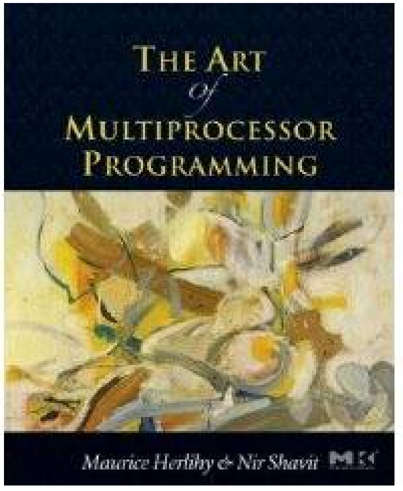
Mutual Exclusion



Companion slides for

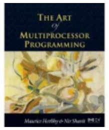
The Art of Multiprocessor Programming

by Maurice Herlihy & Nir Shavit

With some very minor changes by APS

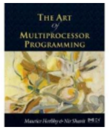
Mutual Exclusion

- We will clarify our understanding of mutual exclusion
- We will also show you how to reason about various properties in an asynchronous concurrent setting



Mutual Exclusion

- **Formal problem definitions**
- **Solutions for 2 threads**
- **Solutions for *n* threads**
- **Fair solutions**
- **Inherent costs**

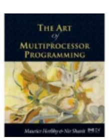


Warning

- You will never use these protocols
  - Get over it
- You are advised to understand them
  - The same issues show up everywhere
  - Except hidden and more complex

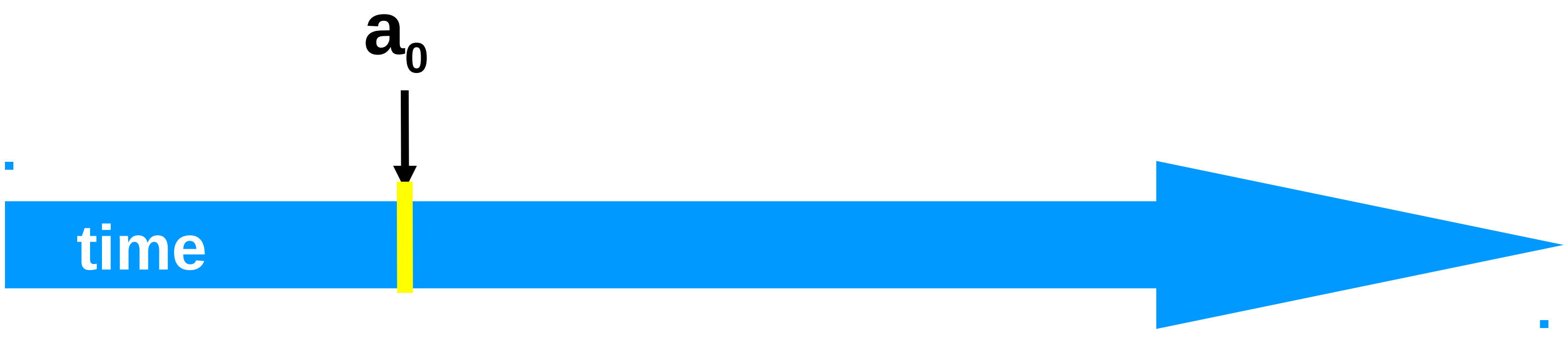
Why is Concurrent Programming so Hard?

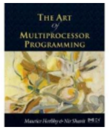
- Try preparing a seven-course banquet
  - By yourself
  - With one friend
  - With twenty-seven friends …
- Before we can talk about programs
  - Need a language
  - Describing time and concurrency



Events

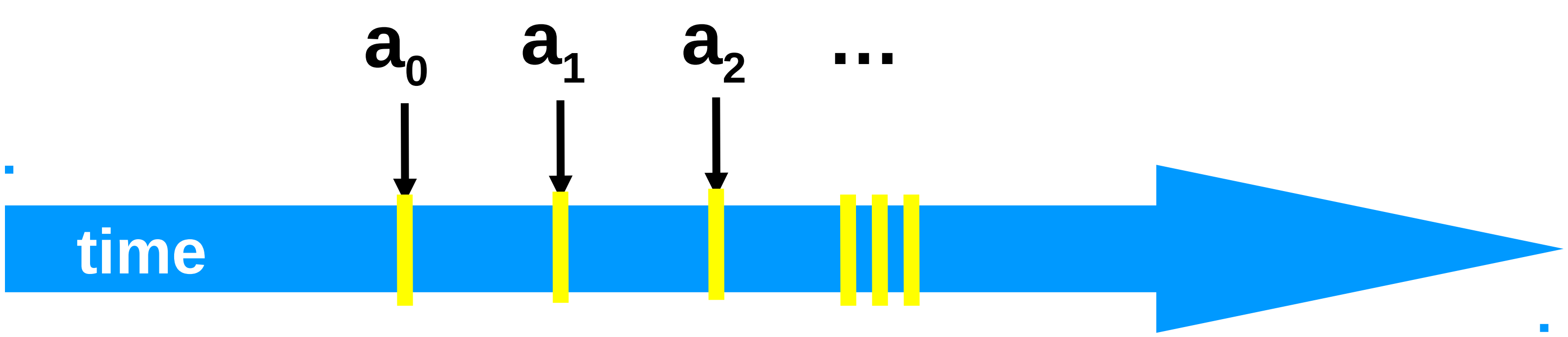
- An *event* a_0 of thread A is
  - Instantaneous
  - No simultaneous events (break ties)





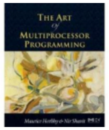
Threads

- A ***thread*** **A** is (formally) a sequence $a_0, a_1, \ldots$ of events
  - "Trace" model
  - Notation: $a_0 \rightarrow a_1$ indicates order

