Distributed Systems

Basic Algorithms

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Distributed Computation

Ref: NL

- How to send messages to all nodes efficiently
- How to compute sums of values at all nodes efficiently

- Network as a graph
- Broadcasting messages
- Computing sums in a tree
- Computing trees in a network
- Communication complexity

Network as a graph

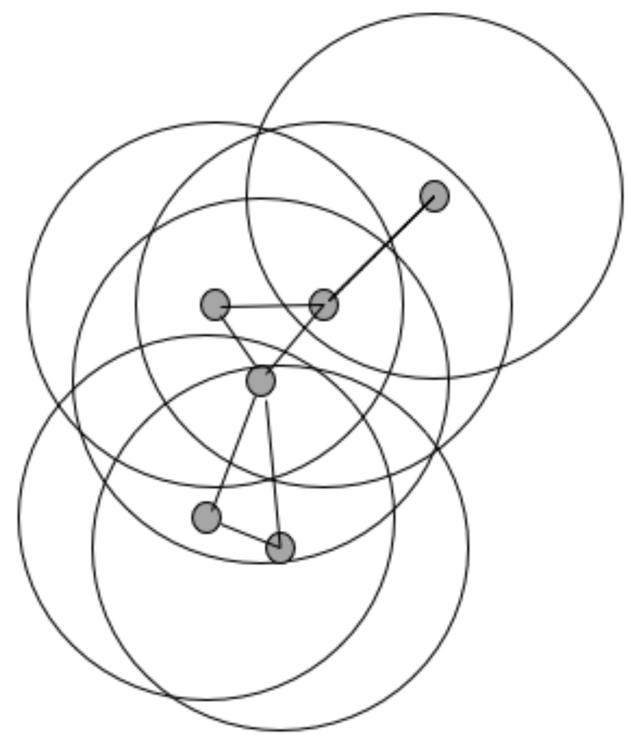
- Network is a graph : G = (V,E)
- Each vertex/node is a computer/process
- Each edge is communication link between 2 nodes
- Every node has a Unique identifier known to itself.
  – Often used 1, 2, 3, … n
- Every node knows its neighbors – the nodes it can reach directly without needing other nodes to route
  – Edges incident on the vertex
  – For example, in LAN or WLAN, through listening to the broadcast medium
  – Or by explicitly asking: Everyone that receives this message, please report back
- But a node *does not* know the rest of the network

Example: Unit disk graphs

- Suppose all nodes are wireless
- Each can communicate with nodes within distance r.
- Say, r = 1

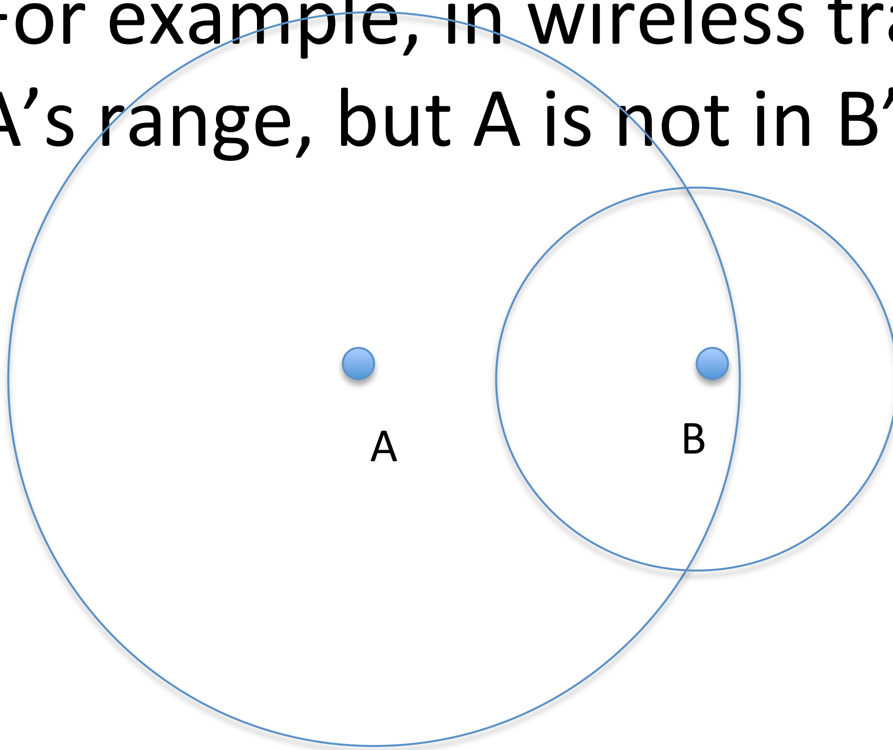
- UDG is a model
- Not perfect

- In general, networks can be any graph



Directed graphs

- When A can send message to B, but B cannot send message to A
- For example, in wireless transmission, if B is in A's range, but A is not in B's range



Directed graphs

- When A can send message to B, but B cannot send message to A
- Or if protocol or technology limitations prevent B from communicating with A

Directed graphs

- Protocols more complex
- Needs more messages

Network as a graph

- Distance/cost between nodes p and q in the network
  - Number of edges on the shortest path between p and q (when all edges are same: unweighted)

