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I-PD Controller Design for Integrating Time Delay Processes Based on Optimum Analytical Formulas

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Abstract: In industrial applications, it is possible to encounter processes that have an integrator in its transfer function. The most widely used controllers in the control of these processes are Proportional-Integral-Derivative (PID) controllers. However, it is well known that PID controllers do not perform well in controlling integrating processes. Hence, in this study, the use of I-PD controllers for controlling integrating processes has been given. Optimal and analytical tuning rules have been derived to identify tuning parameters of the I-PD controller. Simulation examples have been provided to show the use of the proposed optimal I-PD tuning formulas. Comparisons with existing PID and I-PD design methods to control integrating processes have been supplied to illustrate the closed loop performance of the proposed optimal I-PD design approach.

Keywords: PID, I-PD, Integrating process, Time delay, Integral performance indices

1. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers constitute a very large part of the controllers used in the industrial applications (K.J. Åström and Hägglund, 1995a). The most important reason behind this is to have a simple structure and yet to perform quite well and robustly in many control applications. Being the most popular controller, they still attract researchers' attention. An excellent collection on the PID controller design methods can be found in (K. J. Åström and Hägglund, 1995b; O'Dwyer, 2006).

There are very different approaches in the literature for the design of PID controllers. Minimization of the error signal using integral performance criteria has been shown to be one of the very effective approaches for PID controller design. Zhuang and Atherton (Zhuang and Atherton, 1993) obtained tuning rules for a PID controller by minimizing time moment weighted integral performance criterion, assuming a stable first order plus dead time plant transfer function. Visioli (Visioli, 2001) carried out similar calculations based on integral performance indexes in order to achieve optimal PID controller parameters for processes with an integrator and an unstable plant transfer function. Kaya (Kaya, 2001) obtained optimum PI and PID controller settings for a stable first order dead time delay and second order plus dead time delay, where the controllers are used in the Smith predictor structure. Ali and Majhi (Ali and Majhi, 2011) gave tuning formulas for PI/PID controllers for pure integrating plus dead time, integrating plus first order plus dead time and double integrating plus dead time processes. Recently, Kaya and Cengiz (Kaya and Cengiz, 2017a, 2017b) presented optimum analytical PI/PID tuning rules for controlling stable and integrating processes with time delay plus inverse response.

All of the above studies give analytical tuning rules for conventional PI/PID controllers based on integral performance criteria. However, it is well known that due to their structural limitations, PID controllers show poor closed loop performances for open loop unstable processes, integrating processes and processes having poorly located complex poles (Kaya et al., 2006). Therefore, in order to improve closed loop performance of the above cited processes, alternate controller structures have been proposed, including (Atherton and Boz, 1998; Atherton and Majhi, 1999; Kaya, 2003a, 2003b; Majhi and Atherton, 2001). Of course, the efforts in this area are not limited to the studies mentioned, but due to space limitations it is not possible to cite them all. These studies usually suggest the use of PI-PD controller for performance improvement of unstable processes, integrating processes and processes having poorly located complex poles in different control structures. It has been shown that PI-PD controller yields superior closed loop responses over conventional PID controllers. The difficulty with PI-PD controllers is that they have difficulty in design because they have four parameters to be adjusted.

I-PD controller has a similar structure to a PI-PD controller, and performs comparable to PI-PD controllers though it has one less tuning parameter. Recently, Chakraborty et al. (Chakraborty et al., 2017) proposed an I-PD controller for integrating plus time delay processes, where explicit formulas for the design of controllers based on gain and phase margins were derived. However, they used a pure integrator plus dead time model, which may be insufficient to model higher order integrating processes accurately.

This paper provides optimal and simple analytical tuning rules to design an I-PD controller for controlling integrating processes with time delay, by minimizing the error signal

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using time moment weighted integral performance criteria. An integrating plus first order plus dead time model, which can model higher order integrating processes better than the pure integrating plus dead time model used by Chakraborty et al. (Chakraborty et al., 2017), is used to model integrating processes. Simple and analytical expressions, which yield optimum I-PD tuning parameters in the sense of ISTE and IST^{2}E (time moment weighted criteria of the integral of squared error). Simulation results have been carried out in order to illustrate the use of the proposed I-PD controller design approach.

