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Cryptography II

Block ciphers and modes of operations

Block ciphers: getting the concept

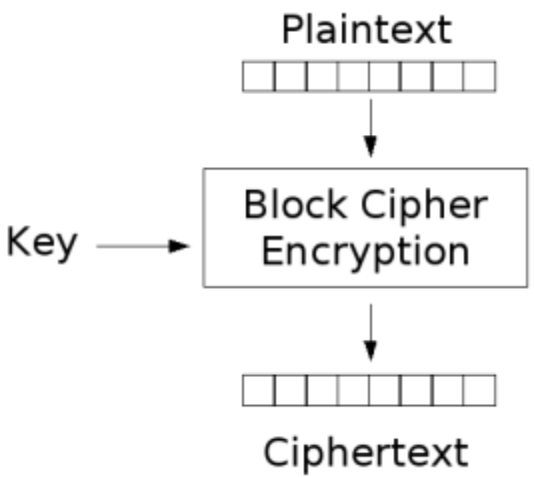
- Stream cipher vs. block cipher: single unit vs. block of units

key	000	001	010	011	100	101	110	111
0	001	111	110	000	100	010	101	011
1	001	110	111	100	011	010	000	101
2	001	000	100	101	110	111	010	011
3	100	101	110	111	000	001	010	011
4	101	110	100	010	011	001	011	111

- Plaintext= 01010011011 1= (010)(100)(110)(111)
  - ➔ Ciphertext = 111 011 000 101 theo key=1
  - ➔ Ciphertext = 100 011 011 111 theo key=4
- There are 5 keys, $2^2 < 5 < 2^3$ ➔ need keys in 3 bits to present ➔ key size= block size= 3.
- Small sizes are dangerous, however: If Eve catches C=001 ➔ can infer P= 000 or 101.

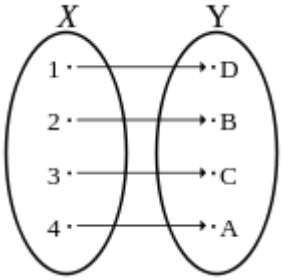
Block cipher: an invertible map

- Map n-bit plaintext blocks to n-bit ciphertext blocks
  - n: block size/length
- For n-bit plaintext and ciphertext blocks and a fixed key, the encryption function is a ***bijection***
- The ***inverse mapping*** is the decryption function,
  - $y = D_k(x)$ denotes the decryption of plaintext x under k.

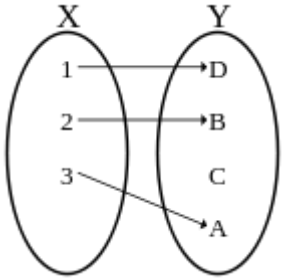


Some math background

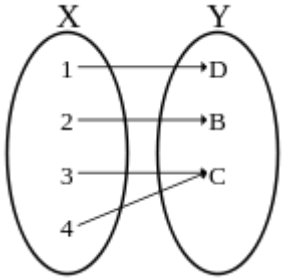
- Map = function
  - A way of associating unique objects to every element in a given set: map $f: A \rightarrow B$ from **A** to ***B*** is a function such that for every $a \in A$, there is a unique object $f(a) \in B$
  - Terms *function, mapping* are synonymous to *map*
- Bijection = Bijective function = both surjective & injective
  - Also, = invertible map, i.e. having a inverse function (map)



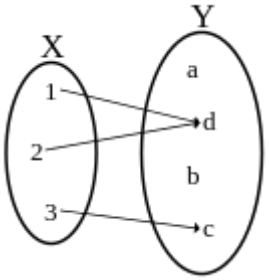
Injective and surjective (bijection)



Injective and non-surjective (injection, or one-to-one)



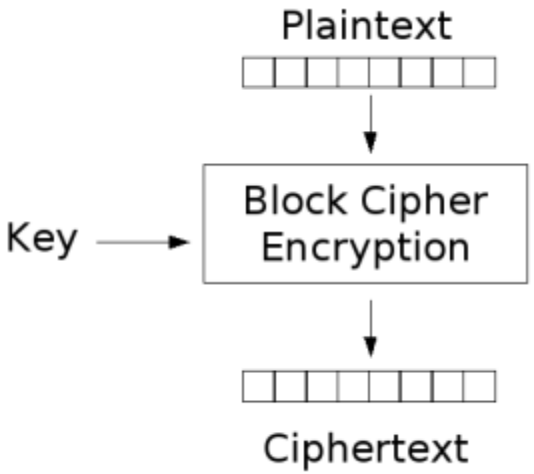
Non-injective and surjective (surjection, or onto)



Non-injective and non-surjective (projection)

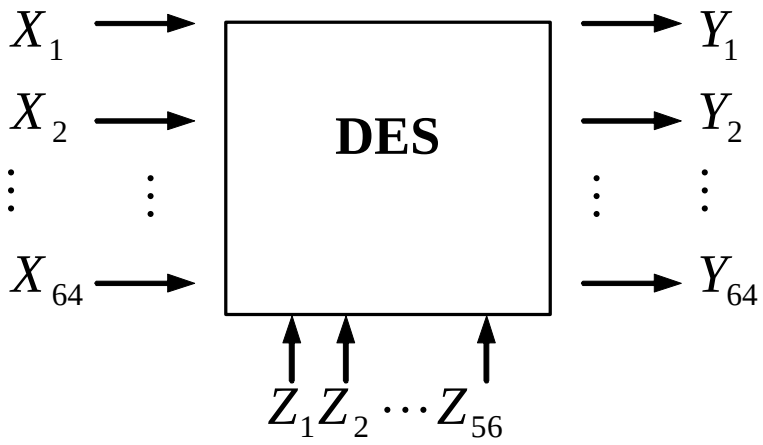
Block cipher: an invertible map

- Map n-bit plaintext blocks to n-bit ciphertext blocks (n: block size/length).
- For n-bit plaintext and ciphertext blocks and a fixed key, the encryption function is a ***bijection***:
  - $E : P_n \times K \rightarrow C_n$ s.t. for all key $k \in K$, $E(x, k)$ is an invertible mapping written $E_k(x)$.
- The inverse mapping is the decryption function, $y = D_k(x)$ denotes the decryption of plaintext x under k.



