

Artificial Intelligence HW2

By 400540213S 林亞辰

Description :

First of all, suppose there is a $(H + 2) \times (W + 2)$ maze with an outer wall, the grid content of * is the wall and cannot be walked into. The grid content 0 ~ 9 indicates the height of the grid, which can be walked into, but if you walk from a grid with height c to the next grid with height d , it costs the robot $10 + (c-d)^2$ in power cost. The robot starts from the top left corner and can move to any of the four adjacent spaces. If it finally reaches the end point in the lower right corner, the task is completed.

Of course, it would be great to find the best solution that costs the least total power!

It will use the following three algorithms to perform the exercise:

1. Uniform-cost search (An algorithm likes Dijkstra's single-source-shortest-path algorithm)
2. Iterative Deepening Depth-First Search (IDS or IDDFS)
3. Iterative Deepening A* (IDA*)

Exercise Essentials

- > How to display the output plate?
- > How the route should be generated?
- > How to implement Frontier?
- > What informations to store at the node?
- > How to distinguish repetition?
- > Will there be infinite loops?
- > Will the memory explode?
- > Will the result be the best solution?
- > Which heuristic result is better to use?
- > How to estimate the Time complexity and Space complexity?

Written Report :

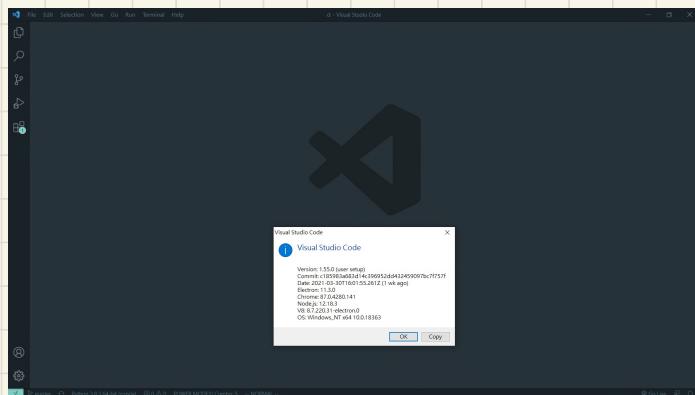
1. Detailed information about the hardware and software specifications of the machine you are using, the operating system, the development software version, and how to execute the program. In addition, please provide your contact phone number for your convenience.

(1) Hardware and software specifications of the machine :

```
□ 命令提示元件
Microsoft Windows [版本 10.0.18363.1379]
(c) 2019 Microsoft Corporation. 權利所有，並保留一切權利。
C:\Users\林亞辰>systeminfo

主機名稱: DESKTOP-70B4947
作業系統名稱: Microsoft Windows 10 家用版
作業系統版本: 10.0.18363 N/A 編號 18363
作業系統製造商: Microsoft Corporation
作業系統設定: 獨立工作站
作業系統組建類型: Multiprocessor Free
註冊公司擁有者: N/A
註冊公司地點: N/A
產品識別碼: 00325-80000-00000-AAOEM
原始安裝日期: 2020/3/22, 11:24:20
系統開機時間: 2021/3/2, 14:40:09
系統製造商: ASUSTeK COMPUTER INC.
系統名稱: X510UNR
系統類型: x64-based PC
處理器: 已安裝 1 處理器。
[0]: Intel® Family 6 Model 142 Stepping 10 GenuineIntel ~1792 Mhz
American Megatrends Inc. X510UNR.309, 2019/5/14
BIOS 版本: C:\WINDOWS\system32
Windows 目錄: C:\WINDOWS\system32
系統目錄: C:\WINDOWS\system32
開機表置: \Device\HarddiskVolume3
系統地區設定: zh-tw;中文 (台灣)
輸入法地區設定: zh-tw;中文 (台灣)
時區: (UTC+08:00) 台北
實體記憶體總計: 16,271 MB
可用實體記憶體: 9,160 MB
虛擬記憶體: 大小上限: 18,703 MB
虛擬記憶體: 可用: 9,984 MB
虛擬記憶體: 使用中: 8,719 MB
分頁檔位置: C:\pagefile.sys
網域: WORKGROUP
登入: \DESKTOP-70B4947
登入手動: 已安裝 18 Hotfix。
Hotfix:
```

(2) Develop Software Version :



(3) How to execute the program :

可以用 make 指令來執行程式，亦可直接下達 "python filename" 來執行程式。(見下頁)

詳細資訊可參照 README.md 的內容：

```
README.md

If you have the make command :

If your device supports the make command, this will be much easier ( because my file name is very long... ).  

You can type make in Terminal to see the output of all Python files directly.  

You can also type in :

>>>make  

// Output all the execution results

>>>make p1  

// Output the execution result of P1_UCS

>>>make p2  

// Output the execution result of P2_IDS

>>>make p3  

// Output the execution result of P3_IDSSTAR


If you do not have the make command :

If your device does not support the make command, there will be a little inconvenience ( because my file name is very long... ).  

You can type in :

>>>python mazeMaker.py  

// If you want to random the looks of maze in input.txt

>>>python P1_UCS.py  

// Output the execution result of P1_UCS

>>>python P2_IDS.py  

// Output the execution result of P2_IDS

>>>python P3_IDSSTAR.py  

// Output the execution result of P3_IDSSTAR
```

(4) Contact phone number : 0972-065282

2. First you set up at least 5 input files for the test. The size of the disk should be large or small, and the difficulty of the solution should be different. Please explain how you create these input files for testing.

(1) 5x5 maze having some ways to the target

```
mazeMaker.py M
input.txt M X

1-4-4-5thGrade(lI) > Artificial Intelligence > HV

1 | *****
2 | *580*
3 | *311*
4 | *244*
5 | *****
6 | |
```

(2) 5x5 maze having no way to the target

```
mazeMaker.py M
input.txt M X

1-4-4-5thGrade(lI) > Artificial Intelligence > HV

1 | *****
2 | *580*
3 | *31**_
4 | *_1*2*
5 | *****
6 | |
```

(3) 8x8 maze having some ways to the target

```
mazeMaker.py M input.txt M X
1-4-4-5thGrade(II) > Artifical Intelligence > HV
1 *****
2 *580311*
3 *244658*
4 *2*0856*
5 *129*38*
6 ***102**
7 **15*683*
8 *****
9
```

(4) 8x8 maze having no way to the target

```
mazeMaker.py M input.txt M X
1-4-4-5thGrade(II) > Artifical Intelligence > HV
1 *****
2 *580311*
3 *244658*
4 *2*0856*
5 *129*38*
6 ***102**
7 **15*6
8 *****
9
```

(5) 10x10 maze having some ways to the target

```
mazeMaker.py M input.txt M X
1-4-4-5thGrade(II) > Artifical Intelligence > HV
1 *****
2 *58031124*
3 *46582*08*
4 *56129*38*
5 ***102*15*
6 **6838*09*
7 **83915*3*
8 *31082350*
9 *61781484*
10 *****
11
```

