Distributed and Parallel Computing

Lecture 12

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Wave Algorithms

A *wave* algorithm sends requests through the whole network to gather information.

Applications include:

- Termination detection
- Routing
- Leader election
- Transaction commit voting in the case of network partitions

To be a wave algorithm, it must meet 3 conditions:

- It must be finite
- It must contain one or more *decide* events
- For each decide event a, and process p, $\exists$ event b in p . $b \prec a$

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  - i.e. every process must participate in each decide event

Traversal Algorithms

A *traversal* algorithm is a type of *wave* algorithm

- An *initiator* sends a *token* to visit each process in the network
- The *token* may collect and/or distribute information on the way
- The *token* eventually returns to the *initiator* with the accumulated information
- The *initiator* makes the *decision*.
- Note that a *token* can only be at one process at any one time

Traversal algorithms can be used to build a *spanning tree* of the network

- The initiator becomes the *root* of the spanning tree
- For every other node, its *parent* is the node from which it received the token for the first time

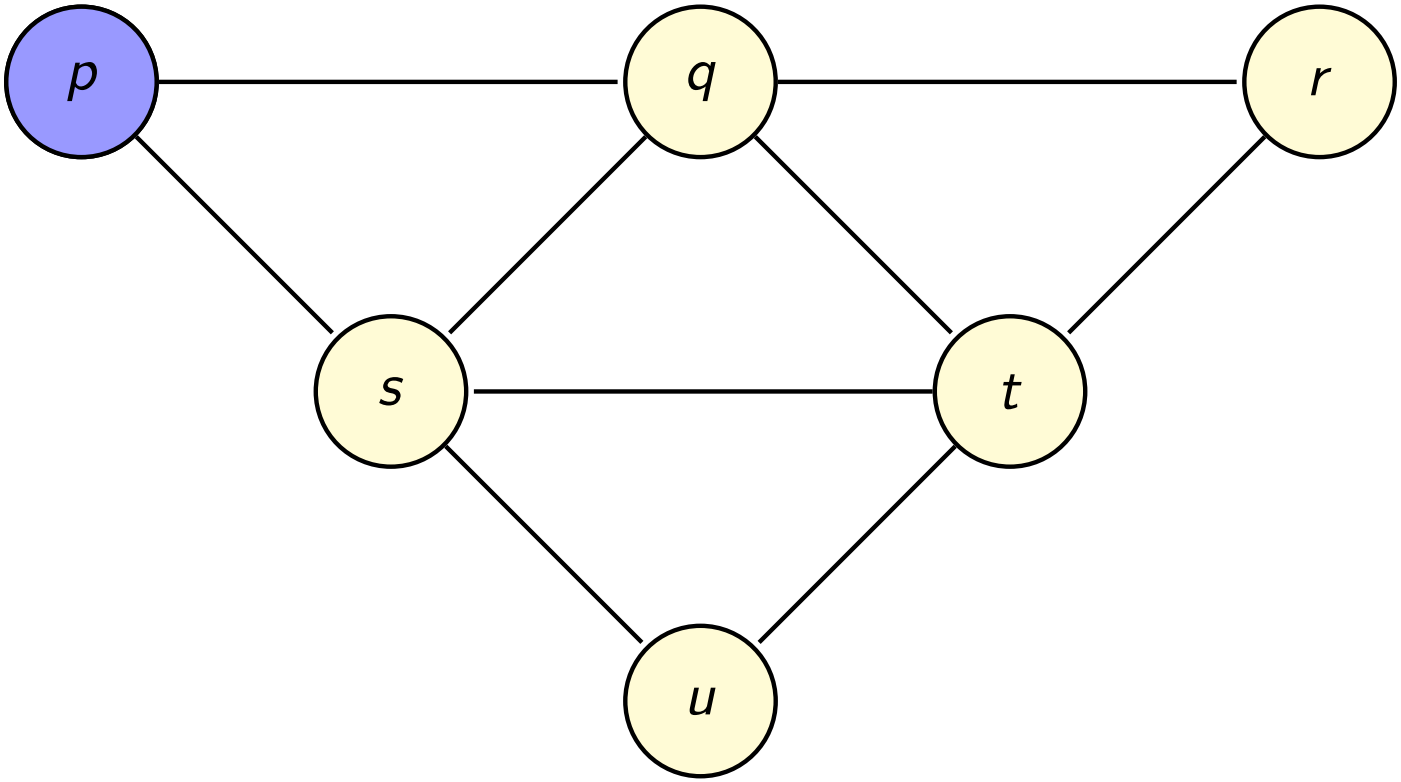
Tarry's Algorithm

Tarry's algorithm is a traversal algorithm for undirected networks. It is based on two rules:

1. A process never forwards the token through the same channel twice
2. A process only forwards the token to its parent when there is no other option

- These rules ensure that the token travels through every channel exactly twice, once in each direction, and ends up back at the initiator
- Note that the rules say how each process acts individually: the global properties of the algorithm have to be proven from these local actions.

Example Execution of Tarry's Algorithm



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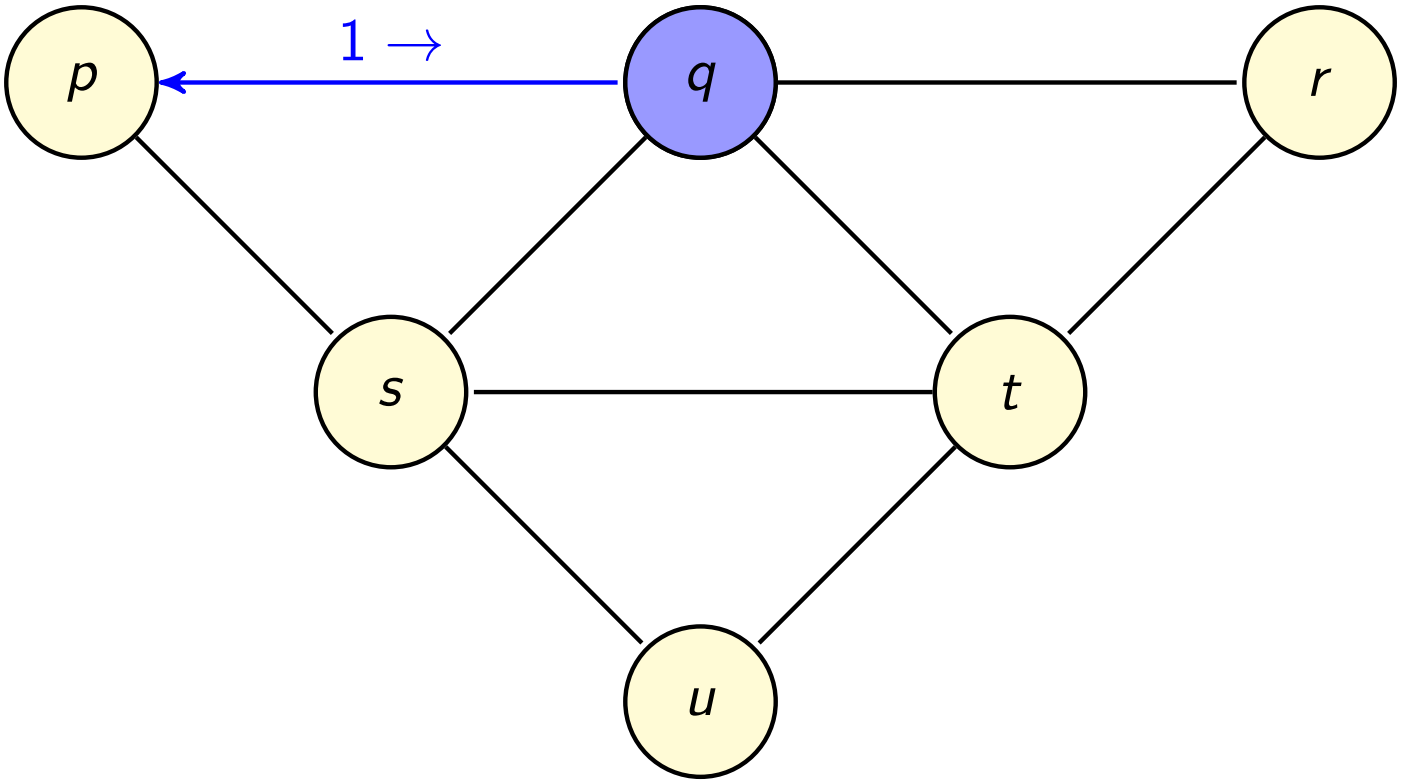