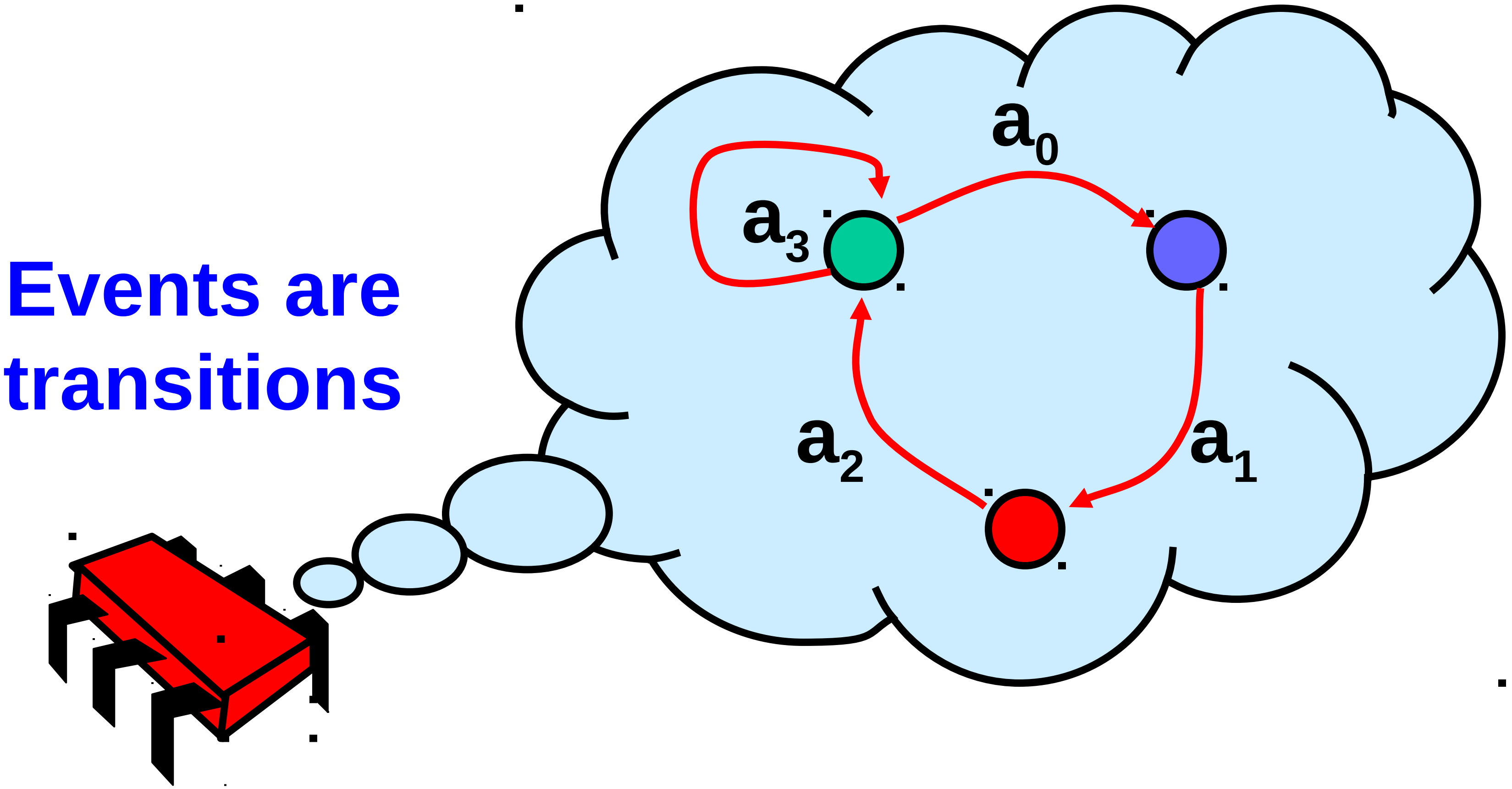


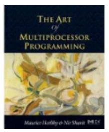
Example Thread Events

- Assign to shared variable
- Assign to local variable
- Invoke method
- Return from method
- Lots of other things …



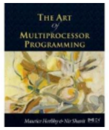
Threads are State Machines





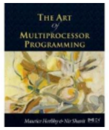
States

- Thread State
  - Program counter
  - Local variables
- System state
  - Object fields (shared variables)
  - Union of thread states



Concurrency

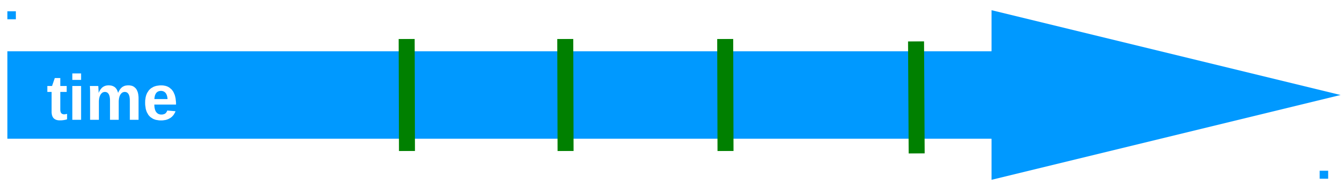
- **Thread A**



Concurrency

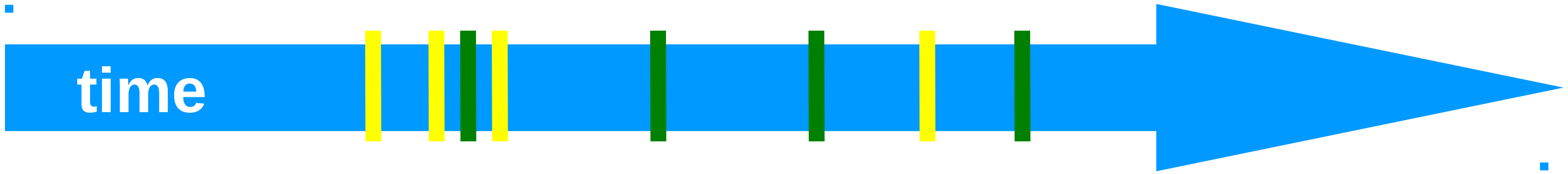
- **Thread A**

- **Thread B**



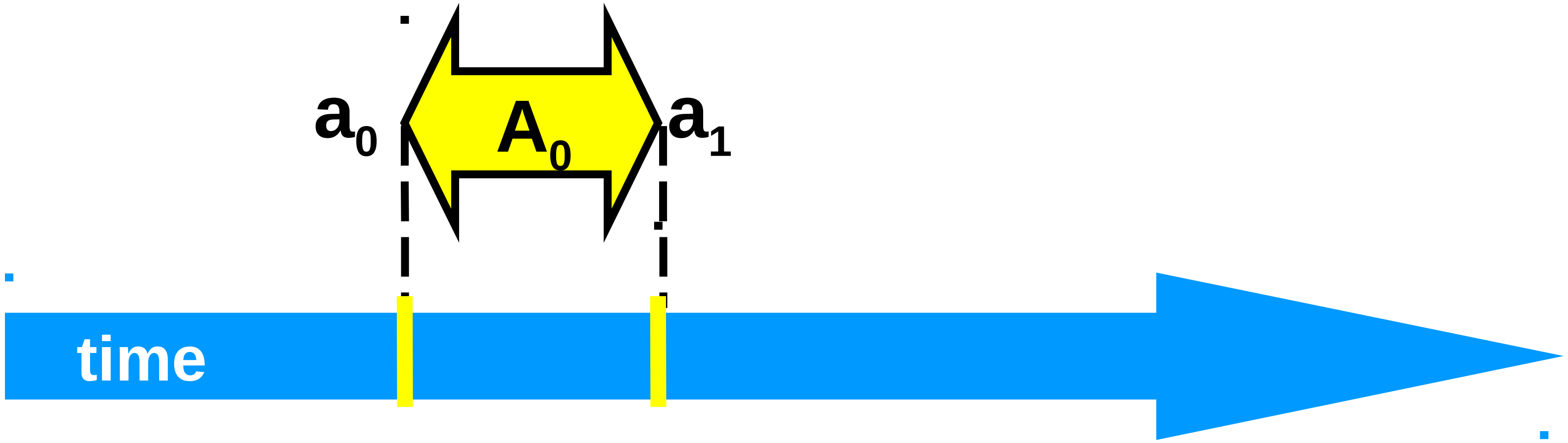
Interleavings

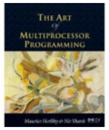
- Events of two or more threads
  - Interleaved
  - Not necessarily independent (why?)