(6) 10x10 maze having no way to the target

```
mazeMaker.py M input.txt M X
1-4-4-5thGrade(II) > Artifical Intelligence > HV
1 *****
2 *58031124*
3 *46582*08*
4 *56129*38*
5 ***102*15*
6 **6838*09*
7 **83915*3*
8 *3108235**
9 *061781*4*
10 *****
11
```

(7) 15x15 maze having some ways to the target

```
mazeMaker.py M input.txt M X
1-4-4-5thGrade(II) > Artifical Intelligence > HV
1 *****
2 *5803112446582*
3 **0856129*38****
4 *102*15*6838*0*
5 *9*83915*33108*
6 *2350617814840*
7 *676*7*11941*4*
8 *98*7943*5*771*
9 ***5642*60736*4*
10 ***70*8*21*8099*
11 ***810002*66*13*
12 *7*609294*8610*
13 *10128600*6238*
14 *26**257910057*
15 *****
16
```

* How to create these input files for testing:

我主要是寫另一支程式叫 mazeMaker.py 來製作測試用迷宮(詳細可見 code)，其中最核心的建構部分為 20~34 行的 def MazeMaker(maze, h, w) 這個 function。

```

1-4-5thGrade(lj) > Artificial Intelligence > HW2 > Artificial-Intelligence_Path-Finding-of-Maze > 🐍 mazeMaker.py
1 import numpy as np
2 import sys
3
4 # Global Variable
5 np.random.seed(1)
6 # needs to small than 1000 x 1000
7 height = 15
8 width = 15
9 maze = [[0] * width for i in range(height)]
10
11
12 > def PrintMaze(m):
13     # end func PrintMaze
14
15
16
17 def MazeMaker(maze, h, w):
18     for i in range(h):
19         for j in range(w):
20             if i == 0 or j == 0 or i == h-1 or j == w-1:
21                 maze[i][j] = '*'
22
23             # Set no solution statement
24             # elif (i == h - 2 and j == w-2) or (i == h - 2 and j == w-3):
25             #     maze[i][j] = '**'
26             else:
27                 tmp = int(np.random.random()*12)
28                 if tmp >= 10:
29                     maze[i][j] = '**'
30                 else:
31                     maze[i][j] = str(tmp)
32
33     # end func MazeRandCreate
34
35
36
37 > if __name__ == '__main__':
38
39
40

```

23~24 行為設製迷宮牆的部分。

26~27 行為設製無解的迷宮(利用將 target 封住的方式)

29~33 行利用亂數設值，並保設計約 1/4 的机率生成 '*'，也就是牆的部分。

3. three programs (please name them P1_UCS, P2_IDS, P3_IDSSTAR) should be commented in the source code, please explain how to execute these three codes

詳見 README.md

README.md

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```

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You can type in :

```

>>> python mazeMaker.py
// If you want to random the looks of maze in input.txt

>>> python P1_UCS.py
// Output the execution result of P1_UCS

>>> python P2_IDS.py
// Output the execution result of P2_IDS

>>> python P3_IDSSTAR.py
// Output the execution result of P3_IDSSTAR

```

4. Please explain what method, what data structure, and what technique you used to solve this problem with the program P1_UCS, and explain how you performed when you tested some plates, how much time and space you used horizontally (assuming plate $(H+2) \times (W+2)$), and how large a plate your program can solve the problem? Please use some examples to help illustrate.

我主要參考講義上有關 Uniform-cost search algorithm 的敘述去查看要如何實作，理所當然用 Dijkstra's algorithm 來實作。一開始用陣列來實作，但發現存取資料相當不容易，因此改採用 class 的 data structure 來建構。不久再次遇到瓶頸，即 Graph 的部分邏輯不足，因此參考網路上的 Graph 如何建立，經修改過後方完成 UCS。
路線的產生方式由 def ShortestPath(v, path) 此 function 來找尋，其主要理念為：由 target 回推 .previous 的座標紀錄在 path 中；

```

122     def ShortestPath(v, path):
123         # make shortest path from v.previous
124         if v.previous:
125             path.append(v.previous.id)
126             shortestPath(v.previous, path)
127         return
128     # end func shor

```

```

18     class Vertex:
19         def __init__(self, node):
20             self.id = node
21             self.adjacent = {}
22             # set distance to infinity for all nodes
23             self.distance = sys.maxsize
24             # Mark all nodes unvisited
25             self.visited = False
26             # Predecessor
27             self.previous = None
28
29     def add_neighbor(self, neighbor, weight=0):
30         self.adjacent[neighbor] = weight
31
32     def get_weight(self, neighbor):
33         return self.adjacent[neighbor]
34
35     def set_distance(self, dist):
36         self.distance = dist
37
38     def set_visited(self):
39         self.visited = True
40
41     def set_previous(self, prev):
42         self.previous = prev
43
# end class Vertex

```

class Vertex 中包含了節點所紀錄的資料；
利用 priority queue 的方式存取接下來要走的節點；
最後完成的程式不会有無窮迴圈，測試下來亦沒有記憶體不足的問題。

実測結果：

5x5 maze with path

```

python mazeMaker.py
The maze is :
* * * *
* 5 8 0 *
* * V 1 *
* 2 4 4 *
* * * *
python P1_UCS.py
=====
P1_UCS =====
The shortest path in maze is :
* 5 8 0 *
* * V 1 *
* 2 4 4 *
* * * *
The shortest path in a list is :
['1', '2', '3', '4', '5']
The cost is : 49
Execution time = 0.010924 sec

```

5x5 maze without path

```

python mazeMaker.py
The maze is :
* * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 5 * 0 2 * *
* * * 1 0 V * *
python P1_UCS.py
=====
P1_UCS =====
No solution ! ( There is no route from the start-position to the
end-position )
Execution time = 0.012919 sec

```

8x8 maze with path

```

python mazeMaker.py
The maze is :
* * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 5 * 0 V * *
* * * 1 0 6 V T *
* * * * * * * *
python P1_UCS.py
=====
P1_UCS =====
The shortest path in maze is :
* * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 V 6 *
* 1 2 9 * V 8 *
* 1 5 * 0 V * *
* * * 1 0 6 V T *
* * * * * * * *
The shortest path in a list is :
['1', '2', '3', '4', '5', '6', '7', '8', '9', '10', '11', '12', '13', '14', '15', '16', '17', '18']
The cost is : 184
Execution time = 0.011013 sec

```

8x8 maze without path

```

python mazeMaker.py
The maze is :
* * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 5 * 0 2 * *
* * * 1 0 6 0 *
* * * * * * * *
python P1_UCS.py
=====
P1_UCS =====
No solution ! ( There is no route from the start-position to the
end-position )
Execution time = 0.009937 sec

```

10x10 maze with path

```
python mazeMaker.py
The maze is :
+-----+
| * * * | * * * *
| * 8 5 | 8 2 * 0 8 *
| 5 6 1 | 2 9 * 3 8 *
| * * 1 | 0 2 * 1 5 *
| 6 8 3 | 8 0 9 *
| * 8 1 | 5 4 * 6 4 *
| * 3 0 | 8 2 3 5 0 *
| 6 1 7 8 | 1 4 8 4 *
+-----+
```

python P1_UCS.py

===== P1_UCS =====

```
The shortest path in maze is :
+-----+
| 5 8 0 3 1 1 2 4 *
| v > > 8 2 * 0 8 *
| 5 6 1 2 9 * 3 8 *
| * 8 1 v > 2 3 5 *
| * 6 8 v 8 * 0 9 *
| * 8 3 v 1 5 * 3 *
| * 3 1 0 8 2 3 5 * 4 *
| * 0 6 1 7 8 1 4 8 *
+-----+
```

