Institute of Technology of Cambodia
Mechanic and Industrial Engineer

Group: I4-GIM (Mechanic)

Assignment: Constructions Mechanic

TOPIC: KUKA KR 10 R1100-2

Lecturer: M. VOEURN YONG ANN

Students:	ID:
• LAI KIMLA	e20180478
• LIM BUNHONG	e20180507
• LY KEAHENG	e20180555
• MOR LENG	e20180624

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Contents:

I. Introduction:

The KUKA KR 10 R1100-2 robot is a 6-axis robot arm, it offers a 11 kg payload and 110n om reach. The repeatability of the KUKA KR 10 R1100-2 robot is 002 mm. Common applications of the KUKA KR 10 R1100- 2 robot include 3d printing, arc welding dispensing remote tcp and spot welding. The KUKA KRTOR1100 2 is manufactured by **KUKA.**

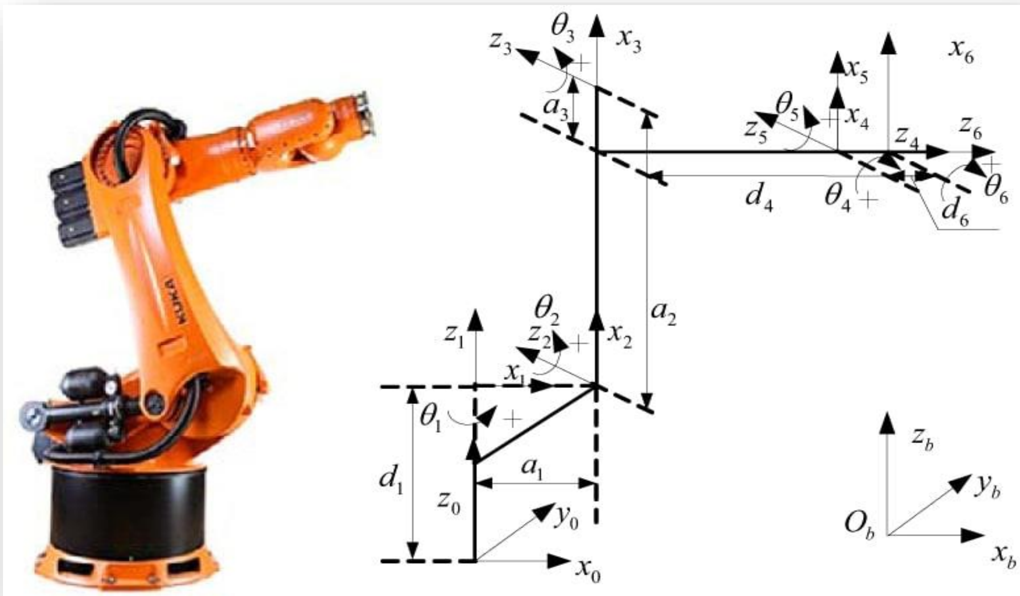
II. Objective:

After this assignment you will learn how to develop kinematic model of robot, solve forward kinematics problem and inverse kinematics problem.

- Forward Kinematic Analysis: means that the location and pose of the end of manipulator in a given references coordinates system can be worked out with the given geometry parameters of the links and the variables of the joints for a robot.
- Inverse Kinematic Analysis: is the opposite of the forward kinematic analysis. The corresponding variables of each joint could found with the given location requirement of the end of the manipulator in the given references coordinates system.

III. Kinematic Model of the Robot:

_Here is the model that we develop: KUKA KR 10 R1100-2



Here is some basic data of KR 10 R1100-2

	KR 10 R1100-2
Number of axes	6
Number of controlled axes	6
Volume of working envelope	5.2 m³
Pose repeatability (ISO 9283)	± 0.02 mm
Weight	approx. 56.5 kg
Rated payload	5 kg
Maximum payload	10.9 kg
Maximum reach	1101 mm
Protection rating (IEC 60529)	IP65 / IP67
Protection rating, in-line wrist (IEC 60529)	IP65 / IP67
Sound level	< 57 dB (A)

Mounting position	Floor; Ceiling; Wall; Desired angle
Footprint	208 mm x 208 mm
Hole pattern: mounting surface for kinematic system	C246
Permissible angle of inclination	-
Default color	Base frame: gray aluminum (RAL 9007); Moving parts: traffic white (RAL 9016)
Controller	KR C4 smallsize-2; KR C4 compact
Transformation name	KR C4: KR10R1100_2 C4SR

Protection classification IP67 can only be assured with a compressed air connection (venting connection PURGE) of 0.3 bar.