The rest of paper is organized as follows: In section 2, a short review of integral performance criteria is given as it has been used to obtain optimal I-PD tuning rules. Optimal tuning rules for an I-PD controller to tune integrating processes plus dead time are derived in Section 3. Simulation examples are provided in Section 4, followed by conclusions given in section 5.

2. INTEGRAL PERFORMANCE CRITERIA

Here, time moment weighted integral performance criteria will be used to achieve optimum tuning rules. Time domain Integral of Squared Error (ISE) criterion is given by

$$J_0 = \int_0^\infty e^2(t)dt. \tag{1}$$

The s-domain calculation of ISE criterion is as the following:

$$J_0 = \frac{1}{2\pi j}\int_0^\infty E(s)E(-s)ds \tag{2}$$

In (2), $E(s)$ is the error signal which is assumed to be given by $E(s) = A(s)/B(s)$. Numerator and denominator of the error function are polynomials with real coefficients given by

$$A(s) = a_0 s^m + a_1 s^{m-1} + \ldots + a_{m-1}s + a_m$$

$$B(s) = b_1 s^{m-1} + \ldots + b_{m-1}s + b_m$$

Aström's recursive algorithm (Åström, 1970) can effectively be used to calculate the integral given in (2). Time moment weighted version of the ISE criterion given by

$$J_n = \int_0^\infty [t^n e(t)]^2 dt \tag{3}$$

can also be evaluated by using Aström's recursive algorithm, since $L\{tf(t)\} = -dF(s)/ds$. Here, L denotes the Laplace transform and $L\{f(t)\} = F(s)$. Taking $n = 0$ in (3) gives the ISE criterion. $n = 1$, and $n = 2$ corresponds to the ISTE and IST^{2}E criteria, which are time moment weighted criteria of the ISE. Increasing n improves the closed loop performance in the sense of responses with less oscillations, smaller overshoots and short settling times. Therefore, tuning formulas will only be determined only for the ISTE and IST^{2}E criteria.

3. OPTIMUM I-PD CONTROLLER DESIGN

The I-PD controller structure is illustrated in Fig. 1. In the figure, $G(s)$ is the transfer function of the integrating process. $G_{c1}(s)$ and $G_{c2}(s)$ are *I* and *PD* controller transfer functions, respectively.

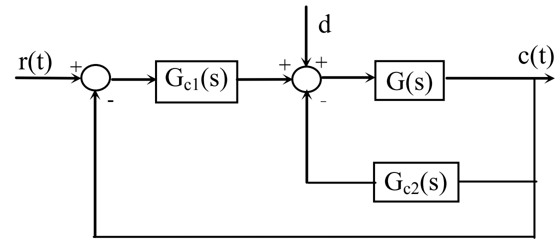

Fig. 1. I-PD Control Structure.

The process transfer function is assumed to be

$$G(s) = \frac{Ke^{-\theta s}}{s(Ts+1)}. \tag{4}$$

Controllers $G_{c1}(s)$ and $G_{c2}(s)$ are assumed to have the following ideal transfer functions, respectively.

$$G_{c1}(s) = \frac{K_c}{T_i s} \tag{5}$$

$$G_{c2}(s) = K_c(1 + T_d s) \tag{6}$$

The error function of Fig. 1 is given by

$$E(s) = \frac{R(s)}{1 + G(s)[G_{c1}(s) + G_{c2}(s)]} \tag{7}$$

Repeated optimizations were carried out on this error function for a unit step input, $R(s)$, and different values of normalized dead time $\theta_n = \theta/T$. Relations between the normalized dead time, $\theta_n = \theta/T$, and KK_cT, T_i/T and T_d/T for ISTE and IST^{2}E criteria are shown Fig. 2, Fig. 3 and Fig. 4. In the figures, asterisks correspond to values obtained from the optimizations and solid lines correspond to values achieved from curve fitting formulae for KK_cT and T_i/T and T_d/T. It is clear from the figures that quite satisfactory fittings have been achieved.