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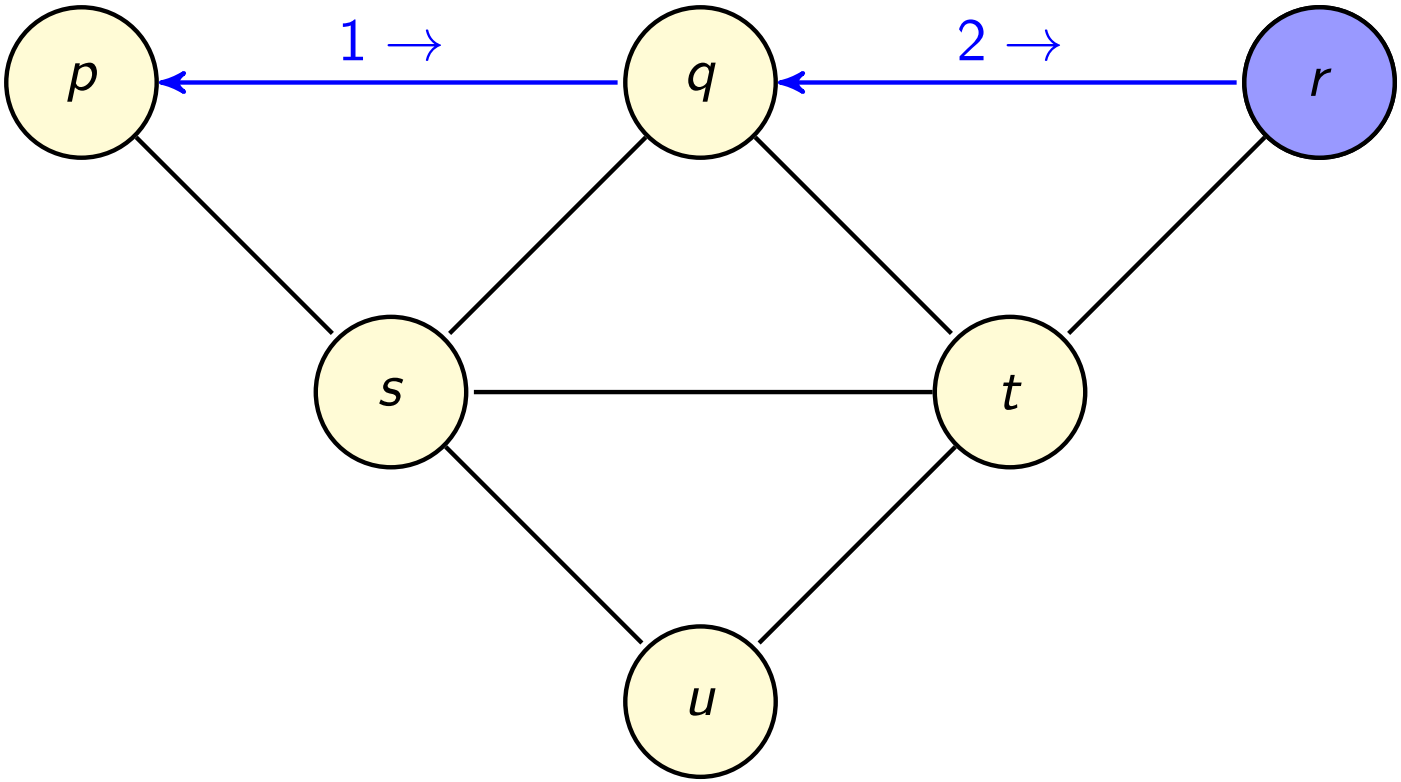

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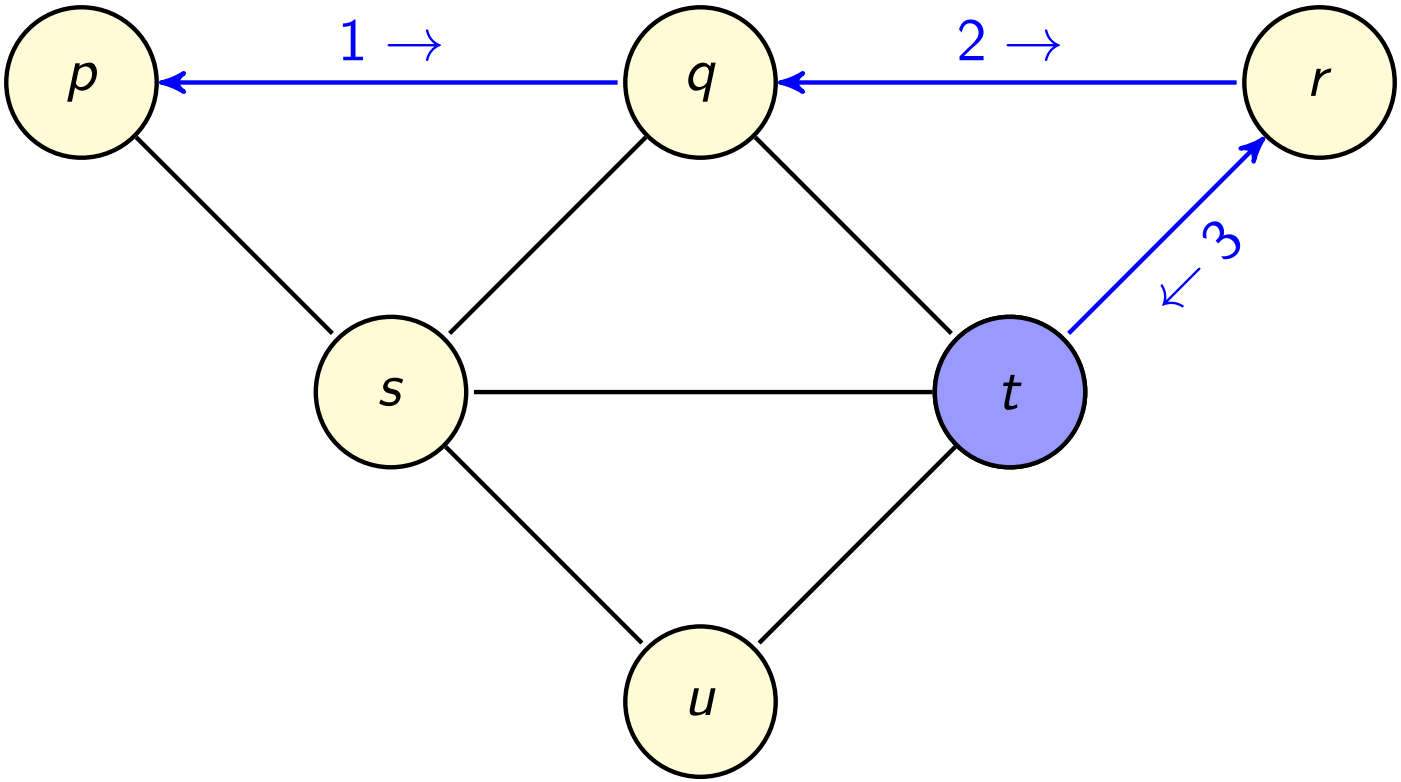

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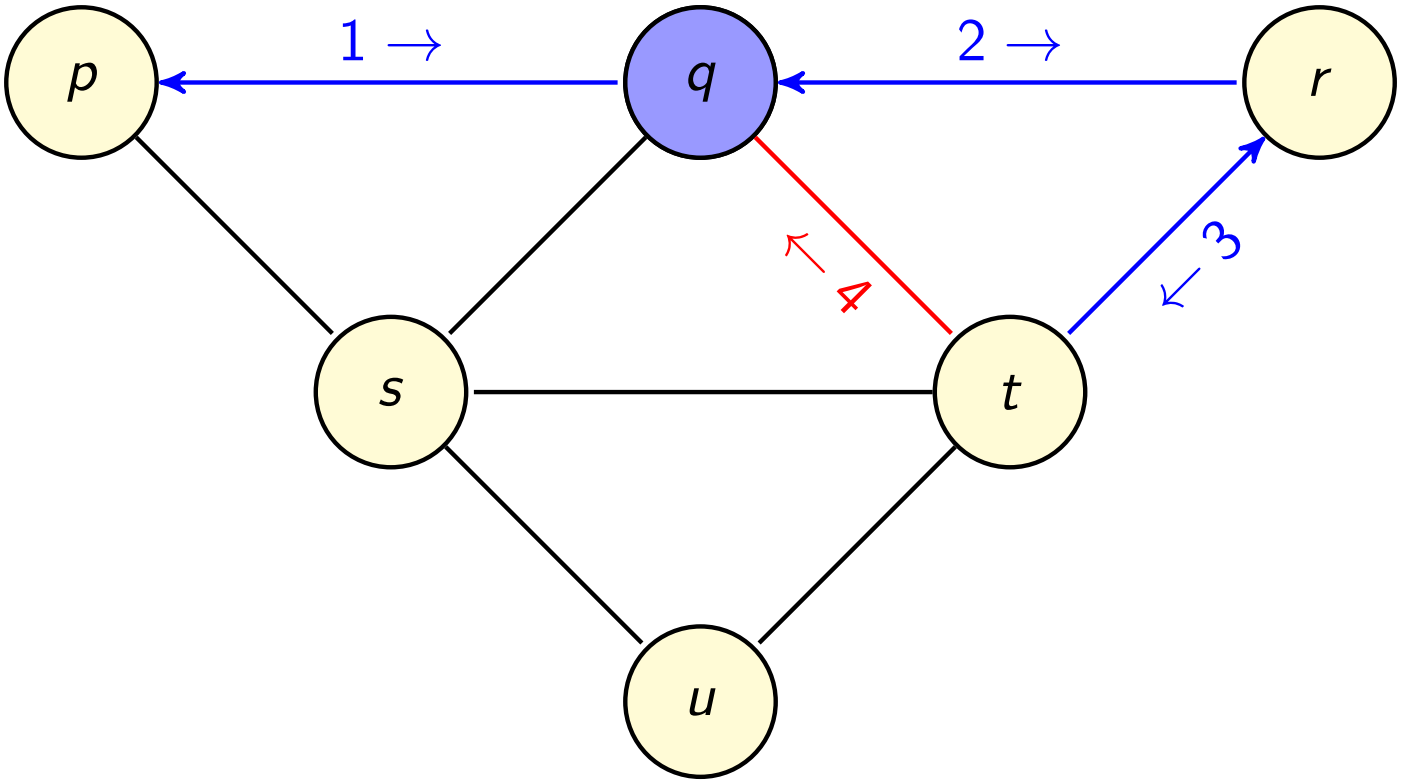