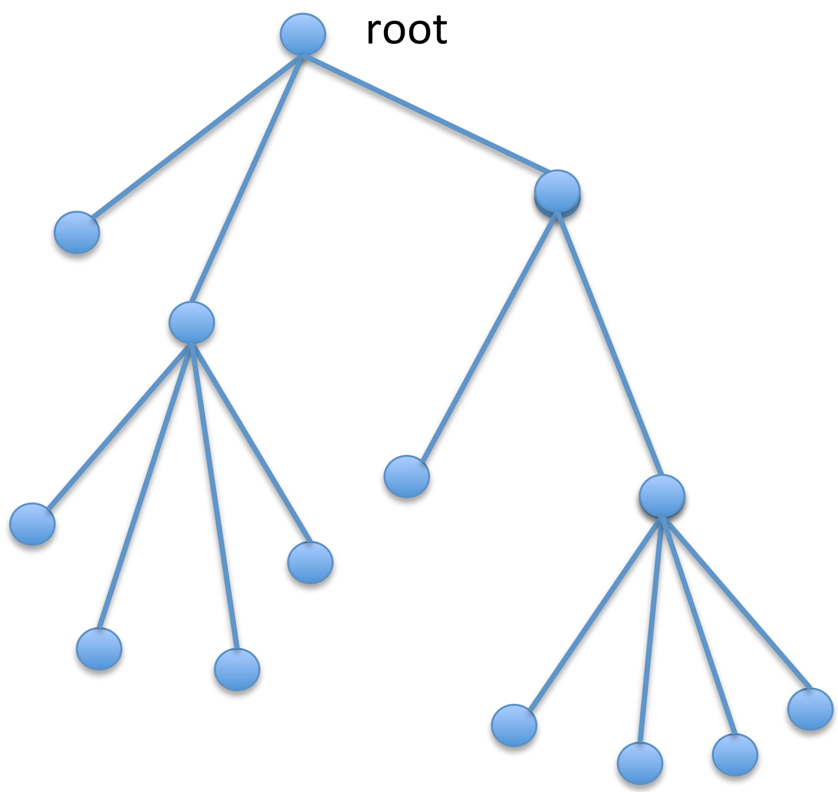
- Sometimes, edges can be weighted
  - Each edge e = (a,b) has a weight *w(e)*
  - w(e) is the cost of using the communication link e (may be length e)
  - Distance/cost between p and q is total weight of edges on the path from p to q with least weight

Network as a graph

- Diameter
  - The maximum distance between 2 nodes in the network
- Radius
  - Half the diameter
- Spanning tree of a graph:
  - A subgraph which is a tree, and reaches all nodes of the graph
  - If network has n nodes
    - How many edges does a spanning tree have?