Following tuning formulae were found from the curve fitting method for the ISTE criterion:

$$K_c = \frac{0.3373 + 0.8669(\frac{\theta}{T}) + 0.08409(\frac{\theta}{T})^2}{KT\left\{0.001124 - 0.02375(\frac{\theta}{T}) + (\frac{\theta}{T})^2\right\}} \tag{8}$$

$$T_i = T\left\{\begin{array}{l} 0.02798 + 3.579(\frac{\theta}{T}) - 0.8116(\frac{\theta}{T})^2 \\ +0.2323(\frac{\theta}{T})^3 - 0.02669(\frac{\theta}{T})^4 \end{array}\right\} \tag{9}$$

$$T_d = T\left\{\frac{-0.0001746+1.948(\frac{\theta}{T})+1.158(\frac{\theta}{T})^2+0.428(\frac{\theta}{T})^3}{0.8444+1.809(\frac{\theta}{T})+(\frac{\theta}{T})^2}\right\} \quad (10)$$

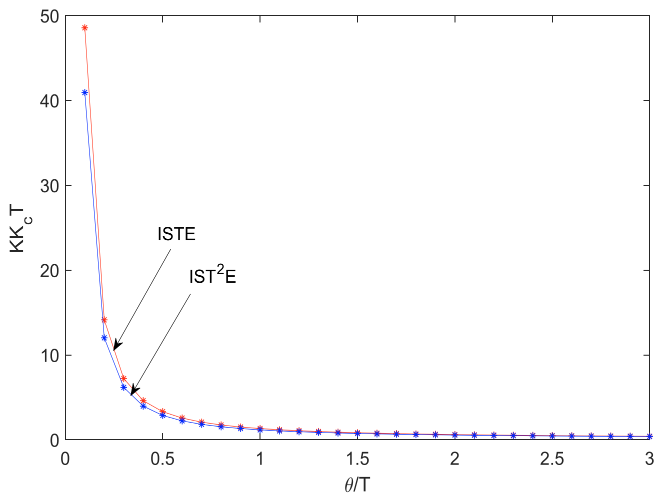

Fig. 2. KK_cT values for range of $0.1 \le \theta_n \le 3.0$

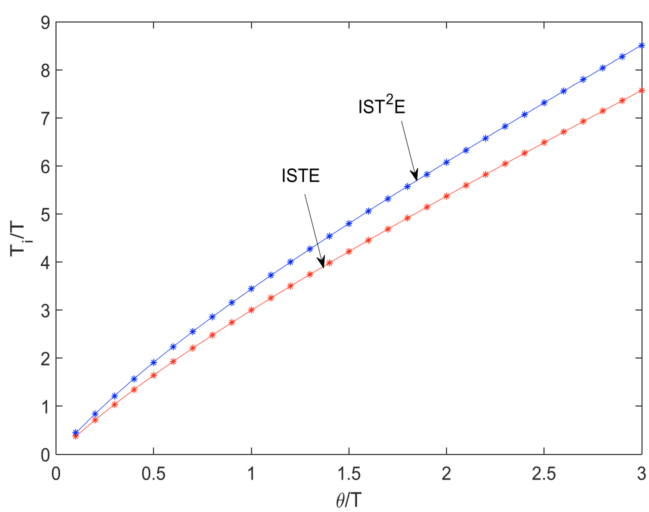

Fig. 3. T_i / T values for range of $0.1 \le \theta_n \le 3.0$

For the IST^{2}E criterion, following tuning formulae were achieved:

$$K_c = \frac{0.2682+0.7817(\frac{\theta}{T})+0.08211(\frac{\theta}{T})^2}{KT\left\{0.001349-0.02865(\frac{\theta}{T})+(\frac{\theta}{T})^2\right\}} \quad (11)$$

$$T_i = T\left\{\begin{array}{l}0.04005+4.201(\frac{\theta}{T})-1.075(\frac{\theta}{T})^2\\ +0.315(\frac{\theta}{T})^3-0.03662(\frac{\theta}{T})^4\end{array}\right\} \quad (12)$$