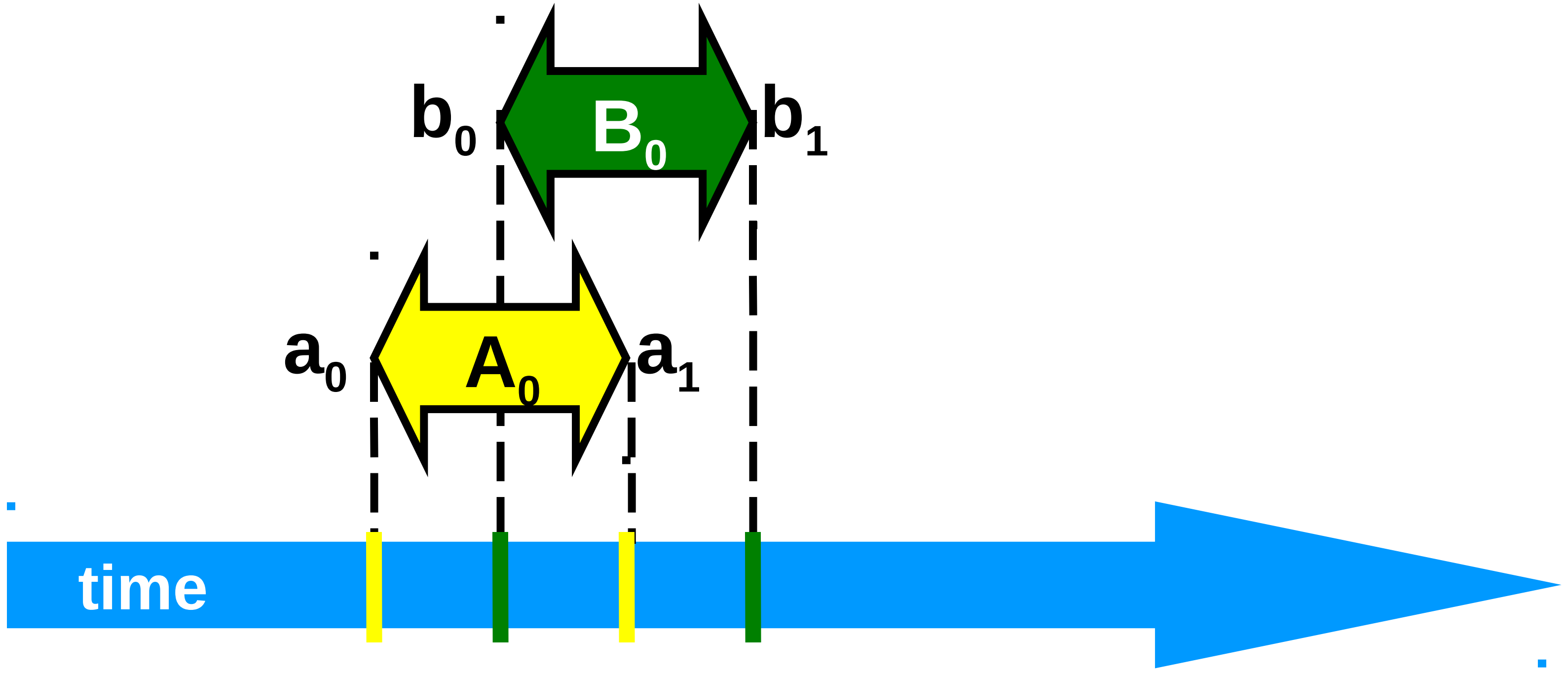
Intervals

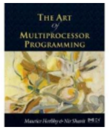
- An ***interval*** $A_0 = (a_0, a_1)$ is
  - Time between events a_0 and a_1



