Shortcoming of Non-Elitist MOEAs

- Elite-preservation is missing
- Elite-preservation is important for proper convergence in SOEAs
- Same is true in MOEAs
- Three tasks
  - Elite preservation
  - Progress towards the Pareto-optimal front
  - Maintain diversity among solutions

Elitist MOEAs

Elite-preservation:

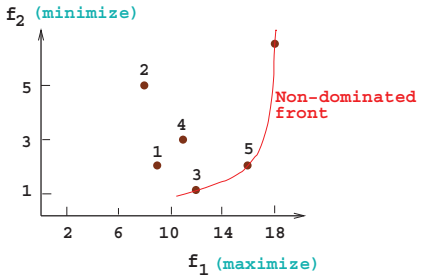
- Maintain an archive of non-dominated solutions

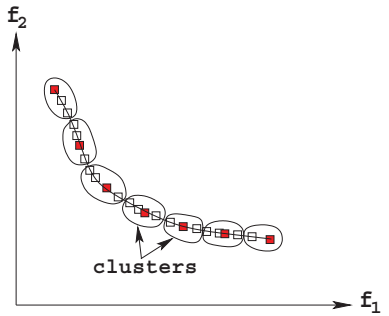
Progress towards Pareto-optimal front:

- Preferring non-dominated solutions

Maintaining spread of solutions:

- Clustering, niching, or grid-based competition for a place in the archive





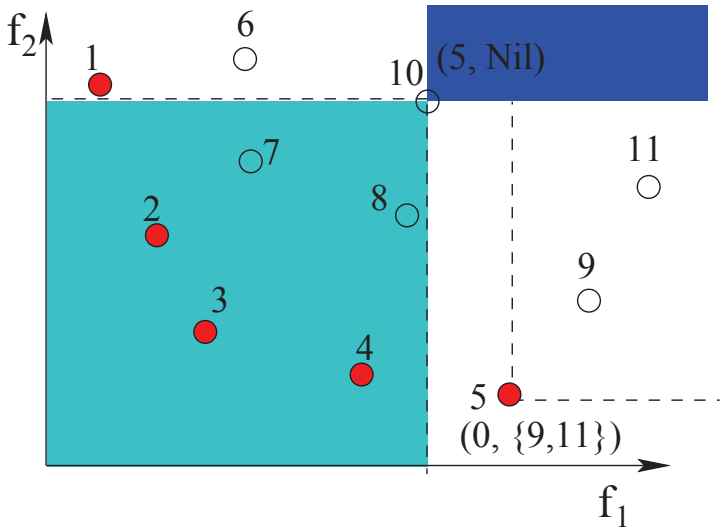
Elitist MOEAs (cont.)

- Distance-based Pareto GA (DPGA) (Osyczka and Kundu, 1995)
- Thermodynamical GA (TDGA) (Kita et al., 1996)
- Strength Pareto EA (SPEA) (Zitzler and Thiele, 1998)
- Non-dominated sorting GA-II (NSGA-II) (Deb et al., 1999)
- Pareto-archived ES (PAES) (Knowles and Corne, 1999)
- Multi-objective Messy GA (MOMGA) (Veldhuizen and Lamont, 1999)
- Other methods: Pareto-converging GA, multi-objective micro-GA, elitist MOGA with coevolutionary sharing

Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

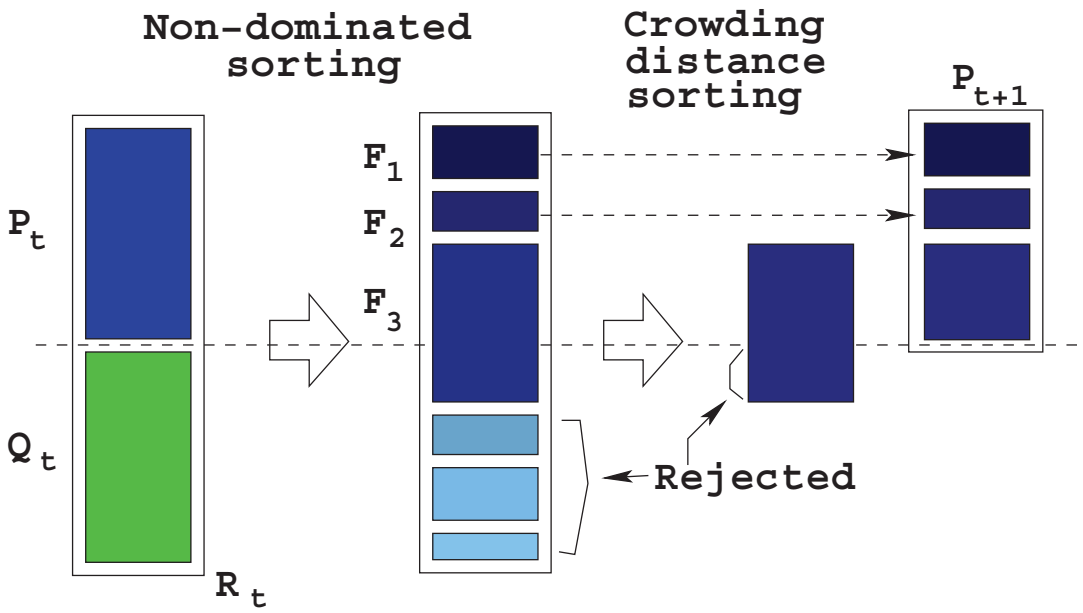
Non-dominated sorting: $O(MN^2)$

- Calculate (n_i, S_i) for each solution i
- n_i: Number of solutions dominating i
- S_i: Set of solutions dominated by i



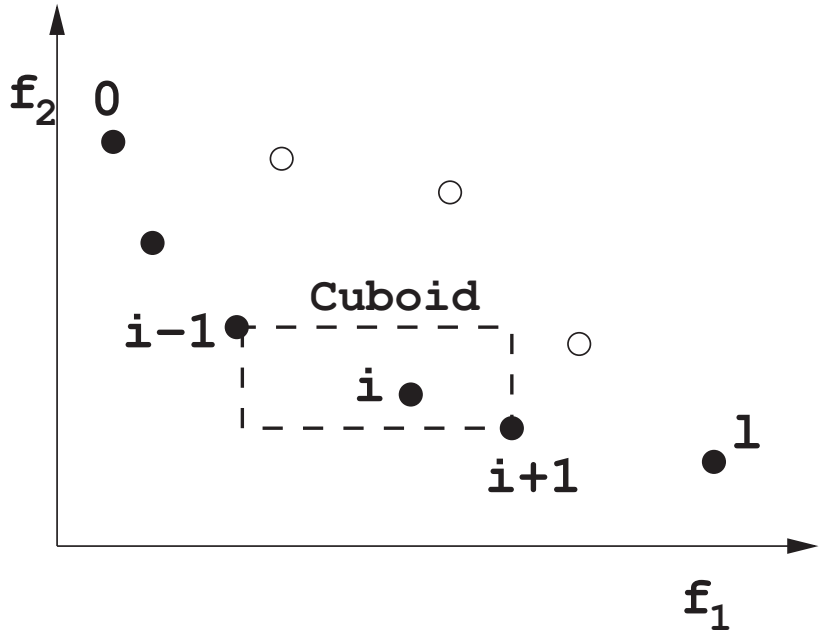
NSGA-II (cont.)

Elites are preserved



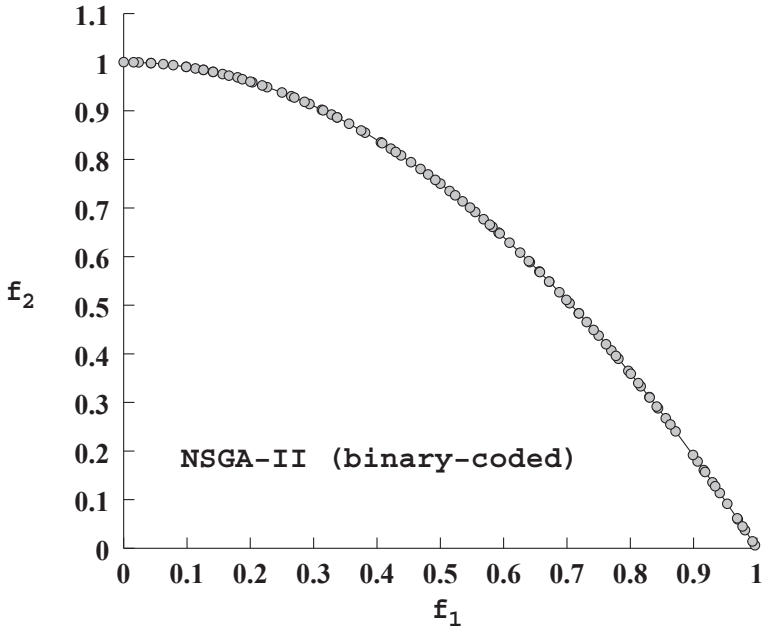
NSGA-II (cont.)

