-test

- Participants are assigned to *only* one trial
- If you have the same number of participants per trial (equal sample sizes):

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\left(\frac{{s_1}^2}{N_1} + \frac{{s_2}^2}{N_2}\right)}}$$

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# Cohen's d for independent samples

$$d = \frac{|m_1 - m_2|}{pooled\ variance}$$

$$pooled\ variance = \sqrt{\left(\frac{(s_1{}^2 + \ s_2{}^2)}{2}\right)}$$

- Why you don't use this for paired samples
  - The effect size assumes a differences between sets of independent scores
  - The paired t-test corrects for correlation, r, into account to correct for pooled variance

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# Calculating an Odds Ratio

- An odds ratio indicates the strength of effect for one condition relative to another
- We are calculating the probability of a result given condition and contrasting this with a the probability of a contrasting result
- The ‘odds’ are the likelihood an event, or p(e) / 1-p(e)
- So, we need to describe our experiment in a 2 x 2 matrix
- Odd Ratio = (a*d) / (b*c)

| DV | IV1 | IV2 |
|---|---|---|
| i | A | B |
| ii | C | D |

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# A Cautionary tale – or completely made-up example

- In a class of 60 students, 37 regularly attended lectures and 23 did not
- Of the regular attenders: 32 passed the exam and 5 failed
- Of the non-attenders, 6 passed the exam and 17 failed
- Odds ratio = (17*32) / (5*6) = 18.13
- So, 18.13 more likely to pass exams if you attend lectures

| | **No lectures** | **Lectures** | |
|---|---|---|---|
| fail | 17 | 5 | 22 |
| pass | 6 | 32 | 38 |
| | 23 | 37 | |

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