Overpressure in the robot	0.03 MPa (0.3 bar)
Compressed air	Oil-free, dry, filtered in accordance with: ISO 8573.1-1, 1.2 to 16.2
Air consumption	0.1 m^3/h
Air line connection	Plug-in connection for hose, standard outside diameter 4 mm

Ambient conditions

Humidity class (EN 60204)	-
Classification of environmental conditions (EN 60721-3-3)	3K3
Ambient temperature	
During operation	0 °C to 45 °C (273 K to 318 K)
During storage/transportation	-40 °C to 60 °C (233 K to 333 K)

For operation at low temperatures, it may be necessary to warm up the robot.

Connecting cables

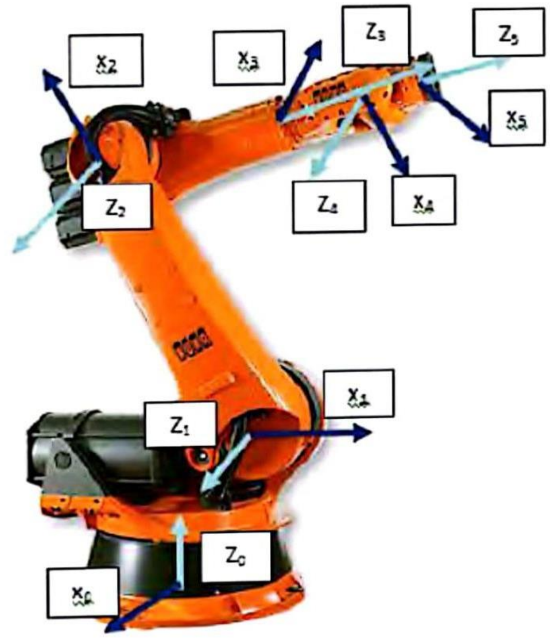
Cable designation	Connector designation robot controller - robot	Interface with robot
Motor cable	X20 - X30	Han Yellock 30
Data cable	X21 - X31	Han Q12
Ground conductor, equipotential bonding (can be ordered as an option)		M4 ring cable lug

	Cable lengths
Standard	1 m, 4 m, 7 m, 15 m, 25 m

For further connecting cables and detailed specifications, see “Description of the connecting cables”.

❖ Mastering positions

Axis	Θ	d	a	α
1	Θ1	0	a 1	90
2	Θ2	0	a 2	0
3	Θ3	d 3	0	90
4	Θ4	0	0	-90
5	Θ5	0	0	90
6	Θ6	0	0	0



○ Load center of gravity:

For all payloads, the load center of gravity refers to the distance from the face of the mounting flange on axis 6.

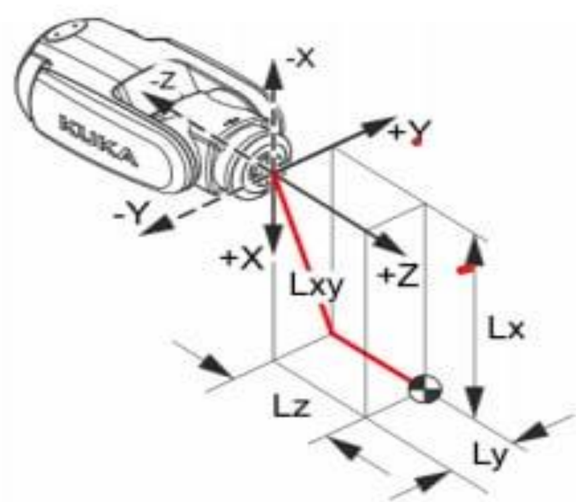


Fig. 4-32: Load center of gravity

○ Payload diagram:

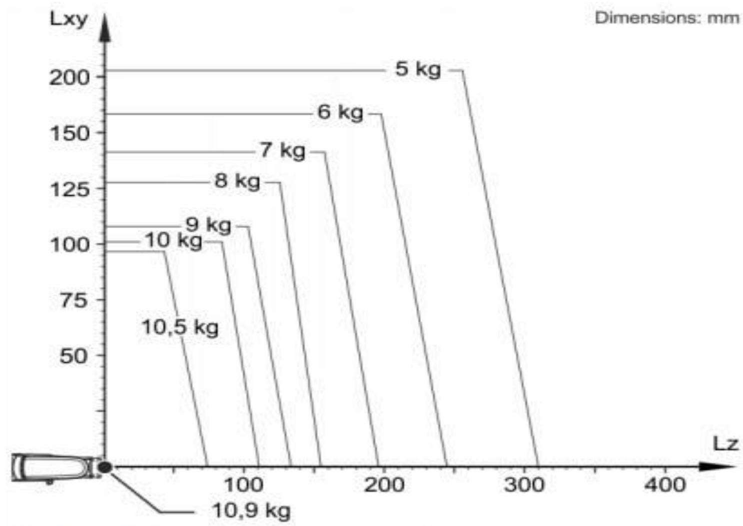


Fig. 4-33: KR 10 R1100-2, payload diagram

- **Supplementary load, reach R1100**

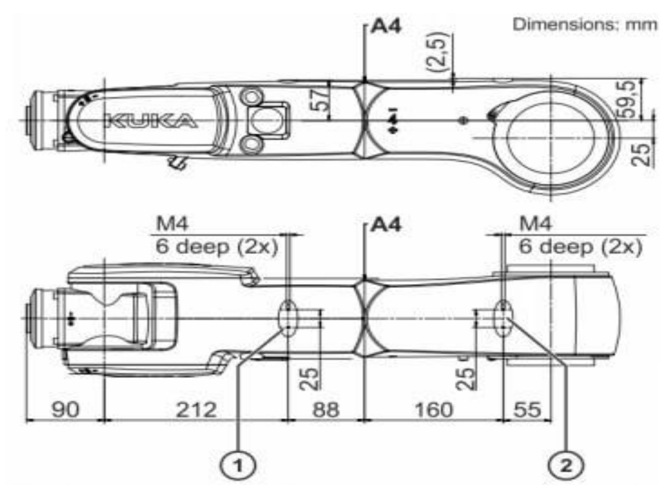


Fig. 4-41: Installation options on arm and in-line wrist

1. Fastening holes, in-line wrist

2 Support for supplementary load, arm

1 Support for supplementary load, link arm

2 Fastening holes, link arm 3 Support for supplementary load, rotating column for the fastening holes on the link arm, the following must be observed:

• Screw without supplementary load: 2x M4x8-8.8-A2K

• Screw with supplementary load: 8 mm + part thickness of supplementary load + max. 1 mm

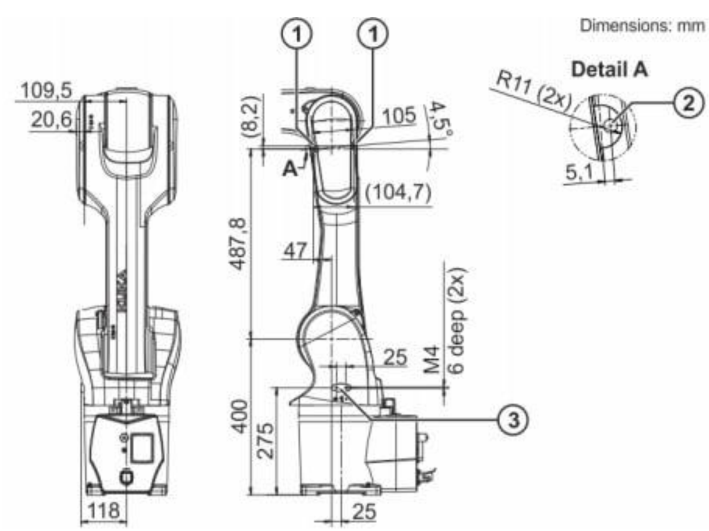


Fig. 4-42: Installation options on link arm and rotating column

IV. Forward Kinematics:

❖ Working envelope:

The following diagrams show the shape and size of the working envelope for these variants of this product family

