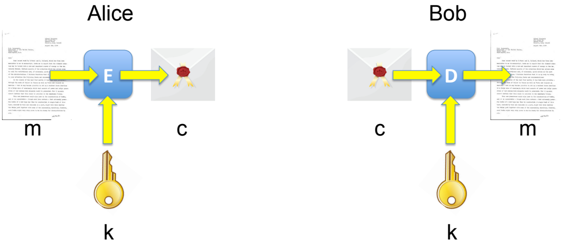
Cryptography

Myrto Arapinis

October 4, 2016

Symmetric ciphers

- encryption algorithm $E : \mathcal{K} \times \mathcal{M} \to \mathcal{C}$
  decryption algorithm $D : \mathcal{K} \times \mathcal{C} \to \mathcal{M}$
  st. $\forall k \in \mathcal{K}$, and $\forall m \in \mathcal{M}$, $D(k, E(k, m)) = m$

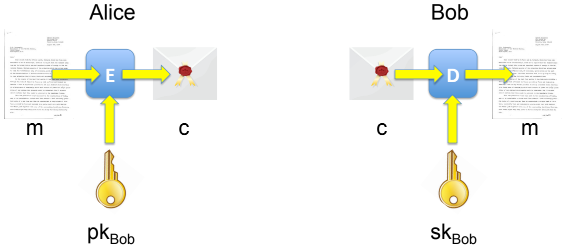


- same key k to encrypt and decrypt
- the key k is secret: only known to Alice and Bob

Examples: One-time pad, DES, AES, ...

Asymmetric ciphers

- key generation algorithm: $G : \to \mathcal{K} \times \mathcal{K}$
  encryption algorithm $E : \mathcal{K} \times \mathcal{M} \to \mathcal{C}$
  decryption algorithm $D : \mathcal{K} \times \mathcal{C} \to \mathcal{M}$
  st. $\forall (sk, pk) \in G$, and $\forall m \in \mathcal{M}$, $D(sk, E(pk, m)) = m$

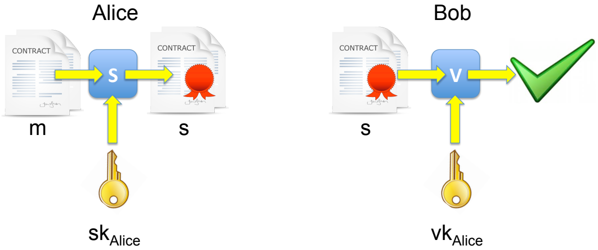


- the decryption key sk is secret (only known to Bob). The encryption key pk is known to everyone. And $sk \neq pk$

Examples: RSA, ElGamal, Diffie-Hellman, ...

Digital signatures

- key generation algorithm: $G : \to \mathcal{K} \times \mathcal{K}$
  signing algorithm $S : \mathcal{K} \times \mathcal{M} \to \mathcal{S}$
  verification algorithm $V : \mathcal{K} \times \mathcal{S} \to \{\top, \bot\}$
  st. $\forall (sk, vk) \in G$, and $\forall m \in \mathcal{M}$, $V(vk, S(sk, m)) = \top$

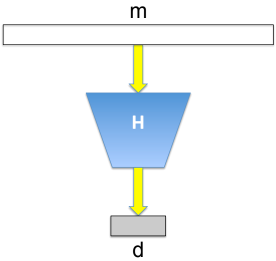


- the signing key sk is secret (only known to Alice). The verification key vk is known to everyone. And $sk \neq vk$

Examples: RSA based, ElGamal based, Schnorr, ...

Hashes

- hash algorithm $H: \ \mathcal{M} \to \mathcal{D}$



  - **preimage resistant**: given a digest d, it is computationally infeasible to find any message m such that $H(m) = d$
  - **collision resistance**: it is hard to find two different messages $m_1 \neq m_2$ such that $H(m_1) = H(m_2)$
- applications: commitment schemes, signature schemes, MACs, key derivation algorithms, ...

Examples: MD5, SHA-1, ...

Many more crypto primitives

- Message Authentication Codes (MACs)
- Zero Knowledge Proofs (ZKPs)
- Fully Homomorphic Encryption (FHE)
- ...

