Sampling & Reconstruction, Pulse Amplitude Modulation (PAM)

Dr. B. Sainath

EEE Dept., BITS PILANI

Sep., 2017

BITS Pilani

Instantaneous Sampling

- Sampling of a finite-energy signal $g(t)$

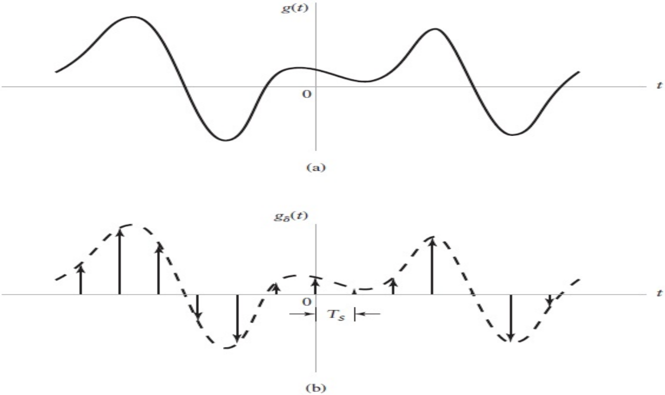
Instantaneous Sampling

- Sampling of a finite-energy signal $g(t)$
  - Mathematical operation to obtain discrete-time sequence $g[nT_s]$ or $g[n]$, where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$
- Instantaneous sampling or ideal sampling

BITS Pilani

Instantaneous Sampling

- Sampling of a finite-energy signal $g(t)$
  - Mathematical operation to obtain discrete-time sequence $g[nT_s]$ or $g[n]$, where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$
- Instantaneous sampling or ideal sampling
  - Take a sample for every T_s seconds $\Leftarrow$ sampling period or interval
  - Uniform sampling rate $f_s = \frac{1}{T_s}$



Sampling Theorem for Lowpass Signals

- Serves as basis for interchangeability of analog signals & discrete-time sequences
  - So important in digital communication systems

- Let $g(t)$ is a lowpass signal & band-limited
- Max. frequency component of $g(t)$ is W
- $g(t)$ completely described by samples taken for each $T_s = \frac{1}{2W}$
- Signal $g(t)$ may be completely recovered from knowledge of its samples taken at Nyquist rate $f_N = 2W$

- In practice, $f_s > f_N$
  - To assure physical realizability of reconstruction filter

BITS Pilani

Interpolation Formula

- For reconstructing $g(t)$ from its samples

$$g(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\text{sinc}\left(\frac{t}{T_s} - n\right)$$
$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\text{sinc}\left(2Wt - n\right), -\infty < t < \infty$$

- In the reconstruction process

BITS Pilani

Interpolation Formula

- For reconstructing $g(t)$ from its samples

$$g(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\text{sinc}\left(\frac{t}{T_s} - n\right)$$
$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\text{sinc}\left(2Wt - n\right), -\infty < t < \infty$$

- In the reconstruction process
  - Each sample is multiplied by its respective interpolation function & all resulting waveforms are added to get $g(t)$

BITS Pilani

- This is analog version of PAM
- Carrier signal $c(t)$ is a pulse train expressed in terms of pulse $p(t)$
- Amplitudes of regularly spaced pulses are varied in direct proportion to the instantaneous sample values of $g(t)$
- PAM wave:

Pulse Amplitude Modulation (PAM)

- This is analog version of PAM
- Carrier signal $c(t)$ is a pulse train expressed in terms of pulse $p(t)$
- Amplitudes of regularly spaced pulses are varied in direct proportion to the instantaneous sample values of $g(t)$
- PAM wave: convolution of instantaneous sampled version of $(1 + k_a g(nT_s))$ & pulse $p(t)$

$$s(t) = \sum_{n=-\infty}^{\infty} (1 + k_a g(nT_s))\, p(t - nT_s)$$

  - Choose k_a to maintain single polarity
  - Ensure that $(1 + k_a T_s) > 0$, for all n
  - Sampling rate $f_s > f_N$
- Exercise: Derive $S(f)$.

