Lecture 12: Decision Trees

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Learning Outcomes

By the end of this lecture you should be able to:

- Understand the concept of a decision tree
- Appreciate that decision trees can be constructed in different ways
- Understand and be able to apply the concept of information entropy to construct a decision tree
- Appreciate some of the limitations of decision trees.

Introduction

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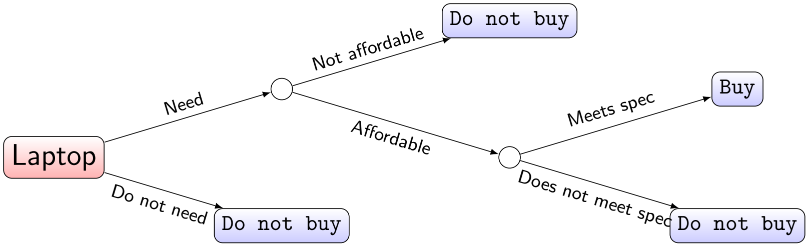
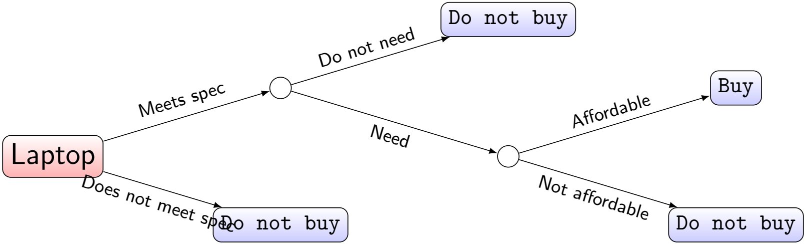
- A (trivial) example: buying a new laptop.
  - Do I need a new laptop?
  - Can I afford it?
  - Does it meet my specication?
- In a decision tree we apply each of these questions in turn to arrive at a final decision.

A Simple Decision Tree



Why not a different tree?



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- How to determine the feature order?

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- Key quantity: Entropy

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- Organised states possible, but highly unlikely
- Information Entropy has a similar interpretation

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- Homogeneous sequences have low entropy
- Random Sequences have high entropy
- We use this to select the feature that gives the biggest gain in information

Information Gain

- Given $S = -\sum_i p(i) \ln p(i)$ we calculate

$$G(P,C) = S(P) - S(C) \tag{3}$$

$$- \sum_{i \in P} p(i) \ln p(i) - \sum_{c \in C} p(c) \sum_{i \in c} -p(i|c) \ln p(i|c) \tag{4}$$

- Let's do an example...

Selecting features by maximising IG

- Outcomes for buying a laptop. . .

Selecting features by maximising IG

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N	Need	Afford	Spec	Buy
1	T	F	T	F
2	F	T	F	F
3	T	F	T	T
4	T	F	T	T
5	F	T	F	F
6	T	T	T	T
7	F	F	F	F
8	T	T	T	T
9	F	T	T	T
10	T	F	F	F

- What variable should we split on first?

Selecting features by maximising IG

- Parent entropy
- Buy: 4T, 6F

$$S(P) = -\sum_i p(i) \ln p(i)$$
$$= -0.4 \ln 0.4 - 0.6 \ln 0.6 = 0.673$$

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1	T	F	T	F
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Selecting features by maximising IG

- “Need”

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- “Need”
- 6T, 4F

Selecting features by maximising IG

- "Need"
- 6T, 4F
- 6T$\mapsto$ 4T, 2F

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- "Need"
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- 4F$\mapsto$ 0T, 4F

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$$
\begin{aligned}
S(C) &= \sum_{c \in C} p(c) \sum_{i \in c} -p(i|c) \ln p(i|c) \\
&= \left[p(\mathrm{Need}) \times \sum_{i \in \mathrm{Need}} -p_i \ln p_i \right] + \left[p(\neg\mathrm{Need}) \times \sum_{i \in \neg\mathrm{Need}} -p_i \ln \right. \\
&= 0.6 \times \left(-\frac{4}{6} \ln \frac{4}{6} - \frac{2}{6} \ln \frac{2}{6} \right) + 0.4 \times (-1 \ln 1 - 0 \ln 0) \\
&= 0.382
\end{aligned}
$$

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- “Afford”

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- “Afford”
- 5T, 5F

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- “Afford”
- 5T, 5F
- 5T$\mapsto$ 2T, 3F

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- 5T$\mapsto$ 2T, 3F
- 5F$\mapsto$ 2T, 3F

Selecting features by maximising IG

- "Afford"
- 5T, 5F
- 5T$\mapsto$ 2T, 3F
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$$S(C) = 0.5 \times \left(-\frac{2}{5}\ln\frac{2}{5} - \frac{3}{5}\ln\frac{3}{5}\right) + 0.5 \times \left(-\frac{2}{5}\ln\frac{2}{5} - \frac{3}{5}\ln\frac{3}{5}\right) \tag{5}$$

$$= 0.5 \times 0.673 + 0.5 \times 0.673 = 0.673 \tag{6}$$

$x \cdot \ln x$

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- “Spec”

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- “Spec”
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- 6T$\mapsto$ 5T, 1F

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- "Spec"
- 6T, 4F
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$$= 0.451 + 0 = 0.270 \tag{8}$$

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- Spec: $0.673 - 0.270 = 0.403$
- The best initial predictor of purchasing a new laptop is its specification.
- Apply these ideas recursively to each partition to build the tree.

Tips and Tricks for Decision Trees

- Decision trees can always fit their training data exactly: low bias
- But leads to unstable model: high variance
- Homogeneous leaf nodes can lead to serious overfitting, especially on small data
- Can be good to limit tree depth: more robust to model noise
- Model is easy to interpret

Summary

- Decision Trees are an intuitive way to build interpretable classifiers
- But they are unstable
- Uncommon to use a single tree
- Much more common to use them as part of an *ensemble*
- Next lecture: how to use ensembles of weak learners to build a strong learner