Historical ciphers

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Rail fence cipher

- shared secret key $k \in \mathbb{N}$
- Encryption: plaintext written in columns of size k. The ciphertext is the concatenation of the resulting rows.

k=6

m = THIS COURSE AIMS TO INTRODUCE YOU TO THE PRINCIPLES AND TECHNIQUES OF SECURING COMPUTERS

T	O	A	O	O	Y		R	L	D	N		C		T
H	U	I		D	O	T	I	E		I	O	U	C	E
I	R	M	I	U	U	H	N	S	T	Q	F	R	O	R
S	S	S	N	C		E	C		E	U		I	M	S
	E		T	E	T		I	A	C	E	S	N	P	
C		T	R		O	P	P	N	H	S	E	G	U	Q

c = TOAOOY RLDN C THUI DOTIE IOUCEIRMIUUHNSTQFRORSSSNC EC EU IMS E TET IACESNPC TR OPPNHSEGUQ

- Decryption: ciphertext written in rows of size $\frac{|c|}{k}$

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H	U	I		D	O	T	I	E		I	O	U	C	E
I	R	M	I	U	U	H	N	S	T	Q	F	R	O	R
S	S	S	N	C		E	C		E	U		I	M	S
	E		T	E	T		I	A	C	E	S	N	P	
C		T	R		O	P	P	N	H	S	E	G	U	Q

c = TOAOOY RLDN C THUI DOTIE IOUCEIRMIUUHNSTQFRORSSSNC EC EU IMS E TET IACESNPC TR OPPNHSEGUQ

- Decryption: ciphertext written in rows of size $\frac{|c|}{k}$

But small key space size: $k < |c| \Rightarrow$ brute force attack!!

Substitution cipher

- shared secret: a permutation ϖ of the set of characters

  $\varpi =$ $a \mapsto q$ $b \mapsto w$ $c \mapsto e$ $d \mapsto r$ $e \mapsto t$ $f \mapsto y$ $g \mapsto u$ $h \mapsto i$ $i \mapsto o$ $j \mapsto m$ $k \mapsto a$ $l \mapsto s$ $m \mapsto d$ $n \mapsto f$ $o \mapsto g$ $p \mapsto h$ $q \mapsto j$ $r \mapsto k$ $s \mapsto l$ $t \mapsto z$ $u \mapsto x$ $v \mapsto c$ $w \mapsto v$ $x \mapsto b$ $y \mapsto n$ $z \mapsto p$

- Encryption: apply ϖ to each character of the plaintext.

$$E(\varpi, m_1 \dots m_n) \;=\; \varpi(m_1) \dots \varpi(m_n)$$

- Decryption: apply ϖ^{-1} to each character of the plaintext.

$$D(\varpi, c_1 \dots c_n) \;=\; \varpi^{-1}(c_1) \dots \varpi^{-1}(c_n)$$

Substitution cipher: example

$m =$ THIS COURSE AIMS TO INTRODUCE YOU TO THE PRINCIPLES AND TECHNIQUES OF SECURING COMPUTERS AND COMPUTER NETWORKS WITH FOCUS ON INTERNET SECURITY. THE COURSE IS EFFECTIVELY SPLIT INTO TWO PARTS. FIRST INTRODUCING THE THEORY OF CRYPTOGRAPHY INCLUDING HOW MANY CLASSICAL AND POPULAR ALGORITHMS WORK E.G. DES, RSA, DIGITAL SIGNATURES, AND SECOND PROVIDING DETAILS OF REAL INTERNET SECURITY PROTOCOLS, ALGORITHMS, AND THREATS, E.G. IPSEC, VIRUSES, FIREWALLS. HENCE, YOU WILL LEARN BOTH THEORETICAL ASPECTS OF COMPUTER AND NETWORK SECURITY AS WELL AS HOW THAT THEORY IS APPLIED IN THE INTERNET. THIS KNOWLEDGE WILL HELP YOU IN DESIGNING AND DEVELOPING SECURE APPLICATIONS AND NETWORK PROTOCOLS AS WELL AS BUILDING SECURE NETWORKS.