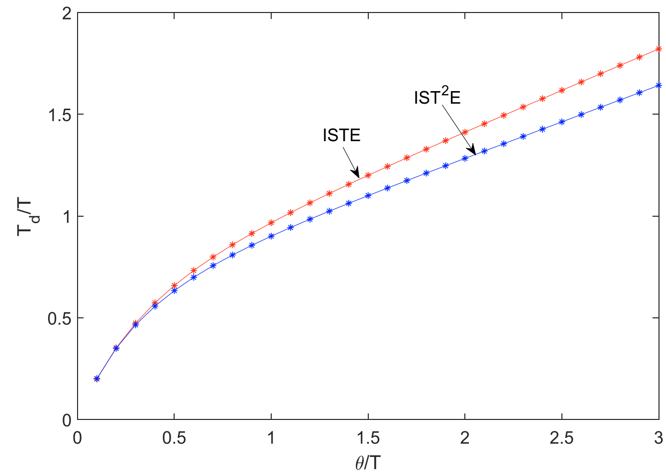

Fig. 4. T_d / T values for range of $0.1 \le \theta_n \le 3.0$

$$T_d = T\left\{\frac{0.000161+1.566(\frac{\theta}{T})+0.9245(\frac{\theta}{T})^2+0.3841(\frac{\theta}{T})^3}{0.663+1.526(\frac{\theta}{T})+(\frac{\theta}{T})^2}\right\} \quad (13)$$

Once the model of the integrating process given by (4) is known, then optimum I-PD settings can be evaluated from (8)-(10) for the ISTE criterion and (11)-(13) for the IST^{2}E criterion.

4. SIMULATION EXAMPLES

Several examples are considered to illustrate the use of the proposed I-PD controller design method. All examples are compared with the design method of Ali and Majhi (Ali and Majhi, 2011) as they also suggest optimum tuning settings for PID controllers to control integrating processes. In addition, comparisons will be performed with design method of Chakraborty et al. (Chakraborty et al., 2017) since they use the I-PD controller for controlling integrating process as well. Relay feedback identification method given in (Kaya, 1999) has been used to find integrating plus first order plus dead time (IFOPDT) plant transfer function, which is needed for the proposed I-PD controller design and for the PID design method of Ali and Majhi (Ali and Majhi, 2011). Pure integrating plus dead time (IPDT) plant transfer function required for I-PD design method of Chakraborty et al. (Chakraborty et al., 2017) has also been determined from relay feedback identification method given in (Kaya, 1999). It should be noted that Ali and Majhi (Ali and Majhi, 2011) suggest to use a set-point filter to reduce large overshoots yielding in their design. Here, this filter will not be used in simulations in order to not distort results of their original design.

4.1 Example 1: A plant transfer function of $G(s) = e^{-3s} / s(s+1)$, which matches the IFOPDT model exactly, is considered here. Equations (8)-(10) and (11)-(13) for the ISTE and IST^{2}E criteria, respectively, were used to calculate optimum I-PD controller tuning parameters. For the ISTE criterion, I-PD controller settings were found to be

$K_p = 0.414$, $T_i = 7.571$ and $T_d = 1.822$. For the IST^{2}E criterion, I-PD controller settings were evaluated as $K_p = 0.376$, $T_i = 8.507$ and $T_d = 1.642$. Design method of Ali and Majhi (Ali and Majhi, 2011) has PID tuning parameters of $K_p = 0.290$, $T_i = 10.440$ and $T_d = 1.670$. I-PD design method of Chakraborty et al. (Chakraborty et al., 2017) needs the IPDT model, which was identified from relay feedback identification method of Kaya (Kaya, 1999) to be $0.826e^{-3.986s}/s$. Using this IPDT model, tuning parameters of I-PD controller for Chakraborty et al. (Chakraborty et al., 2017) design method were obtained as $K_p = 0.256$, $T_i = 14.169$, $T_d = 1.925$. Fig. 5 shows closed loop responses for a unit step input change and a disturbance with magnitude of -0.1 injected into the system at $t = 50$ s for all design methods. Control performances for all design methods are given in Table 1. From Fig. 5 and Table 1, it is seen that design method of Ali and Majhi (Ali and Majhi, 2011) results in a large overshoot in response to a step input change. This is an expected result as it is well known that PID controllers do not perform well for integrating processes. On the other hand, I-PD controller design of Chakraborty et al. (Chakraborty et al., 2017) gives a sluggish response. Proposed optimum I-PD controller designs yield a fast set point tracking and disturbance rejection with reasonable overshoots and settling times. Among the two proposed optimum I-PD design methods, the IST^{2}E criterion yields a smaller overshoot and shorter settling time than the ISTE criterion. This is also predictable because the IST^{2}E criterion tolerates the initial errors but punishes later occurring errors when compared to the ISTE criterion. Fig. 6 depicts control signals for all design methods. Design method of Ali and Majhi (Ali and Majhi, 2011) yields the largest initial control effort. Chakraborty et al. (Chakraborty et al., 2017) design method results in the smallest control signal magnitude.