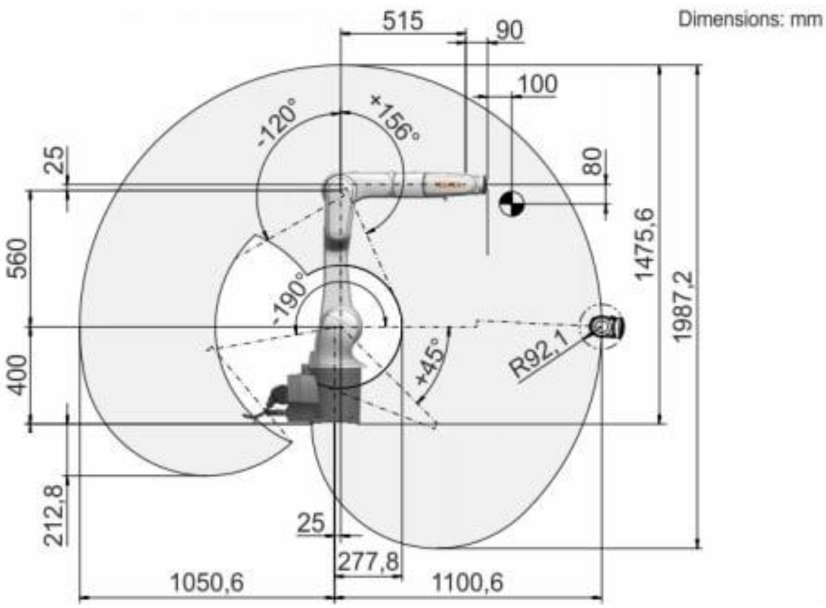


Fig. 4-29: KR 10 R1100-2, working envelope, side view

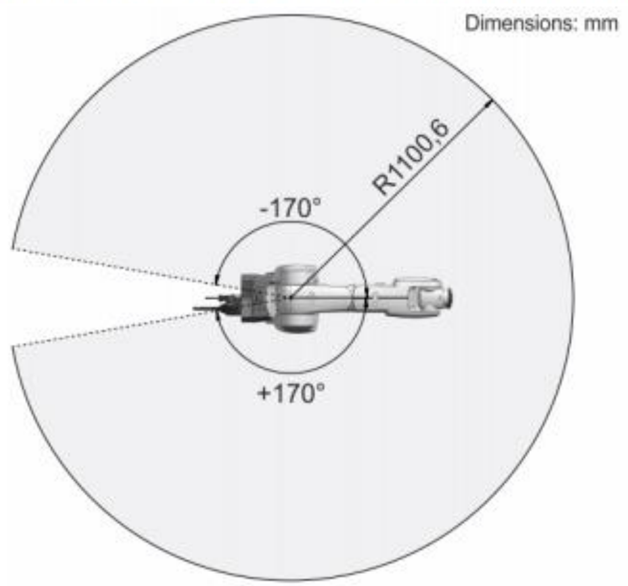
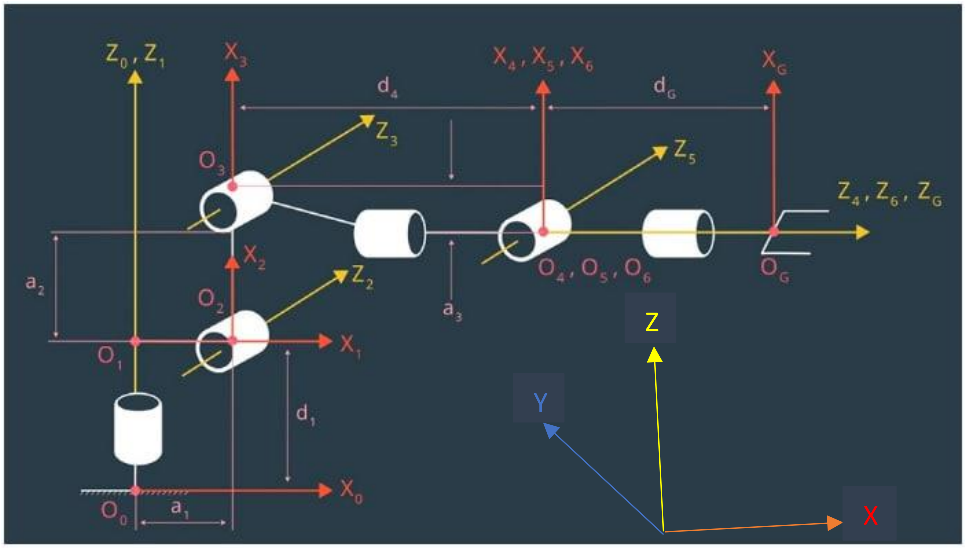


