Binary Phase shift Keying (BPSK)

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Introduction to Monte-Carlo Simulations

- Simulation technique
 - probabilistic system's nature is approximated by random number generator
- Basic idea
 - Generate several (pseudo-) random realizations
 - The number of realizations depend on desired accuracy of output
 - Compute output for each realization
 - Obtain an average value of output (eg. error probability)
- Some Applications
 - Communication systems
 - Queuing systems
 - Statistical physics
 - Signal processing



Problem: Numerical Computation of Error Probability

- Model: a basic communication system comprises of
 - Transmitter
 - transmits BPSK symbols $+\sqrt{E_b}$, $-\sqrt{E_b}$
 - bit '1' $\rightarrow +\sqrt{E_b}$ & bit '0' $\rightarrow -\sqrt{E_b}$, where E_b is the bit energy
 - symbols are equally likely



Figure: BPSK constellation.

- AWGN channel adds randomness to the communication system
- Transmitted symbols are independent of noise
- Receiver model: Coherent & maximum likelihood (ML) detection receiver
- Derive optimum decision rule and symbol (or bit) error probability (SEP/BEP) (in class)



Algorithm to Estimate SEP

- Choose range of SNR values in dB
- M = 2 (binary PSK)
- Initial count = 0, number of realizations = 10⁵
- For each SNR & for each realization
 - Generate transmit symbols which are equiprobable
 - Generate Gaussian noise
 - Add random noise to the transmitted symbol
 - Perform symbol-by-symbol maximum likelihood (ML) detection and compare detected symbol to the transmitted
 - Declare an error if transmitted symbol is not equal to decoded symbol
- Count the number of errors and divide them by the number of realizations



Usefulness of Simulations & Some Remarks

- MC simulations are used to validate theoretical/analytical formulae derived
- Close matching of simulation results with analytical results shows correctness of the analysis
- If some approximations are used in analysis, simulation results will not match with theoretical results
- Researcher often do simulations first. Later they validate simulation results using mathematical or experimental analysis
- MATLAB is widely used programming language for MC simulations
 - since it has strong built-in library functions
 - easy to learn and use
- Drawback of MATLAB is its speed
- Avoiding loops and using vector notations can improve speed



Important Instructions

- Try to complete all tasks within 2 hours. After 2 hrs, evaluation starts.
- For each subtask, create mfiles (eg. CT_HT.m) and save them with suitable name.
- Prepare a word document naming your name and ID. In it, save all results including plots.
- In all plots, put x-label, y-label, legend, font 'Arial' (Size = 10), and, Width '2'.



Useful Commands

- Understand following library functions/commands
 - qfunc
 - erfc
 - rand
 - floor
 - randn
 - abs
 - min
 - semilogy



SNR per Bit

- Let P_s denote transmitted signal power and let T_b denote bit duration
- For ideal BPSK, $T_b = \frac{1}{B}$, where *B* is bandwidth
- Bit energy $E_b = P_s T_b$
- If noise power (N_0B) normalized to unity, SNR per bit = P_s (how ?)





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- If noise power (N_0B) normalized to unity, SNR per bit = P_s (how ?)
 - Normalized *SNR* per bit = $\frac{P_s}{\mathbf{E}[n_0^2]B} = \frac{P_s}{N_0B} = P_s$



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Code Fragments

- ullet Generation of BPSK symbols lpha
 - m = floor(M*rand);
 - $\alpha = \exp(-i2\pi \frac{m}{M});$
- generation of Gaussian noise of unit variance
 - $n_0 = (randn + randn * 1i)/sqrt(2);$
- Received signal $y_0 = \sqrt{E_b}\alpha + n_0$;





Task: SEP Computation & Plot

- Question: Write a program to numerically compute SEP and plot the following
 - SEP as a function of SNR using formula derived in class
 - SEP as a function of SNR using MC simulations
 - Show analytical and simulation curves in single plot. In the plot, provide x-label, y-label, title, and legend.





Question

• Suppose that the noise power is -27 dBm. What is the average probability of error at $P_s = 9$ dB? (Hint: Determine from the SEP plot.)

