PID控制器的计算机仿真与辅助设计

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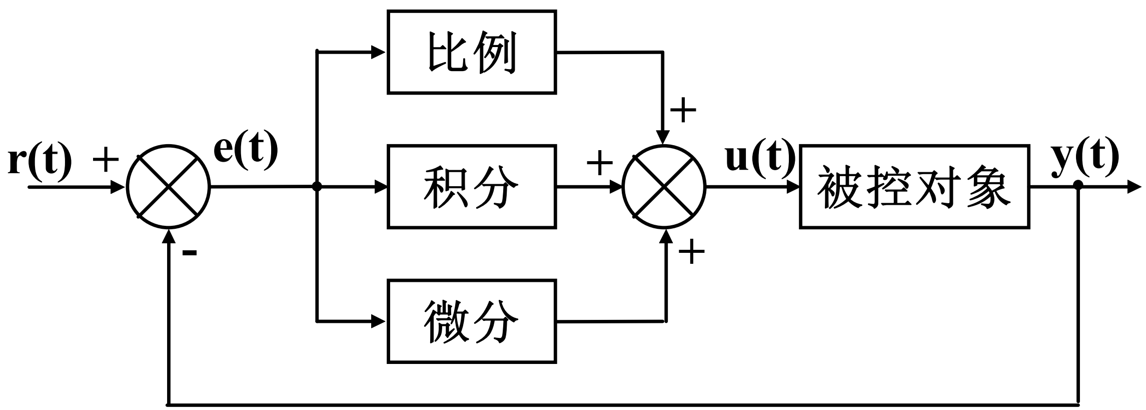
主要内容

- **PID控制器**
  - 标准PID控制算法
  - 标准PID控制算法的改进
  - PID控制器设计原则
- **PID控制器的参数整定**
- **鲁棒PID控制器参数整定**

PID控制器

- **PID控制是比例积分微分控制的简称**，将偏差的比例(P)、积分(I)和微分(D)通过线性组合构成控制量。
- PID控制器以其结构简单、稳定性好、调整方便而成为工业过程中应用最广泛的一类控制器。
- 至今在全世界过程控制中用的84%仍是纯PID调节器，若改进型包含在内则超过90%)。



PID控制器

$$G_c(s)=K_c(1+\frac{1}{T_i s}+T_d s)\quad \text{standard form}$$

$$G_c(s)=K_c(1+\frac{1}{T_i s})(1+T_d s)\quad \text{cascade form}$$

- 标准PID控制算法：

$$u(t)=K_c[e(t)+\frac{1}{T_i}\int_0^t e(t)dt+T_d\frac{de(t)}{dt}]+u_0$$

比例增益　积分时间　微分时间

偏差为0时的调节器输出，称为稳态工作点

> 数字算式

积分用求和代替，微分用有限差分代替

$$u(k)=K_c\left\{e(k)+\frac{T_s}{T_i}\sum_{j=0}^{k}e(j)+\frac{T_d}{T_s}[e(k)-e(k-1)]\right\}+u_0$$

控制周期

位置型PID

给出的是执行机构在采样时刻kT_s时的位置或开度

> **增量型PID算法**

给出的是执行机构在采样时刻kT_s时的增量值

$$\Delta u(k)=u(k)-u(k-1)=K_c\left\{[e(k)-e(k-1)+\frac{T_s}{T_i}e(k)+\frac{T_d}{T_s}[e(k)-2e(k-1)+e(k-2)]\right\}$$

$$u(k)=u(k-1)+\Delta u(k)$$

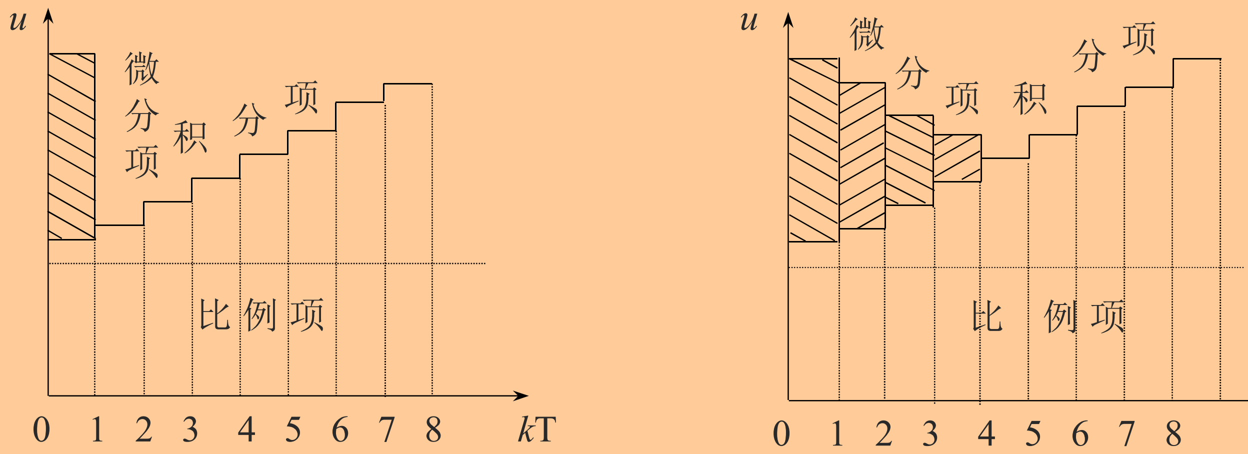
PID控制器（

当偏差的阶跃幅度较大时，受执行机构能力限制，不能在一个周期达到应有的开度，输出将失真，不能充分发挥微分作用

- 标准PID控制算法的改进

标准的微分作用只能维持一个采样周期,且作用很强

实际微分作用能缓慢地保持几个采样周期，使执行机构能较好地跟踪微分作用输出



(A) 理想微分PID　　(B) 实际微分PID

➢ **实际微分PID控制算法**

$$G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{T_d/_N\, s+1}\right)$$

$$G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{T_d s+1}{T_d/_N\, s+1}\right)$$

PID控制器（续）

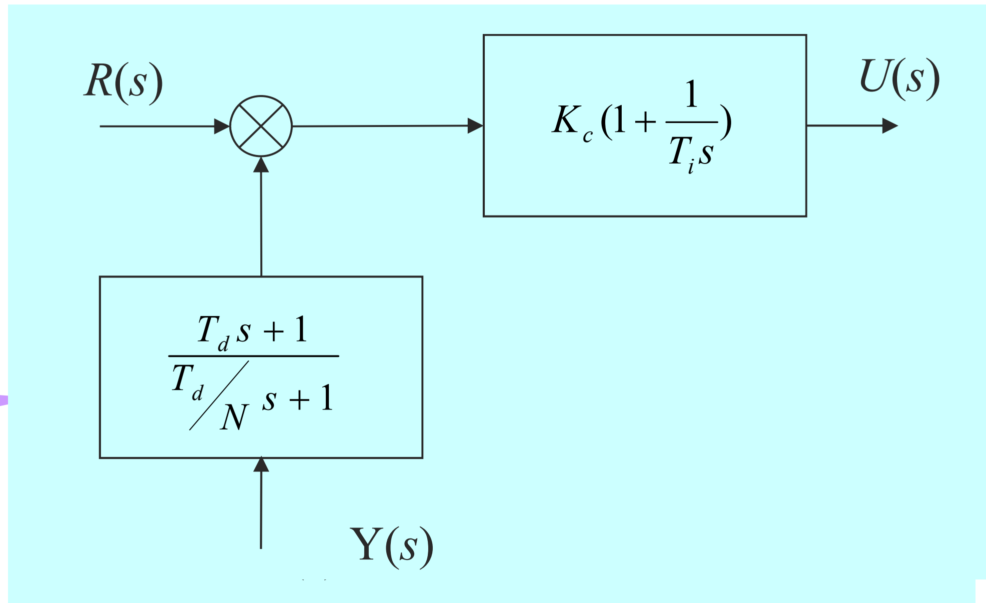
- 标准PID控制算法的改进

  - 在实际运行中，操作工对设定值的调整大多是阶跃形式的，使得微分输出产生极大的突跳(比例输出也会突跳，但没有微分输出严重)，这样不利于生产的稳定操作。

➢ **微分先行PID控制算法**

将控制器的微分部分从误差通道移至测量通道

只对测量值（被控量）进行微分，而不对偏差微分，也即对设定值值无微分作用



PID控制器（续）

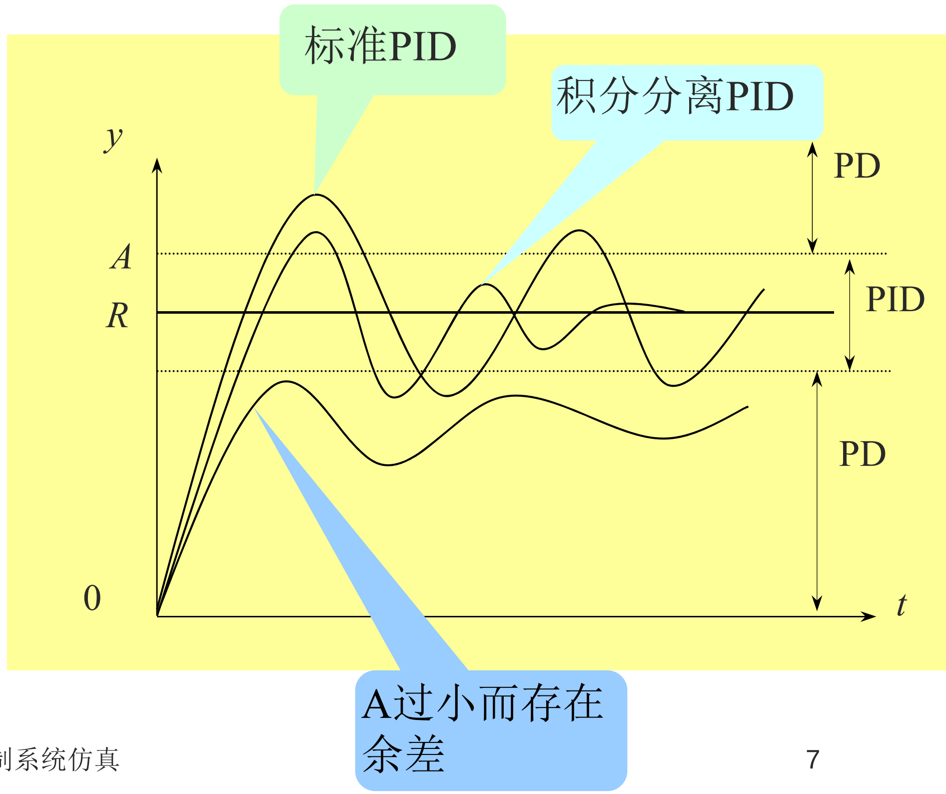
- 标准PID控制算法的改进
  - 采用标准的PID控制算法时，当扰动较大或给定值大幅度变化时，由于产生较大的偏差，加上系统本身的惯性及滞后，在积分作用下，系统往往产生较大的超调和长时间的振荡。