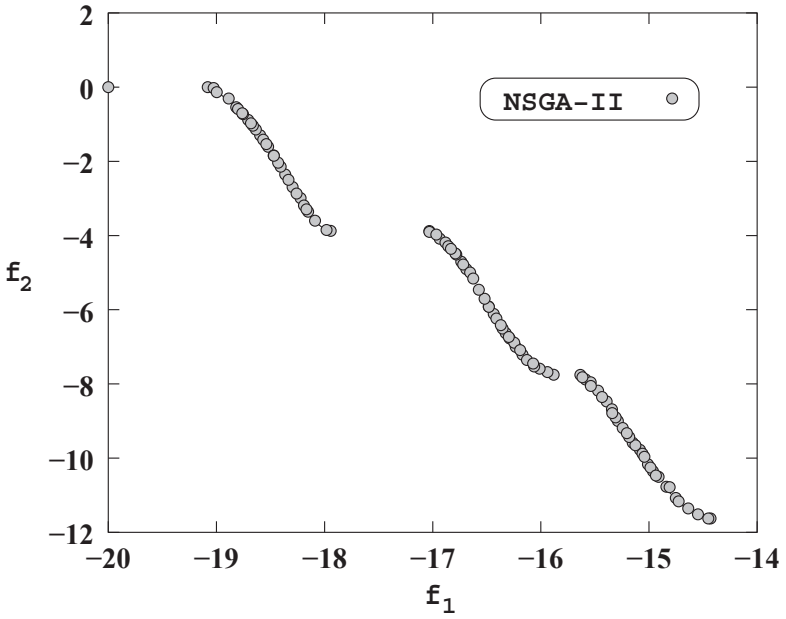
Diversity is maintained: $O(MN \log N)$



Overall Complexity: $O(MN^2)$

NSGA-II Simulation Results



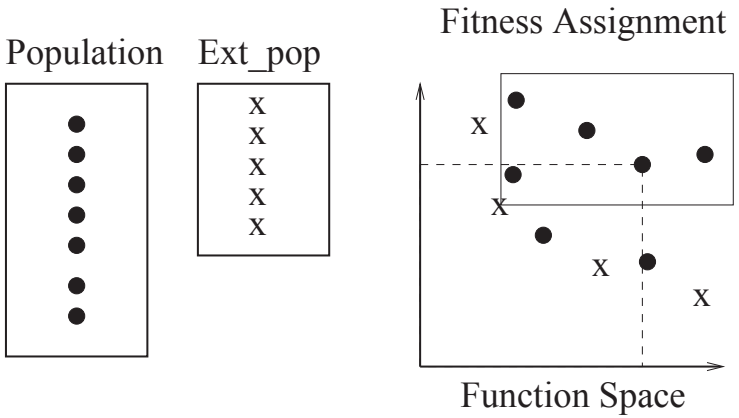


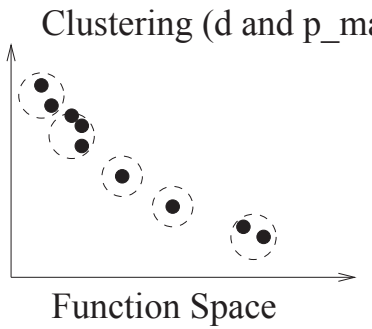
Strength Pareto EA (SPEA)

- Stores non-dominated solutions externally
- Pareto-dominance to assign fitness
  - External members: Assign number of dominated solutions in population (smaller, better)
  - Population members: Assign sum of fitness of external dominating members (smaller, better)
- Tournament selection and recombination applied to combined current and elite populations
- A clustering technique to maintain diversity in updated external population, when size increases a limit

SPEA (cont.)