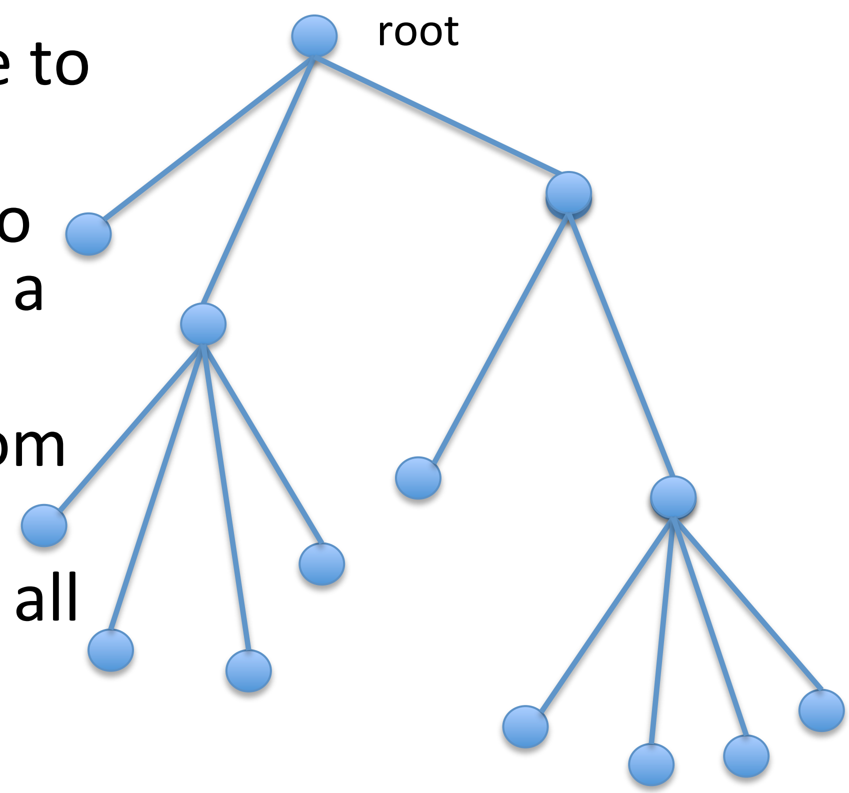
Computing sums in a tree

- Suppose root wants to know sum of values at all nodes



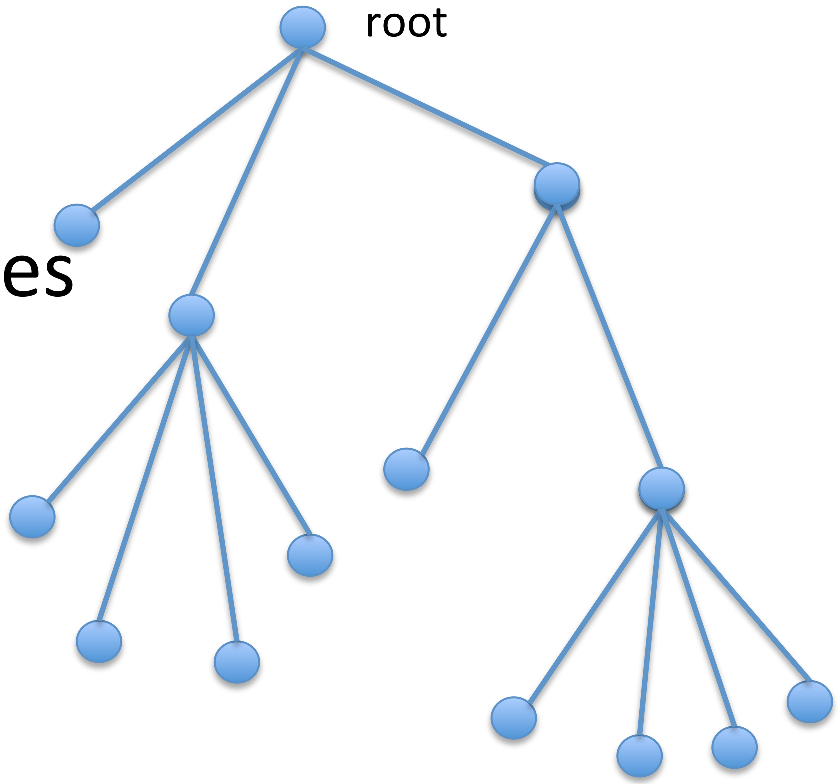
Computing sums in a tree

- Suppose root wants to know sum of values at all nodes
- It sends “compute” message to all children
- They forward the message to all their children (unless it is a leaf node)
- The values move upward from leaves
- Each node adds values from all children and its own value
- Sends it to its parent



Computing sums in a tree

- What can you compute other than sums?
- How many messages does it take?
- How much time does it take?



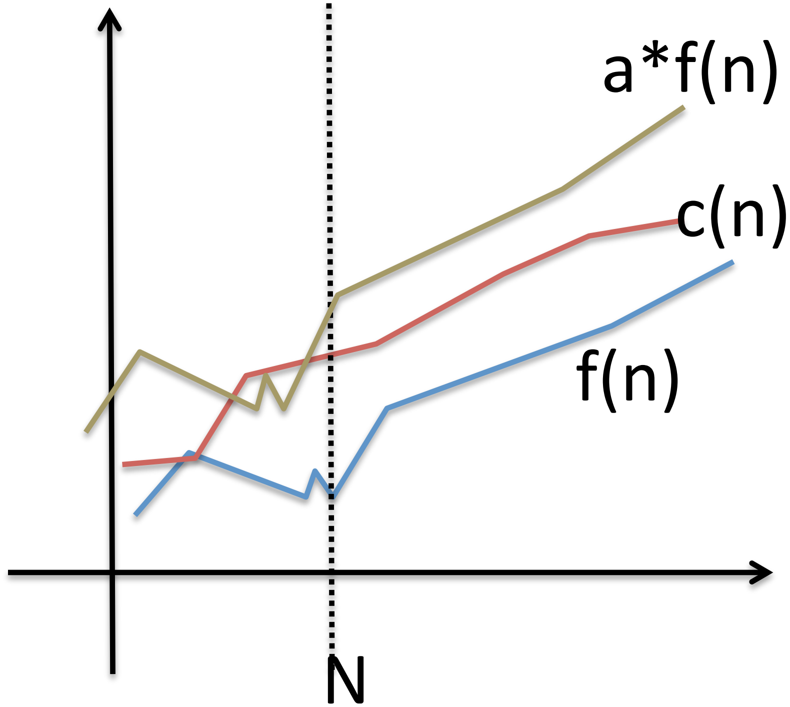
Communication complexity

- Used to represent communication cost for general scenarios
- Called Communication Complexity or Asymptotic communication complexity

- Use big oh notation: O

Big oh – upper bounds

- For a system of n nodes,
- Communication complexity c(n) is O(f(n)) means:
  - There are constants a and N, such that:
  - For n>N: c(n) < a*f(n)



Allowing some initial irregularity, ‘c(n)’ is not bigger than a constant times ‘f(n)’

In the long run, c(n) does not grow faster than f(n)

Examples

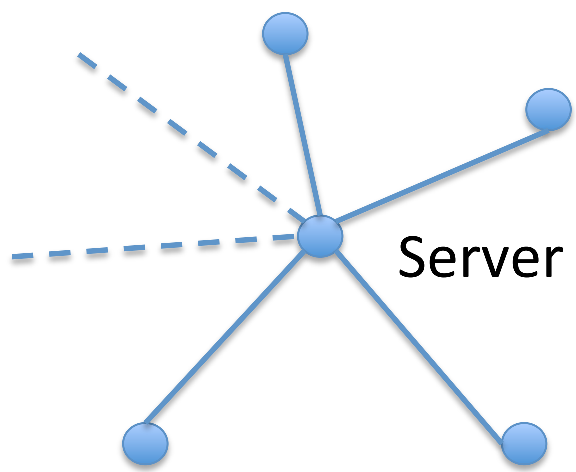
- $3n = O(?)$
- $1000n = O(?)$
- $n^2/5 = O(?)$
- $10\log n = O(?)$
- $2n^3+n+\log n+200 = O(?)$
- $15 = O(?)$