➢ 积分分离PID算法

✓ 在偏差$e(k)$较大时，暂时取消积分作用；当偏差$e(k)$小于某一设定值A时，才将积分作用投入：

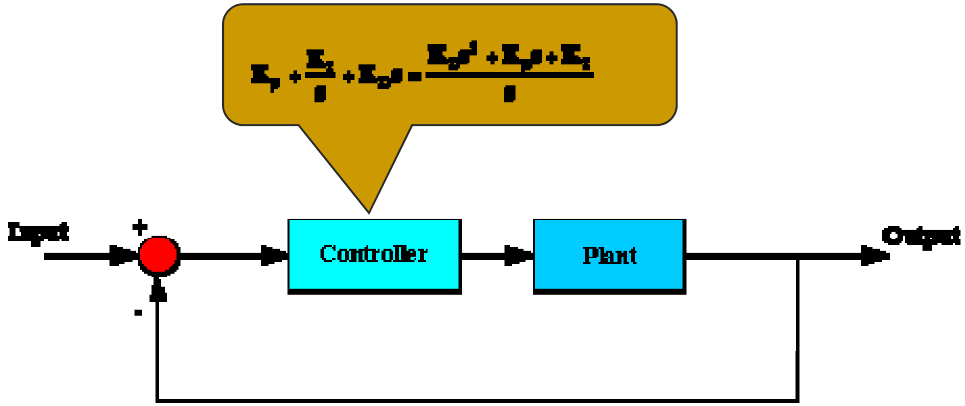
当 $|e(k)| > A$ 时， 用P或PD 控制；
当 $|e(k)| \le A$ 时， 用PI或PID控制。

◆ A过大，起不到积分分离的作用；若A过小，即偏差$e(k)$一直在积分区域之外，长期只有P或PD 控制，系统将存在余差



The characteristics of P, I, and D controllers

- A proportional controller (Kp) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error.



- An integral control (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse.

- A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

The characteristics of P, I, and D controllers

- Effects of each of controllers Kp, Kd, and Ki on a closed-loop system

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

General tips for designing a PID controller

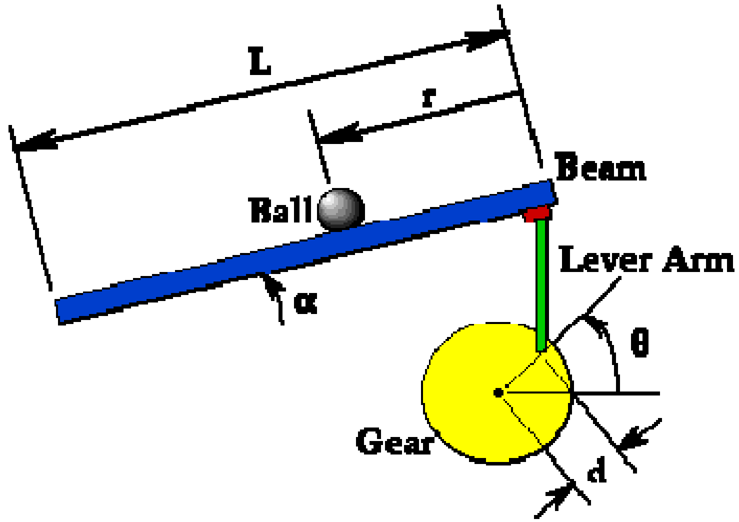
1. Obtain an open-loop response and determine what needs to be improved
2. Add a proportional control to improve the rise time
3. Add a derivative control to improve the overshoot
4. Add an integral control to eliminate the steady-state error
5. Adjust each of Kp, Ki, and Kd until you obtain a desired overall response. You can always refer to the table shown in previous page to find out which controller controls what characteristics.

please keep in mind that you do not need to implement all three controllers (proportional, derivative, and integral) into a single system, if not necessary. For example, if a PI controller gives a good enough response, then you don't need to implement derivative controller to the system. Keep the controller as simple as possible.

Solution to the Ball & Beam Problem Using PID Control

- The open-loop transfer function of the plant

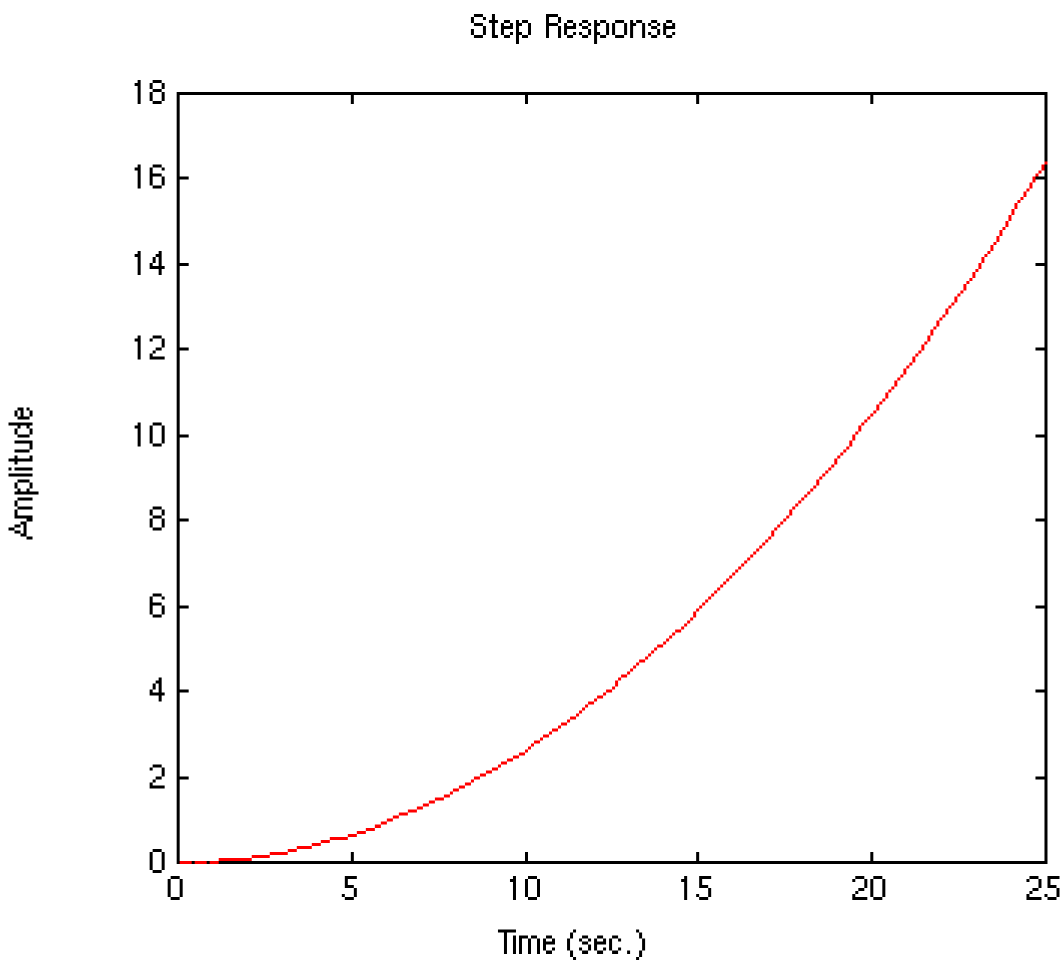
$$\frac{R(s)}{\Theta(s)} = -\frac{mgd}{L\left(\frac{J}{R^2}+m\right)}\frac{1}{s^2}$$



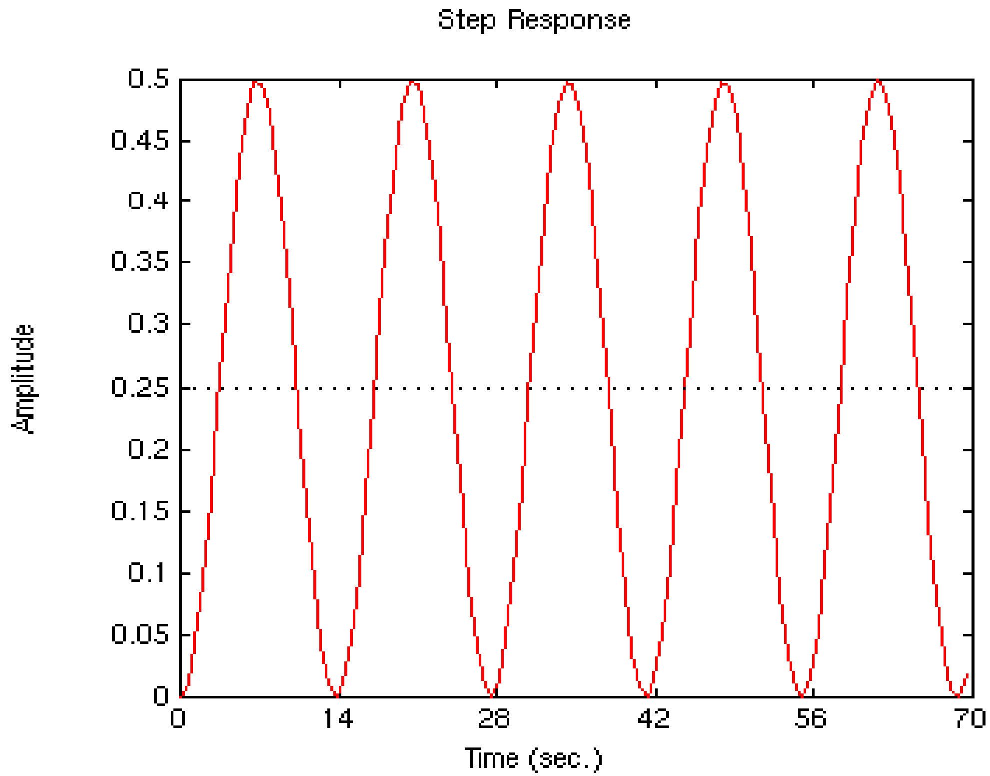
- The design criteria for this problem are:
  - Settling time less than 3 seconds
  - Overshoot less than 5%

Open-loop Response

- the open-loop system is unstable, that causes the ball to roll off from the end of the beam.