The shortest path in a list is :
 ['1', '2', '3', '4', '5', '6', '7', '8', '9', '10', '11', '12', '13', '14', '15', '16', '17', '18', '19', '20', '21', '22', '23', '24', '25', '26', '27', '28', '29', '30', '31', '32', '33', '34', '35', '36', '37', '38', '39', '40', '41', '42', '43', '44', '45', '46', '47', '48', '49', '50', '51', '52', '53', '54', '55', '56', '57', '58', '59', '60', '61', '62', '63', '64', '65', '66', '67', '68', '69', '70', '71', '72', '73', '74', '75', '76', '77', '78', '79', '80', '81', '82', '83', '84', '85', '86', '87', '88', '89', '90', '91', '92', '93', '94', '95', '96', '97', '98', '99', '100']
 The cost is : 275

Execution time = 0.010972 sec

10x10 maze without path

python mazeMaker.py

The maze is :

```
+-----+
| * * * * * * * * *
| * 5 8 0 3 1 1 2 4 *
| * 4 6 5 8 2 * 0 8 *
| * 5 6 1 2 9 * 3 8 *
| * * 1 0 2 * 1 5 *
| * 6 8 3 8 * 0 9 *
| * 8 3 9 1 5 * 3 *
| * 3 1 0 8 2 3 5 * 4 *
| * 0 6 1 7 8 1 4 8 *
+-----+
```

python P1_UCS.py

===== P1_UCS =====

No solution ! (There is no route from the start-position to the end-position)
 Execution time = 0.009974 sec

When I tried to stress test the program...

100x100 maze

only 40 sec...

```
python P1_UCS.py
=====
The shortest path in a list is :
[1, '2', '3', '4', '5', '6', '7', '8', '9', '10', '11', '12', '13', '14', '15', '16', '17', '18', '19', '20', '21', '22', '23', '24', '25', '26', '27', '28', '29', '30', '31', '32', '33', '34', '35', '36', '37', '38', '39', '40', '41', '42', '43', '44', '45', '46', '47', '48', '49', '50', '51', '52', '53', '54', '55', '56', '57', '58', '59', '60', '61', '62', '63', '64', '65', '66', '67', '68', '69', '70', '71', '72', '73', '74', '75', '76', '77', '78', '79', '80', '81', '82', '83', '84', '85', '86', '87', '88', '89', '90', '91', '92', '93', '94', '95', '96', '97', '98', '99', '100]
The cost is : 482
Execution time = 0.017941 sec
```

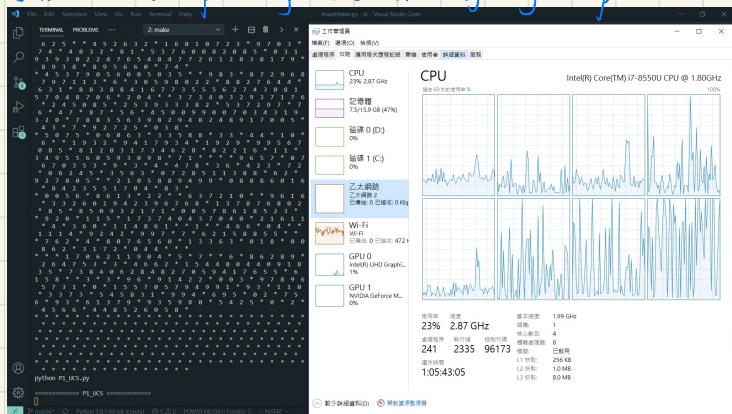
More ☺

150x150 maze

about 14.5 min

```
python P1_UCS.py
=====
The shortest path in a list is :
[1, '2', '3', '4', '5', '6', '7', '8', '9', '10', '11', '12', '13', '14', '15', '16', '17', '18', '19', '20', '21', '22', '23', '24', '25', '26', '27', '28', '29', '30', '31', '32', '33', '34', '35', '36', '37', '38', '39', '40', '41', '42', '43', '44', '45', '46', '47', '48', '49', '50', '51', '52', '53', '54', '55', '56', '57', '58', '59', '60', '61', '62', '63', '64', '65', '66', '67', '68', '69', '70', '71', '72', '73', '74', '75', '76', '77', '78', '79', '80', '81', '82', '83', '84', '85', '86', '87', '88', '89', '90', '91', '92', '93', '94', '95', '96', '97', '98', '99', '100]
The cost is : 1805
Execution time = 88.250256161146 sec
```

But there is still room to spare of the CPU of my computer when running...



5. Please explain what method, what data structure, and what technique you used to solve this problem with the program P2_IDS, and explain how you performed when you tested some plates, how much time and space you used horizontally (assuming plate (H+2)x(W+2)), and how large a plate your program can solve the problem? Please use some examples to help illustrate.

原本是用 recursion 的方式實作，但實在是太慢了，因此嘗試用迴圈的方式實作，分為 def LDS 與 def IDDFS 兩部分。

```
mazeMaker.py M P2_IDS.py M
1-4-4-5thGrade(l) > Artificial Intelligence > HW2 > Artificial-Intelligence_Path-Finding-of-Maze > P2_IDS.py -.

104
105     def LDS(current, target, limit, cost, path):
106         current.reset()
107         while(1):
108             if current.id == '1' and len(current.adjacent) == len(current.visited_adjacent):
109                 return False
110             if current.id == target.id:
111                 cur_cost = current.distance
112                 if cur_cost < cost:
113                     cost = current.distance
114                     ResetPath(target, path)
115                     ShowPath(target, path)
116                     prev = current.previous
117                     current.reset()
118                     current = prev
119                     continue
120             if current.level == limit:
121                 cur = current.previous
122                 current.reset()
123                 current = prev
124                 continue
125             if checkedVisited(current):
126                 cur = current.previous
127                 current.reset()
128                 current = prev
129                 continue
130
131             for next in current.adjacent:
132                 if len(next.visited_adjacent):
133                     current.visited_adjacent.append(next.id)
134                     continue
135
136                 if next.id not in current.visited_adjacent and next.previous == None:
137                     current.visited_adjacent.append(next.id)
138                     new_dist = current.distance + current.get_weight(next)
139                     next.set_distance(new_dist)
140                     next.set_previous(current)
141                     if next.id == target.id:
142                         break
143                     next.visited_adjacent.append(target.id)
144                     next.level = current.level + 1
145                     current = next
146                     break
147
148             return False
149         # end of LDS
```

```
mazeMaker.py M P2_IDS.py M X
1-4-4-5thGrade(l) > Artificial Intelligence > HW2 > Artificial-Intelligence_Path-Finding-of-Maze > P2_IDS.py -.

211
212     def IDDFS(start, target, limit, path, aGraph):
213         num = 0
214         for i in range(1, limit):
215             # Set the distance for the start node to zero
216             start.set_distance(0)
217             start.set_level(0)
218             # print("limit = %d" % (i))
219             DLS(start, target, i, sys.maxsize, path)
220             if len(path) > 1:
221                 num = num + 1
222                 # for v in aGraph:
223                 #     v.reset()
224                 if num >= 5:
225                     break
226             # end func TDDFS
```