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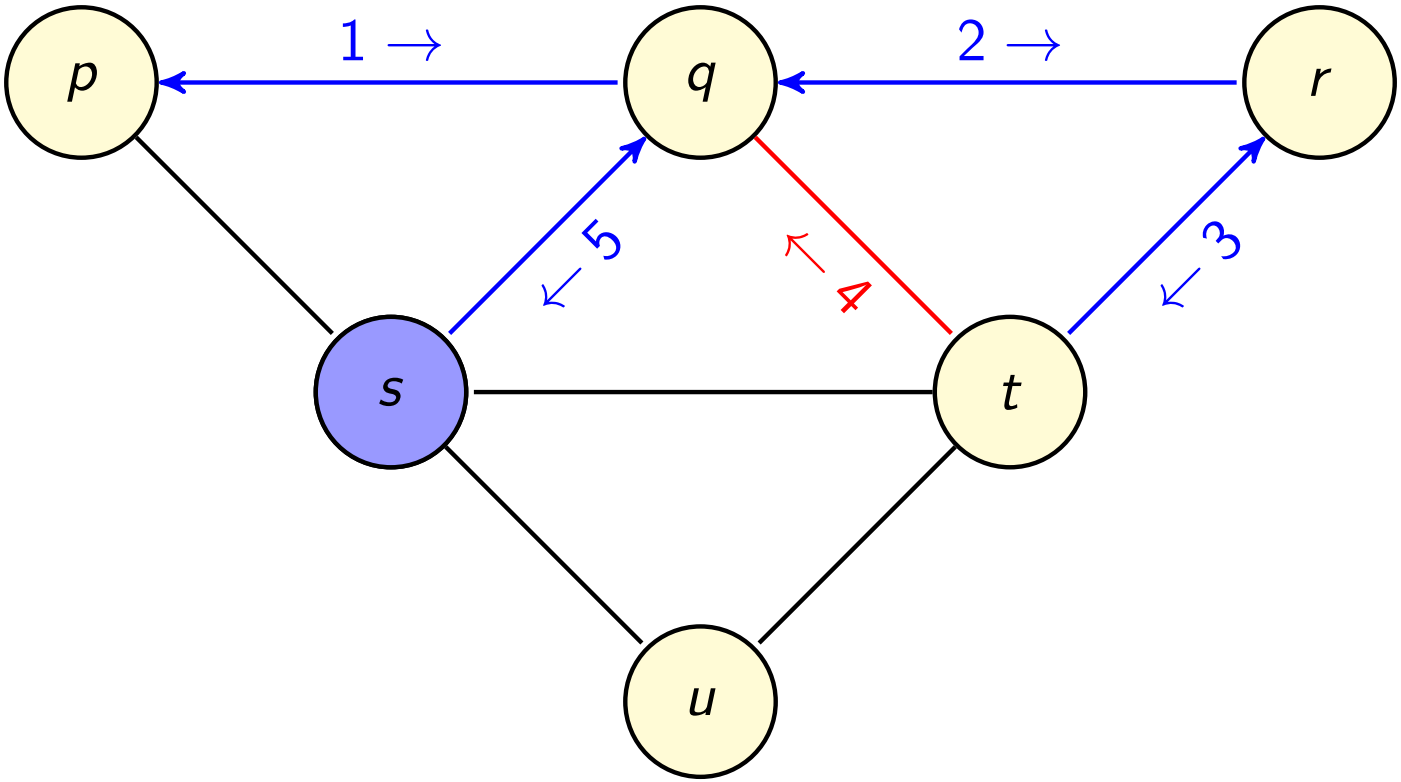

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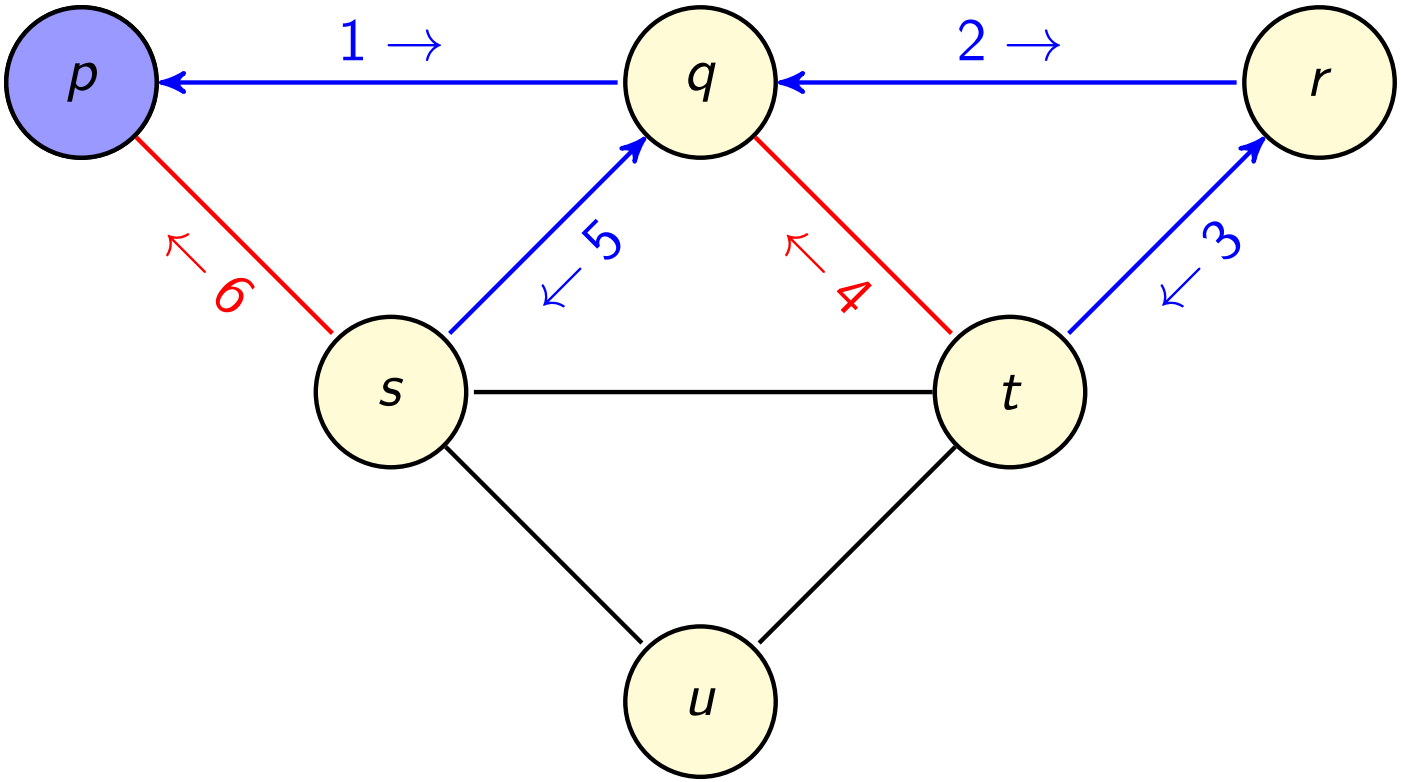