Proportional Control

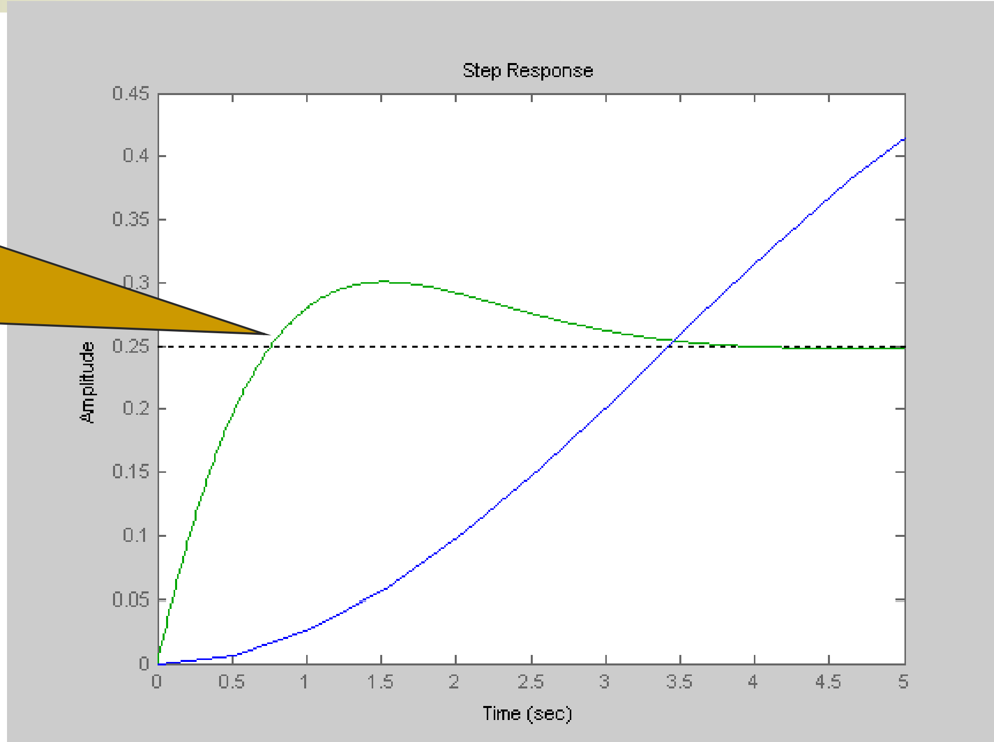


the system remains unstable.

- The closed-loop response to a step input of 0.25 m for proportional control with a proportional gain (kp) equal to 1

Proportional-Derivative Control

the system is stable but the overshoot is much too high and the settling time needs to go down a bit.

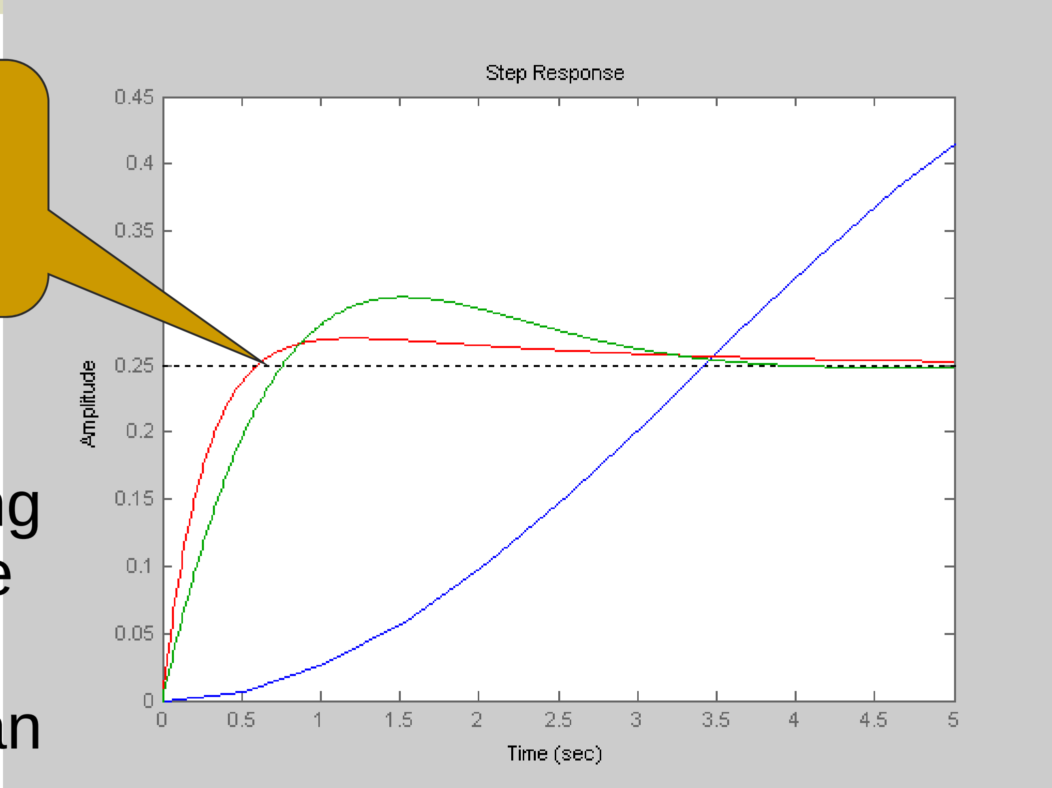


- by increasing kd we can lower overshoot and decrease the settling time slightly.

- Kp=10
- Kd=10

Proportional-Derivative Control

The overshoot criterion is met but the settling time needs to come down a bit.

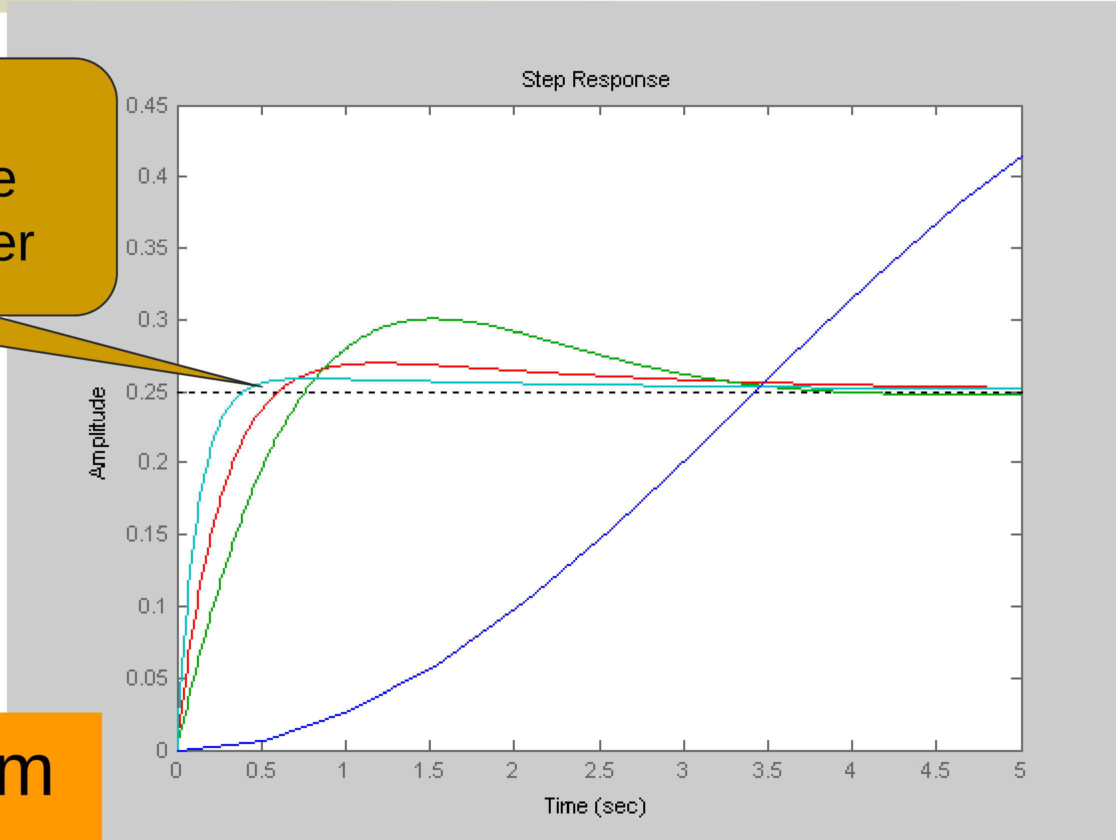


- To decrease the settling time we may try increasing the Kp slightly to increase the rise time.
- The derivative gain Kd can also be increased to take off some of the overshoot that increasing Kp will cause.

- Kp=10
- Kd=20

Proportional-Derivative Control

all the control objectives have been met without the use of an integral controller



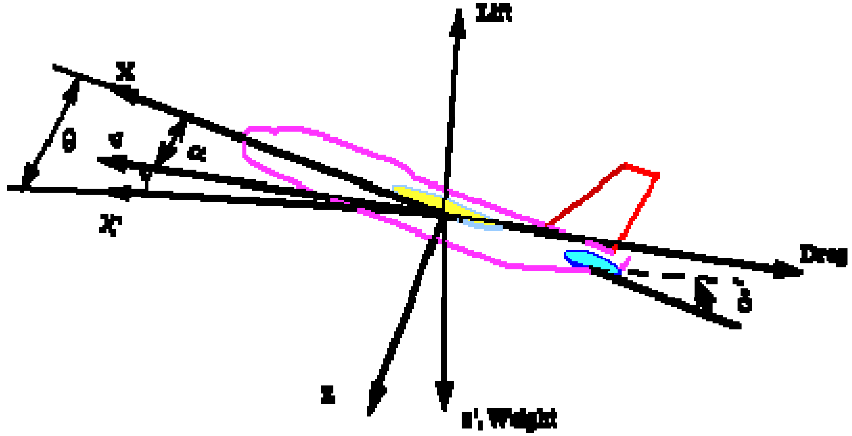
➢For a control problem there is more than one solution for the problem.