BITS Pilani

Pulse Amplitude Modulation (PAM)

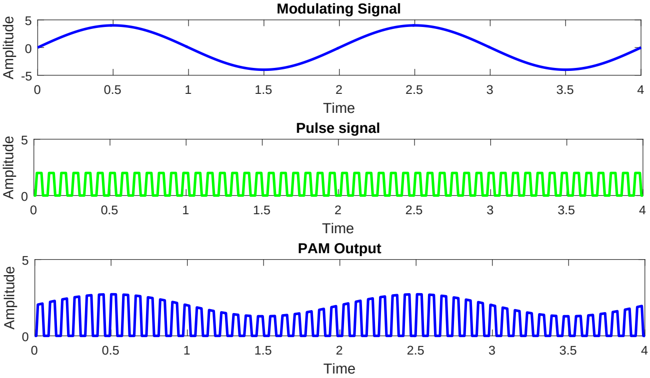
- This is analog version of PAM
- Carrier signal $c(t)$ is a pulse train expressed in terms of pulse $p(t)$
- Amplitudes of regularly spaced pulses are varied in direct proportion to the instantaneous sample values of $g(t)$
- PAM wave: convolution of instantaneous sampled version of $(1 + k_a g(nT_s))$ & pulse $p(t)$

$$s(t) = \sum_{n=-\infty}^{\infty} (1 + k_a g(nT_s))\, p(t - nT_s)$$

  - Choose k_a to maintain single polarity
  - Ensure that $(1 + k_a T_s) > 0$, for all n
  - Sampling rate $f_s > f_N$
- Exercise: Derive $S(f)$.
  - eqvt to passing samples through filter with T/F $P(f)$

BITS Pilani

Modulating Signal, Pulse Train & PAM Signal



Important Instructions for Lab

- Try to complete all tasks within 2 hours. After 2 hrs, evaluation starts.
- For each subtask, create mfiles (eg. *CT_HT*.m) and save them with suitable name.
- Prepare a word document naming your name and ID. In it, save all results including plots.
- In all plots, put x-label, y-label, legend, font 'Arial'(Size = 10), and, Width '2'.

Task 1. (a): Sampling & Reconstruction

- Questions:
  1. A continuous-time signal a $x(t) = \cos(2\pi f_m t)$ is sampled at sample frequency of 1000 Hz. Sampling results in discrete-time sequence $x[n] = 0.5\left(\exp\left(\frac{\pi n}{4}\right) + \exp\left(j\frac{\pi n}{4}\right)\right)$. Determine the following:
     - Sampling period or sampling duration T_s.
     - Three possible values of f_m, say f_1, f_2. (Hint: For f_1, at $t = nT_s$, what is $x(t)$?. Equate it with $x[n]$.)

BITS Pilani

Task 1. (b): Sampling & Reconstruction

- Use the following
  - $f_1 = 125$ Hz
  - $T_s = \frac{1}{1000}$ sec.
  - Observed interval = 0.02 sec.
  - In reconstruction, add 10 past samples and 20 future samples
  - Successive sample separation is $0.01\,T_s$
  - Approximate interpolation formula

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\mathrm{sinc}\left(\frac{t}{T_s} - n\right)$$

- Questions:
- Write a program to plot original message signal $x(t)$, sampled signal $x(nTs)$ and reconstructed signal $x_r(t)$. Show all in single plot. In the plot, provide x-label, y-label, title, and legend.

BITS Pilani

Task 2: Pulse Amplitude Modulation (PAM)

- Use the following
  - $A_c = 2$ volt, $A_m = 4$ volt
  - $f_c = 10000$ Hz, $f_m = 200$ Hz
  - $k_a = 0.09$ /volt
  - Carrier sequence cn = [0 0 1 1 1 1 0 0]
  - $m = \frac{f_c}{f_m}$
  - t=linspace(0,4,m*length(cn))
  - $x(t) = A_m \sin(2\pi f_m t)$
  - PAM signal: $y = A_c(1 + kax(t))$cnx, where cnx is updated carrier sequence
- Questions: Write a program to plot message signal, carrier signal, and, PAM signal in time interval [0, 4] without using MATLAB library function. Give x-label, y-label, title etc. to all subplots.
  - Note that you need to build carrier sequence for the required interval, that is, [0, 4]
- Let $A_{\max}$ denote the maximum amplitude, and $A_{\min}$ denote the minimum amplitude of the PAM wave, respectively. Graphically determine:

$$\frac{A_{\max}}{A_{\min}} (in\ dB).$$