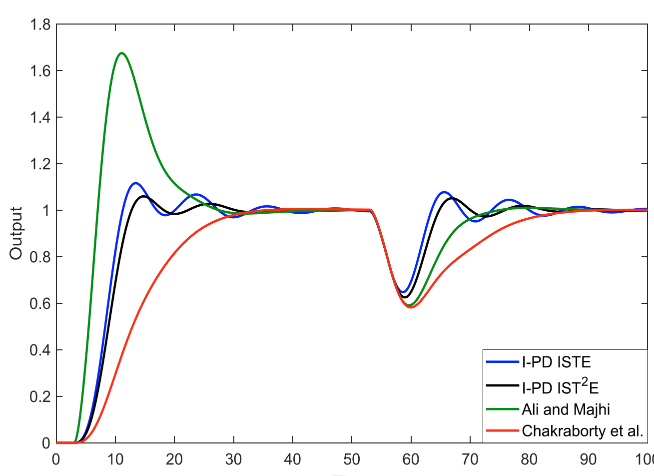

Fig. 5. Step input and disturbance responses for example 1

4.2 Example 2: A third order plant transfer function of $G(s) = e^{-0.2s}/s(0.1s+1)(s+1.2)$ is considered in this example. Relay feedback identification method of Kaya (Kaya, 1999) was used to identify the IFOPDT model as $G(s) = 0.833e^{-0.275s}/s(1.072s+1)$. Equations (8)-(10) were used to calculate I-PD controller tuning parameters for the ISTE criterion as $K_p = 9.073$, $T_i = 1.038$ and $T_d = 0.483$. Similarly, (11)-(13) were used to calculate I-PD controller tuning parameters for the IST^{2}E criterion as $K_p = 7.754$, $T_i = 1.217$ and $T_d = 0.475$. PID controller settings for design method of Ali and Majhi (Ali and Majhi, 2011) were determined to be $K_p = 3.755$, $T_i = 2.302$ and $T_d = 0.701$. IPDT model for the I-PD design method of Chakraborty et al. (Chakraborty et al., 2017) were determined from relay feedback identification of Kaya (Kaya, 1999) to be $0.3655e^{-0.865s}/s$. Hence, tuning parameters of I-PD controller suggested by Chakraborty et al. (Chakraborty et al., 2017) were evaluated as $K_p = 2.668$, $T_i = 3.075$, $T_d = 0.418$. Closed loop responses to a unit step input and a disturbance with magnitude of -0.5 entering the system at $t = 15$ s are shown in Fig. 7 for all design methods. . Control performances for all design methods are summarized in Table 2. Control signals for all design methods are illustrated in Fig. 8. Similar to example 1, design method of Ali and Majhi (Ali and Majhi, 2011) results in a large overshoot and design method of Chakraborty et al. (Chakraborty et al., 2017) yield a sluggish response. Proposed I-PD design methods show the most acceptable responses to both the set point tracking and disturbance rejection.