Fig. 4-30: KR 10 R1100-2, working envelope, top view



This Figure1: Shows the link frames (coordinate systems) choosen according to Modified DH convention discussed next. O_i is the origin for link ii frame, and X_i, Z_i are the XX and ZZ axis correspondingly, and ZZ represents the axis of rotation (translation in case of prismatic joints). Since we are using a right-handed coordinate system, Y_i can be calculated accordingly.
Here is a diagram of the gripper showing the degrees of freedom that the arm has, along with their reference frames. Computing through these transformations can become complex and tedious really fast. So, we use a convention.

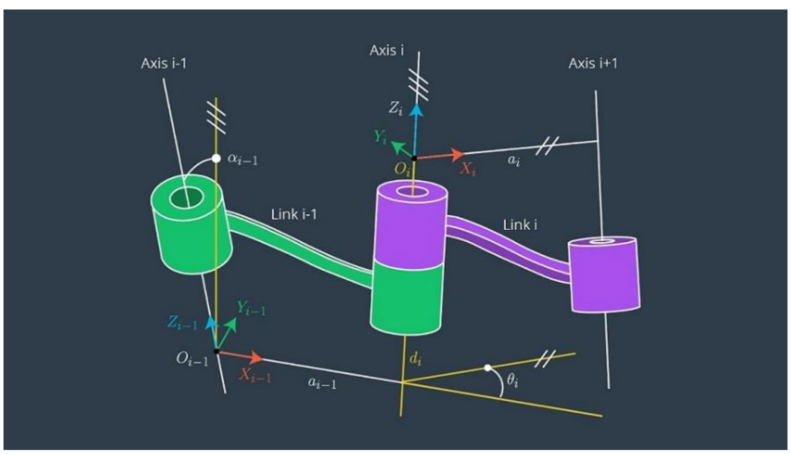


Fig. 2: Modified DH convention axes assignment and parameters.

- d_i: Link offset, distance between $X_{i-1}X_{i-1}$ and X_i, measured along $Z_{i-1}Z_{i-1}$, variable in prismatic joints.
- $\alpha_{i-1}\alpha_{i-1}$: Angle between $Z_{i-1}Z_{i-1}$ and Z_i, measured along X_i.
- $a_{i-1}a_{i-1}$: Link length, distance between $Z_{i-1}Z_{i-1}$ and Z_i, measured along $Z_{i-1}Z_{i-1}$.
- θ_i: Joint angle, Angle between $X_{i-1}X_{i-1}$ and X_i, measured along Z_i, variable in revolute joints.

So, according to this convention, the total transform between links Li−1 and Li can be thought of as a *roation* by αi−1 along Xi−1, *translation* by αi−1 along Xi−1, *rotation* by θi along Zi ,and finally *translation* by di along Zi. That is:

$$A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & a_1S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

At the base of the robot , it can be started with the first joint and then transform to the second joint , then to the third until to the arm - end of the robot , and eventually to the end effectors . The total transformation between the base of the robot and the hand is

At the base of the robot. it can be started with the first

$${}^{R}T_{H} = A_1A_2A_3A_4A_5A_6 \quad (3)$$

$$A_1A_2 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & a_1S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & a_2C_1C_2 + a_1C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & a_2S_1C_2 + a_1S_1 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3A_4 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_3C_4 & -S_3 & -S_4C_3 & 0 \\ S_3C_4 & C_3 & -S_4S_3 & 0 \\ S_4 & 0 & C_4 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5A_6 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_5C_6 & -S_6C_5 & S_5 & 0 \\ S_5C_6 & -S_5S_6 & -C_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$^{R}T_H = A_1A_2A_3A_4A_5A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Calculating overall transformation matrix, you will get the following results:

$$\begin{aligned}
n_x &= C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6) \\
n_y &= S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6) \\
n_z &= S_{23}(C_4C_5C_6 - S_4S_6) + C_{23}S_5C_6 \\
o_x &= C_1[- C_{23}(C_4C_5C_6 + S_4C_6) + S_{23}S_5S_6] + S_1(-S_4S_5S_6 + C_4C_6) \\
o_y &= S_1[- C_{23}(C_4C_5C_6 + S_4C_6) + S_{23}S_5S_6] - C_1(-S_4C_5S_6 + C_4S_6) \\
o_z &= -S_{23}\,(C_4C_5C_6 + S_4C_6) - C_{23}S_5S_6 \\
a_x &= C_1[C_{23}C_4S_5 + S_{23}C_5] - C_1S_4S_5 \\
a_y &= S_1[C_{23}C_4S_5 + S_{23}C_5] - C_1S_4S_5 \\
a_z &= S_{23}C_4S_5 - C_{23}C_5 \\
p_x &= C_1(C_2a_2 + a_1) + S_1d_3 \\
p_y &= S_1(C_2a_2 + a_1) - C_1d_3 \\
p_z &= S_2a_2
\end{aligned} \tag{6}$$

V. Inverse Kinematics:

With inverse kinematic solutions, the value of each joint can be determined in order to place the arm at a desired position and orientation. To decouple the angles, the RTH matrix is routinely pre-multiplied with the individual A_n^{-1} matrices.

To solve for the angle we will successively pre-multiply the two matrices with the A_n^{-1}

$$A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2A_3A_4A_5A_6$$

$$\begin{bmatrix} n_xC_1 + n_yS_1 & o_xC_1 + o_yS_1 & a_xC_1 + a_yS_1 & P_xC_1 + P_yS_1 - a_1(C_1C_1 + S_1S_1) \\ n_z & o_z & a_z & P_z \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & P_xS_1 - P_yC_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{23}(C_4C_5C_6 - S_4C_6) - S_{23}S_5C_6 & C_{23}(-C_4C_5C_6 - S_4C_6) - S_{23}S_5S_6 & C_{23}C_4S_5 + S_{23}C_5 & a_2C_2 \\ S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6 & S_{23}(-C_4C_5C_6 - S_4S_6) - C_{23}S_5S_6 & S_{23}C_4S_5 - C_{23}C_5 & a_2S_2 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_5S_4 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

From (2,4) elements,

$$P_z = a_2S_2 \quad (28)$$

$$S_2 = P_z / a_2 \quad (29)$$

$$\theta_2 = sin^{-1}(P_z/a_2) \quad (30)$$

From (3,4) elements,

$$P_xS_1 - P_yC_1 = d_3 \quad (31)$$

$$S_1 = \frac{d_3 + P_yC_1}{P_x} \quad (32)$$

From (1,4) elements,

$$P_xC_1 + P_yS_1 - a_1 = a_2C_2 \quad (33)$$

From equation 32 and 33,

$$C_1 = \frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2} \quad (34)$$

$$\theta_1 = cos^{-1}\left(\frac{P_x(a_2C_2 + a_1) - P_y d_3}{P_x^2 + P_y^2}\right) \tag{35}$$

Next step is to premultiply by the inverse of A_1 through A_3.

$$A_3^{-1}A_2^{-1}A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_4 A_5 A_6 \tag{36}$$

$$\begin{bmatrix} n_xC_1C_{23} + n_yS_1C_{23} + n_zS_{23} & o_xC_1C_{23} + o_yS_1C_{23} + o_zS_{23} & a_xC_1C_{23} + a_yS_1C_{23} + a_zS_{23} & (P_xC_1 + P_yS_1 - a_1)C_{23} + P_zS_{23} - a_2C_3 \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & P_xS_1 - P_yC_1 - d_3 \\ n_xC_1S_{23} + n_yS_1S_{23} - n_zC_{23} & o_xC_1S_{23} + o_yS_1S_{23} - o_zC_{23} & a_xC_1S_{23} + a_yS_1S_{23} - a_zC_{23} & (P_xC_1 + P_yS_1 - a_1)S_{23} - P_zC_{23} - a_2S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4C_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{37}$$