$c =$ ZIOL EGXKLT QODL ZG OFZKGRXET NGX ZG ZIT HKOFEOHSTL QFR ZTEIFOJXTL GY LTEXKOFU EGDHXZTKL QFR EGDHXZTK FTZVGKAL VOZI YGEXL GF OFZTKFTZ LTEXKOZN. ZIT EGXKLT OL TYYTEZOCTSN LHSOZ OFZG ZVG HQKZL. YOKLZ OFZKGRXEOFU ZIT ZITGKN GY EKNHZGUKQHIN OFESXROFU IGV DQFN ESQLLOEQS QFR HGHXSQK QSUGKOZIDL VGKA T.U. RTL, KLQ, ROUOZQS LOUFQZXKTL, QFR LTEGFR HKGCOROFU RTZQOSL GY KTQS OFZTKFTZ LTEXKOZN HKGZGEGSL, QSUGKOZIDL, QFR ZIKTQZL, T.U. OHLTE, COKXLTL, YOKTVQSSL. ITFET, NGX VOSS STQKF WGZI ZITGKTZOEQS QLHTEZL GY EGDHXZTK QFR FTZVGKA LTEXKOZN QL VTSS QL IGV ZIQZ ZITGKN OL QHHSOTR OF ZIT OFZTKFTZ. ZIOL AFGVSTRUT VOSS ITSH NGX OF RTLOUFOFU QFR RTCTSGHOFU LTEXKT QHHSOEQZOGFL QFR FTZVGKA HKGZGEGSL QL VTSS QL WXOSROFU LTEXKT FTZVGKAL.

Breaking the substitution cipher

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- Key space size: $|\mathcal{K}| = 26!$ ($\approx 2^{88}$) $\Rightarrow$ brute force infeasible!

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- Key space size: $|\mathcal{K}| = 26!$ ($\approx 2^{88}$) $\Rightarrow$ brute force infeasible!
- Exploit regularities of the language
  - Use frequency of letters in english text

    e $>$ t $>$ a $>$ o
  - Use frequency of digrams in english text

    th $>$ he $>$ in $>$ er
  - Use frequency of trigrams in english text

    the $>$ and $>$ ing
  - Use expected words

Breaking the substitution cipher: example

c = ZIOL EGXKLT QODL ZG OFZKGRXET NGX ZG ZIT HKOFEOHSTL QFR ZTEIFOJXTL GY LTEXKOFU EGDHXZTKL QFR EGDHXZTK FTZVGKAL VOZI YGEXL GF OFZTKFTZ LTEXKOZN. ZIT EGXKLT OL TYYTEZOCTSN LHSOZ OFZG ZVG HQKZL. YOKLZ OFZKGRXEOFU ZIT ZITGKN GY EKNHZGUKQHIN OFESXROFU IGV DQFN ESQLLOEQS QFR HGHXSQK QSUGKOZIDL VGKA T.U. RTL, KLQ, ROUOZQS LOUFQZXKTL, QFR LTEGFR HKGCOROFU RTZQOSL GY KTQS OFZTKFTZ LTEXKOZN HKGZGEGSL, QSUGKOZIDL, QFR ZIKTQZL, T.U. OHLTE, COKXLTL, YOKTVQSSL. ITFET, NGX VOSS STQKF WGZI ZITGKTZOEQS QLHTEZL GY EGDHXZTK QFR FTZVGKA LTEXKOZN QL VTSS QL IGV ZIQZ ZITGKN OL QHHSOTR OF ZIT OFZTKFTZ. ZIOL AFGVSTRUT VOSS ITSH NGX OF RTLOUFOFU QFR RTCTSGHOFU LTEXKT QHHSOEQZOGFL QFR FTZVGKA HKGZGEGSL QL VTSS QL WXOSROFU LTEXKT FTZVGKAL.