Example - DES

- DES general structure



General condition in creating secure block ciphers

- The block size has to be large enough to prevent against statistical analysis
  - However, larger block size means slower processing
  - The mentioned example

key	000	001	010	011	100	101	110	111
0	001	111	110	000	100	010	101	011
1	001	110	111	100	011	010	000	101
2	001	000	100	101	110	111	010	011
3	100	101	110	111	000	001	010	011
4	101	110	100	010	011	001	011	111

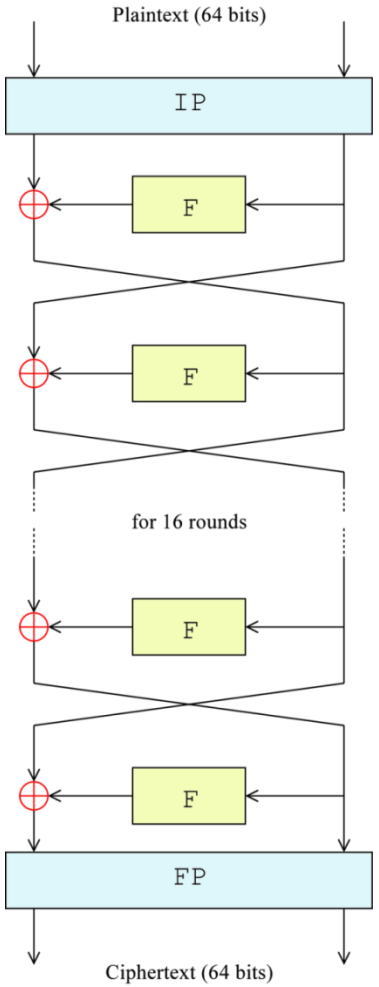
- The key space (then key length) must be large enough to prevent against exhaustive key search
  - However, key length shouldn't be too big that makes key distribution and management more difficult

General principles in designing secure block ciphers

- *Confusion:* As a function, the dependence of the ciphertext on the plaintext should be complex enough so that enemy can't find the rules
  - Note that we only have very basic logic components: AND, OR, XOR, NOT, SHIFT, …
  - The function should be non-linear.
- *Diffusion:* The goal is to spread the information from the plaintext over the entire ciphertext so that changes in plaintext affect many parts in ciphertext
  - This makes it difficult for an enemy to break the cipher by using statistical analysis
- Confusion is made by usings substitutions while *diffusion* by transpositions and/or permutations.

The Feistel structure: processing in rounds

- Block ciphers are usually designed with many rounds where basic round accomplishes the core function f for basic confusion and diffusion.
  - The input of a round is the output of the previous round and a subkey which is generated by a key-schedule algorithm
- The decryption is a reverse process where the sub-keys are handled in the reverse order
  - Because of f as an involution



The overall Feistel structure of DES

Involution

- The core function f usually is an involution,

i.e. $f = f^{-1}$ or $f(f(x)) = x$

  - Example: $x \in$ {binary strings of length 3}

    f: swaps the first two bits and keep the 3rd bit as is, e.g. f(101) = 011

    Then clearly, f is an involution e.g. f(f(101)) = 101

- With f is an involution, the decryption algorithm is almost an inverse process of the encryption algo
  - We will illustrate this with DES later

Block Ciphers Features

- Block size: in general larger block sizes mean greater security.
- Key size: larger key size means greater security (larger key space).
- Number of rounds: multiple rounds offer increasing security.
- Encryption modes: define how messages larger than the block size are encrypted, very important for the security of the encrypted message.

History of Data Encryption Standard (DES)

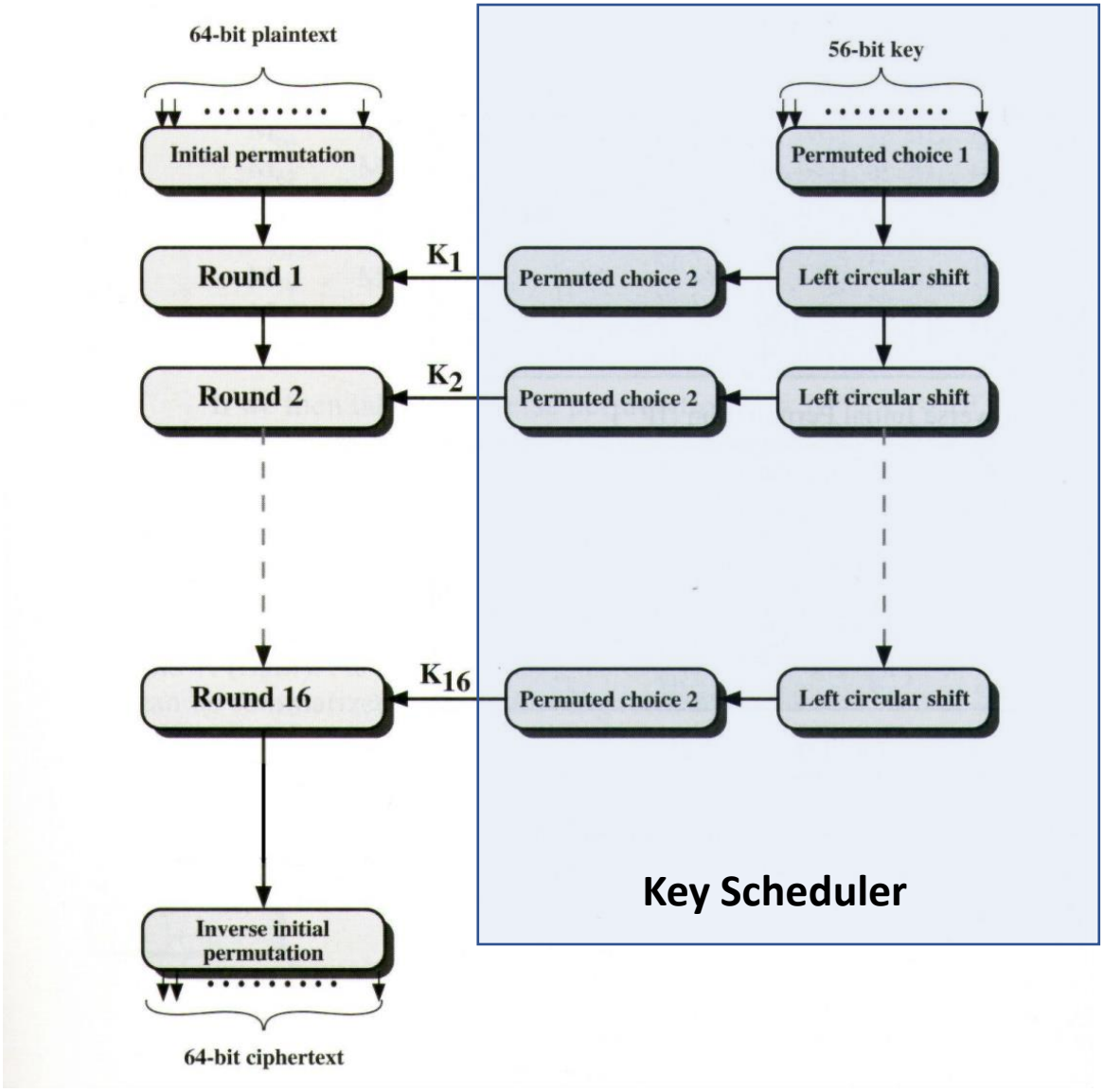
- 1967: Feistel at IBM
  - Lucifer: block size 128; key size 128 bit
- 1972: NBS asks for an encryption standard
- 1975: IBM developed DES (modification of Lucifer
  - block size 64 bits; key size 56 bits
- 1975: NSA suggests modification
- 1977: NBS adopts DES as encryption standard in (FIPS 46-1, 46-2).
- 2001: NIST adopts Rijndael as replacement to DES

