Introduction to quantum computing

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Quantum Hackathon, 13th November 2019

[1]With thanks to Ashley Montanaro, whose slides parts of this talk are based on.

Outline

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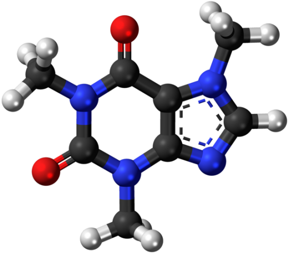
What is quantum physics?

The system of physical laws that govern very small things.



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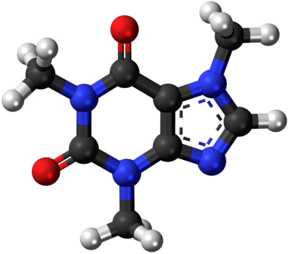

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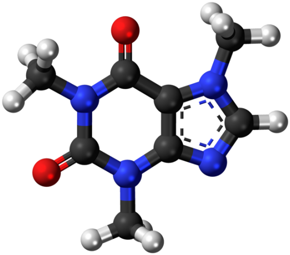

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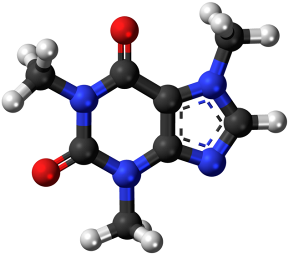

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- Early applications include lasers, LEDs and transistors.
- There are many other quantum phenomena whose technological exploitation is only beginning.

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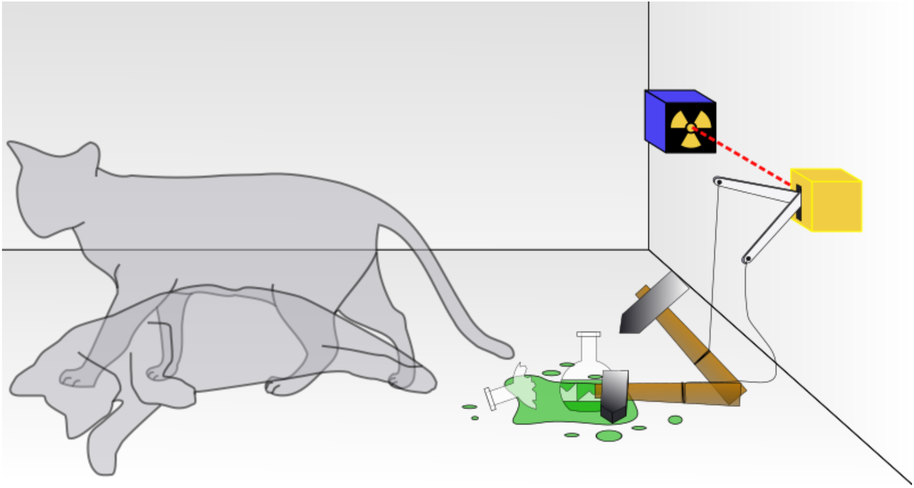
4. **Entanglement**: There exist states of multipartite systems which cannot be described in terms of states of the constituent systems.

Superposition and measurement: Schrödinger's cat



Pic: Wikipedia/Schrodingers_cat

Uncertainty (e.g. of position and momentum)

Pic: anengineersaspect.blogspot.co.uk

'Do you know how fast you were going?'

'No, but I know where I am.'

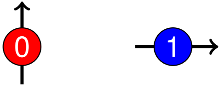
'You were doing 90 miles an hour.'

'Great, now I'm lost.'

The qubit: the basic building-block of a quantum computer

A quantum system with two distinct states is a **qubit.**

For example, a photon – a particle of light – has a property called polarisation which can be vertical or horizontal (↑ or →):



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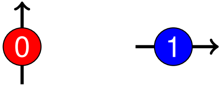
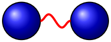


Just as a classical computer operates on bits, a quantum computer operates on qubits.

Entanglement

Imagine we have a pair of entangled qubits:



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Pic: commons.wikimedia.org/wiki/File:Howling_at_the_Moon_in_Mississauga.jpg

- Even if we move one of the qubits to the Moon, the global state of the two qubits cannot be described solely in terms of the individual state of each of them!
- In particular, if we measure one of the qubits, this apparently instantaneously affects the other one.