其中 IDDFS 主要用來控制 limit 故值並設立初值的部分，比較特別的是 223~224 與 225~226 的部分，223~224 用來控制找解的長度，當找到第一組解則再跑下輪復回後，因為大多投 maze 是不會有繞路較省力的事件，若有也就不差一點點，因此儘多 run 与輪提高結束以達到時間上的效益(尤其愈

後來去詢問老師後得知可直接結束，因此有作修改。
 255~256 的部分是來判斷無解情況用的，因為考慮 maze 只有一條最弯曲的路
 eg.  最多為 $W \times H + H$ 步 (經換算後可得 $\frac{W}{2} + \sqrt{W}H$)，因此設此
 限制以縮短執行的時間 (亦可直接放入 for 循環時，但考慮此高法更為清晰易保留)。

DLS 的部分：

- ① 此用來判斷是否尋回根點，即表示此路線無解。
- ② 判斷是否已達到終點，若是則檢查是否比之前紀錄的路徑消耗的能量更少，是則更新 cost 值與 path，接著退前一向繼續找。
- ③ 判斷是否到 limit 值，是則回推
- ④ 若 current 節點的鄰居皆造訪完了則，返回前一點繼續搜尋。

注意：此 ①, ②, ③, ④ 的判斷皆有 current.reset() 指令的為讓下一條路徑再次到此節點時為初始狀態可繼續向下搜尋。

- ⑤ for 迴圈尋找 current 的臨點，並記錄下 cost 值。
- ⑥ len(next.visited_queue) 若不為 0 則表示此點為自己的祖先，所以將其放入 current.visited_queue 以防再次走到形成迴路。
- ⑦ 若 next 未被造訪過，則更新 next 的 distance (也就是 cost) 值，並將 next 設為新的基準點 (current = next)

實測結果：

5x5 maze with path

```
python mazeMaker.py
The maze is :
* * * * *
* 5 8 0 *
* 3 1 1 *
* 2 4 4 *
* * * *
python P2_IDS.py
===== P2_IDS =====
The shortest path in maze is :
* * * * *
* S 8 0 *
* v 1 1 *
* v > T *
* * * *
The shortest path in a list is :
['1', '2', '3', '3', '2', '3', '3']
The cost is : 49
Execution time = 0.010930 sec
```

5x5 maze without path

```
python mazeMaker.py
The maze is :
* * * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 3 0 2 *
* * * * *
python P2_IDS.py
===== P2_IDS =====
no solution ! ( There is no route from the start-position to the end-position )
Execution time = 0.007940 sec
```

8x8 maze with path

```
python mazeMaker.py
The maze is :
* * * * * * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 3 0 2 *
* 1 4 5 * 6 *
* * * * *
python P2_IDS.py
===== P2_IDS =====
The shortest path in maze is :
* * * * * * * *
* 5 8 > 0 3 1 1 *
* v > > > > 8 *
* 2 * 0 8 v 6 *
* 1 2 9 * v 8 *
* 1 3 0 v 7 *
* 1 4 5 * v 7 *
* * * * *
The shortest path in a list is :
['1', '2', '3', '2', '2', '3', '2', '4', '2', '5', '3', '5', '4', '5', '5', '6', '6']
The cost is : 104
Execution time = 0.068840 sec
```

8x8 maze without path

```
python mazeMaker.py
The maze is :
* * * * * * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 3 0 2 *
* 1 4 5 * 6 *
* * * * *
python P2_IDS.py
===== P2_IDS =====
no solution ! ( There is no route from the start-position to the end-position )
Execution time = 0.554768 sec
```

時間有稍微長一點點

10x10 mate with path

```
python mazeMaker.py
The size is :
+-----+
| 5 8 0 3 1 1 2 4 |
| 4 6 5 8 2 * 0 8 |
| 2 6 1 0 2 * 1 3 |
| * 6 8 3 8 * 0 9 |
| 3 8 1 0 2 3 5 0 |
| 6 1 7 8 1 4 8 4 |
| * 0 6 1 7 8 1 4 |
+-----+
python P2_IDS.py
===== P2_IDS =====
The shortest path in maze is :
+-----+
| v > > 8 2 * 0 8 |
| v 6 v 2 0 1 3 |
| * 6 8 v 8 * 0 9 |
| 3 8 1 v 1 5 0 1 |
| 6 1 7 8 1 4 v 1 |
+-----+
the shortest path in a list is :
['1', '2', '3', '1', '2', '2', '3', '4', '3', '4', '4', '5', '4', '6', '4', '7', '5', '7', '6', '8']
the cost is : 98
Execution time = 4.079970 sec
```

10x10 maze without path

```
python mazeMaker.py
The size is :
+-----+
| 5 8 0 3 1 1 2 4 |
| 4 6 5 8 2 * 0 8 |
| 2 6 1 0 2 * 1 3 |
| * 6 8 3 8 * 0 9 |
| 3 8 1 0 2 3 5 0 |
| 6 1 7 8 1 4 8 4 |
| * 0 6 1 7 8 1 4 |
+-----+
python P2_IDS.py
===== P2_IDS =====
No solution ! ( There is no road from the start-position to the end-position )
Execution time = 2089.4299568 sec
```

從 8x8 → 10x10 計算的時間天差地遠,

此次花了將近 35 min !

已經很明顯有停頓的感覺了

15x15 maze with path

```
python P2_IDS.py
===== P2_IDS =====
The shortest path in maze is :
+-----+
| 5 2 3 2 1 1 2 4 |
| 4 0 8 5 9 2 0 8 |
| 3 9 8 0 5 1 0 8 |
| 2 7 6 4 9 1 1 1 0 |
| 1 5 4 3 2 0 7 4 5 |
| 0 6 9 8 6 2 1 3 4 |
| 3 4 5 6 7 2 6 0 1 4 |
| 2 1 0 9 8 5 3 7 6 5 |
| 6 7 8 9 0 2 3 1 0 9 |
| 5 3 4 5 6 7 2 9 8 0 9 |
| 4 0 5 6 7 2 9 8 0 9 |
| 3 2 1 0 9 8 5 3 7 6 5 |
| 2 0 1 0 9 8 5 3 7 6 5 |
| 1 0 9 8 5 3 7 6 5 4 3 |
| 0 9 8 5 3 7 6 5 4 3 |
| 1 0 9 8 5 3 7 6 5 4 3 |
+-----+
the shortest path in a list is :
['1', '1', '2', '3', '1', '4', '5', '1', '2', '6', '7', '8', '9', '10', '11', '12', '13', '14', '15', '16', '17', '18', '19', '20', '21', '22', '23', '24', '25', '26', '27', '28', '29', '30', '31', '32', '33', '34', '35', '36', '37', '38', '39', '40', '41', '42', '43', '44', '45', '46', '47', '48', '49', '50', '51', '52', '53', '54', '55', '56', '57', '58', '59', '60', '61']
the cost is : 682
Execution time = 682.1338937 sec
```