- Kp=15
- Kd=40

PID Design method for the Pitch Controller

- the transfer function was derived as

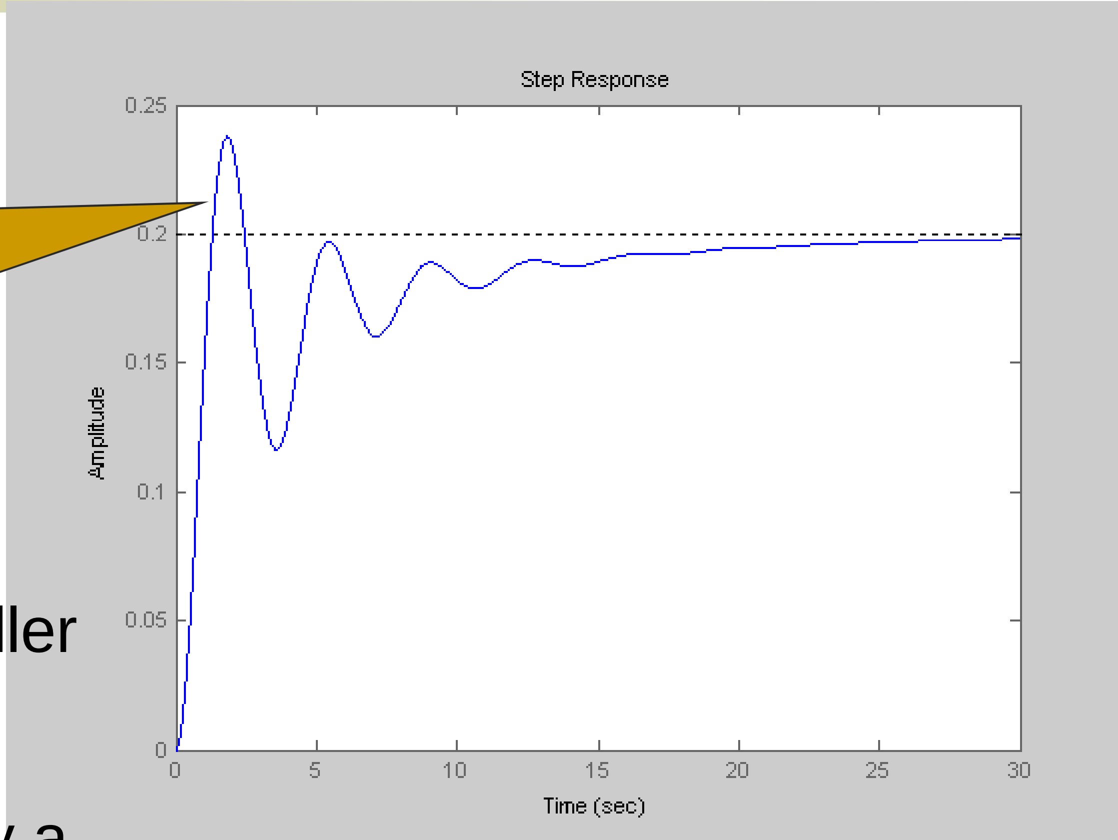
$$\frac{\theta(s)}{\delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s}$$



- Design requirements
  - Overshoot: Less than 10%
  - Rise time: Less than 2 seconds
  - Settling time: Less than 10 seconds
  - Steady-state error: Less than 2%

Proportional control

both the overshoot and the settling time need some improvement.

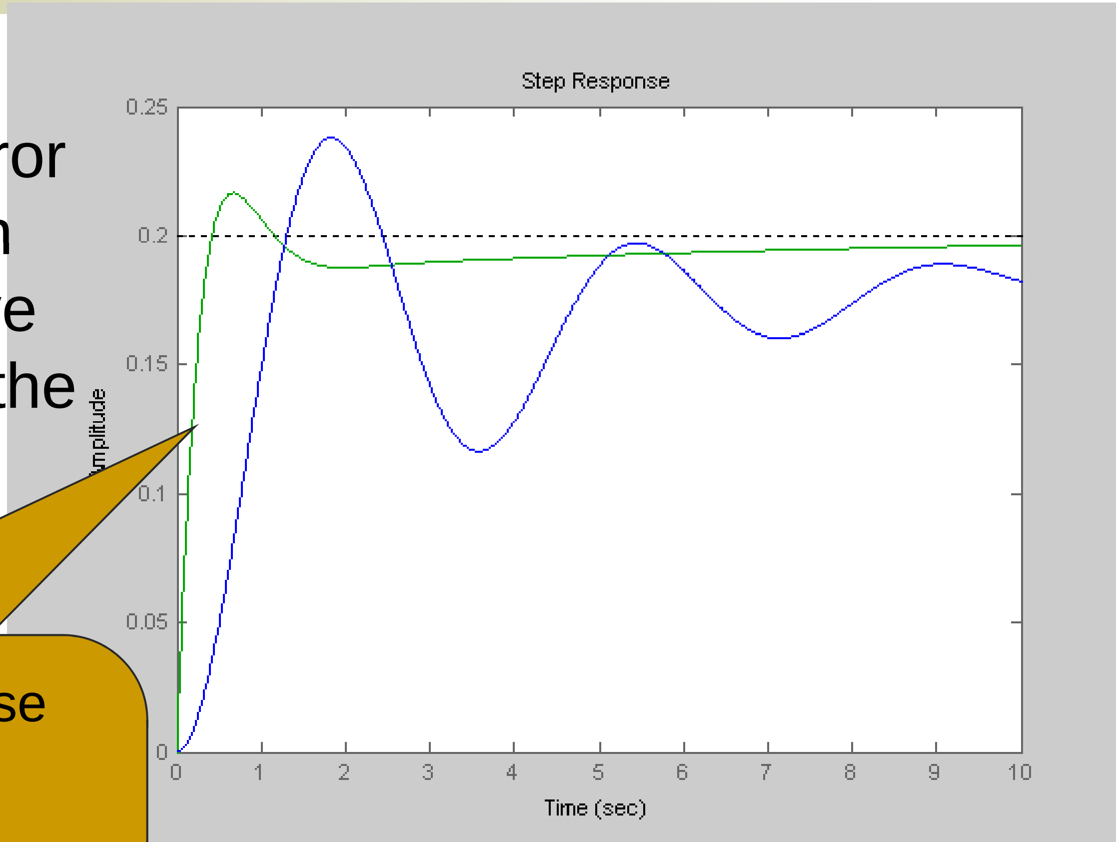


- the derivative controller will reduce both the overshoot and the settling time. Let's try a PD controller.

- Kp=2

PD control

- with several trial-and-error runs, a proportional gain (Kp) of 9 and a derivative gain (Kd) of 4 provided the reasonable response.

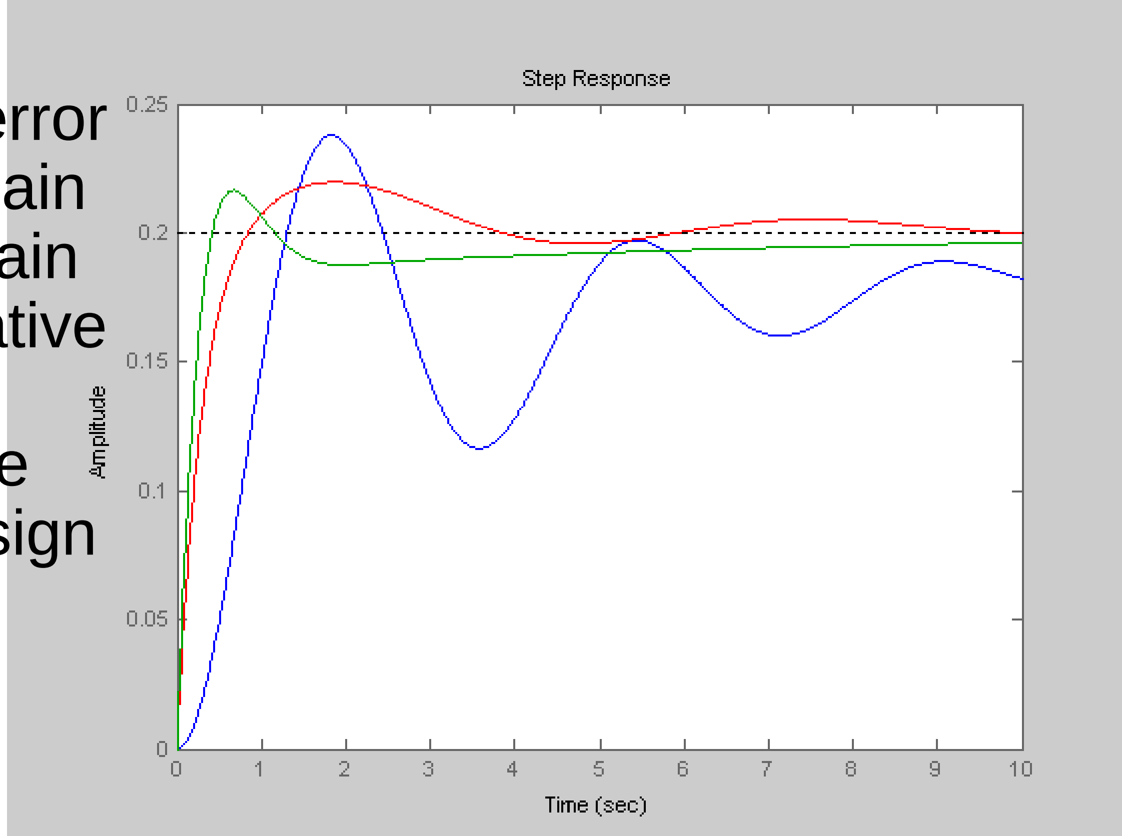


This step response shows the rise time of less than 2 seconds, the overshoot of less than 10%, the settling time of less than 10 seconds, and the steady-state error of less than 2%. All design requirements are satisfied.

the integral controller (Ki) can be added to reduce the sharp peak and obtain smoother response.

PID Control

- After several trial-and-error runs, the proportional gain (Kp) of 2, the integral gain (Ki) of 4, and the derivative gain (Kd) of 3 provided smoother step response that still satisfies all design requirements.



Changing one gain might change the effect of the other two. As a result, you may need to change other two gains as you change one gain.

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  - PID控制器设计
- **PID控制器的参数整定方法**
- **鲁棒PID控制器参数整定**

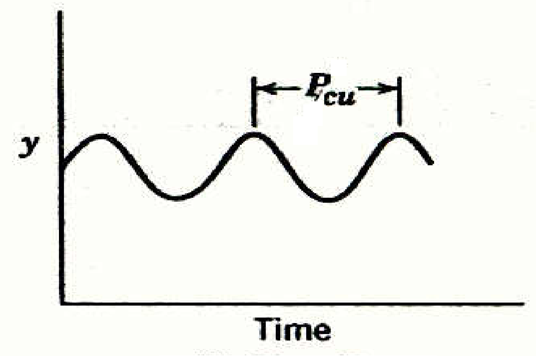
PID控制器的参数自动整定

- 稳定边界法——临界比例度法

  ➢ 在系统闭环情况下，去除积分与微分作用，让系统在纯比例器的作用下产生等幅振荡，利用此时的临界增益Kcu（Critical Gain）和临界振荡周期Pcu（Critical Frequency），根据Z-N经验规则，直接查表得到PID参数。

- 1/4 decay ratio -- too much oscillatory

类型	K_c	τ_I	τ_D
P	$0.5\ K_{CU}$	-	-
PI	$0.45\ K_{CU}$	$P_{CU}/1.2$	-
PID	$0.6\ K_{CU}$	$P_{CU}/2$	$P_{CU}/8$



- Modified Z-N settings for PID control

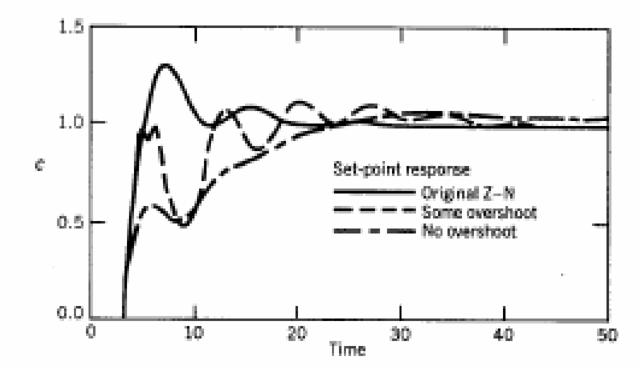
类型	K_c	τ_I	τ_D
original	$0.6\ K_{CU}$	$P_{CU}/2$	$P_{CU}/8$
Some overshoot	$0.33\ K_{CU}$	$P_{CU}/2$	$P_{CU}/3$
No overshoot	$0.2\ K_{CU}$	$P_{CU}/3$	$P_{CU}/2$