Examples

- $3n = O(n)$
- $1000n = O(n)$
- $n^2/5 = O(n^2)$
- $10\log n = O(\log n)$
- $2n^3+n+\log n+ 200 = O(n^3)$
- 15 or any other constant= $O(1)$

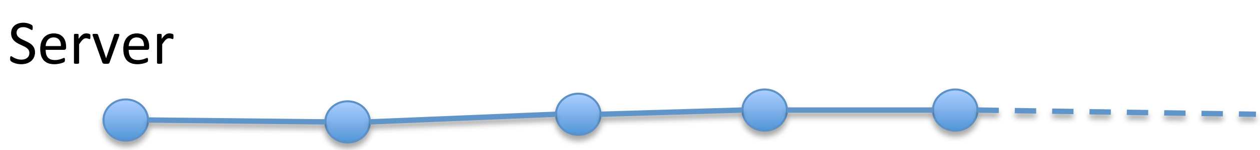
Example 1

- ‘Star’ network
- Computing sum of all values
- Communication complexity: O(n)



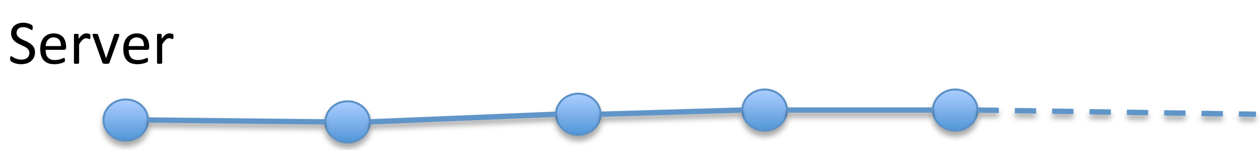
Example 2a

- ‘Chain’ topology network
- Simple protocol where everyone sends value to server
- Communication complexity:?



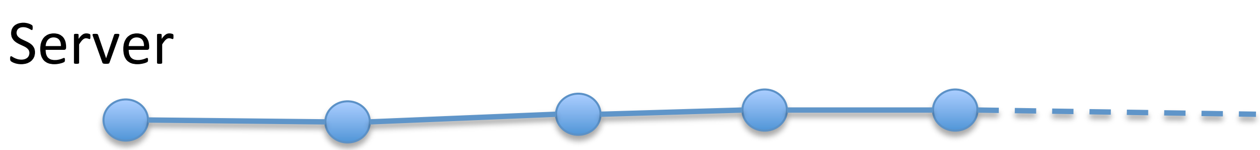
Example 2a

- ‘Chain’ topology network
- Simple protocol where everyone sends value to server
- Communication complexity: 1+2+…+n = $O(n^2)$



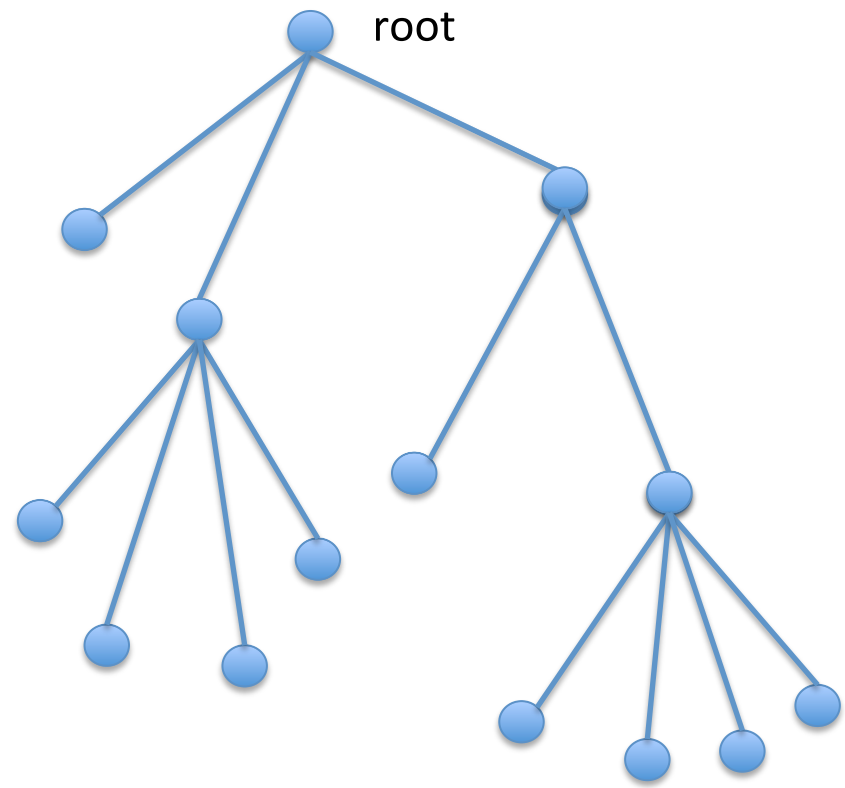
Example 2b

- ‘Chain’ network
- Protocol where each node waits for sum of previous values and sends
- Communication complexity: 1+1+…+1 = O(n)