DES Features

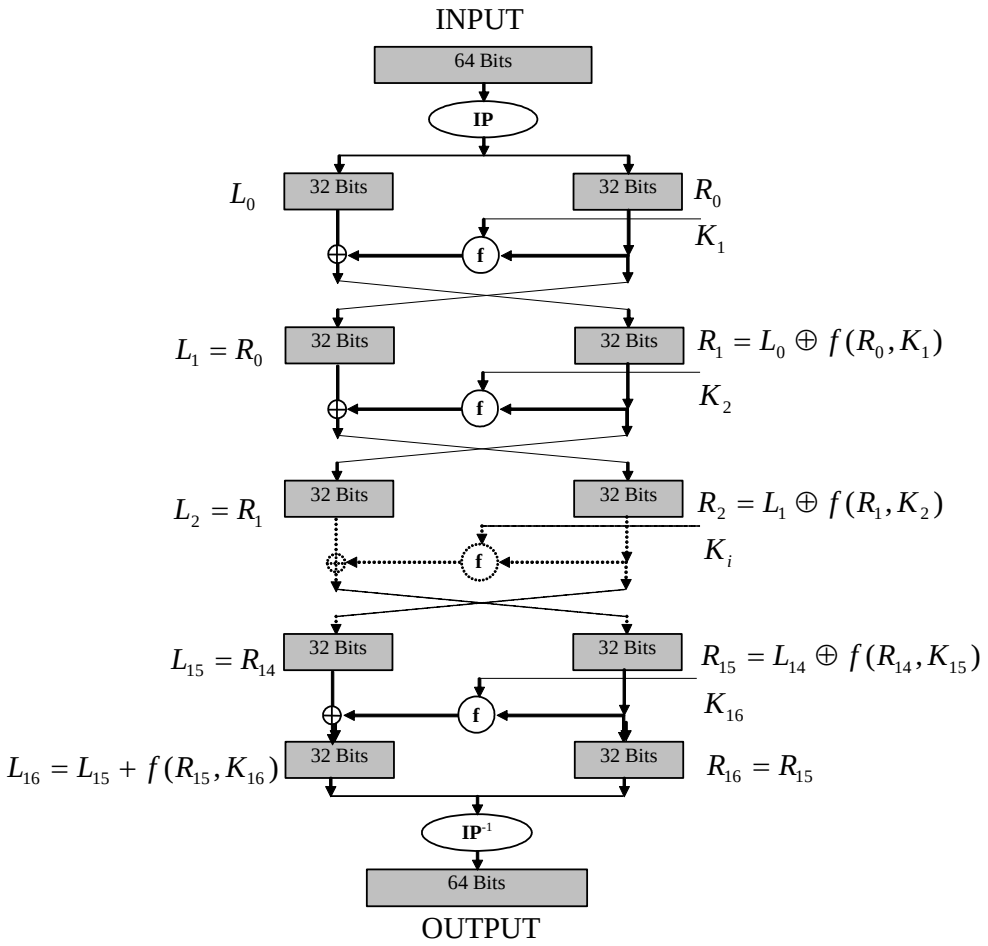
- **Features:**
  - **Block size = 64 bits**
  - **Key size = 56 bits**
  - **Number of rounds = 16**
  - **16 intermediary keys, each 48 bits**

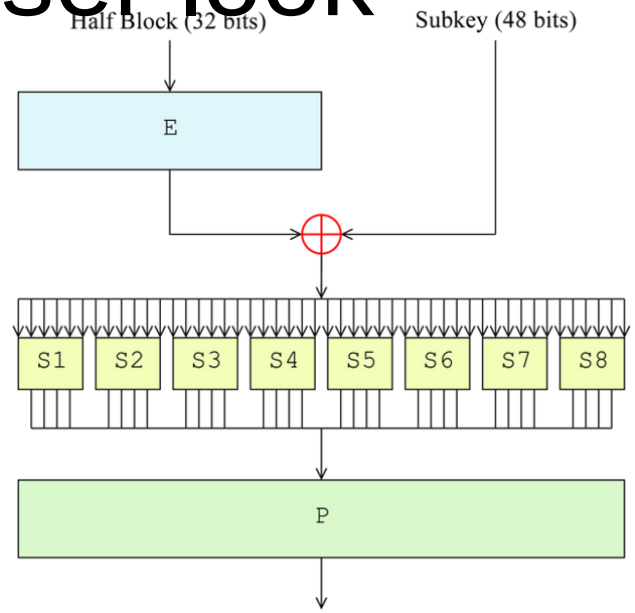
DES Rounds

- the subkeys K_1, K_2, …K_{16} have 48bits each, and are quite different from each other