- Fitness assignment and clustering methods



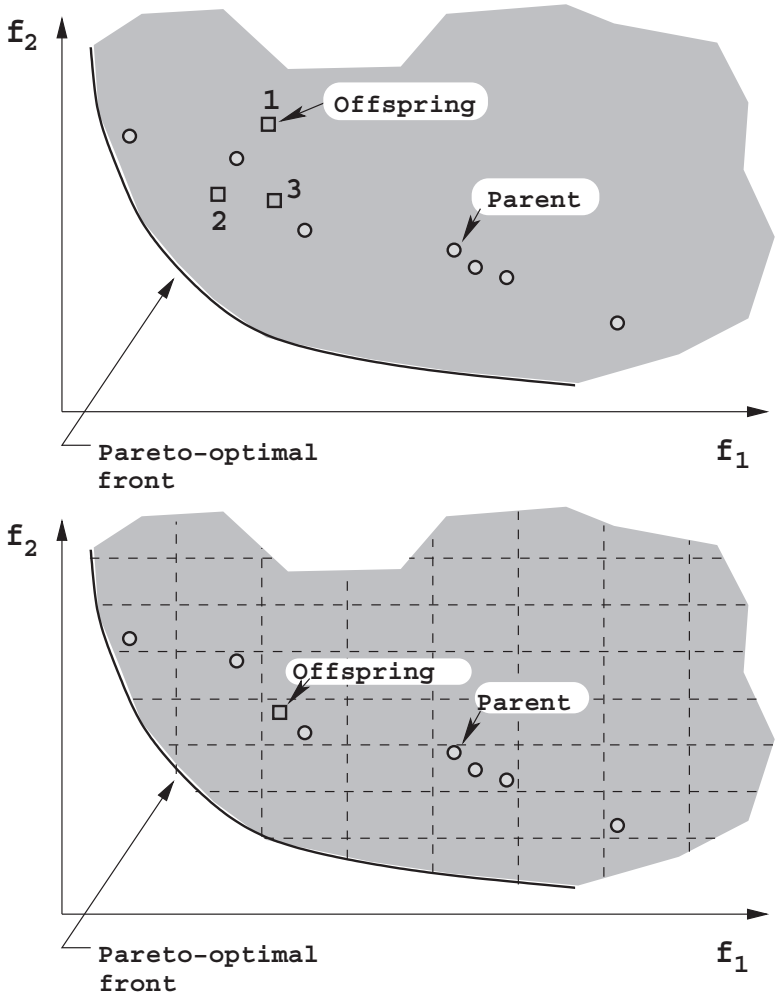


Pareto Archived ES (PAES)

- An (1+1)-ES
- Parent p_t and child c_t are compared with an external archive A_t
- If c_t is dominated by A_t, $p_{t+1} = p_t$
- If c_t dominates a member of A_t, delete it from A_t and include c_t in A_t and $p_{t+1} = c_t$
- If $|A_t| < N$, include c_t and $p_{t+1} = winner(p_t, c_t)$
- If $|A_t| = N$ and c_t does not lie in highest count hypercube H, replace c_t with a random solution from H and $p_{t+1} = winner(p_t, c_t)$.

The winner is based on *least* number of solutions in the hypercube

Niching in PAES-(1+1)



Constrained Handling

- Penalty function approach

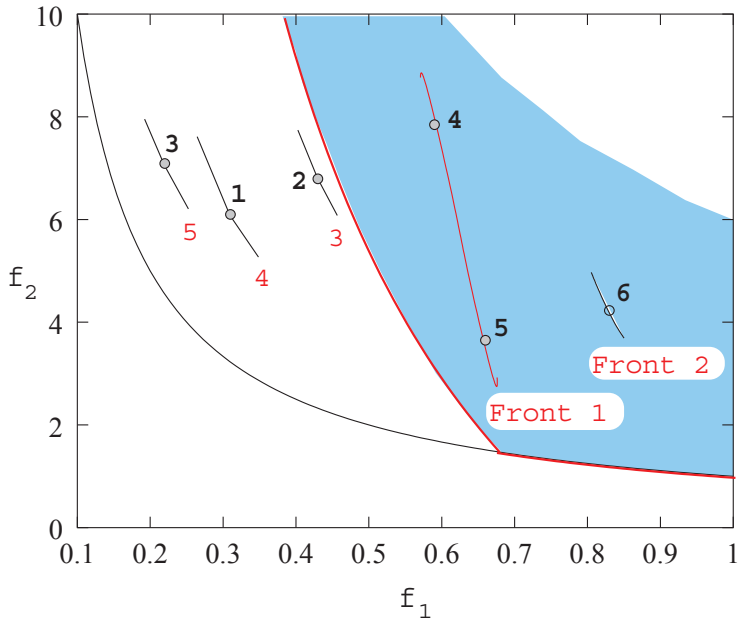
$$F_m = f_m + R_m \Omega(\vec{g}).$$

- Explicit procedures to handle infeasible solutions
  - Jimenez's approach
  - Ray-Tang-Seow's approach
- Modified definition of domination
  - Fonseca and Fleming's approach
  - Deb et al.'s approach