From element (1,4) and (3,4),

$$(P_xC_1 + P_yS_1 - a_1)C_{23} + P_zS_{23} - a_2C_3 = 0 \tag{38}$$

$$(P_xC_1 + P_yS_1 - a_1)s_{23} - P_zC_{23} - a_2S_3 = 0 \tag{39}$$

From equation 38 and 39,

$$C_{23} = \sqrt{\frac{P_z^2 - a_2^2 + [P_xC_1 + P_yS_1 - a_1]^2}{2P_z^2}} \tag{40}$$

$$\theta_{23} = cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + [P_xC_1 + P_yS_1 - a_1]^2}{2P_z^2}}$$

$$\theta_3 = \theta_{23} - \theta_2 \tag{41}$$

From element (1,3) and (2,3),

$$a_xS_1 - a_yC_1 = S_4S_5 \tag{42}$$

$$a_xC_1C_{23} + a_yS_1C_{23} + a_yS_1C_{23} + a_zS_{23} = C_4S_5 \tag{43}$$

From equation 42 and 43,

$$\theta_4 = tan^{-1}\left[\frac{a_x S_1 - a_y C_1}{a_x C_1 C_{23} + a_y S_1 S_{23} + a_z S_{23}}\right] \quad (44)$$

From element (3,1) and (3,2),

$$o_x C_1 S_{23} + o_y S_1 S_{23} - o_z C_{23} = S_5 S_6 \quad (45)$$

$$n_x C_1 S_{23} + n_y S_1 S_{23} - n_z C_{23} = -S_5 S_6 \quad (46)$$

From equation 7 and 8,

$$\theta_6 = tan^{-1}\left[\frac{o_z C_{23} - o_x C_1 S_{23} - o_y S_1 S_{23}}{n_x C_1 S_{23} + n_y S_1 S_{23} - n_z C_{23}}\right] \quad (47)$$

Since,

$$A_4^{-1} A_3^{-1} A_2^{-1} A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_5 A_6 \quad (48)$$

From element (1, 3) and (2, 3),

$$a_x C_1 C_{23} C_4 + a_y S_1 S_{23} C_4 + a_2 S_{23} C_4 + a_x S_1 S_4 - a_y C_1 S_4 = S_5 \quad (49)$$

$$-a_x S_{23} - a_y S_1 S_{23} + a_z C_{23} = -C_5 \quad (50)$$

From equation 9 and 10,

$$\theta_5 = -tan^{-1}\left[\frac{a_x C_1 C_{23} C_4 + a_y S_1 S_{23} C_4 + a_2 S_{23} C_4 + a_x S_1 S_4 - a_y C_1 S_4 = S_5}{-a_x S_{23} - a_y S_1 S_{23} + a_z C_{23}}\right] \quad (51)$$

As a summary,

$$\theta_2 = sin^{-1}(P_z / a_2) \quad (52)$$

$$\theta_1 = cos^{-1}\left(\frac{P_x(a_2 C_2 + a_1) - P_y d_3}{P_x^2 + P_y^2}\right) \quad (53)$$

$$\theta_{23} = cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + [P_xC_1 + P_yS_1 - a_1]^2}{2P_z^2}} \quad (54)$$

$$\theta_3 = \theta_{23} - \theta_2 \quad (55)$$

$$\theta_4 = tan^{-1}\left[\frac{a_xS_1 - a_yC_1}{a_xC_1C_{23} + a_yS_1S_{23} + a_zS_{23}}\right] \quad (56)$$

$$\theta_6 = tan^{-1}\left[\frac{o_zC_{23} - o_xC_1S_{23} - o_yS_1S_{23}}{n_xC_1S_{23} + n_yS_1S_{23} - n_zC_{23}}\right] \quad (57)$$

$$\theta_5 = -tan^{-1}\left[\frac{a_xC_1C_{23}C_4 + a_yS_1S_{23}C_4 + a_zS_{23}C_4 + a_xS_1S_4 - a_yC_1S_4 = S_5}{-a_xS_{23} - a_yS_1S_{23} + a_zC_{23}}\right] \quad (58)$$

VI. Conclusion:

The purpose of the study was to design a three degrees of freedom robotic arm that is capable of picking and placing lightweight objects from one to another specific location. The optimal cross-section for the links was found to be a set of two rectangular areas as it has the highest structural strength to weight ratio when compared to solid and hollow circular beams. Additionally, the forward and inverse kinematic analyses were carried out and a mathematical equation was created to represent and plot the arm's workspace. Lastly, repeatability and accuracy tests were conducted and the error values for the camera and end effector were found. For future work, the team will focus on recalibrating the robotic arm to improve its positioning accuracy and setting up a wireless connection for the control system.

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