DES encryption: A closer look





Decryption uses the same algorithm as encryption, except that the subkeys K_1, K2, …K16 are applied in reversed order

The structure of DES round

- Each DES round computation can be represented as: $(L_i,R_i) = (R_{i-1}, L_{i-1} \oplus f(R_{i-1},K_i))$
- This could also be written as

$$(L_i,R_i) = T \bullet F\ (L_{i-1}, R_{i-1})$$

  - where F is replacing L_{i-1} with $L_{i-1} \oplus f(R_{i-1},K_i)$
  - and T is swapping L (left) and R (right).
  - Thus the whole function of each DES round can be presented as a product of funtions F & T (excep the final round - without T).
- Thus the whole function of **DES** can be represented as

$$DES = (IP)^{-1} \bullet F_{16} \bullet T \bullet F_{15} \bullet T \bullet \ldots \bullet F_2 \bullet T \bullet F_1 \bullet (IP)$$

The DES decryption function

- It has all the elements of the encryption function but the subkeys are used in the reverse order

$$DES^{-1} = (IP)^{-1} \bullet F_1 \bullet T \bullet F_2 \bullet T \bullet \ldots \bullet F_{15} \bullet T \bullet F_{16} \bullet (IP)$$

- Noth that either T of F is an involution ($f=f^{-1}$) ➔ the computing the product $DES \bullet DES^{-1}$ results in the indentity function.
  - Again, it shows how the DES encryption and decryption functions are almost the same albeit the subkeys are being used in reverse order.

Structure of the S-Box

- Each S-box contains 4 conversion operation, each converts a 4-bit string input into a 4-bit output string
  - The 4-bit input is the bit 2-5 of the mentioned 6-bit string
  - Each conversion can be seen as a permutation of the 4-bit string "*alphabet*" (of size 16).
  - The bit no. 1 và 6 combined is used to determine which of the 4 conversion row wo be used
    - Thus, they are named CL and CR (left control và right control bit).

S_5		Middle 4 bits of input															
		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Outer bits	00	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

Properties of S-Box

- The design principles of the 8 S-boxes belong to 'Classified information' in the U.S.
- Initially, NSA revealed 3 properties in designing S-boxes, which support making confusion & diffusion
  1. The dependence of the output bit on the input bits is non-linear
  2. Modification of a single input will lead to changes in at least 2 output bits.
  3. If one fix a bit unchanged but vary the remaining 5 input bits then the S-boxes features a special property called uniform distribution: the number of the 0's and 1's in output are almost always equal.
     - This property makes sure that statistical analysis would be of no value for attacking DES.

Properties of S-Box

- These 3 properties is the base of confusion & diffusion in DES
  - After 8 round all the 64 output bits are dependent on all the input bits and all the key bits.
- However, the structure of S-boxes had been a source of controversies wherein people question if NSA (National Security Agency) had still hidden some property of the S-boxes or had left trapdoor inside such that they can break a DES-based ciphertext easier than other people.

DES weakness

- Complement Property

Ký hiệu $\bar{u}$ là phần bù của u (e.g. 0100101 và 1011010 là bù của nhau) thì DES có tính chất sau:

$$y = DES_z(x) \Rightarrow \bar{y} = DES_{\bar{z}}(\bar{x})$$

Cho nên nếu biết MÃ y được mã hóa từ TIN x với khóa z thì ta suy ra $\bar{y}$ được mã hóa từ TIN $\bar{x}$ với khóa $\bar{z}$.

- Tính chất này chính là một điểm yếu của DES bởi vì nhờ đó kẻ địch có thể loại trừ một nửa số khóa cần phải thử khi tiến hành phép thử-giải mã theo kiểu vét cạn (tiếp)

Weak keys

- Weak keys are ones wherein all the sub-keys created by the key scheduler are the same

$$Z_1 = Z_2 = Z_3 = ... = Z_{15} = Z_{16}$$

Which will make the encryption and description functions to be the same

$$DES_z = DES^{-1}_z$$

- There are only 4 such weak keys :

1) [00000001 00000001 ... ... 00000001]

2) [11111110 11111110 ... ... 11111110]

3) [11100000 11100000 11100000 11100000 11110001 11110001 11110001 11110001]

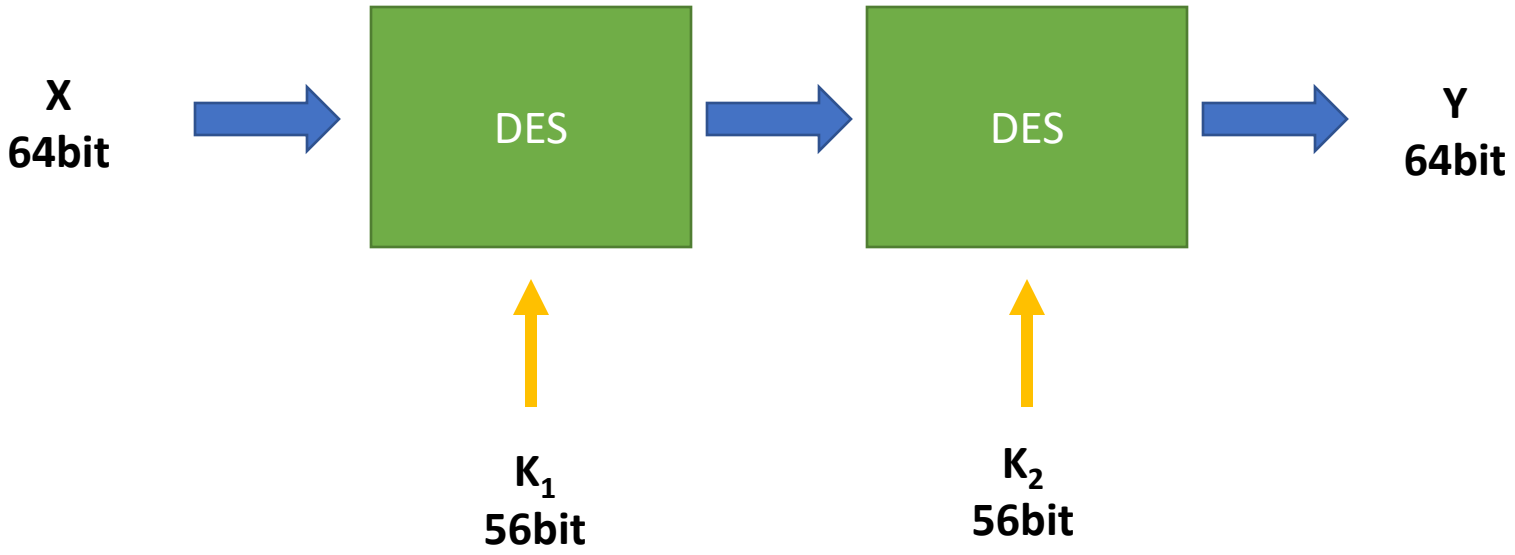
4) [00011111 00011111 00011111 00011111 00001110 00001110 00001110 00001110]