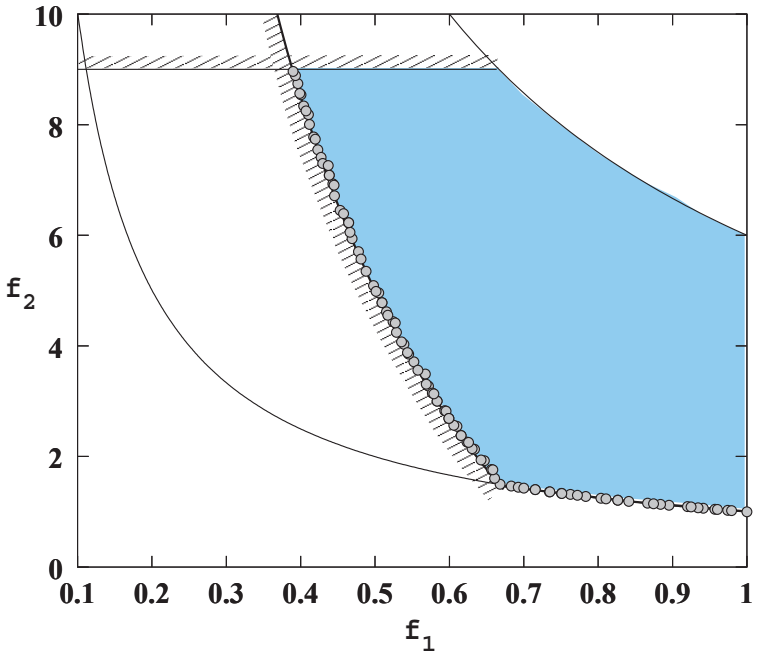
Constrain-Domination Principle

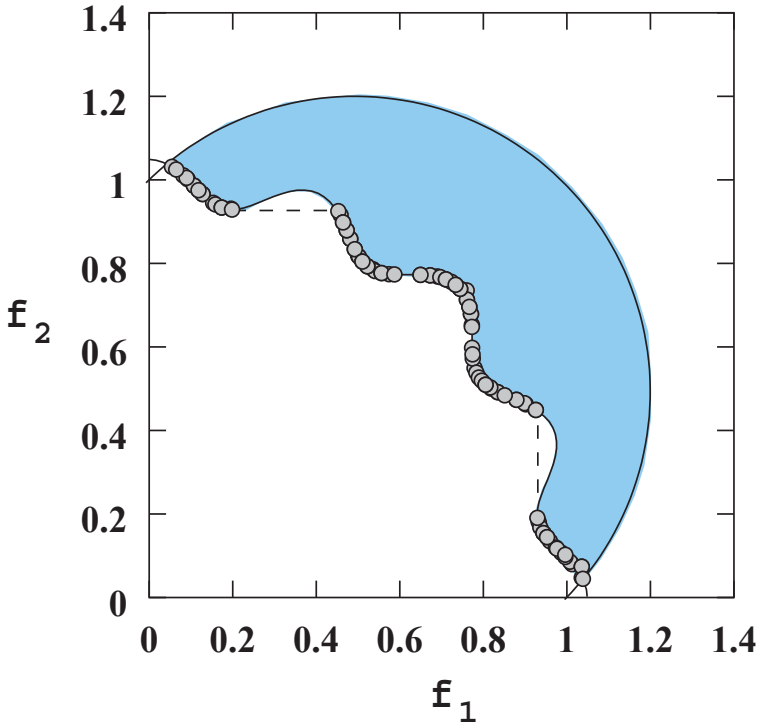
A solution i constrained-dominates a solution j, if any is true:

1. Solution i is feasible and solution j is not.
2. Solutions i and j are both infeasible, but solution i has a smaller overall constraint violation.
3. Solutions i and j are feasible and solution i dominates solution j.



Constrained NSGA-II Simulation Results





Applications of MOEAs

- Space-craft trajectory optimization
- Engineering component design
- Microwave absorber design
- Ground-water monitoring
- Extruder screw design
- Airline scheduling
- VLSI circuit design
- Other applications (refer Deb, 2001 and EMO-01 proceedings)

Conclusions

- Ideal multi-objective optimization is generic and pragmatic
- Evolutionary algorithms are ideal candidates
- Many efficient algorithms exist, more efficient ones are needed
- With some salient research studies, MOEAs will revolutionize the act of optimization
- EAs have a definite edge in multi-objective optimization and should become more useful in practice in coming years