⇒ 有鑑於 10x10 without path 已經跑了 34 分鐘了，因此 15x15 without path 就不做測試了。(肯定很恐怖)

另一方面，我想 11 min 已經滿久了，而且之後的時間花費會有極大提升，因此 IDDFS 的極限可能差不多就在這裡了(更何況是 Recursion method)

補充：

前面忘記提及此 program 的 class Vertex 是有稍作更動的：

```
mazeMaker.py M > P2_IDS.py M X
1-4-4-5thGrade(l) > Artificial Intelligence > HW2 > Artificial-Intelligence_Path-Finding
17
18     class Vertex:
19         def __init__(self, node):
20             self.id = node
21             self.adjacent = {}
22             self.visited_adjacent = []
23             # Set distance to infinity for all nodes
24             self.distance = sys.maxsize
25             # Mark all nodes unvisited
26             self.visited = False
27             # Predecessor
28             self.previous = None
29             # Level
30             self.level = int(0)
31
32             def add_neighbor(self, neighbor, weight=0):
33                 self.adjacent[neighbor] = weight
34
35             def get_weight(self, neighbor):
36                 return self.adjacent[neighbor]
37
38             def set_distance(self, dist):
39                 self.distance = dist
40
41             def set_visited(self):
42                 self.visited = True
43
44             def set_unvisited(self):
45                 self.visited = False
46
47             def set_previous(self, prev):
48                 self.previous = prev
49
50             def set_level(self, level):
51                 self.level = level
52
53             def reset(self):
54                 for i in range(len(self.visited_adjacent)):
55                     self.visited_adjacent.pop()
56                     self.adjacent_num = 0
57                     self.visited = False
58                     self.level = 0
59                     if self.id != '1_1':
60                         self.previous = None
61
62             # end class Vertex
```

↑ 增加限制
紀錄已造訪矣

重設初值用

6. Please explain what method, what data structure, and what technique you used to solve this problem with the program P3_IDSSTAR, and explain how you performed when you tested some plates, how much time and space you used horizontally (assuming plate $(H+2) \times (W+2)$), and how large a plate your program can solve the problem? Please use some examples to help illustrate.

至於 IDA* 的運作方式，類似於前兩種 Algorithm 相加，因此萬起來較為順利。

IDA*的實作由於一方面要計算“最短距離+weight”，另一方面也要紀錄實際的耗能，(若不另外增加一個儲存空間，它可能會忽略繞遠路反而耗能低的可能性，測試時即有出現此現象)，因此 class Vertex的部分再加入變數 cost 與 distance 作區別。

```
1 class Vertex:
2     def __init__(self, node):
3         self.id = node
4         self.adjacent = {}
5         self.visited = []
6
7         # Set distance to infinity for all nodes
8         self.distance = sys.maxsize
9
10        # Set cost to infinity for all nodes
11        self.cost = sys.maxsize
12
13        # Mark all nodes unvisited
14        self.visited = False
15
16        # Predecessor
17        self.previous = None
18
19        # Level
20        self.level = int(0)
21
22
23    def add_neighbor(self, neighbor, weight=0):
24        self.adjacent[neighbor] = weight
25
26
27    def get_weight(self, neighbor):
28        return self.adjacent[neighbor]
29
30
31    def set_distance(self, dist):
32        self.distance = dist
```

```
42
43     def set_cost(self, costs):
44         self.cost = costs
45
46     def set_visited(self):
47         self.visited = True
48
49     def set_unvisited(self):
50         self.visited = False
51
52     def set_previous(self, prev):
53         self.previous = prev
54
55     def set_level(self, level):
56         self.level = level
57
58     def reset(self):
59         for i in range(len(self.visited_adjacent)):
60             self.visited_adjacent.pop()
61         self.distance = sys.maxsize
62         self.cost = sys.maxsize
63         self.adjacent_num = 0
64         self.visited = False
65         self.level = 0
66         if self.id != '_1_1':
67             self.previous = None
```

Algorithm 寫作的 function 則分 def DLSSTAR 和 def IDDFSSTAR：

```
mazeMaker.py M P3_IDSSTAR.py M x
1-4-5thGrade(l) > Artificial Intelligence > HW2 > Artificial-Intelligence_Path-Finding-of-Maze
163
164     def DLSSTAR(self, start, target, limit, cost, path):
165         ④ unvisited_queue = [(start.distance, start.id, start)]
166
167         while len(unvisited_queue):
168             ② heapq.heapify(unvisited_queue)
169             uv = heapq.heappop(unvisited_queue)
170             current = uv[2]
171             current.set_visited()
172
173             ③ if current.id == target.id:
174                 cur_cost = current.cost
175                 if cur_cost < cost:
176                     cost = cur_cost
177                     ResetPath(target, path)
178                     ShortestPath(target, path)
179
180             ④ if current.level == limit:
181                 continue
182
183             ⑤ for next in current.adjacent:
184                 ⑥ if next.visited:
185                     continue
186
187                 ⑦ next_id = next.id.split('_')
188                 id_y = int(next_id[0])
189                 id_x = int(next_id[1])
190
191                 new_dist = current.distance + current.get_weight(
192                     next) + ((width-2) - id_x) + ((height-2) - id_y)
193
194                 new_cost = current.cost + current.get_weight(next)
195
196                 ⑧ if new_cost < next.cost:
197                     next.set_distance(new_dist)
198                     next.set_cost(new_cost)
199                     next.set_previous(current)
200                     next.level = current.level + 1
201                     heapq.heappush(unvisited_queue,
202                                     (next.distance, next.id, next))
203
204     # end func DLSSTAR
```

```

203 def IDDFSSTAR(aGraph, start, target, limit, path):
204     cost = sys.maxsize
205     for i in range(1, limit):
206         # Set the distance for the start node to zero
207         start.set_distance(0)
208         # Set the cost for the start node to zero.
209         # Because we use ".distance" to filter the future path to take,
210         # so the cost must be calculated separately, in case the long
211         # way around but lower energy consumption is ignored.
212         start.set_cost(0)
213         start.set_level(0)
214         DLSSTAR(start, target, i, cost, path)
215         for v in aGraph:
216             v.reset()
217     # end fund IDDFSSTAR

```

DLASTAR 的部分：

- ① 結合 Dijkstra 的概念，先將起點放進 unvisited-queue 中，並且優先權順序為 distance 低 (近 target) 者
 - ② 設製好後選出 distance 最低者 (heapp, heappop)，並將其設為已走訪
 - ③ 若已找到 target，檢查 "cost" (非 distance) 是否較小，是則更新 shortest path
 - ④ 若違反限步數則回頭
- 注意：有別於 IDS 的是，以上這些操作皆不用 reset，拜訪過則紀錄並不會再走訪 (非 Dijkstra 用)。

⑤ for 週圈尋找臨桌。

⑥ 若已走訪則跳過。

⑦ 計算 "next" 邊桌的 distance (new_dist) 与 cost (new_cost)

⑧ 若新路徑的 cost 比原本紀錄的 cost 還要低，則更新 next桌的值，包含 distance 的部分，並放入 unvisited-queue 中，加入可能走訪的 priority queue。