Cryptanalysis of DES

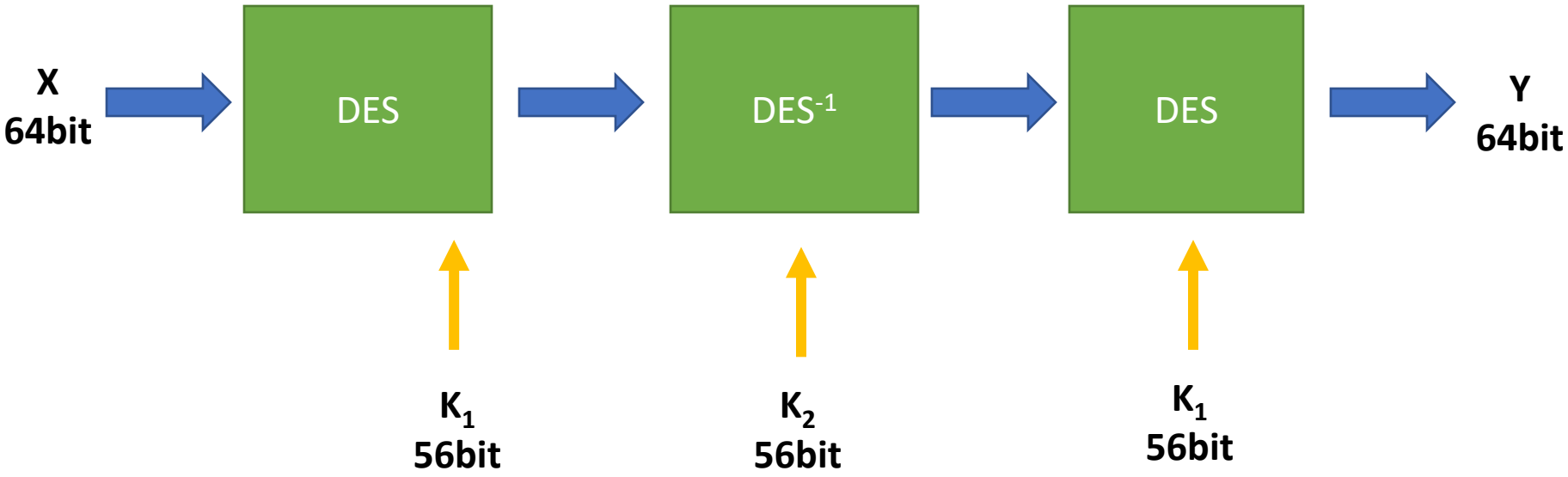
Brute Force:

- Known-Plaintext Attack
- Try all 2^{56} possible keys
- Requires constant memory
- Time-consuming
- DES challenges: (RSA)
  - msg="the unknown message is :xxxxxxxx"
  - CT=" C1 | C2 | C3 | C4"
  - 1997 Internet search: 3 months
  - 1998 EFF machine (costs $250K): 3 days
  - 1999 Combined: 22 hours

2-DES



3-DES



Rijndael Features

- Designed to be efficient in both hardware and
- software across a variety of platforms.
- Uses a variable block size, **128,192, 256-bits**, key size **of 128-, 192-, or 256-bits.**
- 128-bit round key used for each round (Can be precomputed and cached for future encryptions).
- Note: AES uses a 128-bit block size.
- Variable number of rounds (10, 12, 14):
  - 10 if B = K = 128 bits
  - 12 if either B or K is 192 and the other is ≤ 192
  - 14 if either B or K is 256 bits

Rijndael Design

- Operations performed on State (4 rows of bytes).
- The 128 bit key is expanded as an array of 44 32bits words; 4 distinct words serve as a round key for each round; key schedule relies on the S-box
- Algorithms composed of three layers
  - Linear diffusion
  - Non-linear diffusion
  - Key mixing

Decryption

- The decryption algorithm is not identical with the encryption algorithm, but uses the same key schedule.
- There is also a way of implementing the decryption with an algorithm that is equivalent to the encryption algorithm (each operation replaced with its inverse), however in this case, the key schedule must be changed.