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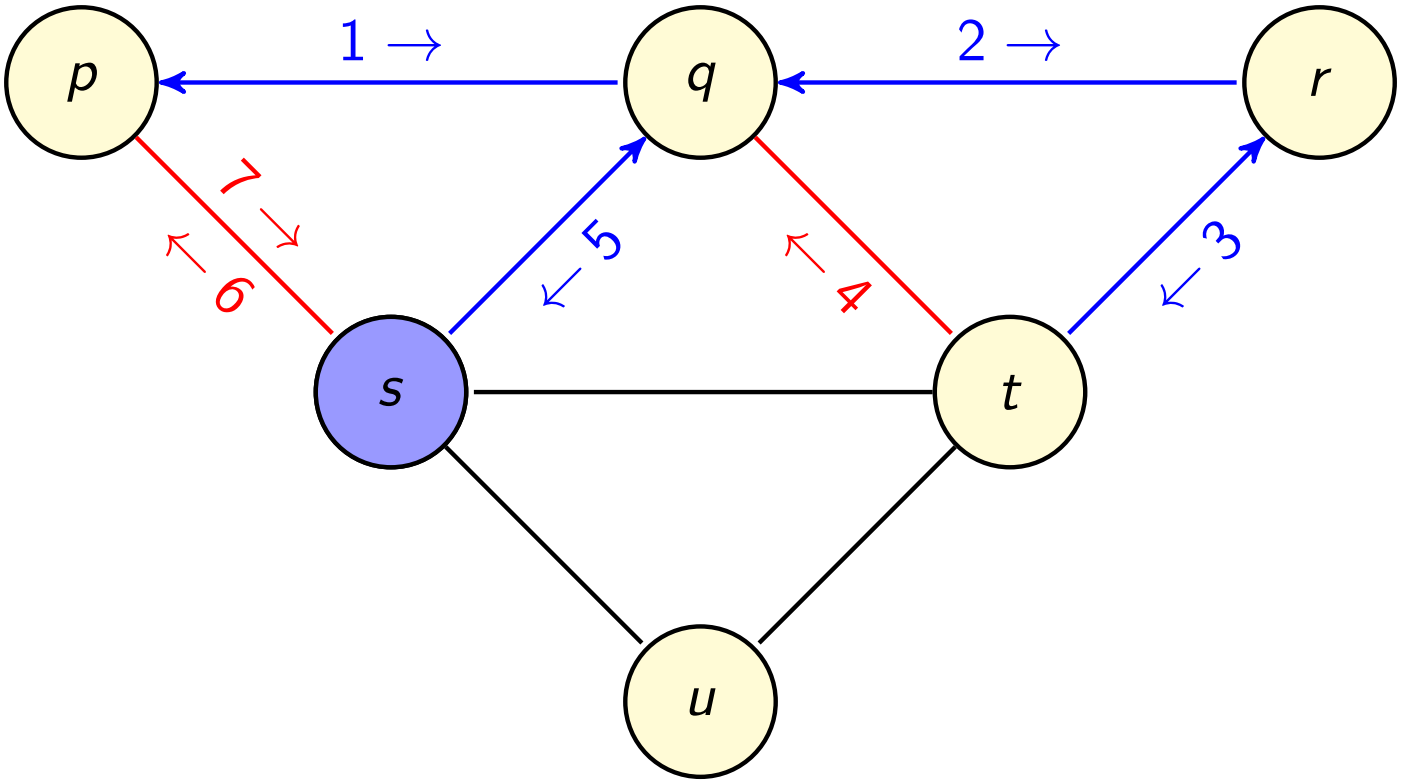

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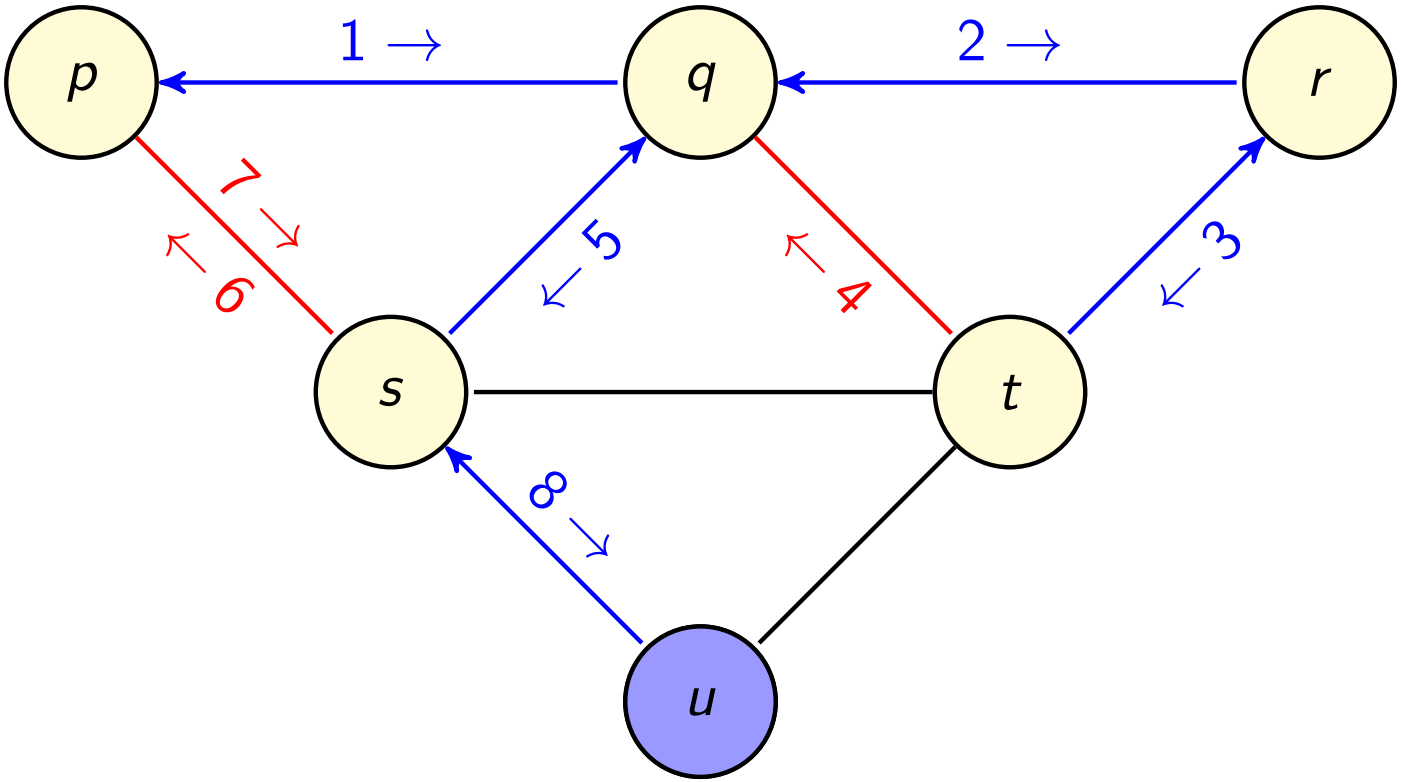

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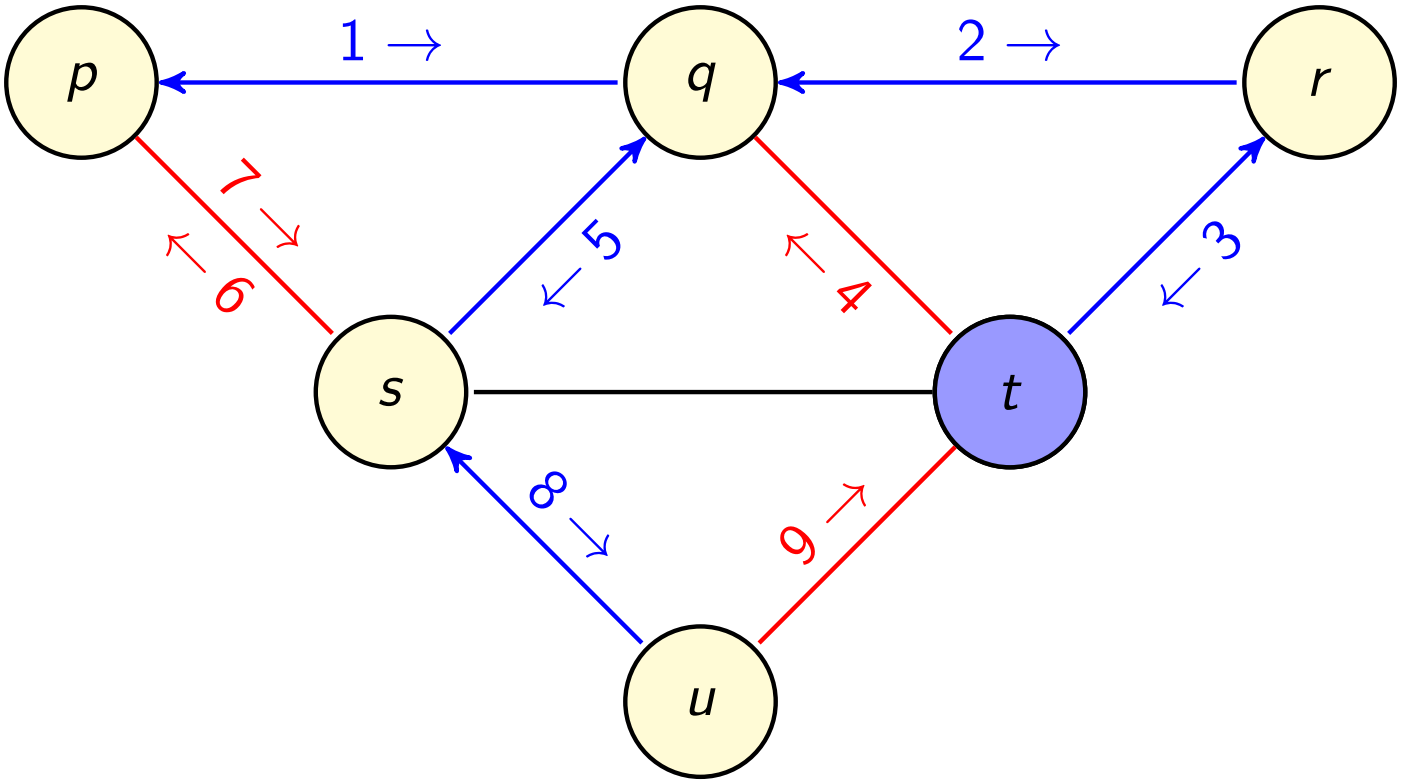