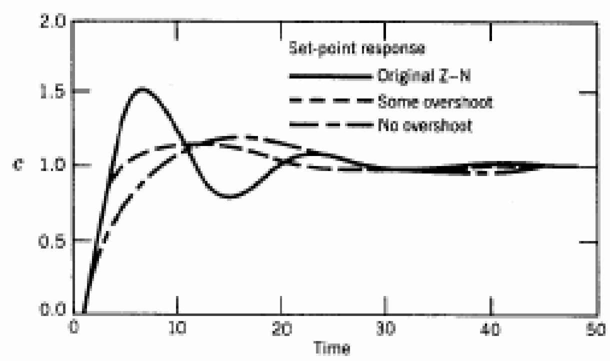
$$G_p(s) = \frac{4e^{-3.5s}}{7s+1} \qquad K_{CU} = 0.95 \qquad P_{CU} = 12$$

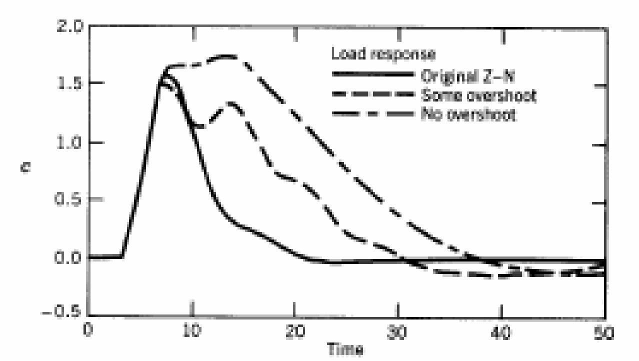
$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)} \qquad K_{CU} = 7.88 \qquad P_{CU} = 11.6$$

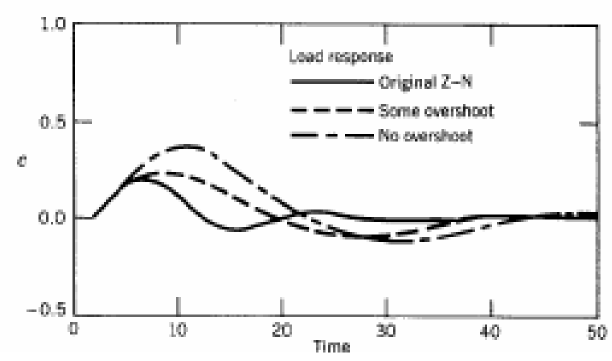
Controller	K_C	τ_I	τ_D
Original	0.57	6.0	1.5
Some overshoot	0.31	6.0	4.0
No overshoot	0.19	6.0	4.0

Controller	K_C	τ_I	τ_D
Original	4.73	5.8	1.45
Some overshoot	2.60	5.8	3.87
No overshoot	1.58	5.8	3.87





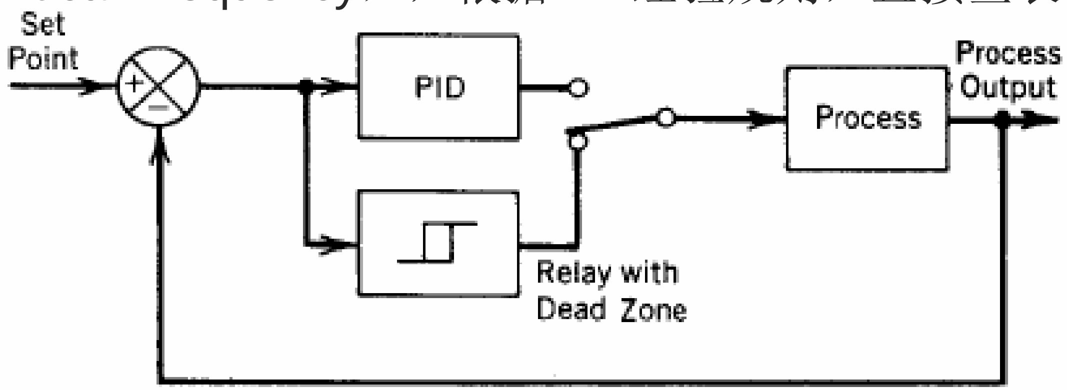


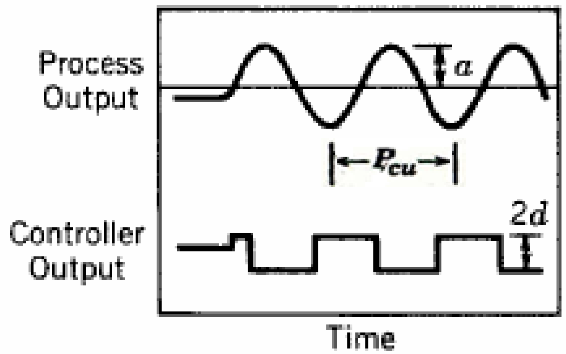


PID控制器的参数自动整定

- 基于继电反馈的参数整定法

➢用具有继电特性的非线性环节代替稳定边界法中的纯比例器，使系统产生稳定极限环振荡，获得所需的临界增益Kcu（Critical Gain）和临界振荡周期Pcu（Critical Frequency），根据Z-N经验规则，直接查表得到PID参数。





d - 继电器幅度

a - 输出等幅震荡的峰值

$$K_{CU} = \frac{4d}{\pi a}$$

PID控制器的参数自动整定

- Cohen-Coon参数整定方法

$$G(s)=\frac{Ke^{-\theta s}}{\tau s+1}$$

Empirical relation for ¼ decay ratio for FOPDT model

Controller	Settings	Cohen–Coon
P	K_c	$\frac{1}{K}\frac{\tau}{\theta}[1+\theta/3\tau]$
PI	K_c	$\frac{1}{K}\frac{\tau}{\theta}[0.9+\theta/12\tau]$
	τ_I	$\frac{\theta[30+3(\theta/\tau)]}{9+20(\theta/\tau)}$
PID	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[\frac{16\tau+3\theta}{12\tau}\right]$
	τ_I	$\frac{\theta[32+6(\theta/\tau)]}{13+8(\theta/\tau)}$
	τ_D	$\frac{4\theta}{11+2(\theta/\tau)}$

PID控制器的参数自动整定

$$IAE = \int_0^\infty |e(t)|\,dt$$

$$ISE = \int_0^\infty e(t)^2\,dt$$

$$ITAE = \int_0^\infty t|e(t)|\,dt$$

- 最小化积分误差

 - 1/4 decay ratio is too much oscillatory

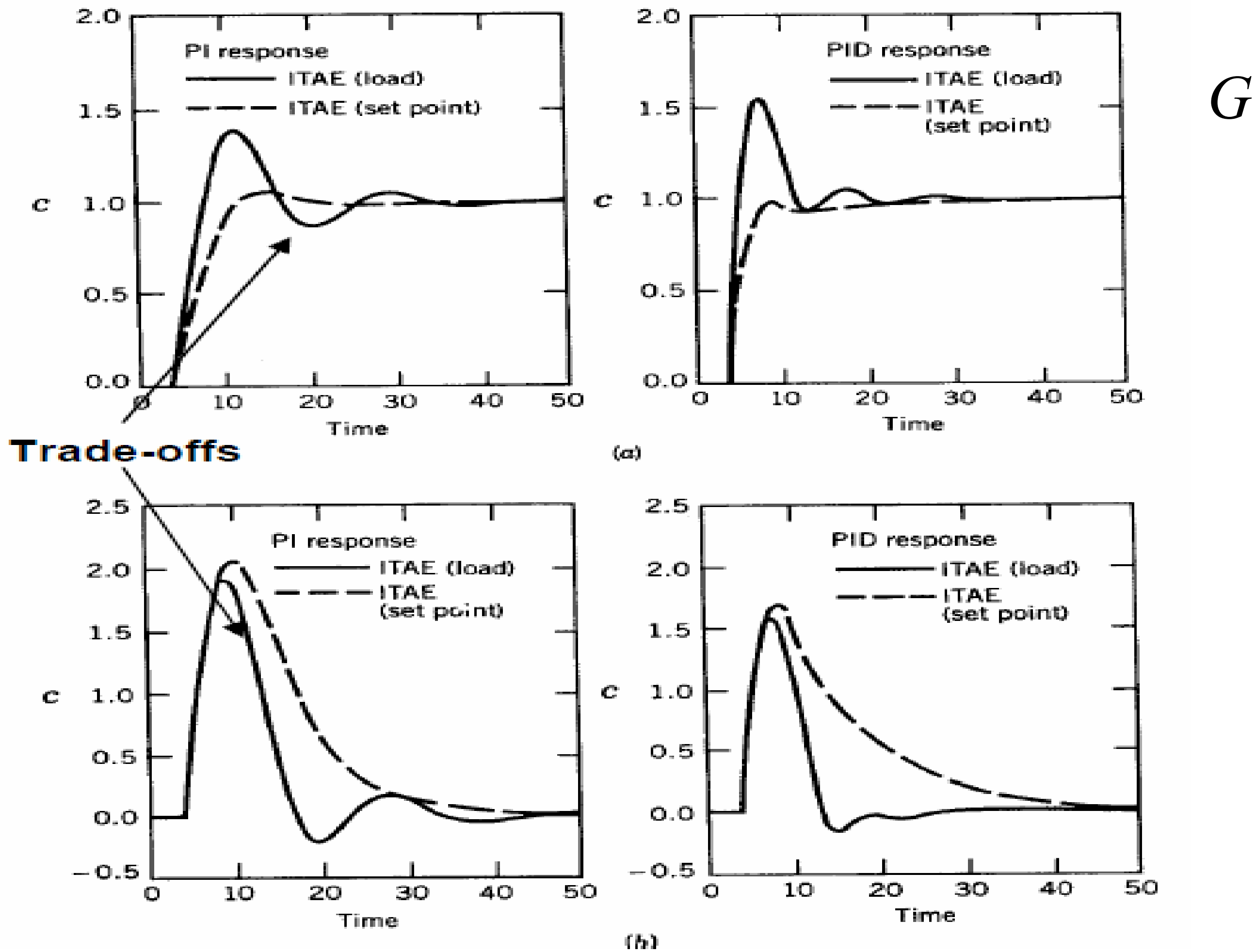
 - Decay ratio concerns only two peak points of the response