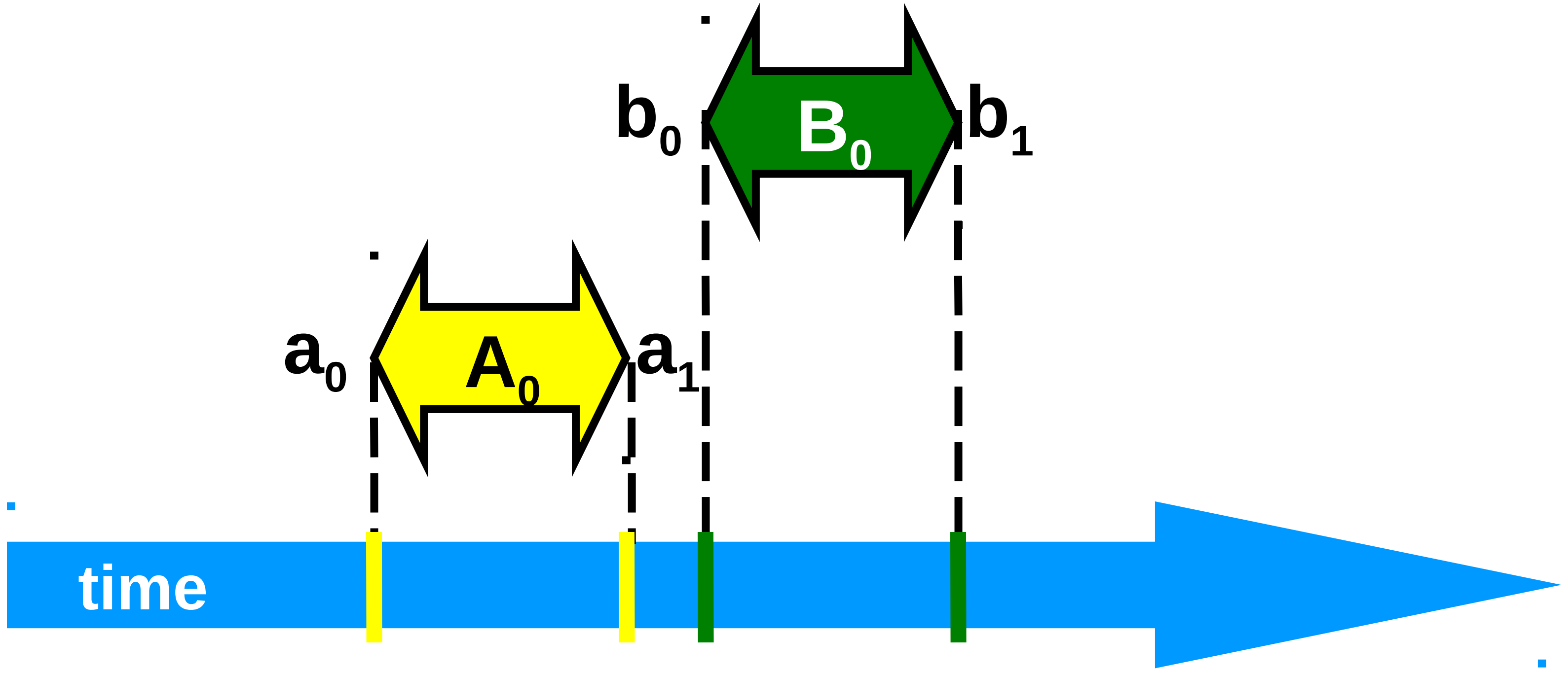


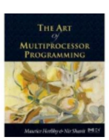
Intervals may Overlap





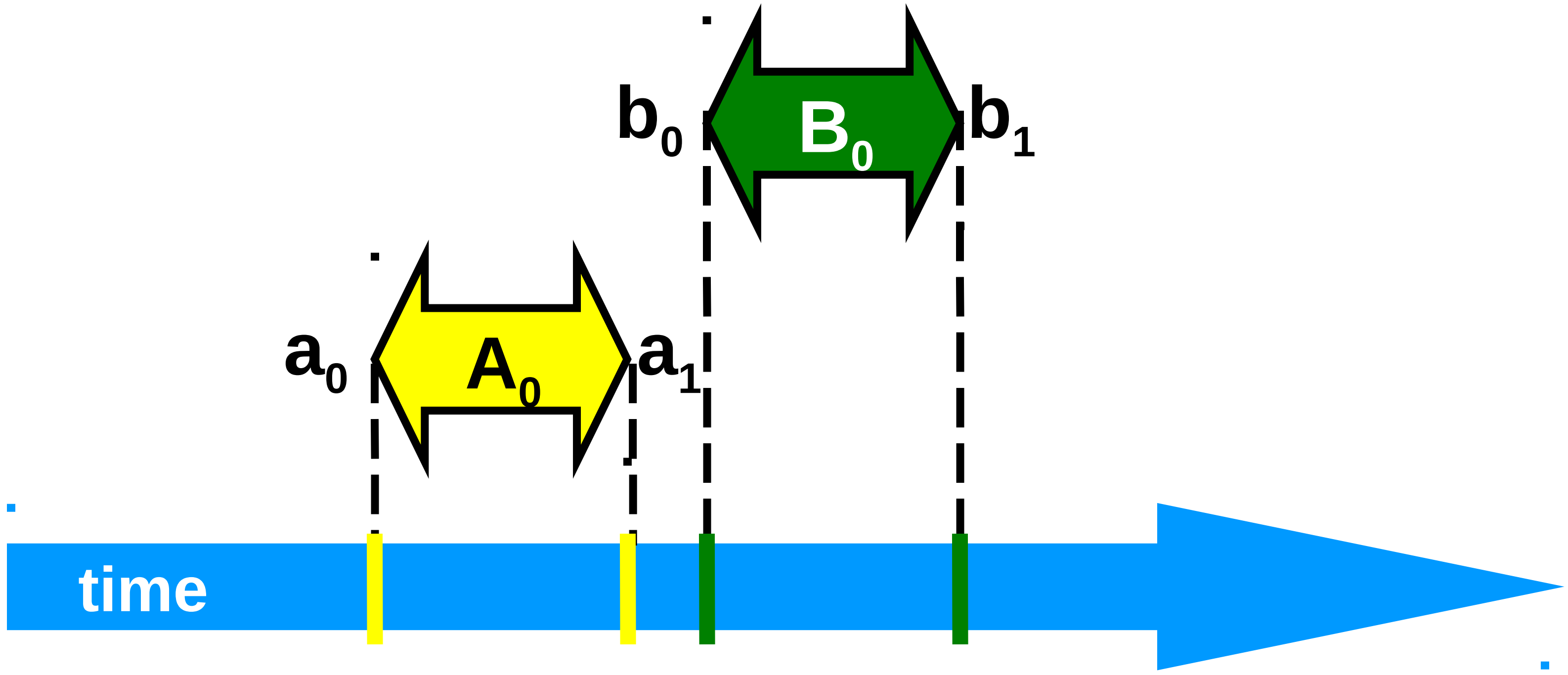
Intervals may be Disjoint



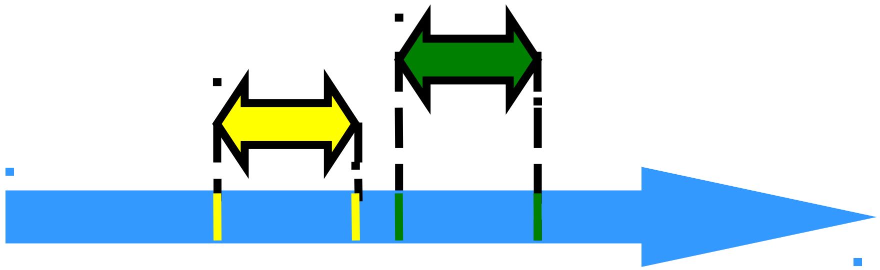


Precedence

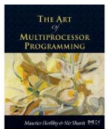
Interval A_0 precedes interval B_0



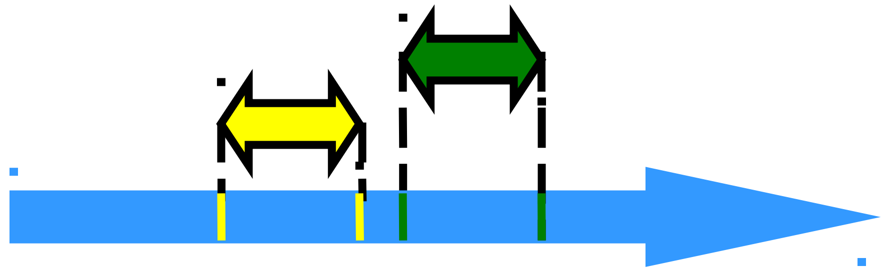
Precedence



- **Notation:** $A_0 \rightarrow B_0$
- **Formally,**
  - End event of A_0 before start event of B_0
  - Also called "happens before" or "precedes"

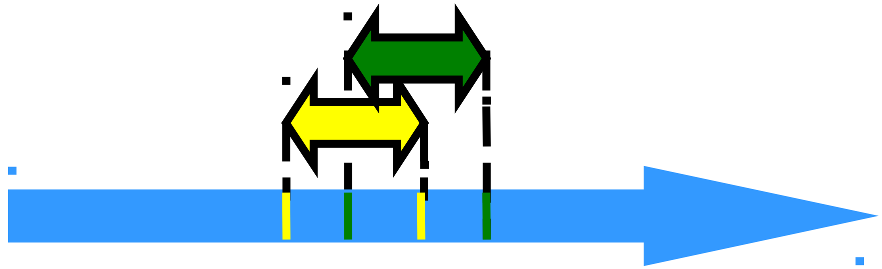


Precedence Ordering

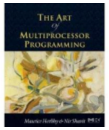


- Remark: $A_0 \rightarrow B_0$ is just like saying
  - 1066 AD $\rightarrow$ 1492 AD,
  - Middle Ages $\rightarrow$ Renaissance,
- Oh wait,
  - what about this week vs this month?

Precedence Ordering



- Never true that A → A
- If A → B then not true that B → A
- If A → B & B → C then A → C
- Funny thing: A → B & B → A might both be false!



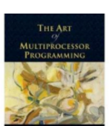
Partial Orders
(review)

- **Irreflexive:**
  - Never true that A $\rightarrow$ A
- **Antisymmetric:**
  - If A $\rightarrow$ B then not true that B $\rightarrow$ A
- **Transitive:**
  - If A $\rightarrow$ B & B $\rightarrow$ C then A $\rightarrow$ C

Total Orders
(review)

- Also
  - Irreflexive
  - Antisymmetric
  - Transitive
- Except that for every distinct A, B,

Either A → B or B → A (Trichotomy)



Repeated Events

```
while (mumble) {
  a0; a1;
}
```

a_0^k

k-th occurrence of event a_0

A_0^k

k-th occurrence of interval $A_0 = (a_0, a_1)$

Implementing a Counter

```
public class Counter {
  private long value;

  public long getAndIncrement() {
    temp  = value;
    value = temp + 1;
    return temp;
  }
}
```

Make these steps *indivisible* using locks

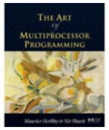
Locks (Mutual Exclusion)

```
public interface Lock {

 public void lock();

 public void unlock();
}
```

Note: `java.util.concurrent.locks` **has more methods**



Locks (Mutual Exclusion)

```
public interface Lock {

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```
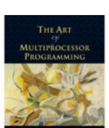

acquire lock

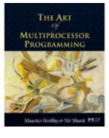


Locks (Mutual Exclusion)

```
public interface Lock {

 public void lock();

 public void unlock();
}
```

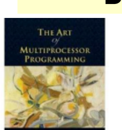
acquire lock

release lock



Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
     lock.unlock();
   }
   return temp;
  }}
```



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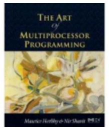
```

acquire Lock



Using Locks

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public class Counter {
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   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
     lock.unlock();
   }
   return temp;
  }}
```

Release lock (no matter what)

Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
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   }
   return temp;
  }}
```

critical section

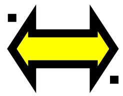
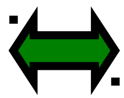
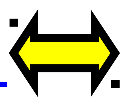
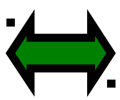
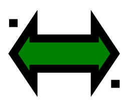
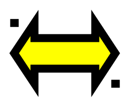
Desired 1: Mutual Exclusion

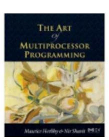
- Let $CS_i^k \Leftrightarrow$ be thread i's k-th critical section execution

Desired 1: Mutual Exclusion

- Let $CS_i^k \Leftrightarrow$ be thread i's k-th critical section execution
- And $CS_j^m \Leftrightarrow$ be thread j's m-th critical section execution

Desired 1: Mutual Exclusion

- Let CS_i^k  be thread i's k-th critical section execution
- And CS_j^m  be j's m-th execution
- Then either
  -   or  



Desired 1: Mutual Exclusion

- Let CS_i^k be thread i's k-th critical section execution
- And CS_j^m be j's m-th execution
- Then either
  - or

$CS_i^k \rightarrow CS_j^m$

Desired 1: Mutual Exclusion

- Let CS_i^k be thread i's k-th critical section execution
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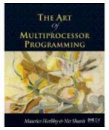
  - or

$CS_i^k \rightarrow CS_j^m$

$CS_j^m \rightarrow CS_i^k$

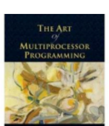


Desired 2: Deadlock-Free

- If ANY thread calls lock(), then SOME thread will acquire it
- If some thread calls lock()
  - And never returns
  - Then other threads must be completing lock() and unlock() calls infinitely often
- System as a whole makes progress
  - Even if individuals starve

Desired 3: Starvation-Free

- If some thread calls lock()
  - It will eventually return
- Individual threads make progress

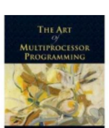


Two-Thread vs *n*-Thread Solutions

- 2-thread solutions first
  - Illustrate most basic ideas
  - Fits on one slide
- Then *n*-thread solutions

Two-Thread Conventions