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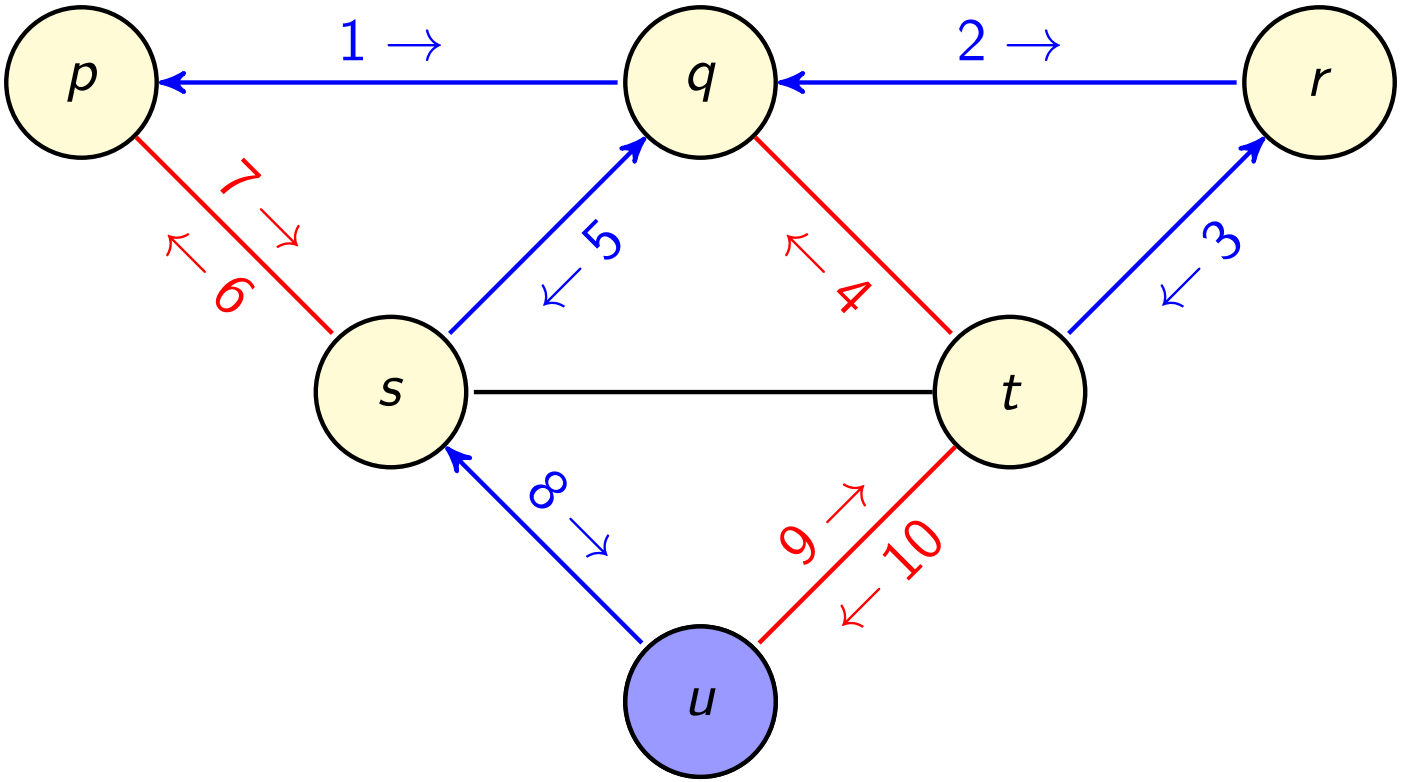

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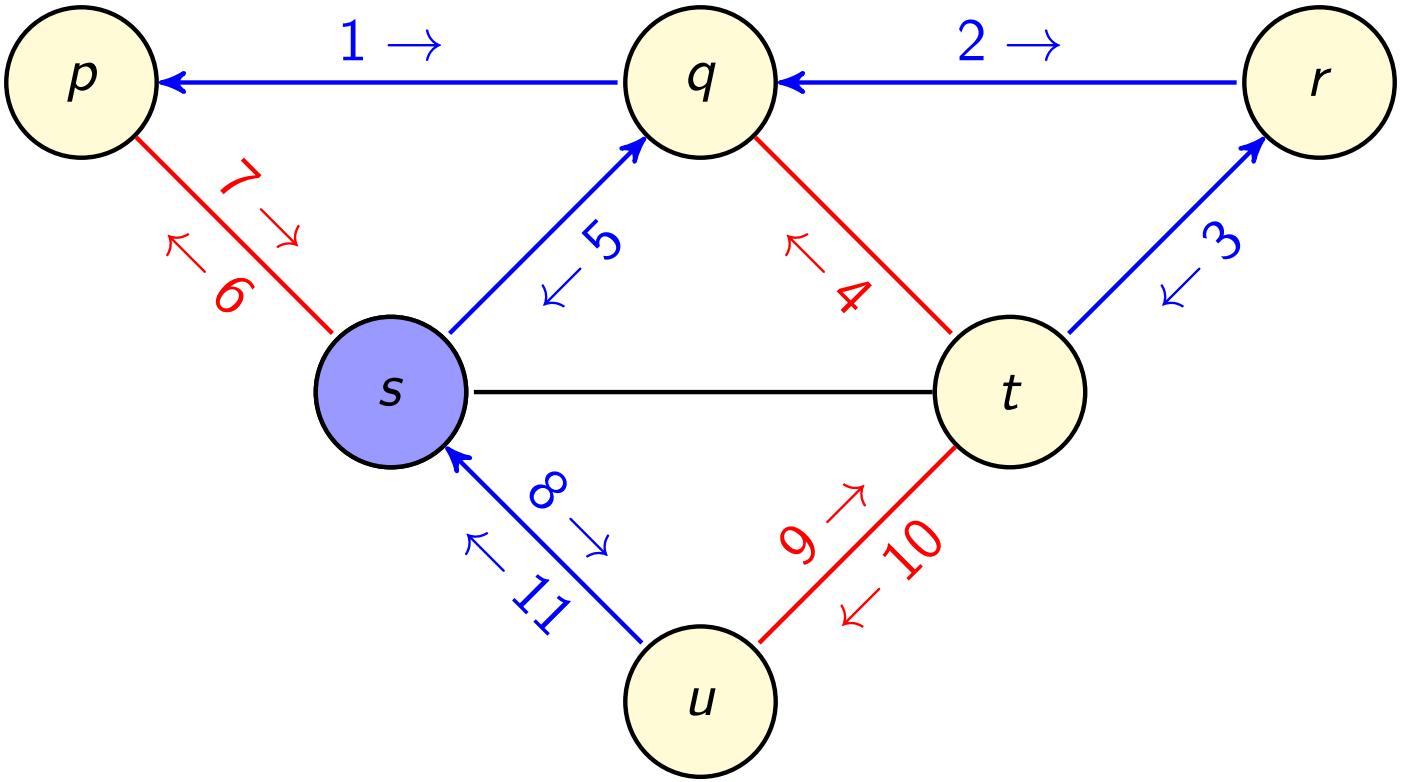