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

PID控制器的参数自动整定



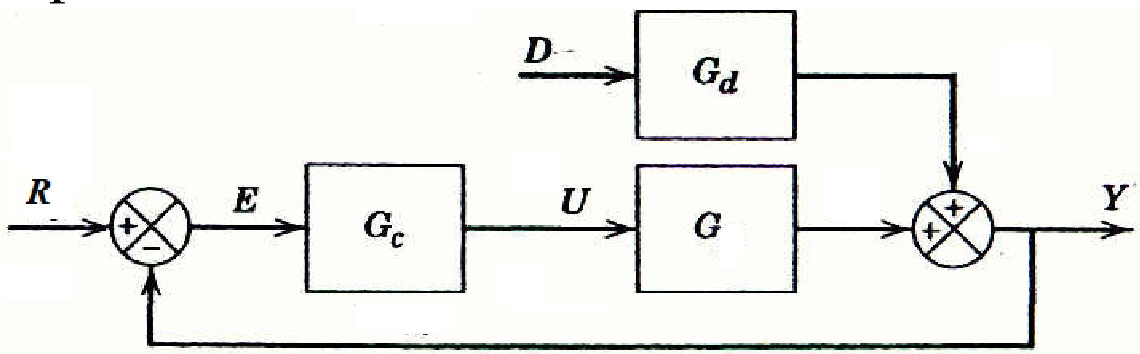
$$G(s)=\frac{4e^{-3.5s}}{7s+1}$$

PID控制器的参数自动整定

- Direct Synthesis Method

  The controller design is based on a process model and a desired closed-loop transfer function.

$$\frac{Y}{R}=\frac{G_c G}{1+G_c G}$$



Specify $\left(\frac{Y}{R}\right)_d \Rightarrow G_C=\frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)$

The specification of $(Y/R)_d$ is the key design decision $\left(\frac{Y}{R}\right)_d=\frac{e^{-\theta s}}{\tau_c s+1}$

PID控制器的参数自动整定

- Examples

1. $G(s)=\dfrac{Ke^{-\theta s}}{(\tau s+1)}$ and $(Y/R)_d=\dfrac{e^{-\theta s}}{\tau_c s+1}$

Physically realizable

$$G_c(s)=\frac{1}{G(s)}\left(\frac{e^{-\theta s}/(\tau_c s+1)}{1-e^{-\theta s}/(\tau_c s+1)}\right)=\frac{\tau s+1}{K}\boxed{\frac{1}{\tau_c s+1-e^{-\theta s}}}\ \text{(not a PID)}$$

With 1st-order Taylor series approx. ($e^{-\theta s}\approx 1-\theta s$)

$$G_c(s)=\frac{\tau s+1}{K}\frac{1}{(\tau_c+\theta)s}=\frac{\tau}{K(\tau_c+\theta)}\left(1+\frac{1}{\tau s}\right)\ \text{(PI)}$$

2. $G(s)=\dfrac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$ and $(Y/R)_d=\dfrac{e^{-\theta s}}{\tau_c s+1}$

$$G_c(s)=\frac{(\tau_1 s+1)(\tau_2 s+1)}{K}\frac{1}{(\tau_c+\theta)s}=\frac{(\tau_1+\tau_2)}{K(\tau_c+\theta)}\left(1+\frac{1}{(\tau_1+\tau_2)s}+\frac{\tau_1\tau_2}{(\tau_1+\tau_2)}s\right)\ \text{(PID)}$$

PID控制器的参数自动整定

- Direct Synthesis Method $$G = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

Consider three values of the desired closed-loop time constant: $\tau_c = 1, 3, \text{and } 10$. Evaluate the controllers for unit step changes in both the set point and the disturbance, assuming that $G_d = G$. Repeat the evaluation for two cases:

a. The process model is perfect ($\tilde{G} = G$).

b. The model gain is $\tilde{K} = 0.9$, instead of the actual value, $K = 2$. Thus,

$$\tilde{G} = \frac{0.9e^{-s}}{(10s+1)(5s+1)}$$

PID控制器的参数自动整定

- Direct Synthesis Method

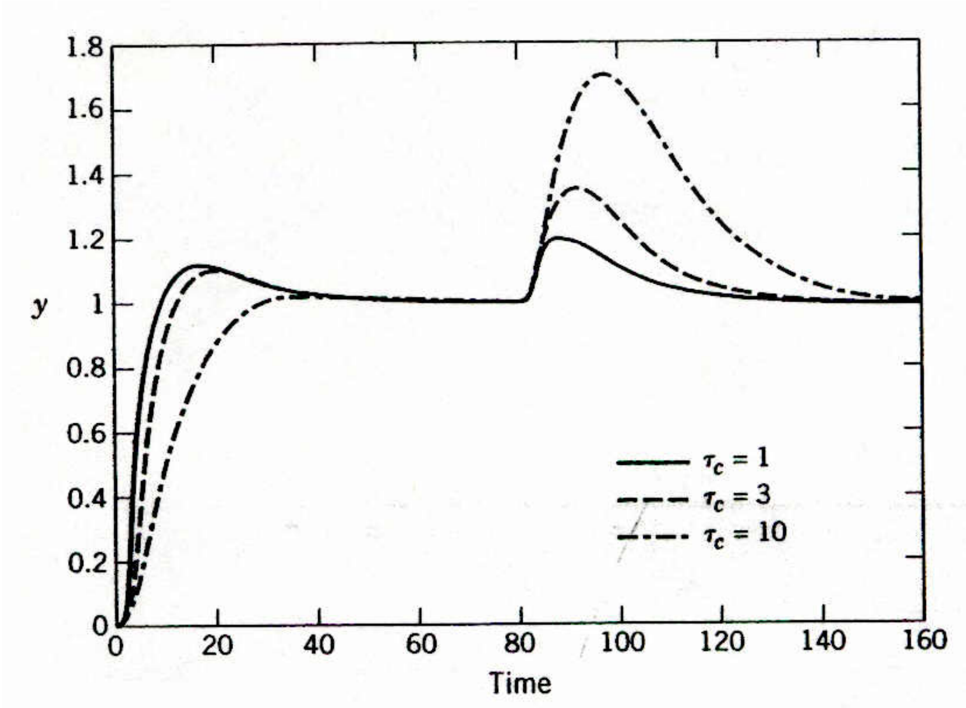
The controller settings for this example are:

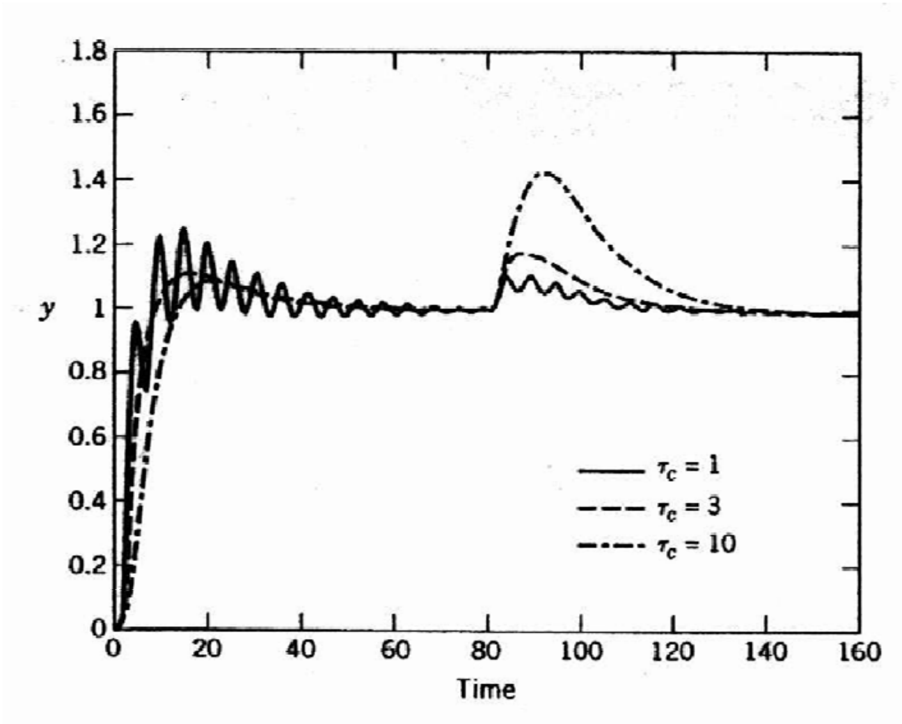
	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$K_c\left(\tilde{K} = 2\right)$	**3.75**	**1.88**	**0.682**
$K_c\left(\tilde{K} = 0.9\right)$	**8.33**	**4.17**	**1.51**
τ_I	**15**	**15**	**15**
τ_D	**3.33**	**3.33**	**3.33**

The values of K_c decrease as τ_c increases, but the values of τ_I and τ_D do not change

PID控制器的参数自动整定

- Direct Synthesis Method

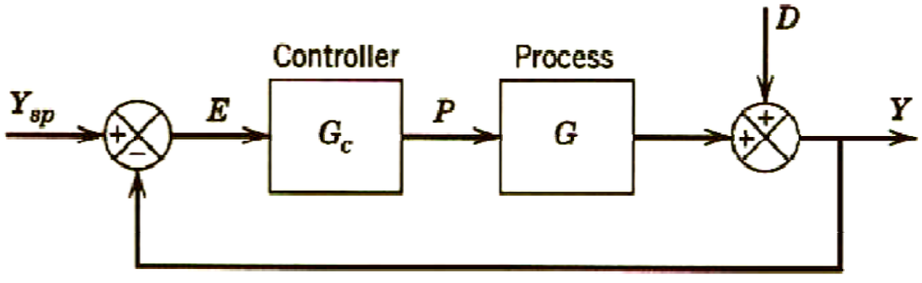




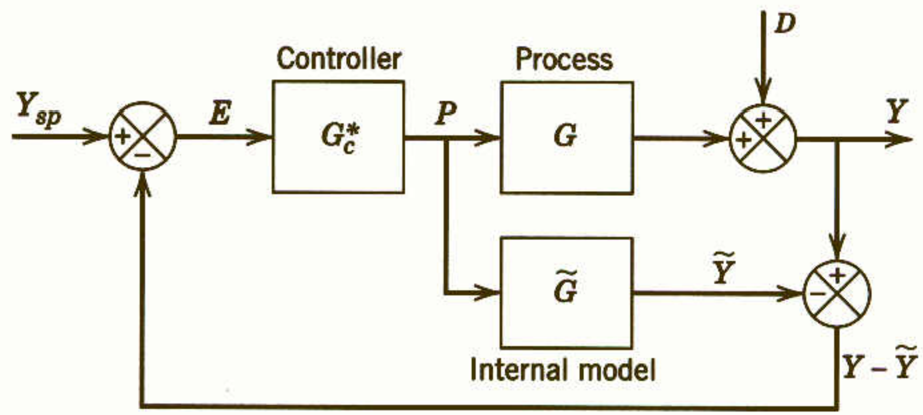
- Gain Correct
- Gain Incorrect

PID控制器的参数自动整定

- IMC-PID Design Method



(a) Classical feedback control



(b) Internal model control

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$$

$$Y = \frac{G_c^* G}{1 + G_c^* \left(G - \tilde{G} \right)} Y_{sp} + \frac{1 - G_c^* \tilde{G}}{1 + G_c^* \left(G - \tilde{G} \right)} D$$

$$Y = G_c^* G Y_{sp} + \left(1 - G_c^* G \right) D \text{ when } \tilde{G} = G$$

PID控制器的参数自动整定

- **Factor the process model as** $\tilde{G} = \tilde{G}_+ \tilde{G}_-$
  - $\tilde{G}_+$ contains any time delays and RHP zeros and is specified so that the steady-state gain is one
  - $\tilde{G}_-$ is the rest of ***G***
- **The controller is specified as**

$$G_c^* = \frac{1}{\tilde{G}_-} f$$

$$Y = \tilde{G}_+ f Y_{sp} + \left(1 - f\tilde{G}_+\right) D \text{ when } \tilde{G} = G$$

- **IMC filter** f is a low-pass filter with a steady-state gain of one
- **Typical IMC filter** $f = \dfrac{1}{\left(\tau_c s + 1\right)^r}$
- The τ_c is the desired closed-loop time constant and parameter r is a positive integer that is selected so that the order of numerator of G_c^* is same as the order of denominator or exceeds the order of denominator by one.