```
class … implements Lock {
  …
  // thread-local index, 0 or 1
  public void lock() {
    int i = ThreadID.get();
    int j = 1 - i;
  …
  }
}
```



Two-Thread Conventions

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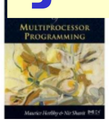
class … implements Lock {
  …
  // thread-local index, 0 or 1
  public void lock() {
    int i = ThreadID.get();
    int j = 1 - i;
  …
  }
}
```

Henceforth: i is current thread, j is other thread

LockOne

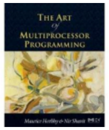
```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  int i = ThreadId.get();
  int j = 1 - i ;
  flag[i] = true;
  while (flag[j]) {}
 }
public void unlock(){
  int i = ThreadId.get();
  flag[i] = false;
  }
}
```



LockOne

```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
 }
```

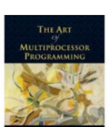
Each thread has flag (initialised to false)



LockOne

```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
 }
```

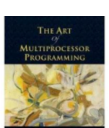
Set my flag



LockOne

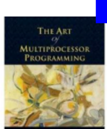
```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
  }
```

Wait for other flag to become false



LockOne Satisfies Mutual Exclusion

- Assume CS_A^j overlaps CS_B^k
- Consider each thread's last (**j**-th and **k**-th) read and write in the **lock()** method before entering
  - Note: only considering 4 events
- Derive a contradiction



From the Code

- $\text{write}_A(\text{flag[A]=true}) \rightarrow \text{read}_A(\text{flag[B]==false}) \rightarrow \text{CS}_A$

- $\text{write}_B(\text{flag[B]=true}) \rightarrow \text{read}_B(\text{flag[A]==false}) \rightarrow \text{CS}_B$

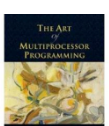
```
class LockOne implements Lock {
…
public void lock() {
  …
  flag[i] = true;
  while (flag[j]) {}
 }
```

From the Assumption

- **read$_A$(flag[B]==false) $\rightarrow$ write$_B$(flag[B]=true)**

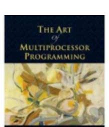
- **read$_B$(flag[A]==false) $\rightarrow$ write$_A$(flag[A]=true)**

```
class LockOne implements Lock {
…
public void lock() {
   …
   flag[i] = true;
   while (flag[j]) {}
  }
```



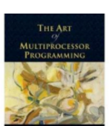
Combining

- **Assumptions:**
  - read$_A$(flag[B]==false) $\rightarrow$ write$_B$(flag[B]=true)
  - read$_B$(flag[A]==false) $\rightarrow$ write$_A$(flag[A]=true)
- **From the code**
  - write$_A$(flag[A]=true) $\rightarrow$ read$_A$(flag[B]==false)
  - write$_B$(flag[B]=true) $\rightarrow$ read$_B$(flag[A]==false)



Combining

- Assumptions:
  - read$_A$(flag[B]==false) → write$_B$(flag[B]=true)
  - read$_B$(flag[A]==false) → write$_A$(flag[A]=true)
- From the code
  - write$_A$(flag[A]=true) → read$_A$(flag[B]==false)
  - write$_B$(flag[B]=true) → read$_B$(flag[A]==false)



Combining

- Assumptions:
  - read$_A$(flag[B]==false) $\rightarrow$ write$_B$(flag[B]=true)
  - read$_B$(flag[A]==false) $\rightarrow$ write$_A$(flag[A]=true)
- From the code
  - write$_A$(flag[A]=true) $\rightarrow$ read$_A$(flag[B]==false)
  - write$_B$(flag[B]=true) $\rightarrow$ read$_B$(flag[A]==false)

Combining

- Assumptions:
  - $\text{read}_A(\text{flag[B]==false}) \rightarrow \text{write}_B(\text{flag[B]=true})$
  - $\text{read}_B(\text{flag[A]==false}) \rightarrow \text{write}_A(\text{flag[A]=true})$
- From the code
  - $\text{write}_A(\text{flag[A]=true}) \rightarrow \text{read}_A(\text{flag[B]==false})$
  - $\text{write}_B(\text{flag[B]=true}) \rightarrow \text{read}_B(\text{flag[A]==false})$

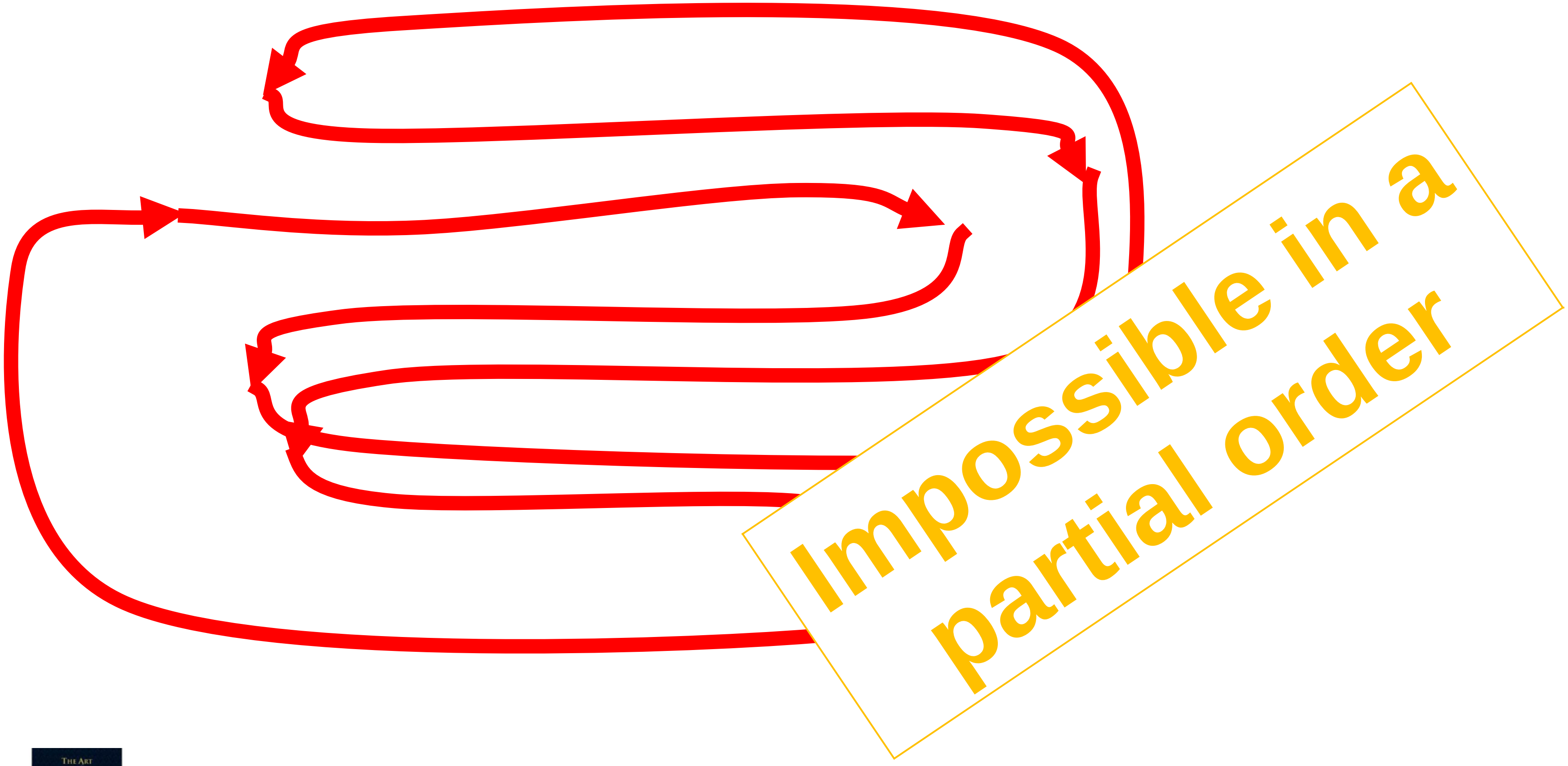
Combining

- Assumptions:
  - read$_A$(flag[B]==false) → write$_B$(flag[B]=true)
  - read$_B$(flag[A]==false) → write$_A$(flag[A]=true)
- From the code
  - write$_A$(flag[A]=true) → read$_A$(flag[B]==false)
  - write$_B$(flag[B]=true) → read$_B$(flag[A]==false)

Combining

- Assumptions:
  - $\text{read}_A(\text{flag}[B]==\text{false}) \rightarrow \text{write}_B(\text{flag}[B]=\text{true})$
  - $\text{read}_B(\text{flag}[A]==\text{false}) \rightarrow \text{write}_A(\text{flag}[A]=\text{true})$
- From the code
  - $\text{write}_A(\text{flag}[A]=\text{true}) \rightarrow \text{read}_A(\text{flag}[B]==\text{false})$
  - $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow \text{read}_B(\text{flag}[A]==\text{false})$

Cycle!