Rijandel Cryptanalysis

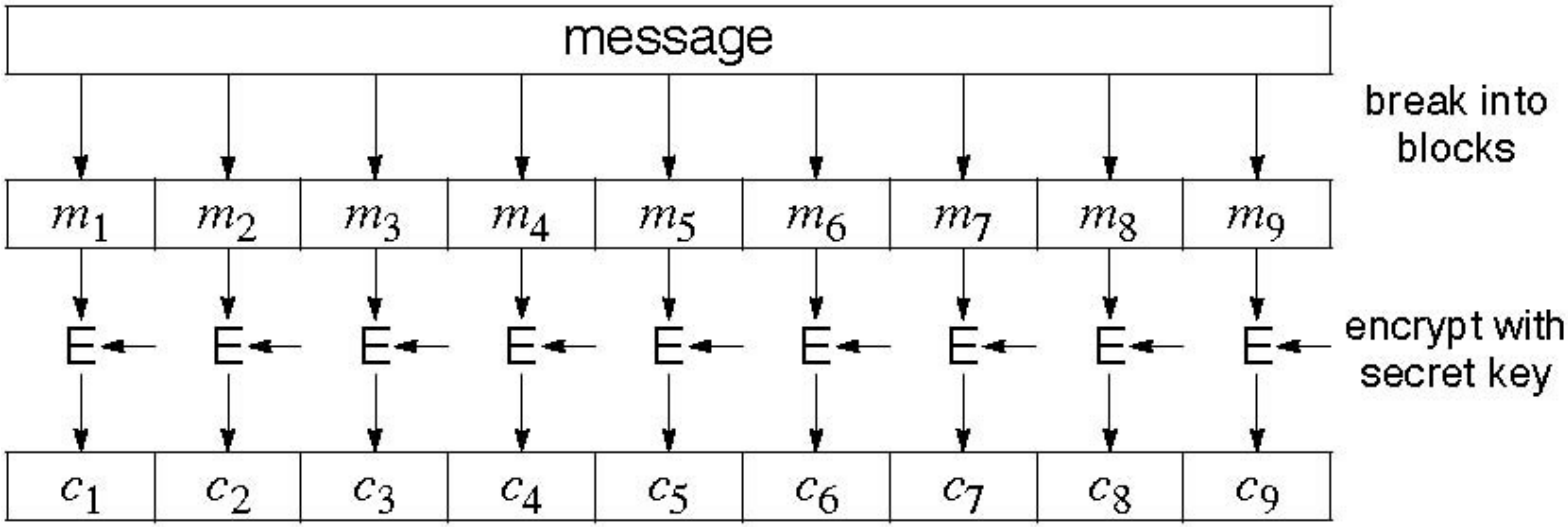
- Academic break on weaker version of the cipher, 9 rounds
- Requires 2^{224} work and 2^{85} chosen *related-key* plaintexts.
- Attack not practical.

Modes of Operation (Encryption modes)

- Mode of operation (or encryption mode):
  - A block cipher algorithm takes on a fixed-length input, i.e. a block, and produce an output, usually a block of the same fixed-length.
  - In practice, we want to encrypt files of various length ➔ need to divide a file into block of that given fixed length ➔ then call the encryption algorithms several times
  - Operation mode: the manner and structure in which we feed the encryption algorithm (several times) with blocks of the plaintext file and concatenate the resulted blocks to produce the ciphertext file.
- The popular modes:
  - ECB, CBC, OFB, CFB, CTR
- We now overview the properties of certain modes (privacy, integrity) and potential attacks against them.

Electronic Code Book (ECB)

- Each block is independently encoded



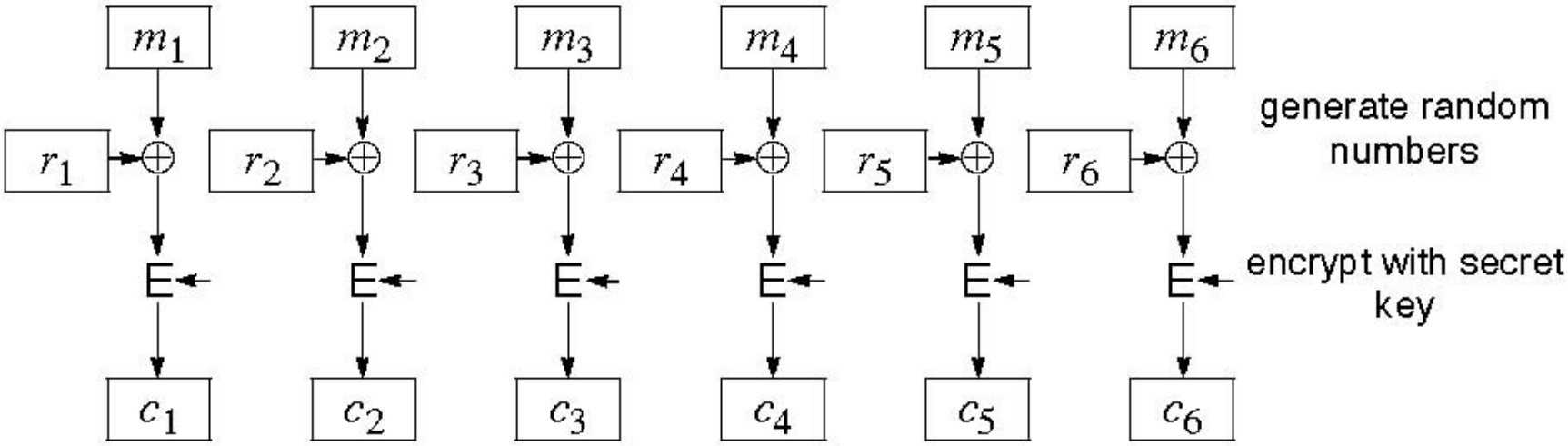
- Problem:
  - Identical Input➔ Identical Output
    - Deterministic: the same data block gets encrypted the same way, reveals patterns of data when a data block repeats.

ECB critics

- Weakness: Replay/Manipulation attack
  - Can insert encoded blocks
  - Reordering ciphertext results in reordered plaintext.
- Strength:
  - Errors in one ciphertext block do not propagate.
- Usage:
  - not recommended to encrypt more than one block of data
  - Encryption in database