Breaking the substitution cipher: example

c = TIOL EGXKLE QODL TG OFTKGRXEE NGX TG TIE HKOFEOHSEL QFR TEEIFOJXEL GY LEEXKOFU EGDHXTEKL QFR EGDHXTEK FETVGKAL VOTI YGEXL GF OFTEKFET LEEXKOTN. TIE EGXKLE OL EYYEETOCESN LHSOT OFTG TVG HQKTL. YOKLT OFTKGRXEOFU TIE TIEGKN GY EKNHTGUKQHIN OFESXROFU IGV DQFN ESQLLOEQS QFR HGHXSQK QSUGKOTIDL VGKA E.U. REL, KLQ, ROUOTQS LOUFQTXKEL, QFR LEEGFR HKGCOROFU RETQOSL GY KEQS OFTEKFET LEEXKOTN HKGTGEGSL, QSUGKOTIDL, QFR TIKEQTL, E.U. OHLEE, COKXLEL, YOKEVQSSL. IEFEE, NGX VOSS SEQKF WGTI TIEGKETOEQS QLHEETL GY EGDHXTEK QFR FETVGKA LEEXKOTN QL VESS QL IGV TIQT TIEGKN OL QHHSOER OF TIE OFTEKFET. TIOL AFGVSERUE VOSS IESH NGX OF RELOUFOFU QFR RECESGHOFU LEEXKE QHHSOEQTOGFL QFR FETVGKA HKGTGEGSL QL VESS QL WXOSROFU LEEXKE FETVGKAL.

Most common letters in c: t $>$ z $>$ ...

Breaking the substitution cipher: example

c = THOL EGXKLE QODL TG OFTKGRXEE NGX TG THE HKOFEOHSEL QFR TEEHFOJXEL GY LEEXKOFU EGDHXTEKL QFR EGDHXTEK FETVGKAL VOTH YGEXL GF OFTEKFET LEEXKOTN. THE EGXKLE OL EYYEETOCESN LHSOT OFTG TVG HQKTL. YOKLT OFTKGRXEOFU THE THEGKN GY EKNHTGUKQHHN OFESXROFU HGV DQFN ESQLLOEQS QFR HGHXSQK QSUGKOTHDL VGKA E.U. REL, KLQ, ROUOTQS LOUFQTXKEL, QFR LEEGFR HKGCOROFU RETQOSL GY KEQS OFTEKFET LEEXKOTN HKGTGEGSL, QSUGKOTHDL, QFR THKEQTL, E.U. OHLEE, COKXLEL, YOKEVQSSL. HEFEE, NGX VOSS SEQKF WGTH THEGKETOEQS QLHEETL GY EGDHXTEK QFR FETVGKA LEEXKOTN QL VESS QL HGV THQT THEGKN OL QHHSOER OF THE OFTEKFET. THOL AFGVSERUE VOSS HESH NGX OF RELOUFOFU QFR RECESGHOFU LEEXKE QHHSOEQTOGFL QFR FETVGKA HKGTGEGSL QL VESS QL WXOSROFU LEEXKE FETVGKAL.

Most common digrams in c: of $>$ zi $>$ ...

t$\mapsto$z suggests h$\mapsto$i

Breaking the substitution cipher: example

c = THIL EGXKLE QIDL TG INTKGRXEE NGX TG THE HKINEIHSEL QNR TEEHNIJXEL GY
LEEXKINU EGDHXTEKL QNR EGDHXTEK NETVGKAL VITH YGEXL GN INTEKNET
LEEXKITN. THE EGXKLE IL EYYEETICESN LHSIT INTG TVG HQKTL. YIKLT
INTKGRXEINU THE THEGKN GY EKNHTGUKQHHN INESXRINU HGV DQNN ESQLLIEQS
QNR HGHXSQK QSUGKITHDL VGKA E.U. REL, KLQ, RIUITQS LIUNQTXKEL, QNR
LEEGNR HKGCIRINU RETQISL GY KEQS INTEKNET LEEXKITN HKGTGEGSL,
QSUGKITHDL, QNR THKEQTL, E.U. IHLEE, CIKXLEL, YIKEVQSSL. HENEE, NGX
VISS SEQKN WGTH THEGKETIEQS QLHEETL GY EGDHXTEK QNR NETVGKA
LEEXKITN QL VESS QL HGV THQT THEGKN IL QHHSIER IN THE INTEKNET. THIL
ANGVSERUE VISS HESH NGX IN RELIUNINU QNR RECESGHINU LEEXKE
QHHSIEQTIGNL QNR NETVGKA HKGTGEGSL QL VESS QL WXISRINU LEEXKE
NETVGKAL.