PID控制器的参数自动整定

- IMC-PID Design Method
  - **Example**
    - FOPDT model with 1/1 Pade approximation

$$G(s) = \frac{Ke^{-\theta s}}{(\tau s+1)}$$

$$\tilde{G} = \frac{K(1-\theta s/2)}{(1+\theta s/2)(\tau s+1)}$$

$$\tilde{G}_+ = 1-\theta s/2 \qquad \tilde{G}_- = \frac{K}{(1+\theta s/2)(\tau s+1)}$$

$$G_c^* = \frac{1}{\tilde{G}_-} f = \frac{(1+\theta s/2)(\tau s+1)}{K} \frac{1}{(\tau_c s+1)}$$

$$G_c = \frac{G_c^*}{1-G_c^*\tilde{G}} = \frac{(1+\theta s/2)(\tau s+1)}{K(\tau_c+\theta/2)s} \quad \text{(PID)}$$

$$K_c = \frac{1}{K}\frac{(\tau+\theta/2)}{(\tau_c+\theta/2)} \qquad \tau_I = \tau+\theta/2 \qquad \tau_D = \frac{\tau\theta/2}{\tau+\theta/2}$$

F

IM

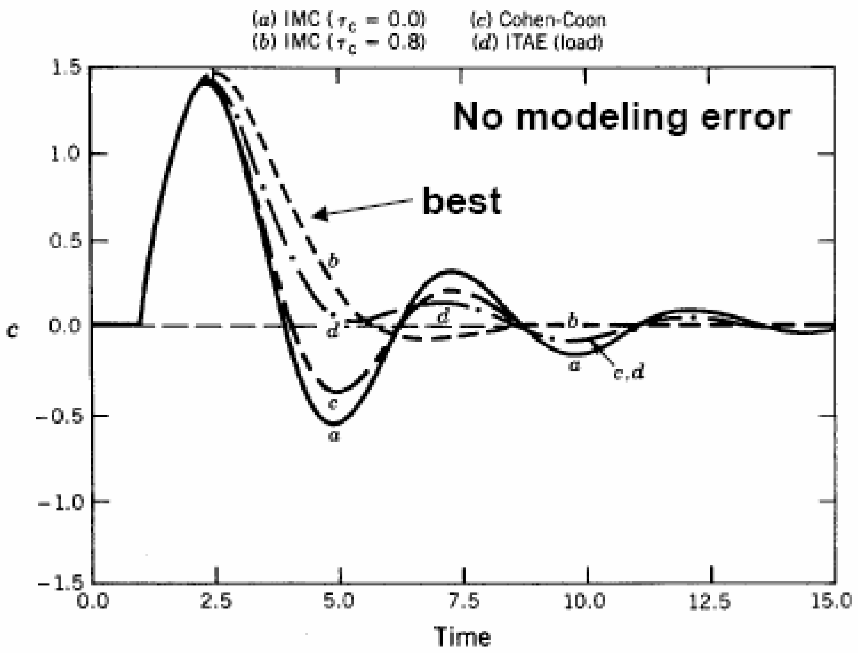
Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

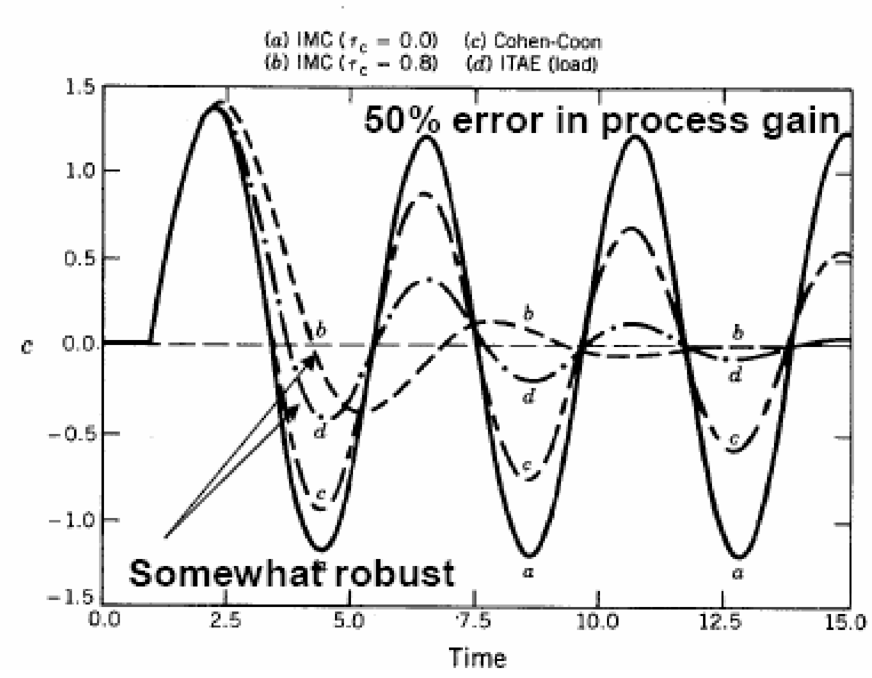
Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s+1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
C	$\frac{K}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$, $\beta>0$	$\frac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s+1)}$	$\frac{2\tau_c+\tau}{\tau_c^2}$	$2\tau_c+\tau$	$\frac{2\tau_c\tau}{2\tau_c+\tau}$
G	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau}{\tau_c+\theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$	$\tau+\frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$
I	$\frac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\frac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\frac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\frac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$
K	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau_1\tau_2}{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
L	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_e+\theta}$	$2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau^2}{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{\left(\tau_c+\frac{\theta}{2}\right)^2}$	$2\tau_c+\theta$	$\frac{\tau_c\theta+\frac{\theta^2}{4}}{2\tau_c+\theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\tau+\theta$	$\frac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$

PID控制器的参数自动整定

- IMC-PID Design Method

$$G(s) = \frac{2e^{-s}}{s+1}$$





主要内容

- **PID控制器**
  - 标准PID控制算法
  - 标准PID控制算法的改进
  - PID控制器设计
- **PID控制器的参数自动整定**
- **鲁棒PID控制器参数整定**

鲁棒PID控制器参数整定思想

- 常规整定方法：通过某种方式获取系统的模型参数，再按照某种规则，由模型参数定出PID参数。
  - 过程条件件发生变化，如原料的性质、处理量的变化、设备故障、环境条件的变化等，均会导致工艺过程模型发生变化

导致控制品质变坏，甚至出现振荡或者发散现象

重新整定控制器参数以适应新的工况

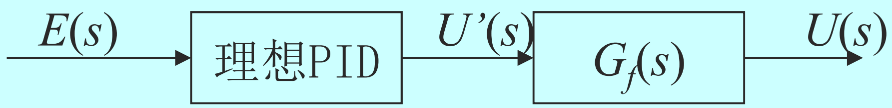
鲁棒PID控制器的参数整定方法

过程模型常在一定范围内变动

- **基于极小极大原理， 寻找一组合理的PID参数，使控制器的性能对于模型的不确定性不敏感，并且在模型的一定变化范围内保证控制器有良好的控制性能。**

鲁棒PID控制器参数整定算法

- PID控制器形式



$$G_0 = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\left(\frac{1}{T_f s+1}\right)$$

实际微分PID，对控制器的输出做滤波处理，防止控制动作过大，给控制系统引入大的振荡。

- 性能指标
  - ISE （Integral Squared Error）

    $$ISE = \int_0^{\infty} e(t)^2\,dt$$

  - ITSE（Integral Time Squared Error）

    $$ITSE = \int_0^{\infty} t e(t)^2\,dt$$

  - IAE（Integral Average Error）

    $$IAE = \int_0^{\infty} |e(t)|\,dt$$

  - ITAE（Integral Time Average Error）

    $$ITAE = \int_0^{\infty} t\,|e(t)|\,dt$$

不同的判据对系统性能“何为最优”有不同的判断。

鲁棒PID控制器参数整定算法（续）

- 过程模型的表示形式

$$G_p = \frac{K}{Ts+1}e^{-\tau s}$$

- 鲁棒PID控制器参数整定的设计思想

$$\underset{K_c,T_i,T_d,T_f}{Min}\ \underset{K,T,\tau}{Max}\ ISE(K_c,T_i,T_d,T_f,K,T,\tau)$$

在最坏的工艺情况下寻找最佳的控制性能。

➢ 考虑给定值单位阶跃变化下，计算性能指标ISE

$$ISE = \int_0^{\infty} e(t)^2\,dt = \sum_{k=1}^{N}(sp(k)-y(k))^2 T_s$$

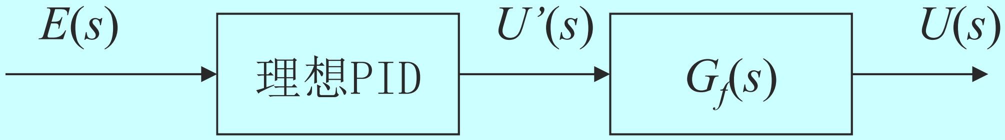
$$sp(k) = \begin{cases} 1, k \ge 0 \\ 0, k \le 0 \end{cases}$$

