
Algorithm 1 Non-dominated Sorting

Require: A population of individuals P

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for each individual  $i \in P$  do
    Set  $S_i := \emptyset$  and  $n_i := 0$ 
end for
for all pairs  $i, j \in P, i \neq j$  do
    if  $j$  dominates  $i$  then
         $S_j := S_j \cup \{i\}$ 
    else if  $i$  dominates  $j$  then
         $n_j := n_j + 1$ 
    end if
end for
for each  $i \in P$  do
    If  $n_i = 0$ , keep  $i$  in the first non-dominated front  $P_1$ 
end for
Set  $k = 1$ 
while  $P_k \neq \emptyset$  do
    for each  $i \in P_k$  and  $j \in S_i$  do
        Set  $n_j := n_j - 1$ 
        if  $n_j = 0$  then
            Update  $Q := Q \cup \{j\}$ 
        end if
    end for
    end for
    Set  $k = k + 1$  and  $P_k = Q$  and update  $Q := \emptyset$ .
end while
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Algorithm 2 Crowding Distance¹

Require: A set $\mathcal{F} = \{(f_1^i, \dots, f_n^i)\}_{i \in [\ell]}$ of ℓ objective vectors

for each $i \in [\ell]$ **do**

 Set $d_i := 0$

 ▷ initialise distances to 0

end for

for each objective $m \in [M]$ **do**

 Sort the set \mathcal{F} according to objective f_m s.t.

$$f_m^{(I_1^m)} \leq f_m^{(I_2^m)} \leq \dots \leq f_m^{(I_\ell^m)}$$

 Set $d_{I_1^m} := \infty$ and $d_{I_\ell^m} := \infty$

 ▷ so that boundary points are selected

for $j \in \{2, \dots, \ell - 1\}$ **do**

 ▷ for all other points

$$d_{I_j} := d_{I_j} + \frac{f_m^{(I_{j+1}^m)} - f_m^{(I_{j-1}^m)}}{f_m^{\max} - f_m^{\min}}$$

end for

end for

return the “crowding distances” (d_1, \dots, d_ℓ)

¹We use the notation $[n] := \{1, 2, \dots, n\}$.

Algorithm 3 NSGA-II (one generation)

Require: A parent population P_t of size N

Require: An offspring population Q_t of size N

- 1: Sort $R_t := P_t \cup Q_t$ into non-dominated fronts $\mathcal{F}_1, \mathcal{F}_2, \dots$
 - 2: Set $i := 1$ and $P_{t+1} := \emptyset$.
 - 3: **while** $|P_{t+1}| + |\mathcal{F}_i| < N$ **do**
 - 4: Set $P_{t+1} = P_{t+1} \cup \mathcal{F}_i$ ▷ add the i -th non-dominated front to the parent pop
 - 5: Set $i := i + 1$
 - 6: **end while**
 - 7: Perform “crowding sort” on the individuals in front \mathcal{F}_i
 - 8: Add the $N - |P_{t+1}|$ most widely spread solutions by crowding distance to P_{t+1}
 - 9: Create an offspring population Q_{t+1} from P_{t+1} using crowded tournament selection, crossover, and mutation operators.
 - 10: Return P_{t+1} and Q_{t+1}
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