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## Symmetric encryption schemes

#### A symmetric cipher consists of two algorithms

- encryption algorithm  $E: \mathcal{K} \times \mathcal{M} \to \mathcal{C}$
- decryption algorithm  $D: \mathcal{K} \times \mathcal{C} \to \mathcal{M}$

st. 
$$\forall k \in \mathcal{K}$$
, and  $\forall m \in \mathcal{M}$ ,  $D(k, E(k, m)) = m$ 

#### Kerckhoff's principle

- ▶ The encryption (E) and decryption (D) algorithms are public
- ▶ The security relies entirely on the secrecy of the key

### Adversarie's capabilities - threat model

#### The attacker may have access to :

- ightharpoonup some ciphertexts  $c_1, \ldots, c_n$
- ▶ some plaintext/ciphertext pairs  $(m_1, c_1), \ldots, (m_n, c_n)$ st.  $c_i = E(k, m_i)$
- ▶ an encryption oracle he can maybe trick a user to encrypt messages  $m_1, \ldots, m_n$  of his choice
- ▶ a decryption oracle he can maybe trick a user to decrypt ciphertexts  $c_1, \ldots, c_n$  of his choice
- ▶ unlimited, or polynomial, or realistic ( $\leq 2^{80}$ ) computational power
- A cryptographic scheme is secure under some assumptions, that is against a certain type of attacker
- A cryptographic scheme may be vulnerable to certain types of attacks but not others

### What is a good encryption scheme?

An encryption scheme is secure against a given adversary, if this adversary cannot

- recover the secret key k
- recover the plaintext m underlying a ciphertext c
- recover any bits of the plaintext m underlying a ciphertext c
- **.** . . .

$$\blacktriangleright \mathcal{M} = \mathcal{C} = \mathcal{K} = \{0,1\}^n$$

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▶ Consistency:  $D(k, E(k, m)) = k \oplus (k \oplus m) = m$ 

### Perfect secrecy

#### Definition

A cipher (E,D) over  $(\mathcal{M},\mathcal{C},\mathcal{K})$  satisfies perfect secrecy if for all messages  $m_1,m_2\in\mathcal{M}$  of same length  $(|m_1|=|m_2|)$ , and for all ciphertexts  $c\in\mathcal{C}$ 

$$|Pr(E(k, m_1) = c) - Pr(E(k, m_2) = c)| \le \epsilon$$

where  $k \stackrel{r}{\leftarrow} \mathcal{K}$  and  $\epsilon$  is some "negligible quantity".

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$$|Pr(E(k, m_1) = c) - Pr(E(k, m_2) = c)| \le \left| \frac{1}{\#K} - \frac{1}{\#K} \right| = 0$$

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  - ▶ OTP is malleable given the ciphertext c = E(k, m) with  $m = to\ bob: m_0$ , it is possible to compute the ciphertext c' = E(k, m') with  $m' = to\ eve: m_0$   $c' := c \oplus "to\ bob: 00...00" \oplus "to\ eve: 00...00"$

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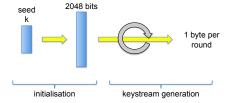
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- Stream ciphers are malleable

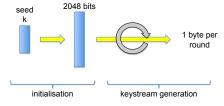
### RC4

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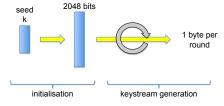


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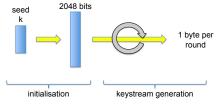
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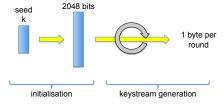
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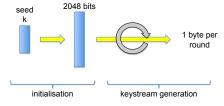
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  - subject to related keys attacks
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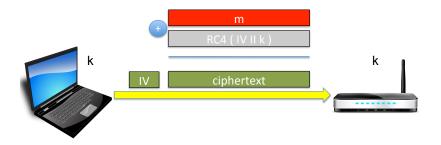
#### RC4: initialisation

```
for i := 0 to 255 do S[i] := i end j := 0 for i := 0 to 255 do j := (j + S[i] + K[i(\text{mod } |K|)])(\text{mod } 256) swap(S[i], S[j]) end i := 0 j := 0
```

### RC4: key stream generation

```
while generatingOutput i := i + 1 \pmod{256} j := j + S[i] \pmod{256} swap(S[i], S[j]) output(S[S[i] + S[j] \pmod{256}]) end
```

#### WEP uses RC4



Initialisation Vector (IV): 24-bits long string

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#### Weaknesses of TLS

MUST READ THESE TEN CITIES ARE HOME TO THE BIGGEST BOTNETS

# RC4 NOMORE crypto exploit used to decrypt user cookies in mere hours

Websites using RC4 encryption need to change their protocols as exploits using design flaws are now far easier to perform.





 $ightharpoonup K = \{0,1\}^s$ 

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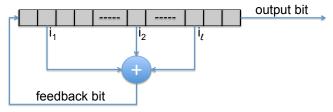
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feedback bit:  $R[i_1] \oplus R[i_2] \oplus \cdots \oplus R[i_\ell]$ 

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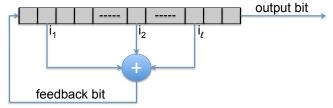


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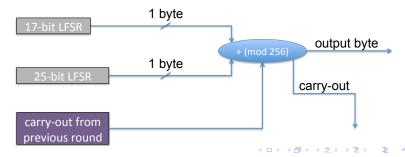
- ▶ Broken LFSR-based stream ciphers:
  - ► DVD encryption: CSS (2 LFSRs)
  - GSM encryption: A5 (3 LFSRs)
    - Bluetooth encryption: E0 (4 LFSRs)

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Can be broken in time  $2^{17}$ . The idea of the attack is as follows:

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- Given output of 17 bit LFSR, can deduce output of 25 bit LFSR by subtraction
- ► Hence try all 2<sup>17</sup> possibilities for 17 bit LFSR and if generated 25 bit LFSR produces observed keystream, cipher is cracked

#### Android BitCoin attack



### Modern stream ciphers

Project eStream: project to "identify new stream ciphers suitable for widespread adoption", organised by the EU ECRYPT network

→ HC-128, Rabbit, Salsa20/12, SOSEMANUK,

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#### Conjecture

These eStream stream ciphers are "secure"

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