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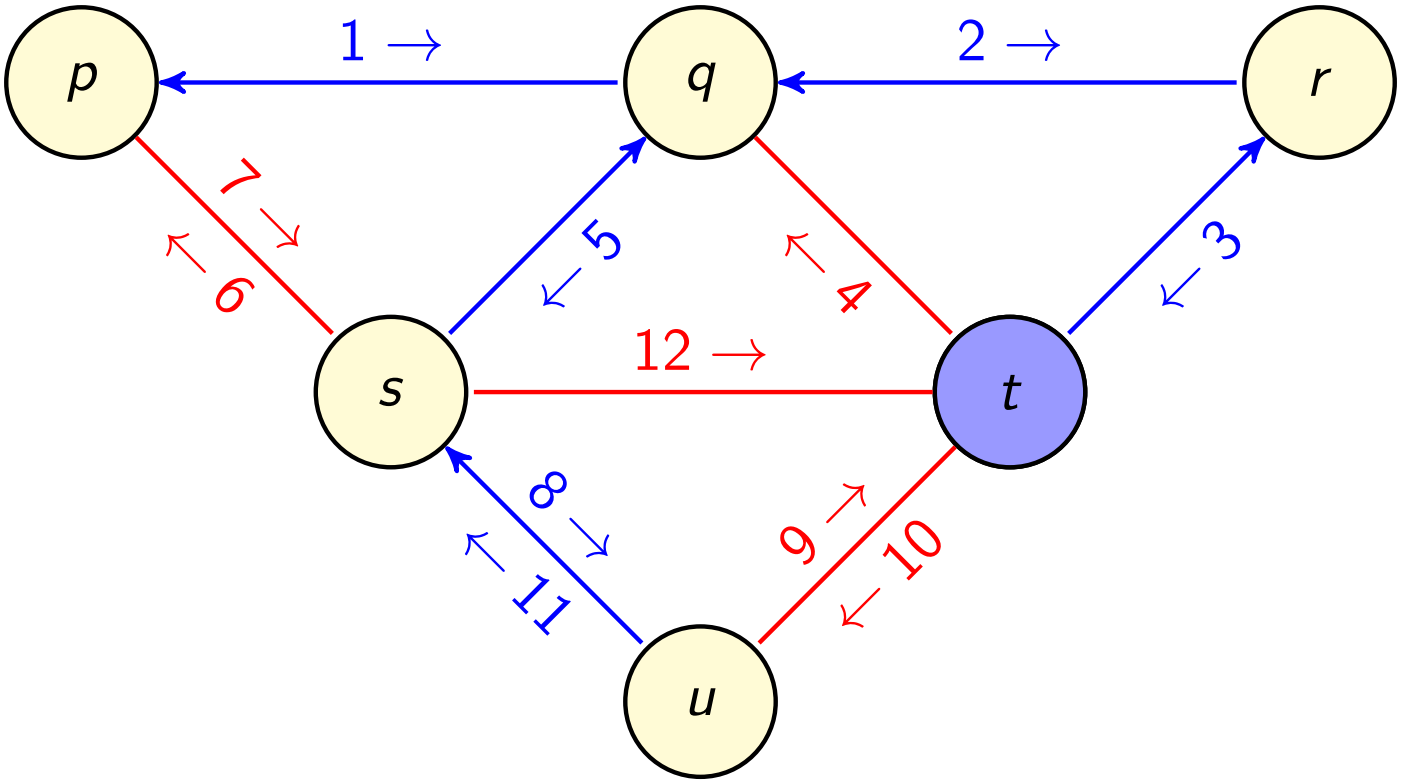

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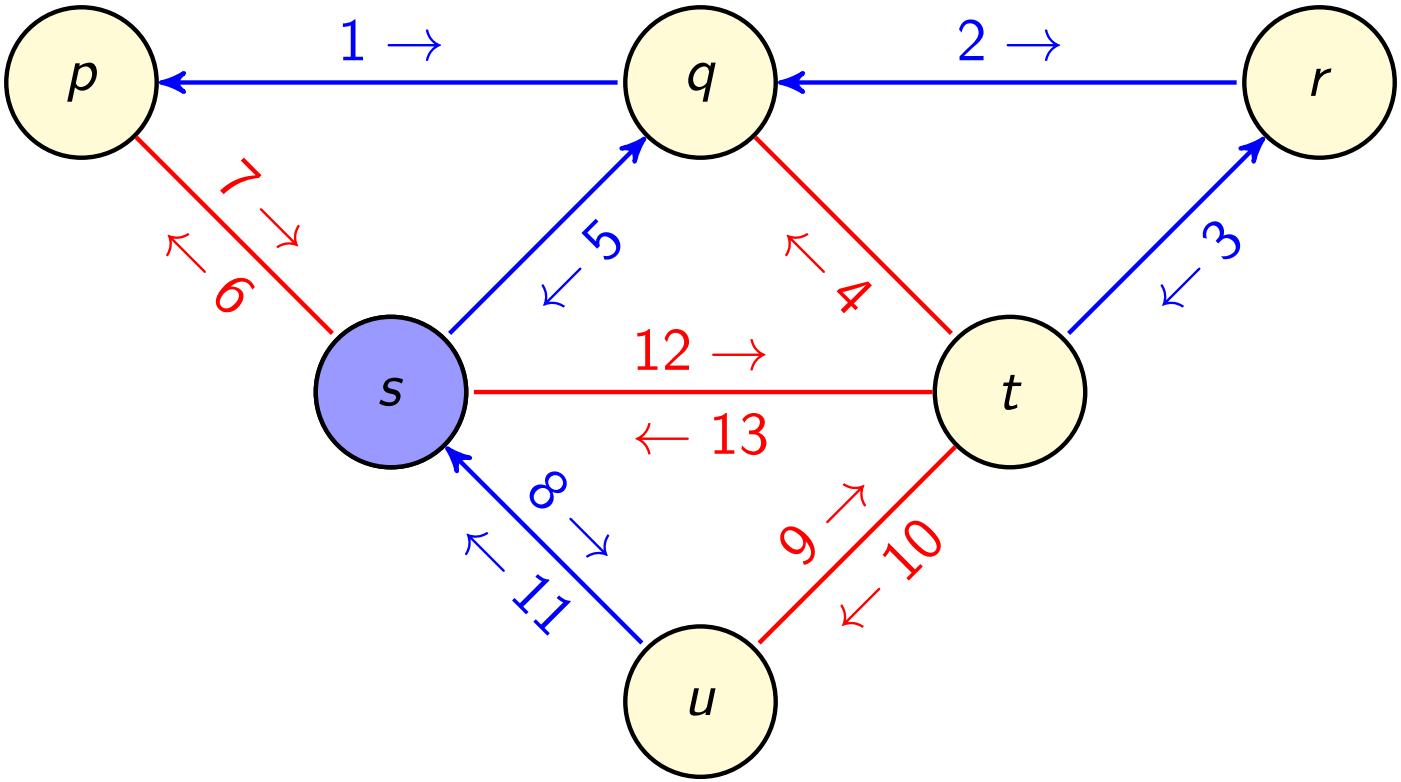

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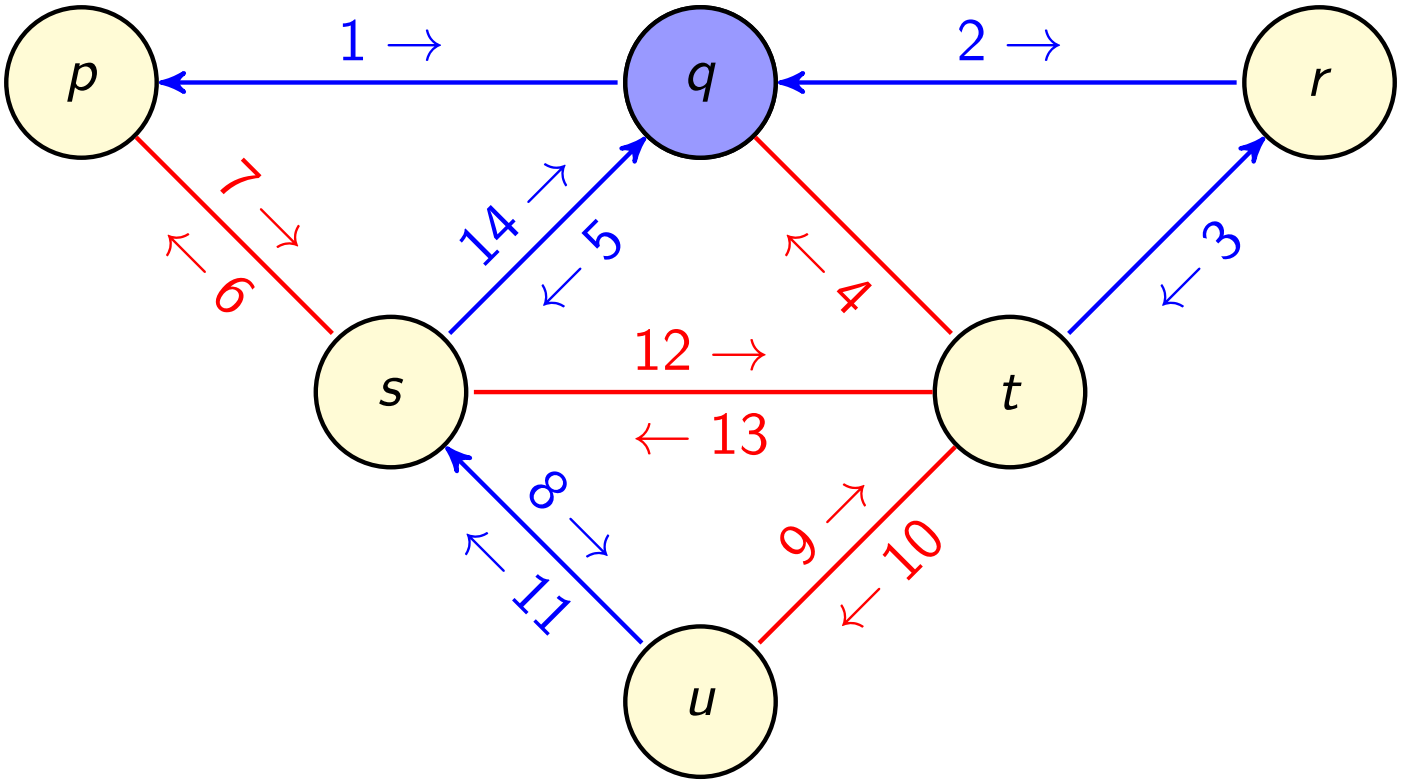