Most common digrams in c: of > zi > . . .

we guess in$\mapsto$of

Breaking the substitution cipher: example

c = THIL EGXKLE QIDL TG INTKGRXEE NGX TG THE HKINEIHSEL QNR TEEHNIJXEL GY LEEXKINU EGDHXTEKL QNR EGDHXTEK NETVGKAL VITH YGEXL GN **INTEKNET** LEEXKITN. THE EGXKLE IL EYYEETICESN LHSIT INTG TVG HQKTL. YIKLT INTKGRXEINU THE THEGKN GY EKNHTGUKQHHN INESXRINU HGV DQNN ESQLLIEQS QNR HGHXSQK QSUGKITHDL VGKA E.U. REL, KLQ, RIUITQS LIUNQTXKEL, QNR LEEGNR HKGCIRINU RETQISL GY KEQS INTEKNET LEEXKITN HKGTGEGSL, QSUGKITHDL, QNR THKEQTL, E.U. IHLEE, CIKXLEL, YIKEVQSSL. HENEE, NGX VISS SEQKN WGTH THEGKETIEQS QLHEETL GY EGDHXTEK QNR NETVGKA LEEXKITN QL VESS QL HGV THQT THEGKN IL QHHSIER IN THE INTEKNET. THIL ANGVSERUE VISS HESH NGX IN RELIUNINU QNR RECESGHINU LEEXKE QHHSIEQTIGNL QNR NETVGKA HKGTGEGSL QL VESS QL WXISRINU LEEXKE NETVGKAL.

We identify in c the word INTEKNET

suggests $r \mapsto k$

Breaking the substitution cipher: example

c = THIS EGXRSE QIDS TG INTRGRXEE NGX TG THE HRINEIHSES QNR TEEHNIJXES GY SEEXRINU EGDHXTERS QNR EGDHXTER NETVGRAS VITH YGEXS GN INTERNET SEEXRITN. THE EGXRSE IS EYYEETICESN SHSIT INTG TVG HQRTS. YIRST INTRGRXEINU THE THEGRN GY ERNHTGURQHHN INESXRINU HGV DQNN ESQSSIEQS QNR HGHXSQR QSUGRITHDS VGRA E.U. RES, RSQ, RIUITQS SIUNQTXRES, QNR SEEGNR HRGCIRINU RETQISS GY REQS INTERNET SEEXRITN HRGTGEGSS, QSUGRITHDS, QNR THREQTS, E.U. IHSEE, CIRXSES, YIREVQSSS. HENEE, NGX VISS SEQRN WGTH THEGRETIEQS QSHEETS GY EGDHXTER QNR NETVGRA SEEXRITN QS VESS QS HGV THQT THEGRN IS QHHSIER IN THE INTERNET. THIS ANGVSERUE VISS HESH NGX IN RESIUNINU QNR RECESGHINU SEEXRE QHHSIEQTIGNS QNR NETVGRA HRGTGEGSS QS VESS QS WXISRINU SEEXRE NETVGRAS.

The first word is THIL

suggests s$\mapsto$l

Breaking the substitution cipher: example

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Going back to letter frequency and a few more guesses!!

Vigenere cipher

- shared secret key: a word w over the english alphabet

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- Encryption: break the plaintext $m = m_1 \dots m_n$ in $\frac{|m|}{|w|}$ blocks, and encrypt each block as follows

$$\begin{array}{llcl} & m_{i+1} & \dots & m_{i+|w|} \\ + & w_1 & \dots & w_{|w|} \\ \hline & \underbrace{m_{i+1} + w_1 (\mathrm{mod}\ 26)}_{c_{i+1}} & \dots & \underbrace{m_{i+|w|} + w_{|w|} (\mathrm{mod}\ 26)}_{c_{i+|w|}} \end{array}$$