其中，采样周期为T_s，仿真时间取NT_s（N为仿真次数）

鲁棒PID控制器参数整定算法（续）

- 将控制器中的后置滤波器并入对象传递函数一并离散化，广义的过程模型

$$\frac{y(s)}{u(s)}=\frac{ke^{-\tau s}}{Ts+1}\times\frac{1}{T_f s+1}$$



➤ 对应的微分方程（初值条件为零）为

$$TT_f\ddot{y}(t)+(T+T_f)\dot{y}(t)+y(t)=Ku(t-\tau)$$

$$TT_f\dddot{y}(t)+(T+T_f)\ddot{y}(t)+\dot{y}(t)=K\dot{u}(t-\tau)$$

$$\frac{dy}{dt}\doteq\frac{\Delta y}{\Delta t}=\frac{y(k)-y(k-1)}{T_s}$$

$$\frac{d^2y}{dt^2}\doteq\frac{y(k)-2y(k-1)+y(k-2)}{T_s^2}$$

$$\frac{d^3y}{dt^3}\doteq\frac{y(k)-3y(k-1)+3y(k-2)-y(k-3)}{T_s^3}$$

$$\left[\frac{TT_f}{KT_s^2}+\frac{T+T_f}{KT_s}+\frac{1}{K}\right]y(k)+\left[-\frac{3TT_f}{KT_s^2}-\frac{2(T+T_f)}{KT_s}-\frac{1}{K}\right]y(k-1)+\left[\frac{3TT_f}{KT_s^2}+\frac{T+T_f}{KT_s}\right]y(k-2)+\left[-\frac{TT_f}{KT_s^2}\right]y(k-3)=\Delta u(k-\tau/T_s)$$

鲁棒PID控制器参数整定算法（续）

- 数字PID增量算式：

$$\Delta u(k)=K_c\left\{\left[e(k)-e(k-1)\right]+\frac{T_s}{T_i}e(k)+\frac{T_d}{T_s}\left[e(k)-2e(k-1)+e(k-2)\right]\right\}$$

e(k)=$sp(k)$-$y(k)$
$sp(k+i)$=const, i=1, …

$$\Delta u(k)=K_c\left[(1+\frac{T_s}{T_i}+\frac{T_d}{T_s})sp(k)-(1+\frac{2T_d}{T_s})sp(k-1)+(\frac{T_d}{T_s})sp(k-2)\right]-K_c\left[(1+\frac{T_s}{T_i}+\frac{T_d}{T_s})y(k)-(1+\frac{2T_d}{T_s})y(k-1)+(\frac{T_d}{T_s})y(k-2)\right]$$

$$\begin{aligned}y(k)=&\frac{1}{\frac{TT_iT_f}{T_s^3}+\frac{T_i(T+T_f)}{T_s^2}+\frac{T_i}{T_s}}\cdot\Bigg\{KK_c\left[\left(1+\frac{T_i}{T_s}+\frac{T_iT_d}{T_s^2}\right)sp(k-p)-\left(\frac{T_i}{T_s}+\frac{2T_iT_d}{T_s^2}\right)sp(k-p-1)+\frac{T_iT_d}{T_s^2}sp(k-p-2)\right]\\&+\left[\frac{3TT_iT_f}{T_s^3}+\frac{2T_i(T+T_f)}{T_s^2}+\frac{T_i}{T_s}\right]y(k-1)-\left[\frac{3TT_iT_f}{T_s^3}+\frac{T_i(T+T_f)}{T_s^2}\right]y(k-2)+\frac{TT_iT_f}{T_s^3}y(k-3)\\&-KK_c\left[\left(1+\frac{T_i}{T_s}+\frac{T_iT_d}{T_s^2}\right)y(k-p)-\left(\frac{T_i}{T_s}+\frac{2T_iT_d}{T_s^2}\right)y(k-p-1)+\frac{T_iT_d}{T_s^2}y(k-p-2)\right]\Bigg\}\end{aligned}$$

$$p=\tau/T_s+1$$

鲁棒PID控制器参数整定算法（续）

$$ISE = \int_0^\infty e(t)^2\,dt = \sum_{k=1}^{N} (sp(k) - y(k))^2 T_s$$

$$\min_{K_c,T_i,T_d,T_f} \; \max_{K,T,\tau} ISE(K_c,T_i,T_d,T_f,K,T,\tau)$$

可以任意选取，一般可选取对象特性为标准值时所确定的控制器最佳整定值

- 第一步优化命题

$$\max_{K,T,\tau} ISE(K_c,T_i,T_d,T_f,K,T,\tau)$$

$$s.t. \quad 0.4K_0 \le K \le 1.6K_0,$$
$$0.4T_0 \le T \le 1.6T_0,$$
$$0.4\tau_0 \le \tau \le 1.6\tau_0$$

对于给定的PID参数，求取性能指标最差的模型参数

测试得到的标称模型参数

- 第二步优化命题

$$\min_{K_c,T_i,T_d,T_f} ISE(K_c,T_i,T_d,T_f,K,T,\tau)$$

对第一步得到的性能最差的模型计算最优的PID参数

- 返回第一步，对第二步优化得到的结果重新计算最差的情形

- 收敛准则 $\|\Delta ISE\| \le \varepsilon$ $\quad \|\Delta X\| \le \varepsilon$

$$X = (K_c,T_i,T_d,T_f,K,T,\tau)^T$$

鲁棒PID控制器示例

- 有一供气压力系统，经实测后得到其近似的一阶惯性加纯滞后过程模型

$$G(s)=\frac{0.8}{1.07s+1}e^{-2.1s}$$

在不确定度达40%的条件下（模型参数在原值的40%以内波动），用鲁棒PID参数整定方法来整定PID控制器的参数。

$$\begin{aligned}&\min_{K_c,T_i,T_d,T_f}\ \max_{K,T,\tau} ISE(K_c,T_i,T_d,T_f,K,T,\tau)\\ &s.t.\qquad 0.6K_0\le K\le 1.4K_0\\ &\qquad\quad 0.6T_0\le T\le 1.4T_0\\ &\qquad\quad 0.6\tau_0\le \tau\le 1.4\tau_0\end{aligned}$$

采用设定值单位阶跃输入，采样周期0.2s，仿真次数500次（即仿真时间为100s）。

鲁棒PID控制器示例（续）

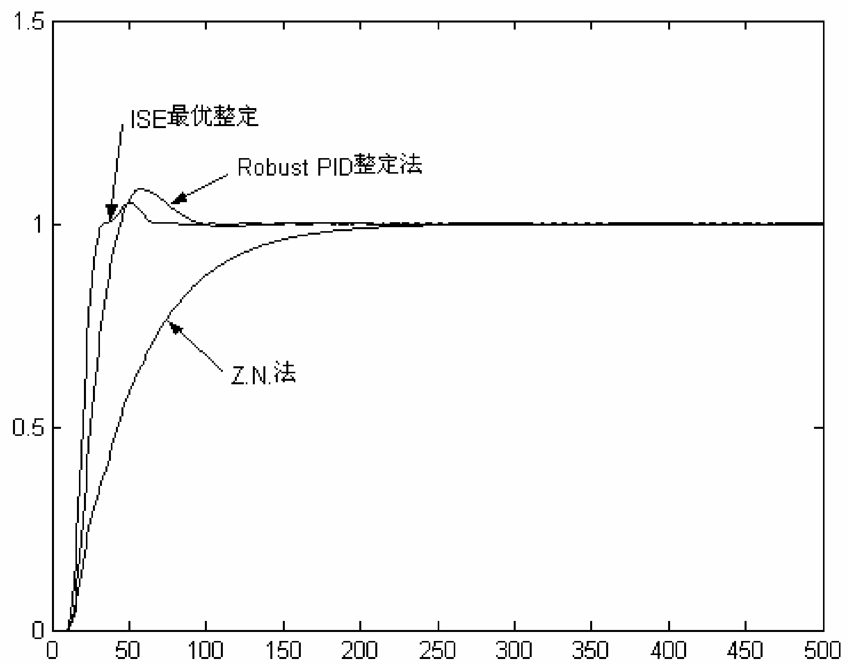
PID 算法整定结果（不确定度为 40%）

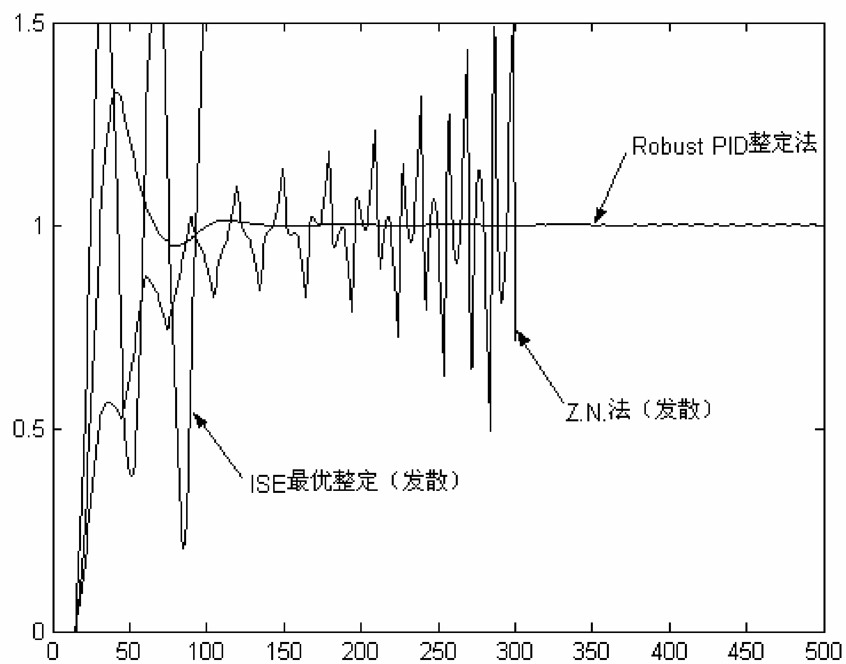
算 法	PID 参数				正常情况下 ISE 值（100 秒内）	最坏工况下的 ISE 值(100 秒内)	最坏工况下的对象特性参数		
	K_c	T_i	T_d	T_f			K	T	τ
Zieglor-Nichlos 参数整定方法	0.764	4.20	1.05	/	6.637	发散	1.12 ($1.4K_0$)	0.642 ($0.6T_0$)	2.94 ($1.4\tau_0$)
ISE 指标最佳参数整定	0.984	1.46	1.05	/	3.3366	发散	1.12	0.642	2.94
Robust PID	0.499	1.38	1.41	0.246	4.1894	4.5179	1.12	0.642	2.94

- Zieglor-Nichlos方法：$K_c = 1.2T/K\tau,\quad T_i = 2\tau,\quad T_d = 0.5\tau$
- ISE指标最优参数整定：$K_c = \dfrac{a_1}{K}\left[\dfrac{\tau}{T}\right]^{b_1}$ $T_i = \dfrac{T}{a_2 + b_2(\tau/T)}$ $T_d = a_{3T}\left[\dfrac{\psi}{T}\right]^{b_3}$

鲁棒PID控制器示例（续）

- 系统对单位阶跃的仿真响应曲线
  - 正常情况
  - 最坏工况





总结

- **PID控制器**
  - 标准PID控制算法
  - 标准PID控制算法的改进
  - PID控制器设计
- **PID控制器的参数自动整定**
- **鲁棒PID控制器参数整定**

The End

谢谢！