Quadrature Amplitude Modulation

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Oct., 2017

Problem: Numerical Computation of SEP

- Model: a basic communication system comprises of
  - Transmitter
    - transmits QAM symbols $+\sqrt{E_s}$, $-\sqrt{E_s}$
    - E_s is the symbol energy
    - symbols are equally likely

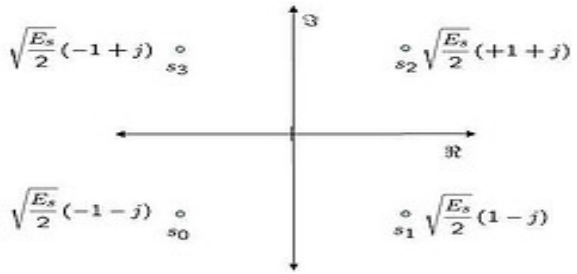

Figure: 4QAM constellation.

  - AWGN channel adds randomness to the communication system
  - Transmitted symbols are independent of noise
  - Receiver model: Coherent & maximum likelihood (ML) detection receiver

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Algorithm to Estimate SEP

- Choose range of SNR values in dB
- M = 4 (4-QAM)
- Initial count = 0, number of realizations = 10^5
- For each SNR & for each realization
  - Generate transmit symbols which are equiprobable
  - Generate Gaussian noise
  - Add random noise to the transmitted symbol
  - Perform symbol-by-symbol maximum likelihood (ML) detection and compare detected symbol to the transmitted
  - Declare an error if transmitted symbol is not equal to decoded symbol
- Count the number of errors and divide them by the number of realizations

Usefulness of Simulations & Some Remarks

- MC simulations are used to validate theoretical/analytical formulae derived
- Close matching of simulation results with analytical results shows correctness of the analysis
- If some approximations are used in analysis, simulation results will not match with theoretical results
- Researcher often do simulations first. Later they validate simulation results using mathematical or experimental analysis
- MATLAB is widely used programming language for MC simulations
  - since it has strong built-in library functions
  - easy to learn and use
- Drawback of MATLAB is its speed
- Avoiding loops and using vector notations can improve speed

Important Instructions

- Try to complete all tasks within 2 hours. After 2 hrs, evaluation starts.
- For each subtask, create mfiles (eg. CT_HT.m) and save them with suitable name.
- Prepare a word document naming your name and ID. In it, save all results including plots.
- In all plots, put x-label, y-label, legend, font 'Arial'(Size = 10), and, Width '2'.

Useful Commands

- Understand following library functions/commands
  - clear
  - qfunc
  - erfc
  - qfunc
  - rand
  - ceil
  - floor
  - randn
  - abs
  - min
  - semilogy

SNR per Symbol

- Let P_s denote transmitted signal power and let T_s denote symbol duration
- Symbol energy $E_s = P_s T_s$
- If noise power ($N_0 B$) normalized to unity, SNR per symbol = P_s (how ?)

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- If noise power ($N_0 B$) normalized to unity, *SNR* per symbol = P_s (how ?)
  - Normalized *SNR* per symbol = $\frac{P_s}{\mathbf{E}\left[n_0^2\right] B} = \frac{P_s}{N_0 B} = P_s$

Code Fragments

- Generation of MQAM symbols α
  - For $k = 1 : \sqrt{M}$, $a_I = 2k - 1 - \sqrt{M}$; $a_Q = 2l - 1 - \sqrt{M}$;
  - $l = 1 : \sqrt{M}$, $\alpha = \sqrt{\frac{1.5}{M-1}}\,(a_I + j a_Q)$;
- generation of Gaussian noise of unit variance
  - $n_0 = \frac{1}{\sqrt{2}}\,(\text{randn} + \text{randn} * 1j)$;
- Received signal $y_0 = \sqrt{E_s}\alpha + n_0$;

SEP Analytical Formula

- SEP Formula: Let $b = \frac{3}{M-1}$. For 4-QAM, M = 4.

$$\text{SEP} = 1 - (1 - \text{SEP}_{\text{sc}})^2$$

  - where SEP_{sc} denotes SEP in per carrier sense.
    $\text{SEP}_{\text{sc}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q(\sqrt{bP_s})$

Task: SEP Computation & Plot

- Question: Write a program to numerically compute SEP of $4-$QAM ($M = 4$) and plot the following
  1. SEP as a function of SNR using formula derived in class
  2. SEP as a function of SNR using MC simulations (Hint: Similar to BPSK, determine error regions and compute symbol error logically)
     - Show analytical and simulation curves in single plot. In the plot, provide x-label, y-label, title, and legend.

Question

- Suppose that the noise power is 33 dBm. What is the average probability of error at $P_s = 9$ dB? (Hint: Determine from the SEP plot.)