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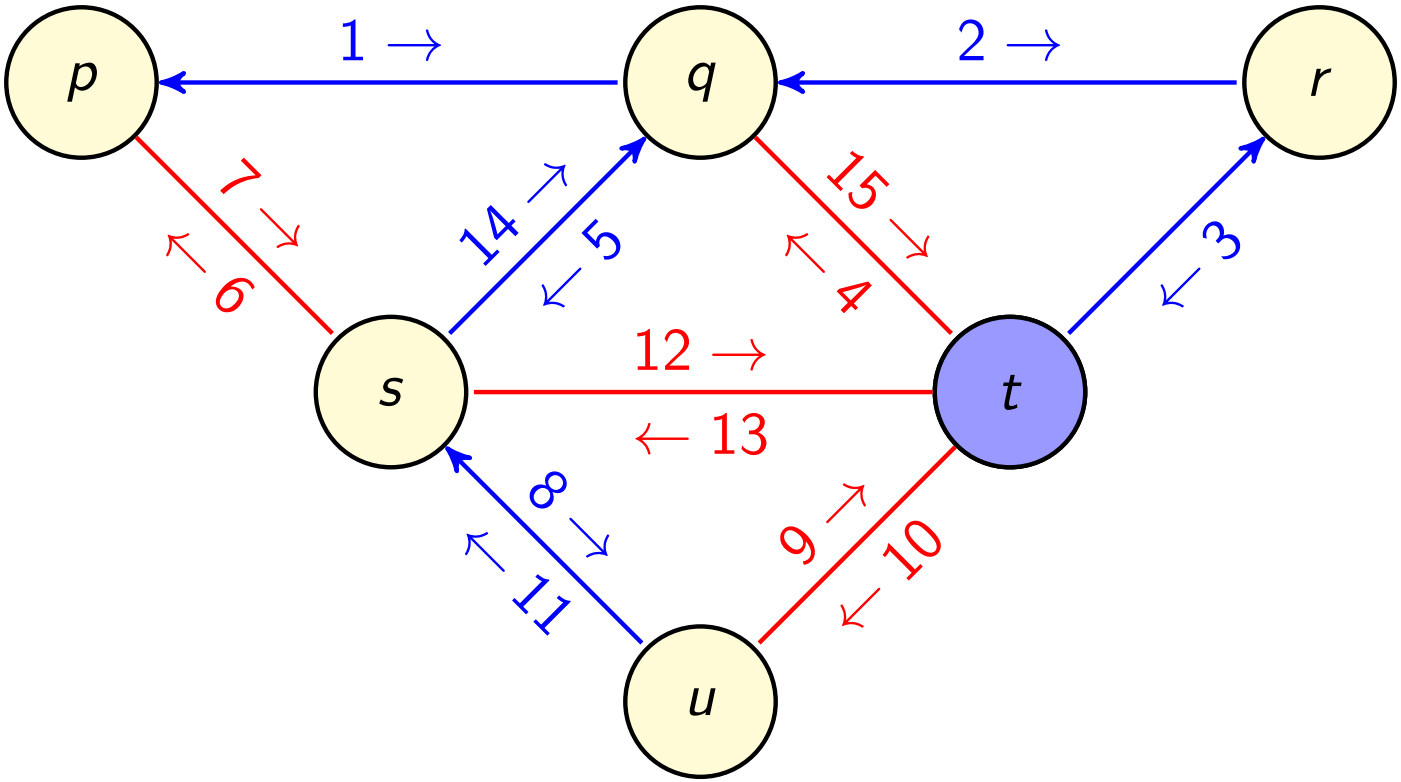

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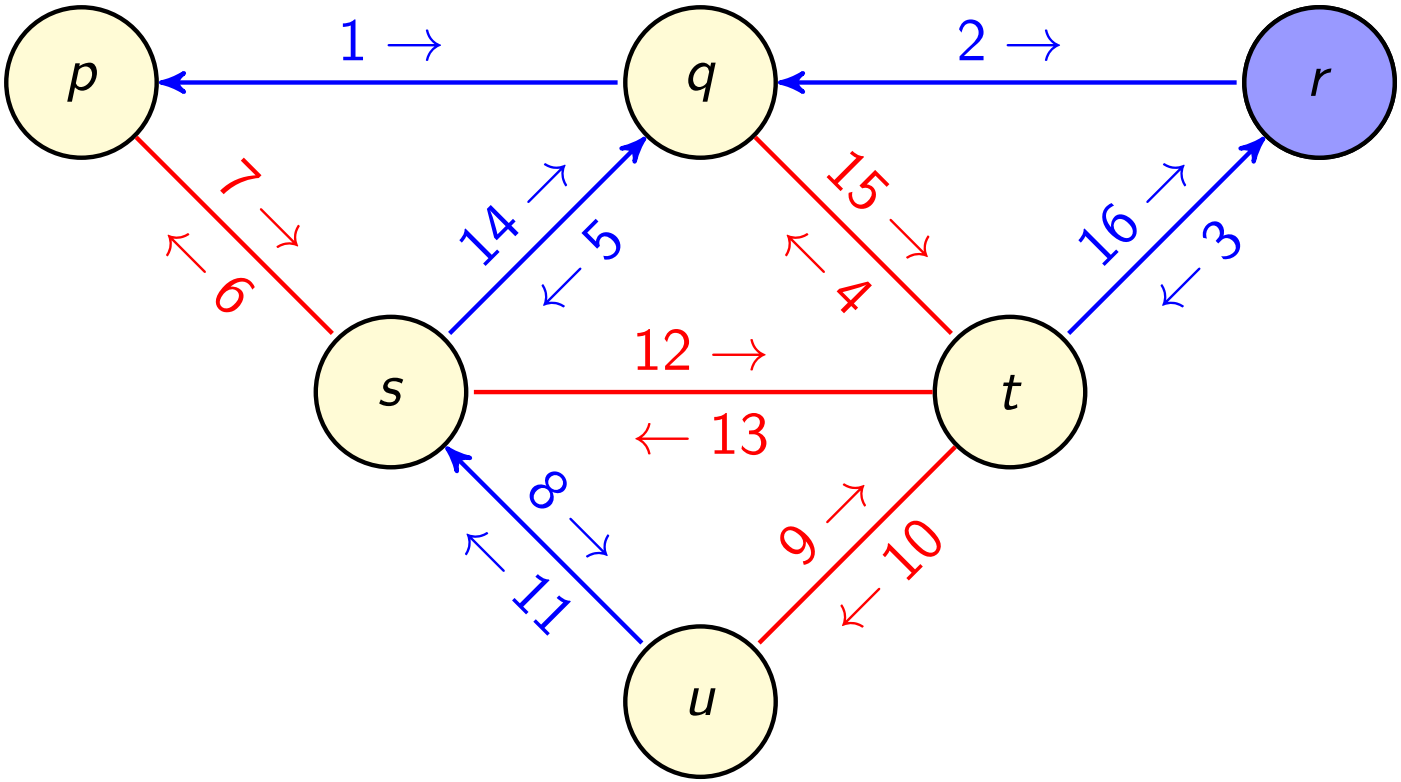