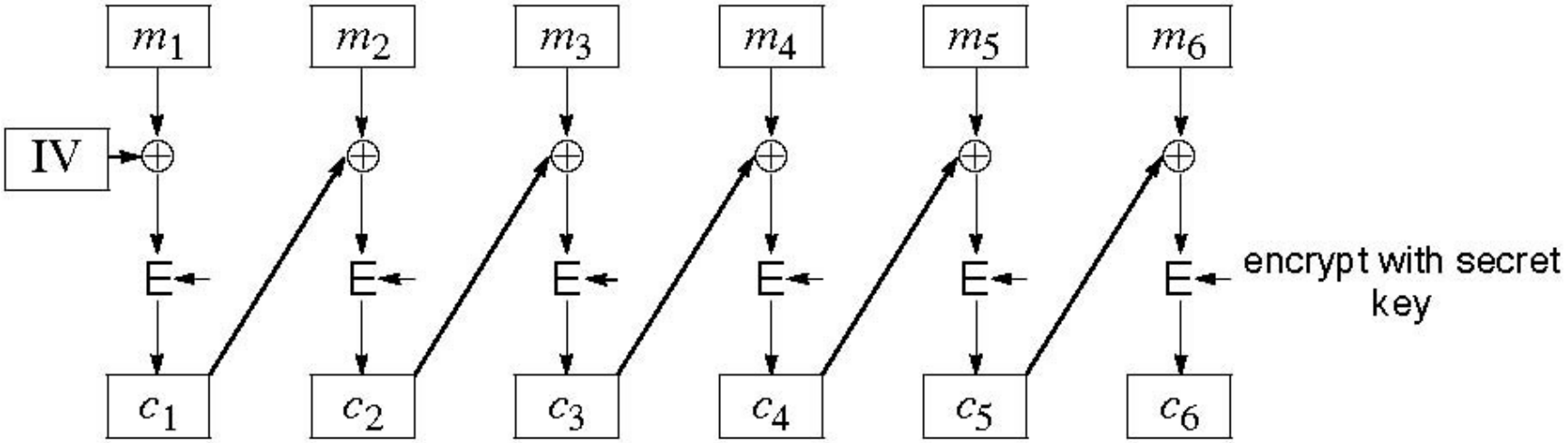
Cipher Block Chaining (CBC)

- Improving on ECB: think of adding a random number before encoding



CBC (cont.)

- The main idea:
  - Use C_i as random number block operation for i+1
  - So, need a so called Initial Value (IV)
    - If no IV, then one can guess changed blocks

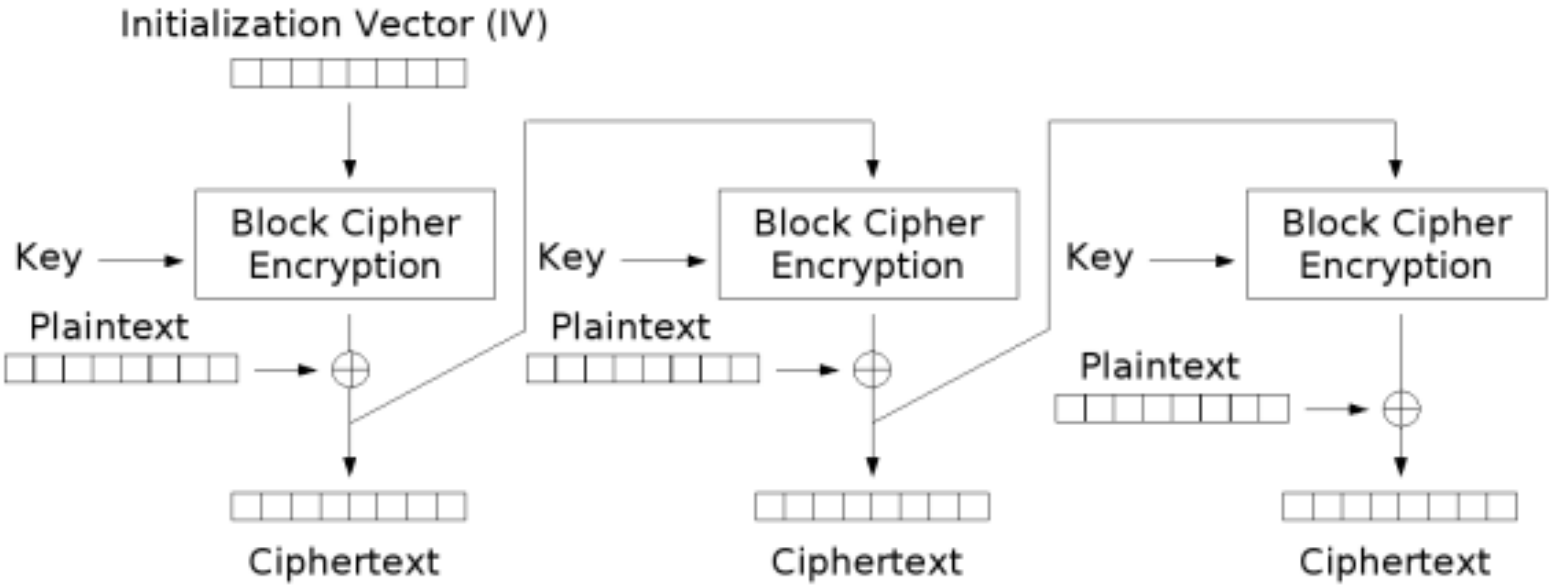


CBC critics

- Good
  - Randomized encryption: repeated text gets mapped to different encrypted data.
    - Can be proven to be “secure” assuming that the block cipher has desirable properties and that random IV’s are used
  - A ciphertext block depends on all preceding plaintext blocks
    - reorder affects decryption
- Bad
  - Errors in one block propagate to two blocks
    - one bit error in C_j affects all bits in M_j and one bit in M_j+1
  - Sequential encryption, cannot use parallel hardware
  - Observation: if $C_i=C_j$ then $E_k(M_i \oplus C_{i-1}) = E_k(M_j \oplus C_{j-1})$; thus $M_i \oplus C_{i-1} = M_j \oplus C_{j-1}$; thus $M_i \oplus M_j = C_{i-1} \oplus C_{j-1}$

Cipher Feedback (CFB)