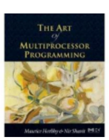


Deadlock Freedom

- **LockOne Fails deadlock-freedom**
  - **Concurrent execution can deadlock**

```
flag[i] = true;     flag[j] = true;
while (flag[j]){}   while (flag[i]){}
```

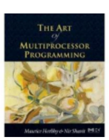
  - **Sequential executions OK**



LockTwo

```
public class LockTwo implements Lock {
 private int victim;
 public void lock() {
  victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```

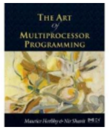


LockTwo

```
public class LockTwo implements Lock {
 private int victim;
 public void lock() {
  victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```

Let other go first

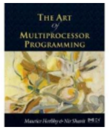


LockTwo

```
public class LockTwo implements Lock {
 private int victim;
 public void lock() {
 victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```

Wait for permission

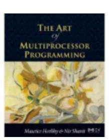


LockTwo

```
public class Lock2 implements Lock {
 private int victim;
 public void lock() {
  victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```

Nothing to do



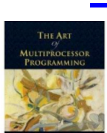
LockTwo Claims

- Satisfies mutual exclusion
  - If thread `i` in CS
  - Then `victim == j`
  - Cannot be both 0 and 1

```
public void LockTwo() {
  victim = i;
  while (victim == i) {};
 }
```

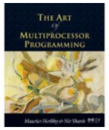
- Not deadlock free
  - Sequential execution deadlocks
    - if one thread never tries to get the lock
  - Concurrent execution does not

Exercise: Prove mutual exclusion



Peterson’s Algorithm

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
}
```



Peterson’s Algorithm

Announce I’m interested

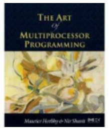
```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
}
```

Peterson's Algorithm

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
}
```

Announce I'm interested

Defer to other



Peterson's Algorithm

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim

Peterson's Algorithm

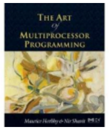
```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
 }
public void unlock() {
 flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim

No longer interested

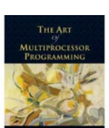


Mutual Exclusion

(1) $\text{write}_B(\text{Flag[B]=true}) \rightarrow \text{write}_B(\text{victim=B})$

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
```

From the Code



Also from the Code

(2) $\text{write}_A(\text{victim=A}) \rightarrow \text{read}_A(\text{flag[B]})$
$\rightarrow \text{read}_A(\text{victim})$

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
```

Assumption

(3) $\text{write}_B(\text{victim}=B) \rightarrow \text{write}_A(\text{victim}=A)$

W.L.O.G. assume A is the last thread to write victim

Combining Observations

(1) write$_B$(flag[B]=true)$\rightarrow$ write$_B$(victim=B)

(3) write$_B$(victim=B)$\rightarrow$ write$_A$(victim=A)

(2) write$_A$(victim=A)$\rightarrow$ read$_A$(flag[B])

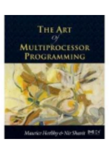
$\rightarrow$ read$_A$(victim)

Combining Observations

(1) $\text{write}_B(\text{flag[B]=true}) \rightarrow$

(3) $\text{write}_B(\text{victim=B}) \rightarrow$

(2) $\text{write}_A(\text{victim=A}) \rightarrow \text{read}_A(\text{flag[B]})$
$\rightarrow \text{read}_A(\text{victim})$



Combining Observations

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i){};
}
```

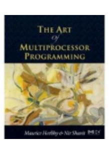
(1) $\text{write}_B(\text{flag[B]=true}) \rightarrow$

(3) $\text{write}_B(\text{victim=B}) \rightarrow$

(2) $\text{write}_A(\text{victim=A}) \rightarrow \text{read}_A(\text{flag[B]})$

$\rightarrow \text{read}_A(\text{victim})$

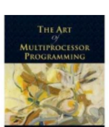
A read flag[B] == true and victim == A, so it could not have entered the CS (QED)



Deadlock Free

```
public void lock() {
  …
  while (flag[j] && victim == i) {};
```

- Thread blocked
  - only at `while` loop
  - only if other’s flag is true
  - only if it is the victim
- Solo: other’s flag is false
- Both: one or the other not the victim



Starvation Free

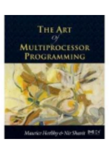
- Thread `i` blocked only if `j` repeatedly re-enters so that

  `flag[j] == true` and `victim == i`
- When `j` re-enters
  - it sets `victim` to `j`.
  - So `i` gets in

```
public void lock() {
  flag[i] = true;
  victim   = i;
  while (flag[j] && victim == i) {};
}

public void unlock() {
  flag[i] = false;
}
```

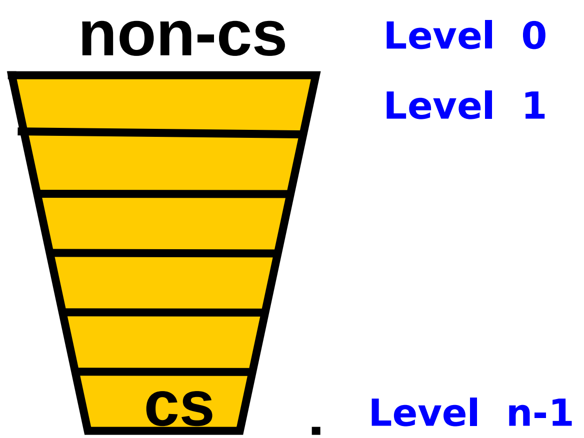
Problem? Only two threads

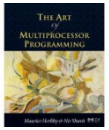


The Filter Algorithm for *n* Threads

There are n-1 "waiting rooms" called levels

- At each level
  - At least one enters level
  - At least one blocked at that level if many try
- Only one thread makes it through
- A thread at level j is also at level j-1,…,0