Table 1. Control performances for example 1

Design Method	Maximum Overshoot (%)	Settling Time (s)	IST^{2}E (x10^4)
I-PD ISTE	11.63	31.577	1.7484
I-PD IST^{2}E	5.940	27.711	1.0284
Ali and Majhi	67.500	25.539	7.4985
Chakraborty et al.	0.450	30.164	10.4520

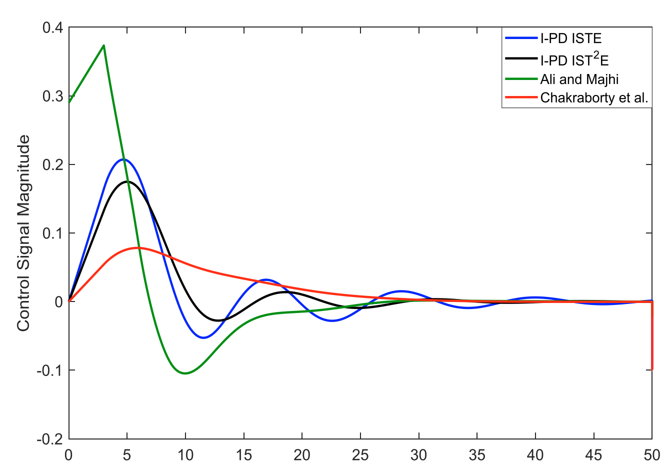

Fig. 6. Control signals for example 1

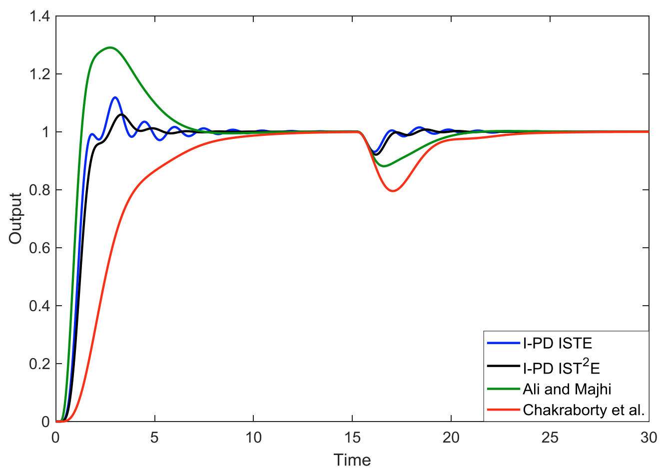

Fig. 7. Step input and disturbance responses for example 2

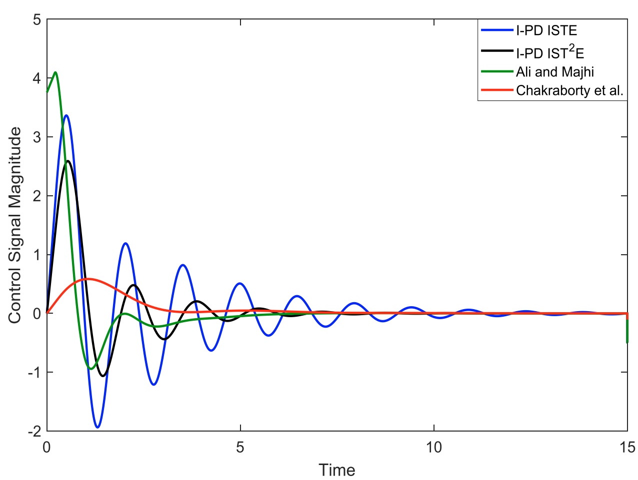

Fig. 8. Control signals for example 2

Table 2. Control performances for example 2

Design Method	Maximum Overshoot (%)	Settling Time (s)	IST^{2}E
I-PD ISTE	11.810	5.467	2.3592
I-PD IST^{2}E	5.900	3.873	0.8456
Ali and Majhi	29.010	6.463	25.9402
Chakraborty et al.	-	9.096	70.6758