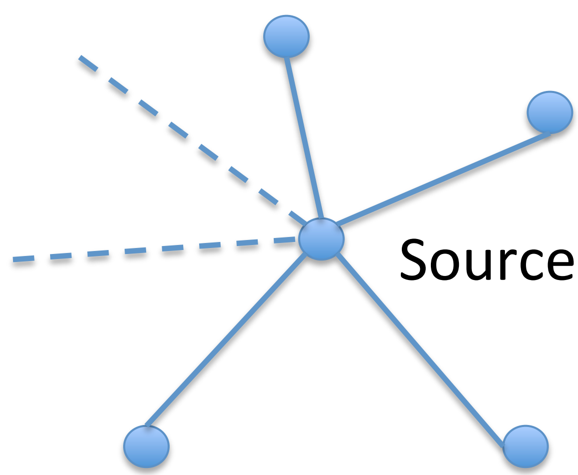
Computing sums in a tree

- How many messages does it take?
- How much time does it take?



Global Message broadcast

- Message must reach *all nodes in the network*
  - Different from broadcast transmission in LAN
  - All nodes in a large network cannot be reached with single transmission



Global Message broadcast

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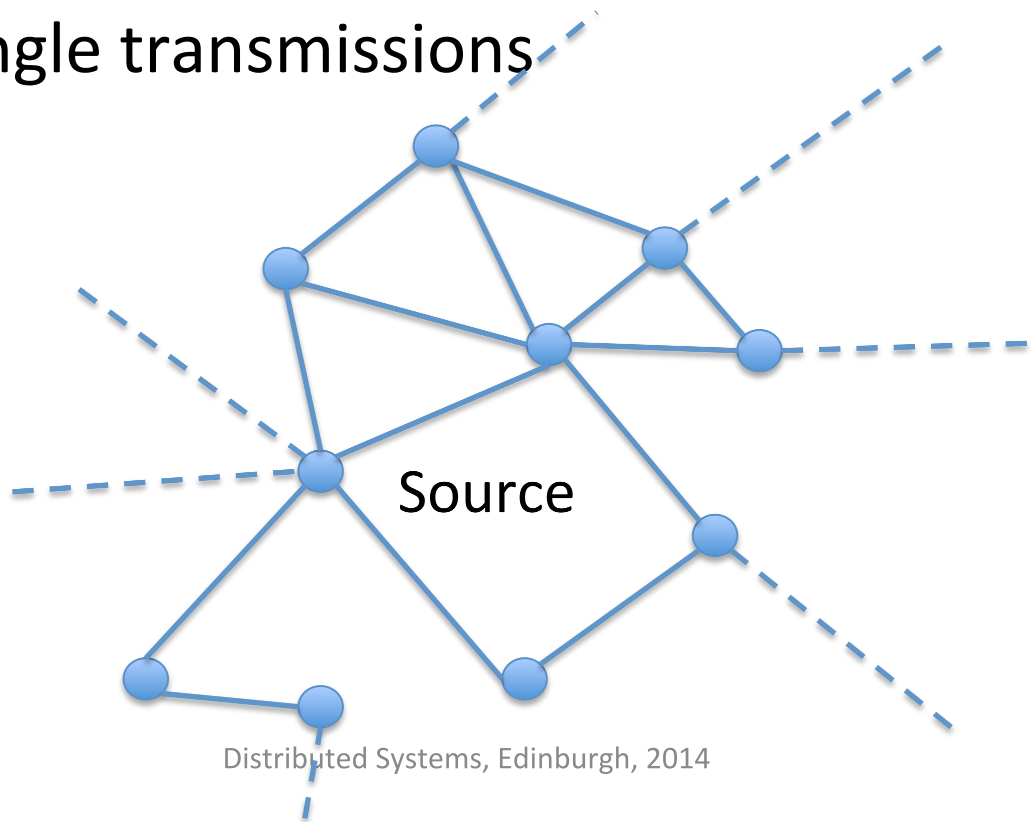


Flooding for Broadcast

- The source sends a *Flood* message to all neighbors

- The message has
  - Type *Flood*
  - *Unique id: (source id, message seq)*
  - *Data*

Flooding for Broadcast

- The source sends a *Flood* message, with a unique message id to all neighbors
- Every node p that receives a flood message m, does the following:
  - *If m.id was seen before, discard m*
  - *Otherwise, Add m.id to list of previously seen messages and send m to all neighbors of p*

Flooding for broadcast

- Storage
  - Each node needs to store a list of flood ids seen before
  - If a protocol requires x floods, then each node must store x ids
    - (there is a way to reduce this. Think!)