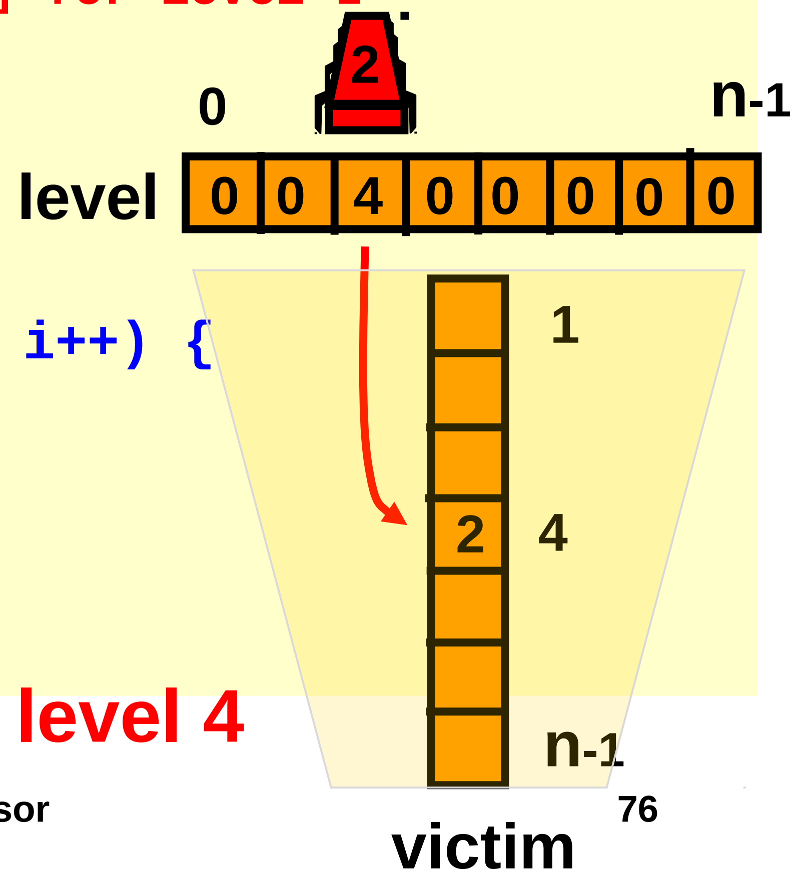




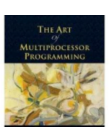
Filter

```
class Filter implements Lock {
   int[] level;  // level[i] for thread i
   int[] victim; // victim[L] for level L

  public Filter(int n) {
      level  = new int[n];
      victim = new int[n];
      for (int i = 1; i < n; i++) {
          level[i] = 0;
      }}
   …
}
```



Thread 2 at level 4



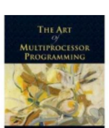
Filter

```
class Filter implements Lock {
  …

  public void lock(){
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;
      while ((∃ k != i level[k] >= L) &&
                 victim[L] == i ) {};
    }}
  public void unlock() {
    level[i] = 0;
  }}
```

thread i trying to enter level L

thread i succeeded in entering level L

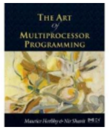


Filter

```
class Filter implements Lock {
  …

  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
                victim[L] == i) {};
    }}
  public void release(int i) {
    level[i] = 0;
  }}
```

One level at a time



Filter

```
class Filter implements Lock {
  …

  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
                victim[L] == i)
    }}
  public void release(int i) {
    level[i] = 0;
  }}
```

Announce intention to enter level L

Filter

```
class Filter implements Lock {
  int level[n];
  int victim[n];
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;

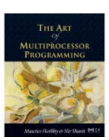
      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i) {};
    }}
  public void release(int i)
    level[i] = 0;
  }}
```

Give priority to anyone but me

Filter

Wait as long as someone else is at same or higher level, and I'm designated victim

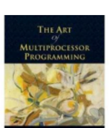
```
public void lock() {
  for (int L = 1; L < n; L++) {
    level[i]  = L;
    victim[L] = i;
    while ((∃ k != i) level[k] >= L) &&
           victim[L] == i) {};
  }}
public void release(int i) {
  level[i] = 0;
}}
```



Filter

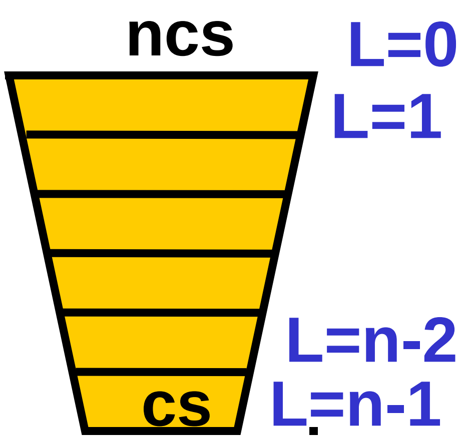
```
class Filter implements Lock {
  int level[n];
  int victim[n];
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i) {};
    }}
```

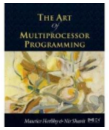
Thread *enters* level L when it completes the loop



Claim

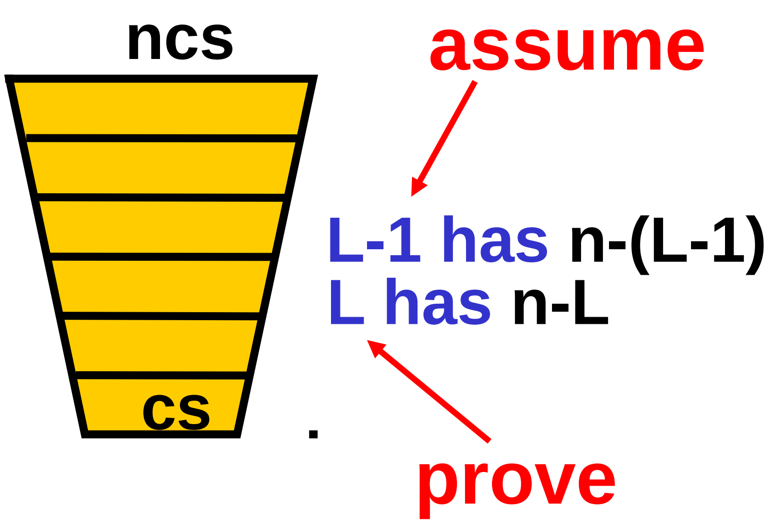
- Start at level L=0
- At most n-L threads enter level L
- Mutual exclusion at level L=n-1



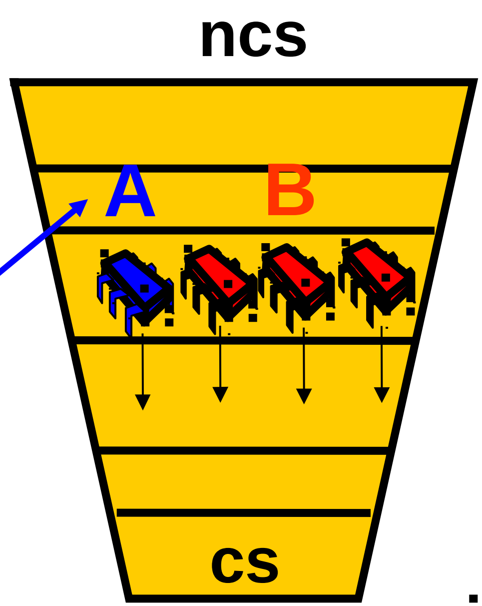


Induction Hypothesis

- No more than n-(L-1) at level L-1
- Induction step: by contradiction
  - Assume all at level L-1 enter level L
  - A last to write victim[L]
  - B is any other thread at level L



Proof Structure



Assumed to enter L-1

n-L+1 = 4

n-L+1 = 4

Last to write victim[L]

By way of contradiction all enter L

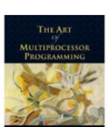
Show that A must have seen B in level[L] and since victim[L] == A could not have entered

Just Like Peterson

(1) $\text{write}_B(\text{level}[B]=L) \rightarrow \text{write}_B(\text{victim}[L]=B)$

```
public void lock() {
 for (int L = 1; L < n; L++) {
   level[i] = L;
   victim[L]  = i;
   while ((∃ k != i) level[k] >= L)
          && victim[L] == i) {};
   }}
```

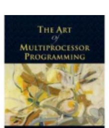
From the Code



From the Code

(2) $\text{write}_A(\text{victim}[L]=A) \Rightarrow \text{read}_A(\text{level}[B]) \Rightarrow \text{read}_A(\text{victim}[L])$

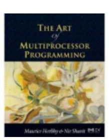
```
public void lock() {
 for (int L = 1; L < n; L++) {
   level[i] = L;
   victim[L]  = i;
   while ((∃ k != i) level[k] >= L)
          && victim[L] == i) {};
   }}
```



By Assumption

(3) $\text{write}_B(\text{victim[L]=B}) \rightarrow \text{write}_A(\text{victim[L]=A})$

By assumption, A is the last thread to write victim[L]



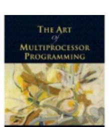
Combining Observations

(1) $\text{write}_B(\text{level}[B]=L) \rightarrow \text{write}_B(\text{victim}[L]=B)$

(3) $\text{write}_B(\text{victim}[L]=B) \rightarrow \text{write}_A(\text{victim}[L]=A)$

(2) $\text{write}_A(\text{victim}[L]=A) \rightarrow \text{read}_A(\text{level}[B])$

$\rightarrow \text{read}_A(\text{victim}[L])$



Combining Observations

(1) $\text{write}_B(\text{level}[B]=L) \rightarrow$

(3) $\text{write}_B(\text{victim}[L]=B) \rightarrow \text{write}_A(\text{victim}[L]=A)$

(2) $) \rightarrow \text{read}_A(\text{level}[B])$

$\rightarrow \text{read}_A(\text{victim}[L])$

Combining Observations

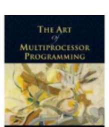
(1) write$_B$(level[B]=L)➔

(3) write$_B$(victim[L]=B)➔write$_A$(victim[L]=A)

(2) … A)➔read$_A$(level[B])

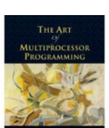
➔read$_A$(victim[L])

A read level[B] ≥ L, and victim[L] = A, so it could not have entered level L!