Concatenate the resulting blocks to obtain the ciphertext

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\end{array}
$$

  Concatenate the resulting blocks to obtain the ciphertext
- Decryption: break the ciphertext $c = c_1 \dots c_n$ in $\frac{|m|}{|w|}$ blocks, and decrypt each block as follows

$$
\begin{array}{cccc}
 & c_{i+1} & \dots & c_{i+|w|} \\
- & w_1 & \dots & w_{|w|} \\
\hline
 & \underbrace{c_{i+1} - w_1 (\text{mod } 26)}_{m_{i+1}} & \dots & \underbrace{c_{i+|w|} - w_{|w|} (\text{mod } 26)}_{m_{i+|w|}}
\end{array}
$$

  Concatenate the resulting blocks to retrieve the message

Vigenere cipher: example

$w = MACBETH$

$m =$ WHEN SHALL WE THREE MEET AGAIN IN THUNDERLIGHTNING OR IN RAIN

$$
\begin{array}{cccccccccccccccccc}
 & M & A & C & B & E & T & H & M & A & C & B & E & T & H & \dots & M & A \\
+ & W & H & E & N & S & H & A & L & L & W & E & T & H & R & \dots & I & N \\
\hline
 & J & I & H & P & X & B & I & Y & M & Z & G & Y & B & Z & \dots & V & O
\end{array}
$$

$c =$ IHGO WAHXL YF XAYQE OFIM HSAKO MG ATUPEIKSUGJVRBUS OT JR KHUN

Breaking the Vigenere cipher

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- Suppose we know the length of the key w. Break the ciphertext in $\frac{|c|}{|w|}$ blocks:

$$c_1 \dots c_{|w|} \mid\mid c_{|w|+1} \dots c_{2|w|} \mid\mid \dots \mid\mid c_{|c|-|w|+1} \dots c_{|c|}$$

  for each position in $\{1, \dots, |w|\}$, consider the characters $c_{j|w|+i}$ for all $j \in \frac{|c|}{|w|}$. All these characters have been encrypted using the same key character w_i. Perform letter frequency analysis on this set of characters.

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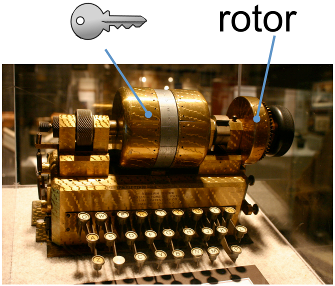
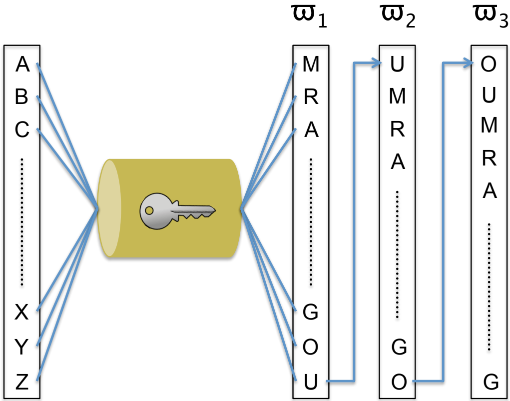
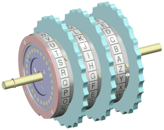
- If the size of w is not known apply Kasiski's method to narrow the possibilities:
  - identify all the sequences of letters of length greater than 4 that occur more than once
  - for each such sequence compute the distance between two of its occurences
  - compute the corresponding possible key-length

Rotor machines: the Herbern machine

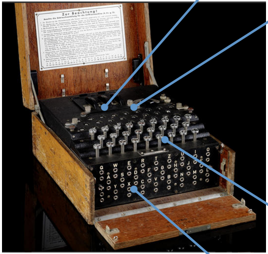


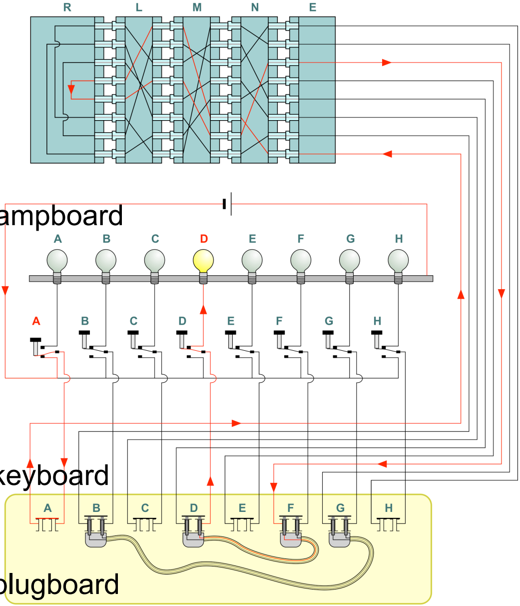


Rotor machines: the enigma machine



Rotors





lampboard

keyboard

plugboard

The One-Time Pad (OTP)