IDDFSSTAR 的部分則較單純，將起點設初值，便丟給 DLSSTAR 執行，回來後重設所有 vertex 的值，再給 limit，再檢測。

実測結果：

5x5 maze with path

```
python mazeMaker.py
The maze is :
* * * * *
* 5 8 0 *
* 3 1 1 *
* 2 4 4 *
* * * * *

python P3_IDSSTAR.py
===== P3_IDSSTAR =====

The shortest path in maze is :
* * * *
* S 8 0 *
* V 1 1 *
* V > T *
* * * *

The shortest path in a list is :
['1_1', '2_1', '3_1', '3_2', '3_3']
The cost is : 49

Execution time = 0.010021 sec
```

5x5 maze without path

```
python mazeMaker.py
The maze is :
* * * * *
* * 8 0 *
* 3 1 1 *
* 1 * 2 *
* * * * *

python P3_IDSSTAR.py
===== P3_IDSSTAR =====

No solution ! ( There is no route from the start-position to the end-position )
Execution time = 0.008976 sec
```

8x8 maze without path

```
python mazeMaker.py
The maze is :
* * * * * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 * 0 2 * * *
* * * 6 * 6 *
* * * * * * *

python P3_IDSSTAR.py
===== P3_IDSSTAR =====

No solution ! ( There is no route from the start-position to the end-position )
Execution time = 0.012965 sec
```

8x8 maze with path

```
python mazeMaker.py
The maze is :
* * * * * * *
* 5 8 0 3 1 1 *
* 2 4 4 6 5 8 *
* 2 * 0 8 5 6 *
* 1 2 9 * 3 8 *
* 1 * 0 2 * * *
* * * 6 * 6 *
* * * * * * *

python P3_IDSSTAR.py
===== P3_IDSSTAR =====

The shortest path in maze is :
* * * * * * *
* V > > > > T *
* 2 * 0 8 V 6 *
* 1 2 9 * 3 8 *
* 1 * 0 V 6 *
* 1 5 * 6 8 3 *
* * * * * * *

The shortest path in a list is :
['1_1', '2_1', '2_2', '2_3', '2_4', '2_5', '3_5', '4_5', '5_5', '6_5', '6_6']

The cost is : 184

Execution time = 0.018958 sec
```

10x10 maze with path

```

python mazeSeker.py
The maze is :
* * * * * * * * *
* 5 8 0 3 1 1 2 4 *
* 4 6 5 8 2 0 0 8 *
* 5 6 1 4 3 2 3 8 *
* 1 0 2 * 3 8 * *
* 6 8 3 8 0 9 * *
* 8 3 9 1 5 * 3 * *
* 3 1 0 8 2 3 5 0 *
* 6 1 7 8 1 4 8 4 *
* * * * * * * * *

python P3_IDSTAR.py

----- P3_IDSTAR -----
The shortest path in maze is :
* * * * * * * * *
* S 8 0 3 1 1 2 4 *
* V > 8 2 * 0 8 *
* 5 6 V 2 9 * 3 8 *
* 1 0 2 * 3 8 * *
* 6 8 V 8 * 0 9 * *
* 8 3 V 5 * 5 * *
* 3 1 V > > > 0 * *
* 6 1 7 8 1 4 V T *
* * * * * * * * *

The shortest path in a list is :
[ '1.1', '2.1', '2.2', '2.3', '3.3', '4.3', '4.4', '5.4', '6.4', '7.4', '7.5', '7.6',
  '7.7', '8.7', '8.8' ]
The cost is : 275

Execution time = 0.824934 sec

```

15x15 maze with path

```

python mazeFaker.py
The maze is :
 *   *   *   *   *   *   *   *   *
 5 8 0 3 1 2 4 4 5 5 8 2
 6 1 2 5 0 3 9 5 3 8 0 *
 1 0 2 * 1 5 8 3 8 0 * 0
 9 * 8 3 9 1 5 * 3 1 0 8 *
 2 3 5 0 6 1 7 8 1 4 8 4 0
 6 7 6 * 7 * 1 1 9 4 1 * 4
 9 8 2 4 3 * 5 6 7 7 4
 8 6 0 2 5 6 0 7 3 6 4 *
 7 0 * 8 * 0 7 * 8 0 9 9 *
 * 8 1 0 0 0 2 * 6 6 0 * 1 3 *
 7 * 6 0 9 2 9 4 * 8 6 1 0 8 *
 1 0 1 2 8 6 0 0 * 6 2 3 8 *
 2 6 * 2 5 7 9 1 0 0 5 7 *
 4 5 * 4 6 7 8 9 0 1 0 0 0 *
 0 1 2 3 4 5 6 7 8 9 0 1 0 0 0

python P3_IDSTAR.py
===== P3_IDSTAR =====

The shortest path in maze is :
 *   *   *   *   *   *   *   *   *
 5 > > > > > > > > > 5 8 2
 0 8 5 6 1 2 9 * > V 8 * * *
 1 0 2 0 * 1 5 * 6 8 V 8 * 0 *
 9 * 8 3 9 1 5 * 3 V 1 0 8 *
 2 3 5 0 6 1 7 8 1 V 8 4 0 *
 6 7 6 * 7 * 1 1 9 4 V > 7 4
 9 8 2 4 3 * 5 6 7 V > 7 4 *
 5 6 4 2 * 4 6 0 < V < V * 4 *
 * 7 0 * 8 * 2 1 * < V < V 0 9 *
 8 1 0 0 0 2 * 6 V 0 * 1 3 *
 7 * 6 0 9 2 9 4 * V > 1 0 0 *
 1 0 1 2 8 6 0 0 * 6 V > 8 0 *
 2 6 * 2 5 7 9 1 0 0 V T *
 4 5 * 4 6 7 8 9 0 1 0 0 0 *

The shortest path in a list is :
['1', '1', '2', '1', '3', '1', '4', '1', '5', '1', '6', '1', '7', '1', '8', '1', '9', '1', '10', '2', '10', '3', '11', '4', '10', '5', '10', '6', '10', '6', '11', '7', '11', '8', '11', '8', '10', '9', '10', '10', '11', '10', '11', '12', '11', '12', '12', '13', '12', '13', '13']

The cost is : 482

Execution time = 0.101768 sec
```

IDA*

雖然皆在1秒內，但實際差有5倍以上的時間。

```

P2_IDS

The shortest path in maze is :
* * * * * * * * * * * * * * *
* 5 > > > > > > > 5 8 2 *
* 0 8 4 6 1 2 9 V V 8 1 0 *
* 1 3 7 5 9 6 8 * * * 0 *
* 9 * 8 3 9 5 * 3 V 1 0 8 *
* 2 3 5 0 6 1 7 8 1 V 8 4 0 *
* 6 7 6 * 7 * 1 1 9 V > * 4 *
* 9 8 * 7 9 4 3 * 5 * V 7 1 *
* 5 6 4 2 * 6 0 7 < V * 4 *
* 7 0 8 * 2 1 V V 0 9 9 *
* 8 1 0 0 0 2 8 5 V > 1 0 *
* 7 0 9 9 2 9 4 * * V > 1 0 *
* 1 0 1 * 2 5 7 9 1 * 6 V > 8 *
* 2 6 * 2 5 7 9 1 0 0 V T *
* * * * * * * * * * * * * * *

The shortest path in a list is :
['1', '8', '1', '2', '1', '3', '1', '4', '1', '5', '1', '6', '1', '7', '1', '8', '1', '9', '1', '10', '2', '10', '3', '10', '4', '10', '5', '10', '6', '10',
'11', '7', '11', '8', '11', '8', '10', '9', '30', '10', '10', '11', '10', '11', '11', '12', '11', '12', '12', '13', '12', '13', '13']

The cost is : 48

Execution time = 682.334937 sec