Assumptions

- We are assuming:
  - Nodes are working in synchronous *communication rounds (e.g. transmissions occur in intervals of 1 second exactly)*
  - Messages from all neighbors arrive at the same time, and processed together
  - In each round, each node can successfully send 1 message to each neighbor
  - Any necessary computation can be completed before the next round

Communication complexity

- The the message/communication complexity is:

Communication complexity

- The the message/communication complexity is:
  - $O(|E|)$

Communication complexity

- The the message/communication complexity is:
  - O(|E|)
  - Worst case: $O(n^2)$

Reducing Communication complexity (slightly)

- Node p need not send message m to any node from which it has already received m
  - Needs to keep track of which nodes have sent the message
  - Saves some messages
  - Does not change asymptotic complexity

Time complexity

- The number of rounds needed to reach all nodes: *diameter of G*

Computing Tree from a network

- BFS tree
  - The Breadth first search tree
  - With a specified root node

BFS Tree

- Breadth first search tree
  - Every node has a *parent* pointer
  - And zero or more child pointers

  - BFS Tree construction algorithm sets these pointers

BFS Tree Construction algorithm

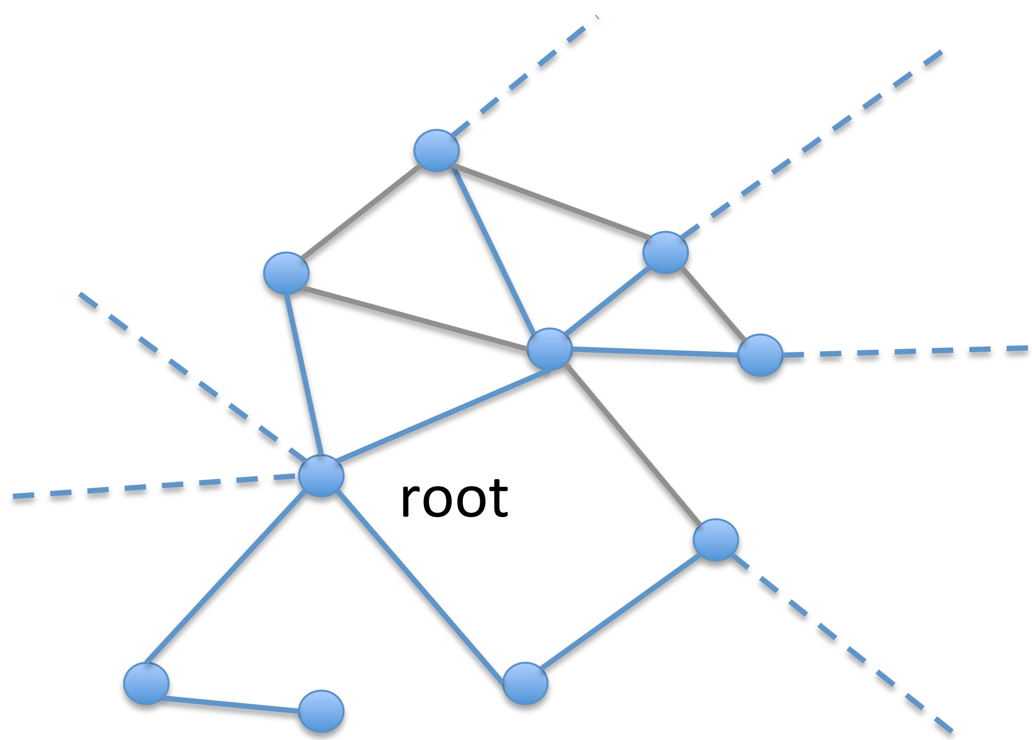
- Breadth first search tree
  - The *root(source)* node decides to construct a tree
  - Uses flooding to construct a tree
  - Every node p on getting the message forwards to all neighbors
  - Additionally, every node p stores *parent* pointer: node from which it first received the message
    - If multiple neighbors had first sent p the message in the same round, choose *parent* arbitrarily. E.g. node with smallest id
  - p informs its parent of the selection
    - Parent creates a child pointer to p

BFS Tree

- Property: BFS tree is a shortest path tree
  - For source s and any node p
  - The shortest path between s and p is contained in the BFS tree

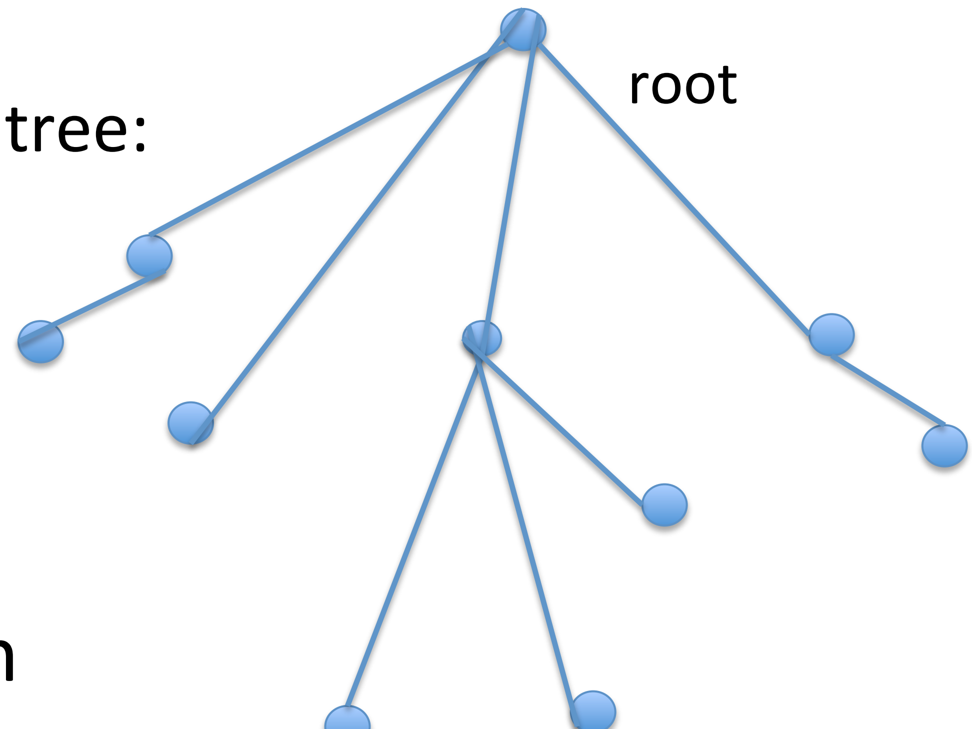
Time & message complexity

- Asymptotically Same as Flooding



Tree based broadcast

- Send message to all nodes using tree
  - BFS tree is a *spanning* tree: connects all nodes
- Flooding on the tree
- Receive message from parent, send to children