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$$
\begin{array}{cccccccccc}
k & = & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
m & = & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
\hline
c & = & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

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- Decryption: $\forall k \in \mathcal{K}.\ \forall c \in \mathcal{C}.\ D(k,c) = k \oplus c$

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$$\begin{array}{ccccccccccc} k & = & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ m & = & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ \hline \\ c & = & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}$$

- Decryption: $\forall k \in \mathcal{K}.\ \forall c \in \mathcal{C}.\ D(k,c) = k \oplus c$

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The One-Time Pad (OTP)

- $\mathcal{M} = \mathcal{C} = \mathcal{K} = \{0,1\}^n$
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- Consistency: $D(k, E(k,m)) = k \oplus (k \oplus m) = m$

Perfect secrecy

Definition

A cipher (E, D) over $(\mathcal{M}, \mathcal{C}, \mathcal{K})$ satisfies perfect secrecy if for all messages $m_1, m_2 \in \mathcal{M}$ of same length ($|m_1| = |m_2|$), and for all ciphertexts $c \in \mathcal{C}$

$$|Pr(E(k, m_1) = c) - Pr(E(k, m_2) = c)| \leq \epsilon$$

where $k \xleftarrow{r} \mathcal{K}$ and ϵ is some "negligible quantity".

OTP satisfies perfect secrecy

Theorem (Shannon 1949)

The One-Time Pad satisfies perfect secrecy

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Proof: We first note that for all messages $m \in \mathcal{M}$ and all ciphertexts $c \in \mathcal{C}$

$$Pr(E(k, m) = c)$$

where $k \xleftarrow{r} \mathcal{K}$.

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Proof: We first note that for all messages $m \in \mathcal{M}$ and all ciphertexts $c \in \mathcal{C}$

$$Pr(E(k, m) = c) \quad = \quad \frac{\#\{k \in \mathcal{K} \colon\ k \oplus m = c\}}{\#\mathcal{K}}$$

where $k \xleftarrow{r} \mathcal{K}$.

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Proof: We first note that for all messages $m \in \mathcal{M}$ and all ciphertexts $c \in \mathcal{C}$

$$\begin{aligned} Pr(E(k,m) = c) &= \frac{\#\{k \in \mathcal{K}:\ k \oplus m = c\}}{\#\mathcal{K}} \\ &= \frac{\#\{k \in \mathcal{K}:\ k = m \oplus c\}}{\#\mathcal{K}} \end{aligned}$$

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Thus, for all messages $m_1, m_2 \in \mathcal{M}$, and for all ciphertexts $c \in \mathcal{C}$

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&= \frac{1}{\#\mathcal{K}}
\end{aligned}
$$

where $k \stackrel{r}{\leftarrow} \mathcal{K}$.

Thus, for all messages $m_1, m_2 \in \mathcal{M}$, and for all ciphertexts $c \in \mathcal{C}$

$$
|Pr(E(k,m_1)=c) - Pr(E(k,m_2)=c)| \leq \left| \frac{1}{\#\mathcal{K}} - \frac{1}{\#\mathcal{K}} \right| = 0
$$

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    English has enough redundancy s.t. $m_1 \oplus m_2 \to m_1, m_2$
  - OTP is malleable
    given the ciphertext $c = E(k, m)$ with $m = to\ bob : m_0$, it is possible to compute the ciphertext $c' = E(k, m')$ with
    $m' = to\ eve : m_0$
    $c' := c \oplus "to\ bob : 00 \ldots 00" \oplus "to\ eve : 00 \ldots 00"$