10.6

$\mu_{\pi} = \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

$r_{\pi} = \frac{1}{3}(1)(1)(0)(2) + \frac{1}{3}(1)(1)(1) = \frac{1}{3}$

$$v_A = (1)(1)\left(0 - \frac{1}{3} + v_B\right) = v_B - \frac{1}{3}$$

$$v_B = (1)(1)\left(0 - \frac{1}{3} + v_C\right) = v_C - \frac{1}{3}$$

$$v_C = (1)(1)\left(1 - \frac{1}{3} + v_A\right) = v_A + \frac{2}{3}$$

These three equations are degenerate, infinite possible solutions.

Clearly series does not converge so unique sum does not exist, we'll use the Cesàrosummation to assign a value. In the following, s_n will be used to denote the sequence of partial sums (up to the nth term)

A: $G_t = 0 - \frac{1}{3} + 0 - \frac{1}{3} + 1 - \frac{1}{3} \ldots$

$s_n = -\frac{1}{3}, -\frac{2}{3}, 0, -\frac{1}{3}, \frac{2}{3}, 0 \ldots$

$\sum_{i=1}^{n} s_i = -\frac{1}{3}, -1, -1, -\frac{4}{3}, -\frac{6}{3}, -\frac{6}{3}, -\frac{7}{3}, -\frac{9}{3}, -\frac{9}{3}$

$\frac{1}{n}\sum_{i=1}^{n} s_i = -\frac{1}{3}, -\frac{1}{2}, -\frac{1}{3}, -\frac{4}{12}, -\frac{6}{15}, -\frac{6}{18}, -\frac{7}{21}, -\frac{9}{24}, -\frac{9}{27}$

$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} s_i = -\frac{1}{3}$

B: $G_t = 0 - \frac{1}{3} + 1 - \frac{1}{3} + 0 - \frac{1}{3} \ldots$

$s_n = -\frac{1}{3}, \frac{1}{3}, 0, -\frac{1}{3}, \frac{1}{3}, 0 \ldots$

$\sum_{i=1}^{n} s_i = -\frac{1}{3}, 0, 0, \frac{1}{3}, 0, 0, \ldots$

$\frac{1}{n}\sum_{i=1}^{n} s_i = -\frac{1}{3}, 0, 0, -\frac{1}{12}, 0, 0, -\frac{1}{21}, 0, 0$

$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} s_i = 0$

C: $G_t = 1 - \frac{1}{3} + 0 - \frac{1}{3} + 0 - \frac{1}{3} \ldots$

$s_n = \frac{2}{3}, \frac{1}{3}, 0, \frac{2}{3}, \frac{1}{3}, 0 \ldots$

$\sum_{i=1}^{n} s_i = \frac{2}{3}, 1, 1, \frac{5}{3}, \frac{6}{3}, \frac{6}{3}, \frac{8}{3}, \frac{9}{3}, \frac{9}{3}$

$\frac{1}{n}\sum_{i=1}^{n} s_i = \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \frac{5}{12}, \frac{6}{15}, \frac{6}{18}, \frac{8}{21}, \frac{9}{24}, \frac{9}{27}$

$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} s_i = \frac{1}{3}$

These values hold to the originally solved relationships

10.7) $r(\pi) = \frac{1}{2}$

A: $G_t = 1 - \frac{1}{2} + 0 - \frac{1}{2} + 1 - \frac{1}{2} + 0 - \frac{1}{2} \ldots$

$s_n = \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \ldots$

$\sum_{i=1}^{n} s_i = \frac{1}{2}, \frac{1}{2}, \frac{2}{2}, \frac{2}{2}, \frac{3}{2}, \frac{3}{2}, \frac{4}{2}, \frac{4}{2}$

$\frac{1}{n}\sum_{i=1}^{n} s_i = \frac{1}{2}, \frac{1}{4}, \frac{2}{6}, \frac{2}{8}, \frac{3}{10}, \frac{3}{12}, \frac{4}{14}, \frac{4}{16}$

$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} s_i = \frac{1}{4}$

B: $G_t = 0 - \frac{1}{2} + 1 - \frac{1}{2} + 0 - \frac{1}{2} + 1 - \frac{1}{2} \ldots$

$s_n = -\frac{1}{2}, 0, -\frac{1}{2}, 0 \ldots$

$\sum_{i=1}^{n} s_i = -\frac{1}{2}, -\frac{1}{2}, -\frac{2}{2}, -\frac{2}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{4}{2}, -\frac{4}{2}$

$\frac{1}{n}\sum_{i=1}^{n} s_i = -\frac{1}{2}, -\frac{1}{4}, -\frac{2}{6}, -\frac{2}{8}, -\frac{3}{10}, -\frac{3}{12}, -\frac{4}{14}, -\frac{4}{16}$

$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} s_i = -\frac{1}{4}$