Tree based broadcast

- Simpler than flooding: send message to all children
- Communication: Number of edges in spanning tree: n-1

Aggregation: Find the sum of values at all nodes

- With BFS tree

- Start from *leaf* nodes
  - Nodes without children
  - Send the value to parent
- Every other node:
  - Wait for all children to report
  - Sum values from children + own value
  - Send to parent

Aggregation

- Without the tree
- Flood from all nodes:
  - O(|E|) cost per node
  - O(n*|E|) total cost: expensive
  - Each node needs to store flood ids from n nodes
    - Requires Ω(n) storage at each node
  - Good fault tolerance
    - If a few nodes fail during operation, all the rest still get some value

Aggregation

- With Tree
- Also called Convergecast

Aggregation

- With Tree

- Once tree is built, any node can use for broadcast
  – Just flood on the tree

- Any node can use for convergecast
  – First flood a message on the tree requesting data
  – Nodes store parent pointer
  – Then receive data
- What is the drawback of tree based aggregation?

Aggregation

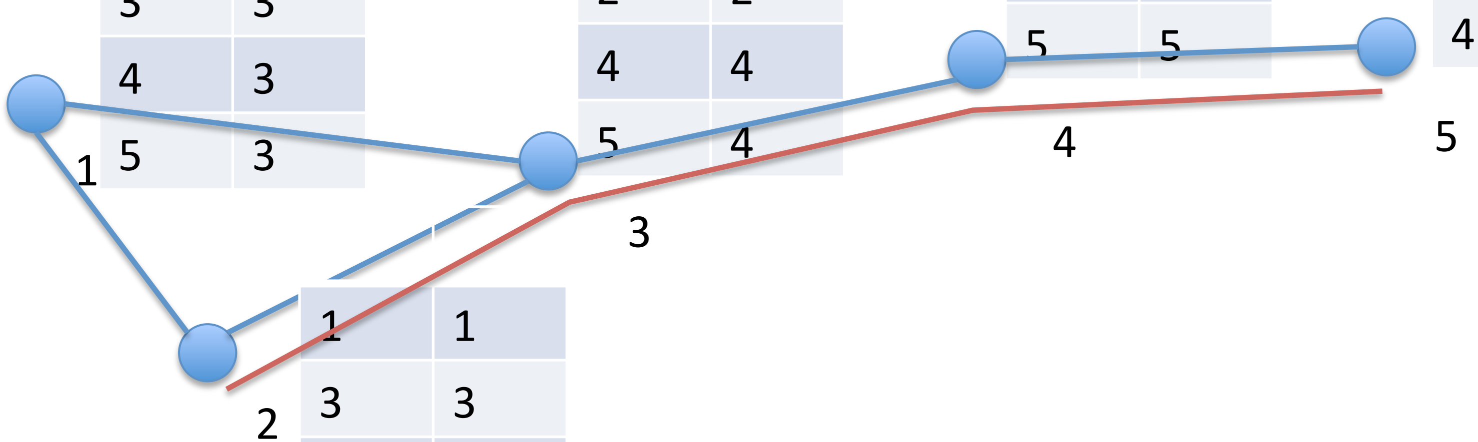
- With Tree

- Once tree is built, any node can use for broadcast
  - Just flood on the tree

- Any node can use for convergecast
  - First flood a message on the tree requesting data
  - Nodes store parent pointer
  - Then receive data
- Fault tolerance not very good
  - If a node fails, the messages in its subtree will be lost
  - Will need to rebuild the tree for future operations

BFS trees can be used for routing

- From each node, create a separate BFS tree
- Each node stores a parent pointer corresponding to each BFS tree
- Acts as routing table



2	2
3	3
4	3
5	3

1	1
3	3
4	3
5	3

1	1
2	2
4	4
5	4

1	3
2	3
3	3
5	5

1	4
2	4
3	4
4	4

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BFS trees can be used for routing