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Quantum simulation

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Simulating quantum physics has applications to drug design, materials science, high-energy physics, ...

Pic: WP/Seth Lloyd

Shor's algorithm for factoring

1994: Peter Shor shows that quantum computers can factorise large integers efficiently.

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Shor's algorithm breaks the RSA public-key cryptosystem on which Internet security is based.

Grover's algorithm for unstructured search

Unstructured search is one of the most basic problems in computer science:

- Imagine we have n boxes, each containing a 0 or a 1. We can look inside a box at a cost of one query.

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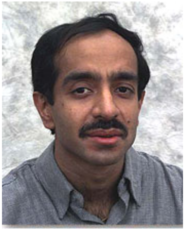

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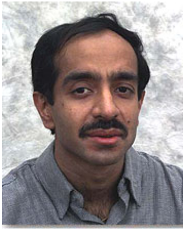

The square-root speedup of Grover's algorithm finds many applications to search and optimisation problems, including in quantum machine learning.

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Solving a system of linear equations: Given a $N \times N$ matrix A and a unit vector $\mathbf{b}$, find the vector $\mathbf{x}$ satisfying $A\mathbf{x} = \mathbf{b}$.

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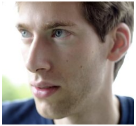

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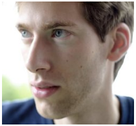

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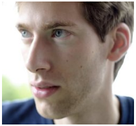

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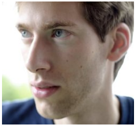

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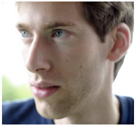
- Applications in science, engineering, machine learning and big data.



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The server learns nothing about the data or the type of computation.

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- probabilistic
- irreversible
- lose ‘quantumness’

Unitary operations

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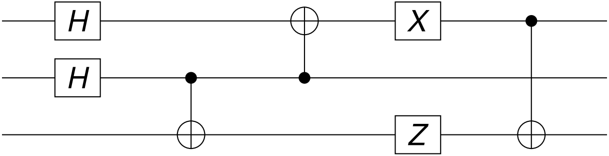

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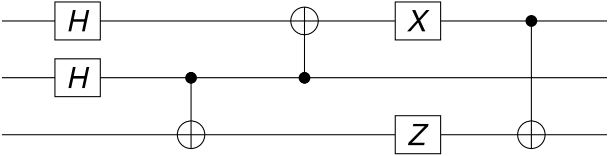

Each horizontal wire represents a qubit, each gate represents an operation on one or more qubits.

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Complex numbers matter: $\frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ give the same probabilities but they are different states.

States of multiple qubits

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For example, the three-qubit state $\left(0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, 0, 0, 0\right)$ has equal probabilities of giving the bit strings 001, 010, or 100 when all qubits are measured.

Reversible logic gates as unitary operations

The NOT gate $-\boxed{X}-$ corresponds to the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix} \qquad \textit{i.e.} \begin{cases} 0 & \mapsto & 1 \\ 1 & \mapsto & 0 \end{cases}$$

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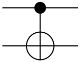
The controlled-NOT gate  corresponds to $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$:

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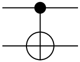
The controlled-NOT gate  corresponds to $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$:

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This is a reversible version of XOR, acting on bits as $(x, y) \mapsto (x, y \oplus x)$

Quantum gates with no classical counterpart

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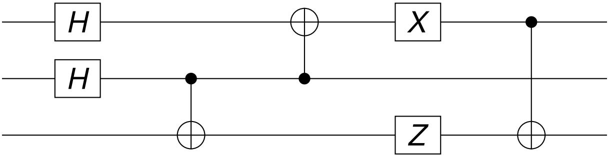
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} a+b \\ a-b \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ -b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} a-b \\ a+b \end{pmatrix}$$

Combining gates into circuits

Connect gates by (arbitrarily long) wires:



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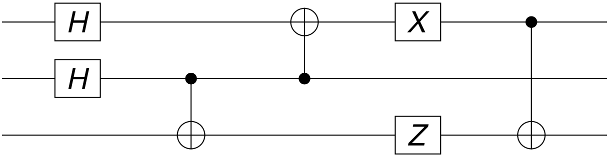

Besides the gates introduced on the previous slides, there are many other gates that are commonly used in quantum circuits in different combinations.

Translating circuits to matrices

Two gates on the same wire correspond to the matrix product:

$$-\boxed{Z}-\boxed{H}- \quad \text{is} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

- Careful about the reversed order!