```

10x10 maze without path

```
python mazeMaker.py
The maze is :
 * * * * * * * * *
 * 5 8 0 3 1 2 4 * *
 * 4 6 5 8 2 * 0 8 *
 * 5 6 1 2 9 * 3 8 *
 * 1 0 2 7 * 1 5 *
 * 6 3 8 0 9 * *
 * 8 3 0 5 5 * *
 * 3 1 0 8 2 3 5 * *
 * 0 6 1 7 8 * 4 *
 * * * * * * * * *

python P3_IDSTAR.py
=====
P3_IDSTAR =====

No solution ! ( There is no route from the start-position to the end-position )
Execution time : 0.82918 sec
```

===== P1_UCS =====

```

The shortest path in maze is :
+-----+
| * * * * |
| * 5 > 6 |
| * 8 5 6 |
| * 7 > 8 |
| * 2 0 2 * |
| * 1 5 * 6 |
| * 8 3 9 1 |
| * 5 6 3 1 |
| * 2 3 5 0 6 |
| * 7 8 1 2 3 |
| * 6 7 6 * 7 |
| * 1 1 9 v > 4 |
| * 9 8 * 7 9 4 3 5 * v 7 |
| * 5 6 4 2 * 6 0 7 < v 4 |
| * 7 0 * 8 * 2 1 * v 0 9 |
| * 8 0 0 0 0 2 6 v * 1 3 |
| * 5 6 0 0 0 0 6 v > 1 3 |
| * 7 0 0 0 0 0 0 v > 1 3 |
| * 6 0 0 0 0 0 0 v > 1 3 |
| * 2 0 0 0 0 0 0 v > 1 3 |
| * 6 0 0 0 0 0 0 v > 1 3 |

```

The shortest path in a list is :
 ['1_1', '1_2', '1_3', '1_4', '1_5', '1_6', '1_7', '1_8', '1_9',
 '1_10', '2_10', '3_10', '4_10', '5_10', '6_10', '6_11', '7_11',
 '8_11', '8_10', '9_10', '10_10', '11_10', '11_11', '12_11', '12_12',
 '13_12', '13_13']
 The cost is : 482

Execution time = 0.017941 sec

Fuchs

相較於 IDS 來說則有顯著的時間壓縮

7

不免俗的，我們還是要來壓力測試一下性能：

⇒ 175.8 sec 感覺尚有發展空間

$\Rightarrow 876.3 \text{ sec} \approx 15 \text{ min } @ @$

相較於當初 P1-UCS 的 150×150 maze 花 "868 sec \(\approx\) 14.5 min" 是差不多的結果，但格叔卻差了 2.25 倍啊，可見在我的 program 中就屬 UCS 最快。

7. Please describe some of the situations and difficulties you have encountered in doing this work

- (1) 其實一開始自己在嘗試寫第一個 program 時方向就已經有點偏了，我忘成 DFS 的程式碼，而且是單純用陣列的方式紀錄資料，這導致當訊息愈多時，就不得不改變儲存方式了。
- (2) 最大的瓶頸在於不知道要如何建立 Graph，包含兩 node 間的 weight 要如何存取等等。因此，上網找資料後才較有頭緒，開始著手題目所要求的項目。
- (3) P1_UCS 基本上就是 Dijkstra's algorithm the shortest path 的應用，因此很快就完成了，難在 P2_IDS 的實作。
IDS 的簡單明瞭版本即依靠 Recursion 的方式計算，但我印象中其速度真的慢上許多，而且當遞迴的參數愈多，再加上內部判斷式拖慢速度，僅僅是 8×8 的迷宮就跑上許久。因此毅然決然將其改為迴圈形式。
- (4) IDS 圍圈化花了我非常多時間思考並修正，因該在每個 vertex 中必須要有取資料的同時也要 reset 走到 limit 的 vertex，讓下次走到此 vertex 時可以以初始狀態繼續探索（達到探索完後此點仍可再次探索，只允許繞遠路）。
- (5) IDA* 的部分延用 UCS 和 IDS 的概念，實作起來基本上沒什麼問題，但測試的時候就會發現不一定會是最佳解，原因在於若依照原本的只紀錄 distance 來當作 priority queue 的資料，那麼若繞遠路的 cost 較小，將可能被忽略。因此才要另設 "cost" 變數去計算並紀錄當前的耗量，比較是否比以往的小，從而更新資料。

8. Please list the sources of your references (including websites) and indicate which parts were used in the assignment.

1. DIJKSTRA'S SHORTEST PATH ALGORITHM :

https://www.bogotobogo.com/python/python_Dijkstras_Shortest_Path_Algorithm.php

⇒ 主要參考他 Graph 和 Vertex 的 class 搭構（但後來發現我的 Dijkstra 和他的幾乎一模一樣，而他的還有註解且命名更清晰，所以也跟著變更了）

2. Iterative deepening depth-first search

https://en.wikipedia.org/wiki/Iterative_deepening_depth-first_search

⇒ 主要參考其 IDS 的 sudo code 概念去實作 IDS。

* Bonus:

Bidirectional Search 的部份經實測後為“非 Optimal”演算法，僅能找“最小步數”的路徑，但權重並非 Minimum。其優點是執行速度非常快。

実測結果：

The screenshot shows the Visual Studio Code interface with the following details:

- File Explorer:** Shows the project structure with files like `mazeMaker.py`, `BiDirectionalSearch.py`, and `BiDirectionalSearch.output.txt`.
- Code Editor:** Displays the `mazeMaker.py` script. The code implements a Bi-directional search algorithm to find the shortest path in a maze. It includes functions for printing the maze, creating a maze, and performing the search.
- Terminal:** Shows the output of the script, which includes the generated maze, the shortest path found, and the execution time.

```
File Edit Selection View Go Run Terminal Help
mazeMaker.py -d - Visual Studio Code
mazeMaker.py M BiDirectionalSearch.py
@) Artificial Intelligence - HW2 / Artificial-Intelligence_Path-Finding-of-Maze > mazeMaker.py > BiDirectionalSearch.output.txt
1-4-8 * 0 5 8 2 7 9 5 6 4 3 3 3 0 3 2 * 7 * 3 0 * * 0 7 4 4 8 8 7 5 7 2 1 * 3 2
* 8 8 0 * 6 9 3 * 4 1 5 4 1 1 7 9 2 0 1 5 9 4 2 3 6 * 1 6 1 0 6 9 2 0 * 4 6 5 2 1 3 1 7
* 0 7 1 1 7 1 * 6 6 * 3 1 5 1 2 9 7 8 6 7 3 8 7 8 7 1 9 0 2 4 3 7 5 8 0 8 5 * 2 7 1 4 8 8
3 0 1 1 6 9 3 1 9 9 5 9 7 2 1 4 * 5 9 2 6 * 3 * 0 * 3 2 5 1 2 8 3 6 9 * 1 * 4 2 1 9 0
8 * 5 3 1 6 7 8 4 * 6 3 8 9 0 2 9 9 5 7 6 7 8 3 * 4 8 5 3 9 4 7 9 6 8 1 5 0 * 7 * 8
*** 2 6 0 4 3 2 7 9 8 2 9 0 * 7 3 8 2 1 3 1 2 * 9 4 9 5 3 7 1 3 * 0 2 9 3 2 0 9 3
* 3 4 6 3 8 * 8 * 6 6 * 0 3 6 2 3 0 8 3 9 9 * 9 8 * 7 * 3 * 5 * 7 5 7 4 * 2 5 2 9 0 8 8
* 1 8 5 0 5 1 6 8 9 * 7 0 2 4 1 9 5 1 5 * 3 0 * 1 2 * 5 7 4 * 2 8 9 8 8 3 6 8 8 3 5 5
* 0 3 7 3 * 2 4 3 0 * 6 * 9 1 * 6 2 1 0 1 6 1 7 3 * 4 0 2 7 * 5 3 2 0 3 9 0 * 6 6 0 0 0 4
0 6 5 * 6 9 5 6 4 4 2 0 2 1 3 0 3 9 9 * 2 0 6 0 2 1 3 0 3 9 9 * 7 6 6 6 6 4 0 0 1 2
0 6 0 1 1 5 1 6 5 4 1 5 4 3 0 5 4 3 0 5 4 3 0 5 4 3 0 5 4 3 0 5 4 3 0 5 4 3 0 5 4 3 0 5 4
* 9 4 * 2 4 3 0 * 7 6 7 8 4 2 9 6 1 6 2 3 2 8 7 3 5 4 * 9 6 7 0 7 * 6 5 9 * 1 * 5 1 6
9 2 0 5 2 5 3 9 8 9 6 3 * 5 9 6 7 3 8 9 * 8 6 4 9 9 3 7 3 4 7 9 8 * 9 5 4 3 7 5 * 5
* 0 9 2 7 1 1 1 8 7 6 * 6 7 4 8 * 1 * 8 3 6 0 * 5 1 8 7 8 * 8 9 9 6 5 3 9 4 0 7 9
* 6 6 8 5 * 4 3 9 0 8 6 2 3 3 2 * 5 6 8 6 0 6 6 9 5 0 4 * 4 0 9 6 9 3 0 8 * 0 * 0 2 3 5
* 1 * 6 * 9 0 2 3 3 5 3 6 5 5 6 3 4 2 3 * 8 6 * 1 1 7 9 7 9 6 3 2 3 * * 2 6 2 8 4 7 5 3 4 2 *
3 8 2 6 2 4 2 3 4 9 0 * 1 0 8 3 9 9 6 0 6 9 9 0 * 4 3 * 9 * 3 * 1 9 7 5 * 9 4 8 2 3 7
* 7 1 4 9 8 1 1 * 3 0 8 0 8 9 7 1 0 1 * 1 * 1 5 8 6 6 4 3 7 * 2 8 5 7 4 * 7 4 * 0 9 2 7 5 6 7 7 7
* 6 3 * 0 * 6 * 4 * 5 * 2 * 1 4 1 4 5 * 8 2 5 7 9 * 0 * 1 2 8 2 5 9 * 5 9 8 1 4 6 4 8 *
4 * 3 6 2 * 6 * 6 9 * 7 * * 5 3 2 5 9 8 4 * 4 * 2 0 8 0 * 7 6 1 * 3 4 * * 8 6 * 5 1 7 5 2
3 * 3 * 0 7 1 8 * 8 0 * 1 7 * 8 8 * 3 7 9 0 * 4 0 8 2 * 7 9 1 7 2 4 1 0 7 9 * 4 3 4 3 2
0 3 * 2 6 6 6 6 3 * 3 0 8 3 9 9 0 * 4 3 * 9 2 9 5 7 * 2 2 9 5 7 * 1 3 * 0 8 2 3 9 3 1 6 5 2 0 9
* 2 4 0 9 8 7 * 3 9 8 * 9 0 8 7 * 3 9 8 * 9 0 8 7 * 3 9 8 * 9 0 8 7 * 3 9 8 * 9 0 8 7 *
2 * 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6 1 0 1 6
* 2 * 5 0 7 3 4 * 7 8 * 4 * 4 9 2 * 2 * 0 2 1 8 8 9 9 6 7 * 6 6 6 5 5 1 6 5 4 4 9 3 2 7
* 9 * 1 4 * 5 * 1 0 9 1 7 9 1 5 5 3 * 0 * 3 2 4 2 0 2 7 5 0 3 6 5 4 * 9 1 7 3 * 3 4 2 3 5 9 8 *
* 8 3 8 7 * 7 6 3 8 3 8 4 2 5 7 * 7 9 4 0 6 6 9 * * 2 2 6 7 4 * 0 * 9 * 1 6 4 3 1 8 6 8 9 * 0 9
* 8 0 1 0 8 1 1 * 0 4 7 4 * 1 2 * 9 9 9 * 3 * 0 * 8 7 4 3 5 7 * * * 0 2 9 * 3 1 1 9 7 6
* 7 * 9 * 2 7 7 0 4 1 8 6 9 9 6 4 0 0 4 7 4 7 1 0 5 * 6 9 8 7 4 4 6 6 5 9 * 7 3 0 0 3 *
* 4 5 9 9 8 2 6 3 9 * 7 5 3 * 3 * 1 6 * 5 7 * 9 * * 6 * 9 4 6 7 7 7 3 9 3 7 7 2
* 0 * 6 3 5 5 9 * 8 2 9 1 6 8 5 2 9 * 8 2 5 9 6 * 8 7 * * 8 1 1 * 5 4 4 6 7 5 * 9 4 2 6
* 0 2 1 * 4 * 9 2 6 4 8 0 4 * 4 * 2 8 2 2 8 2 4 9 5 9 6 8 7 * * 2 0 8 0 8 3 6 5 3 4 2
* 5 * 6 0 7 * 3 * 4 3 1 8 6 9 8 9 7 8 8 * 4 4 * 4 5 3 5 4 7 * 5 1 3 2 3 * * 2 7 3 9 1 2 5 *
```

The shortest path in a list is :
1_1_7_1_2_7_1_2_4_2_1_2_2_5_2_6_1_7_2_2_1_8_2_1_9_2_1_10_2_1_11_2_1_11_3
The execution time = 120.09176135063171 sec

500×500 的迷宮僅花了 2 分鐘的時間，相較前三個浪
鼻法的最佳解 71-UCS, 150×150 maze 就需 15 分鐘了，耗
時大幅縮短！
但代價就是其並非最佳解。Robot 的耗能將十分
恐怖。

~END~