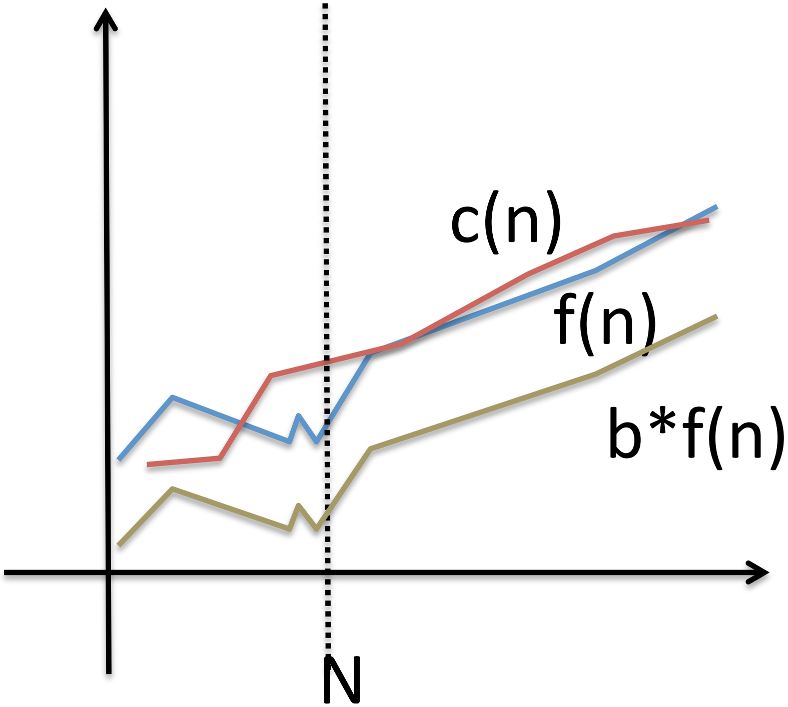
- From each node, create a separate BFS tree
- Each node stores a parent pointer corresponding to each BFS tree
- Acts as routing table
- O(n*|E|) message complexity in computing routing table

Observation on complexity

- Suppose c(n)=n
  - Then c(n) is O(n) and also O(n^2)
  - Although, when we ask for the complexity, we are looking for the tightest possible bound, which is O(n)

Big Ω – lower bounds

- For a system of n nodes,
- Communication complexity c(n) is Ω(f(n)) means:
  - There are constants a and N, such that:
  - For n>N: b*f(n) < c(n)

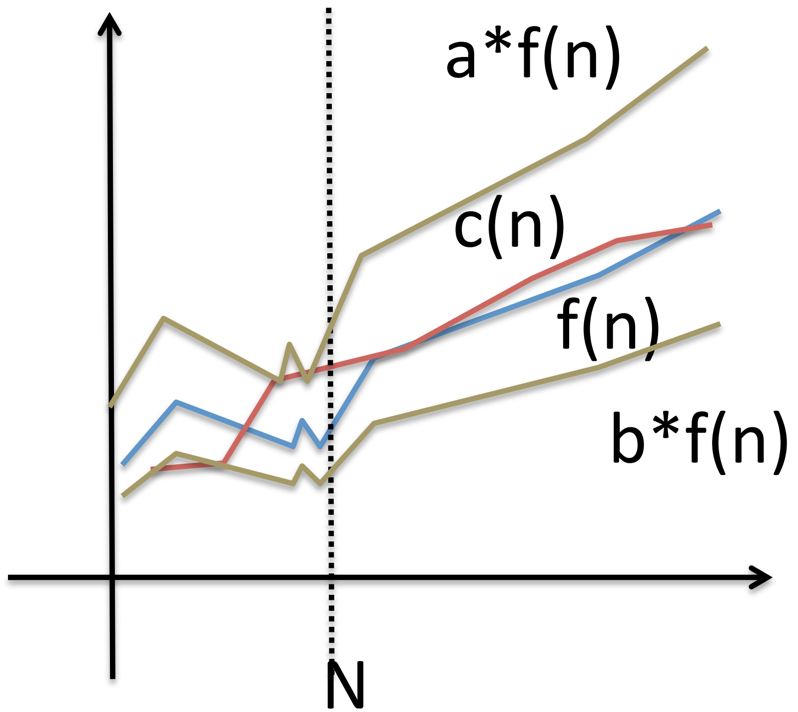


Allowing some initial irregularity, ‘c(n)’ is not smaller than a constant times ‘f(n)’

In the long run, f(n) does not grow faster than c(n)

Big θ – tight bounds: both O and Ω

- For a system of n nodes,
- Communication complexity c(n) is θ(f(n)) means:
  - There are constants a,b and N, such that:
  - For n>N: $b*f(n)<c(n)<a*f(n)$



Allowing some initial irregularity, c(n) and f(n) are Within constant factors of each other.
In the long run, c(n) grows at same rate as f(n), upto constant factors.

Bit complexity of communication

- We have assumed that each communication is 1 message, and we counted the messages
- Sometimes, communication is evaluated by bit complexity – the number of bits communicated
- This is different from message complexity because a message may have number of bits that depend on n or |E|
- For example, node ids in message have size $\Theta(\log n)$

- In practice this is may not be critical since log n is much smaller than packet sizes, so it does not change the number of packets communicated
- But depending on what other data the algorithm is communicating, sizes of messages may matter

Size of ids

- In a network of n nodes
- Each node id needs $\Theta(\log n)$ (that is, both $O(\log n)$ and $\Omega(\log n)$) bits for storage
  - The binary representation of n needs $\log_2 n$ bits

- Ω – since we need at least this many bits
  - May vary by constant factors depending on base of logarithm

Computing Trees:

- What if the edges have weights?

Aggregation using Trees:

- What if the edges have weights?
- The cost may not be O(n) since weights can be higher

- How to get the best performance?

Minimum spanning tree is

- A spanning tree (reaches all nodes)
- With minimum possible total weight

- How can we compute a minimum spanning tree efficiently in a distributed system?
- (remember, a node knows only its neighbors and edge weights)