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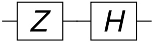

is $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$

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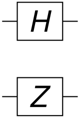
Two gates on parallel wires correspond to the Kronecker product (also called tensor product):



is $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & -1 & 0 & -1\\ 1 & 0 & -1 & 0\\ 0 & -1 & 0 & 1\end{pmatrix}$

- This is not commutative.

Universality

The basic gates CNOT, $-\boxed{H}-$, and $-\boxed{R_{Z,\theta}}-$, corresponding to the matrices

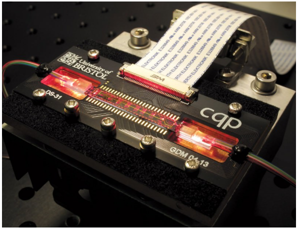
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix},$$

are enough to write down a circuit for any unitary operation on a quantum computer.

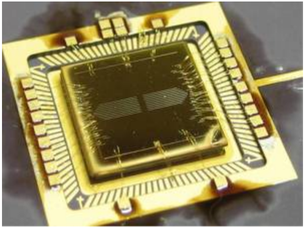
Here, θ is an arbitrary real number, making $e^{i\theta}$ a complex number of absolute value 1.

Outline

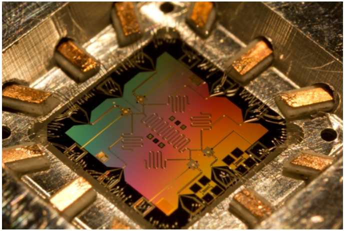
Some approaches to quantum computing



Photonics, Bristol



Ion trap, Oxford



Superconducting electronics, UCSB

Quantum error correction

Building a large-scale quantum computer is extremely challenging because of decoherence.

If a quantum computer interacts with the outside world and is subject to noise, it can lose its ‘quantumness’ and behave like a classical computer.

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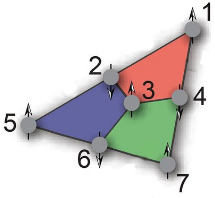

Pic:

DOI:10.1126/science.1253742

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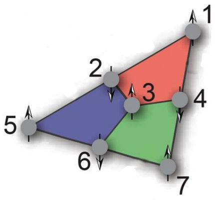

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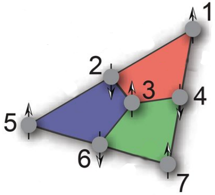
- Quantum error-correcting codes can be used to fight decoherence.
- Optimistic estimates say error rates of up to 1% should be ok.
- Error-correction will massively increase the number of physical qubits needed to implement a given computation (by a factor of 1,000 or more).



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Noisy Intermediate-Scale Quantum Computation

Often abbreviated to NISQ.

- Noisy: does not use error correction.
- Intermediate-scale: about 50-100 qubits.

Computations are kept short to avoid errors accumulating, but are expected to outperform standard computers on certain tasks.

Pic: WP/John Preskill

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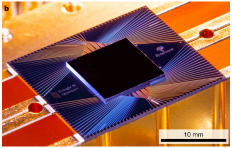

Pic: DOI:10.1038/s41586-019-1666-5

October 2019: Google announces they have performed a computation in 600 seconds on their chip of 53 superconducting ‘transmon’ qubits, which would take 10,000 years on standard computers, or 2.5 days on IBM’s Oak Ridge Summit Supercomputer.

Outline

Summary

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- Quantum computers use these effects to solve certain problems better than standard computers can.
- Quantum algorithms are written down as quantum circuits.
- Theory and implementation of quantum computers for the NISQ era and beyond are being actively developed.
- There are still many interesting open questions about the power and potential of quantum computing to be explored.

Further reading

- Quantum Computing Since Democritus
  Scott Aaronson
  http://www.scottaaronson.com/democritus/
- Introduction to Quantum Computing
  John Watrous
  https://cs.uwaterloo.ca/~watrous/LectureNotes.html
- Quantum Computer Science
  N. David Mermin, Cambridge University Press
- Quantum Computation and Quantum Information
  Michael Nielsen and Isaac Chuang, Cambridge University Press
- Why Google's Quantum Supremacy Milestone Matters
  Scott Aaronson
  https://www.nytimes.com/2019/10/30/opinion/google-quantum-computer-sycamore.html