- The message is XORed with the feedback of encrypting the previous block



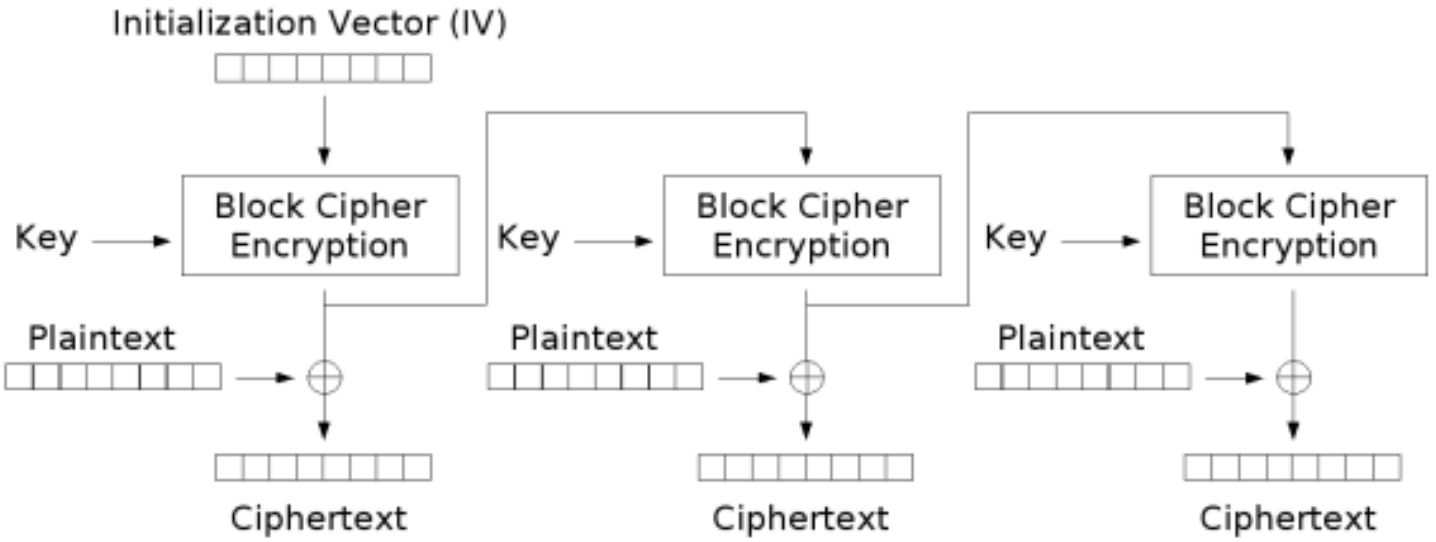
Cipher Feedback (CFB) mode encryption

CFB critics

- Good
  - Randomized encryption
  - A ciphertext block depends on all preceding plaintext blocks; reorder affects decryption
  - Decreased throughput.
    - Can vary the number of bits feed back, trading off throughput for ease of use
- Bad
  - Errors propagate for several blocks after the error, but the mode is self-synchronizing (like CBC).
  - Sequential encryption

Output Feedback (OFB)

- IV is used to generate a stream of blocks
- Stream is used a one-time pad and XOR'ed to plain text



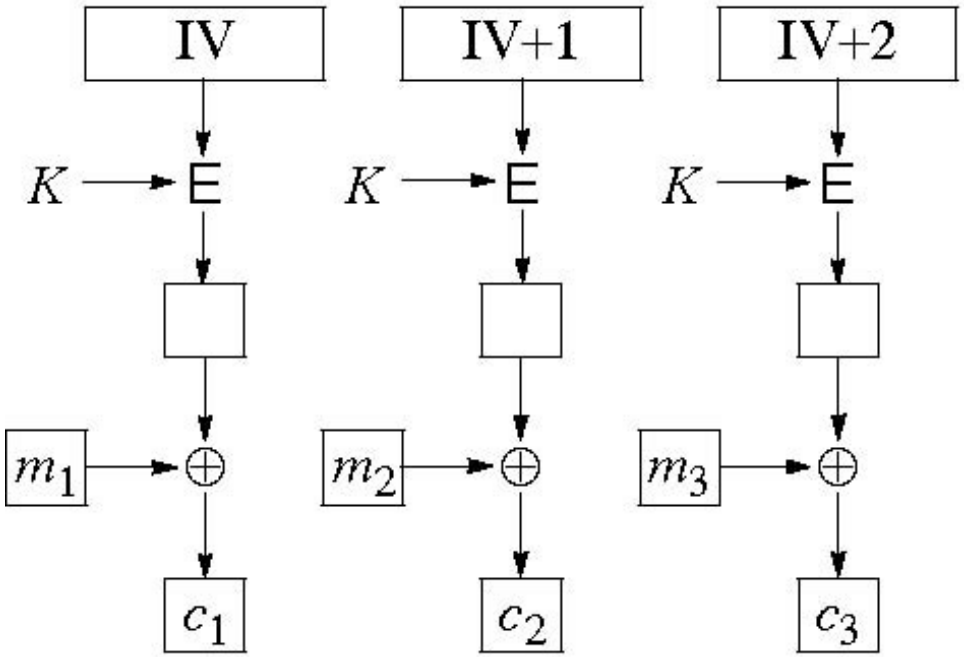
Output Feedback (OFB) mode encryption

OFB critics

- Randomized encryption
- Sequential encryption, but preprocessing possible
- Error propagation limited
- Subject to limitation of stream cipher

Counter Mode (CTR)

- If the same IV and key is used again,
  - XOR of two encrypted messages = XOR of plain text
- IV is incremented and used to generated one-time pad



CTR critics

- Software and hardware efficiency: different blocks can be encrypted in parallel.
- Preprocessing: the encryption part can be done offline and when the message is known, just do the XOR.
- Random access: decryption of a block can be done in random order, very useful for hard-disk encryption.
- Messages of arbitrary length: ciphertext is the same length with the plaintext (i.e., no IV).

Intern student emails

- benseirimane.tiinothe2023t021@sis.hust.edu.vn; didtsch.mlr23t016@sis.hust.edu.vn; fortin.rpp23t015@sis.hust.edu.vn; hanel.ye23t039@sis.hust.edu.vn; herz.j23t036@sis.hust.edu.vn; lecordier.tel23t014@sis.hust.edu.vn; monteiro.a23t012@sis.hust.edu.vn; perez.vbg23t048@sis.hust.edu.vn; sousa.dd23t013@sis.hust.edu.vn

- Please let me know the emails above working for you? MOOC ok? If you need help pls. email me at: vannk@soict.hust.edu.vn

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