4.3 Example 3: A higher order plant transfer function of $G(s)=e^{-5s}/s(10s+1)(s+1)(0.5s+1)(0.25s+1)$, which has a large time delay and time constant, is considered in this example. Again, relay feedback identification method of Kaya (Kaya, 1999) was used to identify the IFOPDT model as $G(s)=e^{-6.667s}/s(10.141s+1)$. Equations (8)-(10) and (11)-(13) were used to calculate I-PD controller tuning parameters for the ISTE and IST^{2}E criteria as $K_p=0.191$, $T_i=24.541$, $T_d=7.433$ and $K_p=0.223$, $T_i=21.205$, $T_d=7.825$, respectively. Settings of PID controller design method of Ali and Majhi (Ali and Majhi, 2011) were found to be $K_p=0.129$, $T_i=35.340$ and $T_d=8.283$. For I-PD design method of Chakraborty et al. (Chakraborty et al., 2017), IPDT model, $0.561e^{-14.802s}/s$, were identified from relay feedback identification method suggested by Kaya (Kaya, 1999). Based on this identified model, tuning parameters of I-PD controller design of Chakraborty et al. (Chakraborty et al., 2017) were evaluated as $K_p=0.102$, $T_i=52.608$, $T_d=7.147$. Comparisons of closed loop performances to a unit step input and a disturbance with magnitude of -0.1 entering the system at $t=150$ s are illustrated in Fig. 9 for all design methods. Fig. 10 depicts control signals of all design methods. Control performances for all design methods are given in Table 3. Similar interpretations to examples 1 and 2 can be derived for this example as well.

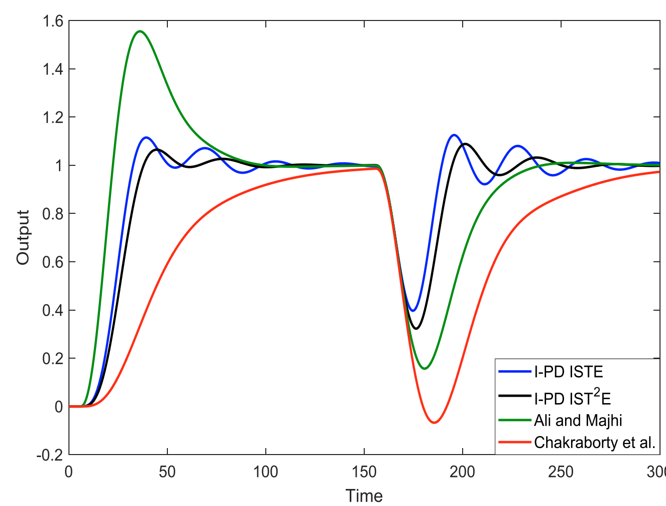

Fig.9. Step input and disturbance responses for example 3

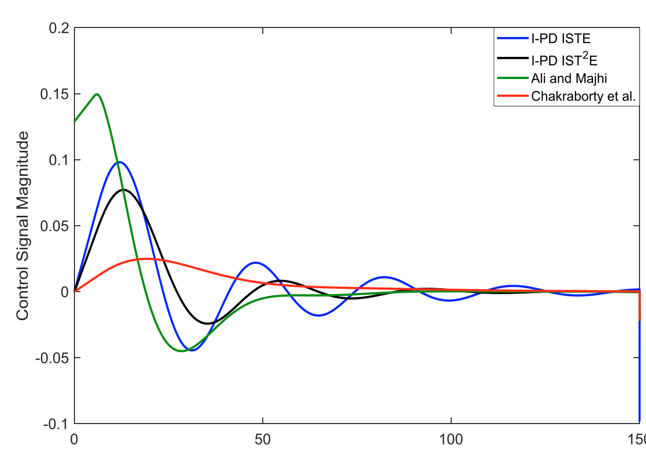

Fig. 10. Control signals for example 3

Table 3. Control performances for example 3

Design Method	Maximum Overshoot (%)	Settling Time (s)	IST^2E ($x10^6$)
I-PD ISTE	11.500	93.090	3.8904
I-PD IST^2E	6.490	83.778	2.3056
Ali and Majhi	55.560	87.491	22.886
Chakraborty et al.	-	144.002	83.212

5. CONCLUSIONS

The paper has provided optimal and analytical tuning formulas for I-PD controllers to control integrating processes with dead time. Time weighted integral performance criteria, namely ISTE and IST^2E, were used to achieve those tuning rules. Several simulation examples have been provided to show the value of the proposed I-PD design method. Simulations have shown that obtained tuning rules result in quite satisfactory closed loop responses when compared to some recently published PID and I-PD design methods, which are also suggested for controlling integrating processes with time delay.

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