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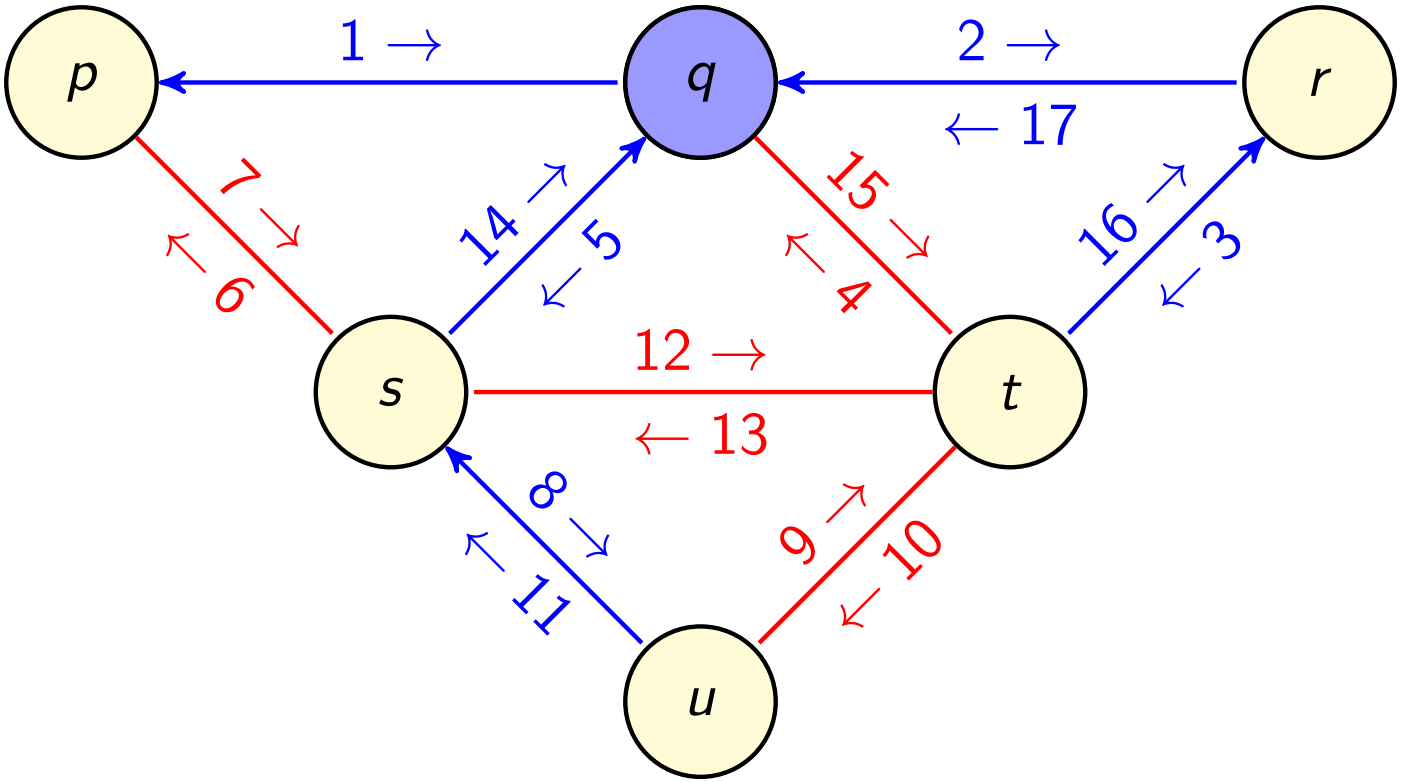

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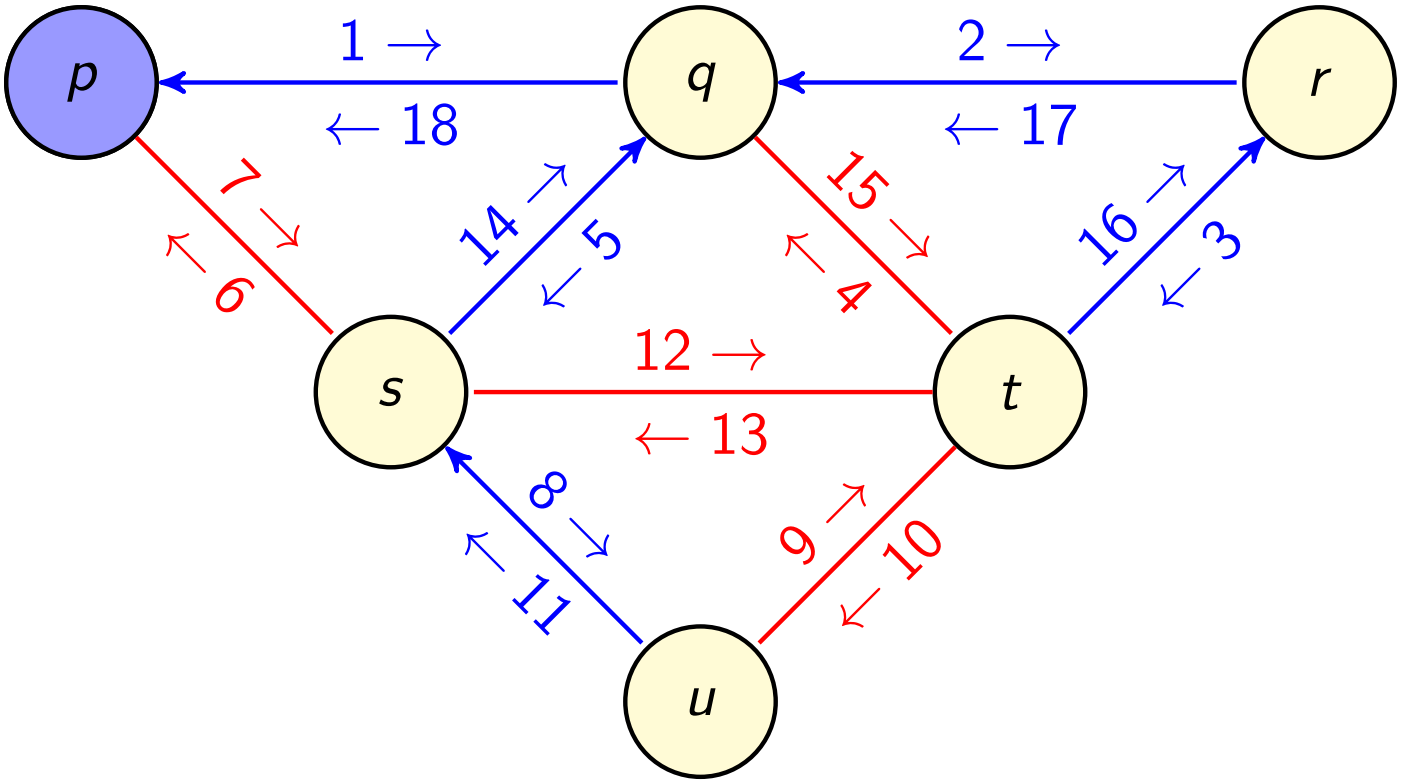

Correctness of Tarry's Algorithm, part 2

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2. The token ends up at the initiator

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- Contradiction

More on Tarry's Algorithm

Performance:

- Number of messages:
- Time to complete:

More on Tarry's Algorithm

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- Number of messages: $2E$
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Performance:

- Number of messages: $2E$
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- Note that this is a serial algorithm: There is only one token and only one process can send the token down one channel at a time — think of football players passing the ball

Tarry's algorithm and Depth First Search

In a depth first search, the token is forwarded to a process that has not yet held the token in preference to one that has.

A depth first spanning tree is one that could have been created by a depth-first search.

A depth first spanning tree will have its frond edges connecting nodes only to their ancestors or descendents in the spanning tree.

- Edges are frond edges if they connect a node to a node that has already been visited.
- In a depth first search, all nodes in a subtree are searched before any nodes in a sibling subtree
- Hence in a depth first search, frond edges will only connect ancestor-descendent pairs

Tarry's algorithm and Depth First Search

We can make Tarry's algorithm generate a depth-first spanning tree by adding an extra rule:

- When a process receives the token, it immediately sends it back through the same channel if allowed by rules 1 and 2.

Note: this does **NOT** make the search in Tarry's algorithm depth-first, but when it diverges off depth-first, it puts it back onto the depth-first track before any more parent-child edges are added to the spanning tree.

There is no extra cost to this change.

Benefits of Depth-First Search

If a depth-first spanning tree is created by the modified Tarry's algorithm:

- We can optimise the algorithm by letting the token carry the information of all processes that have held it:
- Avoid sending the token along frond edges at the cost of extra memory required in the token
- Messages only travel along spanning tree edges, so $2E$ messages reduced to $2N - 2$
- Time complexity reduced from $2E$ to $2N - 2$ time units.
- Bit complexity increases from $O(1)$ to $O(N \log N)$, where $O(\log N)$ bits are needed to represent the process identifiers.