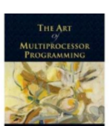
No Starvation

- **Filter Lock satisfies properties:**
  - **Just like Peterson Alg at any level**
  - **So no one starves**
- **But what about fairness?**
  - **Threads can be overtaken by others**



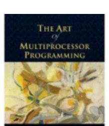
Bounded Waiting

- Want stronger fairness guarantees
- Thread not “overtaken” too much
- If **A** starts before **B**, then **A** enters before **B**?
- But what does “start” mean?
- Need to adjust definitions ….



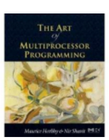
Bounded Waiting

- Divide `lock()` method into 2 parts:
  - Doorway interval:
    - Written D_A
    - always finishes in finite steps
  - Waiting interval:
    - Written W_A
    - may take unbounded steps



First-Come-First-Served

- **For threads A and B:**
  - **If $D_A^k \rightarrow D_B^j$**
    - **A's k-th doorway precedes B's j-th doorway**
  - **Then $CS_A^k \rightarrow CS_B^j$**
    - **A's k-th critical section precedes B's j-th critical section**
    - **B cannot overtake A**

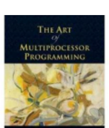


Fairness Again

- Filter Lock satisfies properties:
  - No one starves
  - But very weak fairness
    - Can be overtaken arbitrary # of times
  - That's pretty lame…

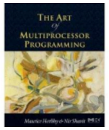
Bakery Algorithm

- **Provides First-Come-First-Served**
- **How?**
  - **Take a "number"**
  - **Wait until lower numbers have been served**
- **Lexicographic order**
  - **(a,i) > (b,j)**
    - **If a > b, or a = b and i > j**



Bakery Algorithm

```
class Bakery implements Lock {
   boolean[] flag;
   Label[] label;
  public Bakery (int n) {
    flag  = new boolean[n];
    label = new Label[n];
    for (int i = 0; i < n; i++) {
       flag[i] = false; label[i] = 0;
    }
  }
 …
```



Bakery Algorithm

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  public Bakery (int n) {
    flag  = new boolean[n];
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    for (int i = 0; i < n; i++) {
       flag[i] = false; label[i] = 0;
    }
  }
 …
```
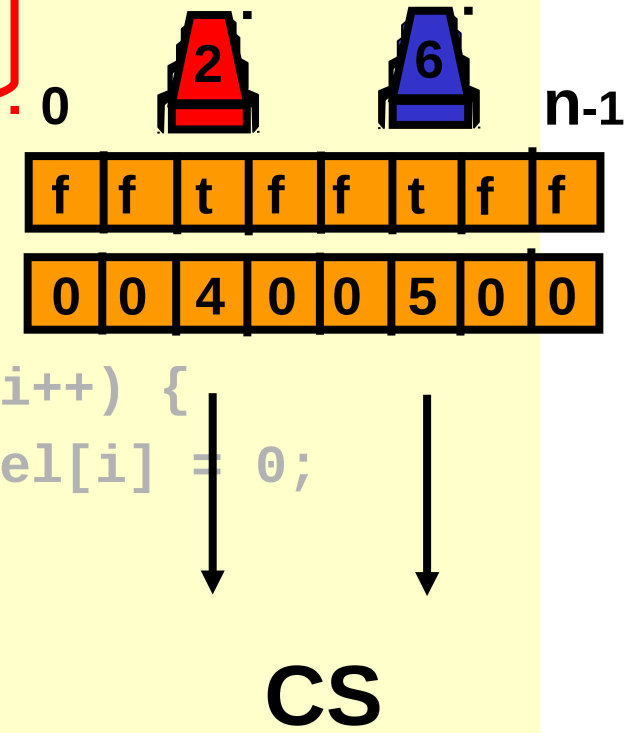
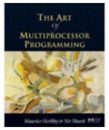
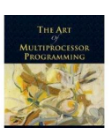





Bakery Algorithm

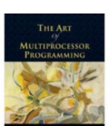
```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```



Bakery Algorithm

```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```

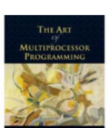
Doorway



Bakery Algorithm

```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```

I’m interested



Bakery Algorithm

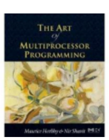
Take increasing label (read labels in some arbitrary order)

```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```

Bakery Algorithm

```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```

Someone is interested

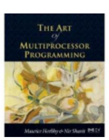


Bakery Algorithm

```
class Bakery implements Lock {
  boolean flag[n];
  int label[n];

 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```

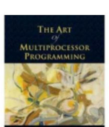
Someone is interested …

… whose (label,i) in lexicographic order is lower



Bakery Algorithm

```
class Bakery implements Lock {

    …

 public void unlock() {
   flag[i] = false;
 }
}
```

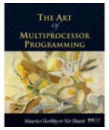


Bakery Algorithm

```
class Bakery implements Lock {

    …

 public void unlock() {
   flag[i] = false;
 }
}
```

No longer interested

labels are always increasing



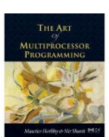
No Deadlock

- **There is always one thread with earliest label**
- **Ties are impossible (why?)**

First-Come-First-Served

- If $D_A \rightarrow D_B$ then
  - A's label is smaller
- And:
  - $\text{write}_A(\text{label[A]}) \rightarrow$
  - $\text{read}_B(\text{label[A]}) \rightarrow$
  - $\text{write}_B(\text{label[B]}) \rightarrow \text{read}_B(\text{flag[A]})$
- So B sees
  - smaller label for A
  - locked out while flag[A] is true

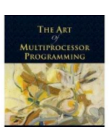
```
class Bakery implements Lock {

public void lock() {
  flag[i]  = true;
  label[i] = max(label[0],
                 …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) >
  (label[k],k));
 }
```



Mutual Exclusion

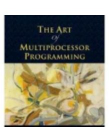
- Suppose **A** and **B** in CS together
- Suppose **A** has earlier label
- When **B** entered, it must have seen
  - **flag[A]** is ***false***, or
  - **label[A]** > **label[B]**

```
class Bakery implements Lock {

public void lock() {
  flag[i]  = true;
  label[i] = max(label[0],
               …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) >
  (label[k],k));
 }
```



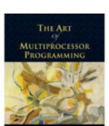
Mutual Exclusion

- Labels are strictly increasing so
- B must have seen flag[A] == false



Mutual Exclusion

- Labels are strictly increasing so
- B must have seen flag[A] == false
- Labeling$_B$ $\rightarrow$ read$_B$(flag[A]) $\rightarrow$ write$_A$(flag[A]) $\rightarrow$ Labeling$_A$



Mutual Exclusion

- Labels are strictly increasing so
- B must have seen flag[A] == false
- $\text{Labeling}_B \rightarrow \text{read}_B(\text{flag[A]}) \rightarrow \text{write}_A(\text{flag[A]}) \rightarrow \text{Labeling}_A$
- Which contradicts the assumption that A has an earlier label