Chapter 1

An Introduction to Model Building

to accompany

Operations Research: Applications and Algorithms

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1.1 An Introduction to Modeling

- **Operations research** (management science) is a scientific approach to decision making that seeks to best design and operate a system, usually under conditions requiring the allocation of scarce resources.
  - ☐ Term coined during WW II when leaders asked scientists and engineers to analyze several military problems.
- A **system** is an organization of interdependent components that work together to accomplish the goal of the system.

- The scientific approach to decision making requires the use of one or more **mathematical models**.

- A mathematical model is a mathematical representation of the actual situation that may be used to make better decisions or clarify the situation.

Example 1: A Modeling Example

- Eli Daisy produces the drug Wozac in batches by heating a chemical mixture in a pressurized container.

- Each time a batch is produced, a different amount of Wozac is produced.
  - The amount produced is the *process yield* (measured in pounds).

- Daisy is interested in understanding the factors that influence the yield of Wozac production process.

- Describe a model-building process for this situation.

Example 1: Solution

- Daisy is first interested in determining the factors that influence the process yield.
  - ☐ This is a *descriptive model* since it describes the behavior of the actual yield as a function of various factors.
- Daisy might determine that the following factors influence yield:
  - ☐ Container volume in liters (V)
  - ☐ Container pressure in milliliters (P)
  - ☐ Container temperature in degrees centigrade (T)
  - ☐ Chemical composition of the processed mixture

5

Ex. 1: Solution continued

- Letting A, B, and C be the percentage of the mixture made up of chemical A, B, and C, then Daisy might find , for example, that:
  Yield = 300 + 0.8V +0.01P + 0.06T + 0.001T*P - $0.01T^2$ – $0.001P^2$ + 11.7A + 9.4B + 16.4C + 19A*B + 11.4A*C – 9.6B*C

- To determine the relationship the yield of the process would have to measured for many different combinations of the factors.

- Knowledge of this equation would enable Daisy to describe the yield of the production process once volume, pressure, temperature, and chemical composition were known.

6

- **Prescriptive models** "prescribe" behavior for an organization that will enable it to best meet its goals.
  - □ Components of this model include:
    - objective function(s)
    - decision variables
    - constraints
- An **optimization model** seeks to find values of the decision variables that optimize (maximize or minimize) an objective function among the set of all values for the decision variables that satisfy the given constraints.

Ex. 1: Solution continued

- The Daisy example seeks to maximize the yield for the production process.
- In most models, there will be a function we wish to maximize or minimize. This function is called the model's *objective function*.
- To maximize the process yield we need to find the values of V, P, T, A, B, and C that make the yield equation (below) as large as possible.

Yield = $300 + 0.8V + 0.01P + 0.06T + 0.001T*P - 0.01T^2 - 0.001P^2$

$+ 11.7A + 9.4B + 16.4C + 19A*B + 11.4A*C - 9.6B*C$

- In many situations, an organization may have more than more objective.
  - For example, in assigning students to the two high schools in Bloomington, Indiana, the Monroe County School Board stated that the assignment of students involve the following objectives:
    - equalize the number of students at the two high schools
    - minimize the average distance students travel to school
    - have a diverse student body at both high schools

- Variables whose values are under our control and influence system performance are called *decision variables*.
  - In the Daisy example, V, P, T, A, B, and C are decision variables.
- In most situations, only certain values of the decision variables are possible.
  - For example, certain volume, pressure, and temperature conditions might be unsafe. Also, A, B, and C must be nonnegative numbers that sum to one.
- These restrictions on the decision variable values are called *constraints*.

Ex. 1: Solution continued

- Suppose the Daisy example has the following constraints:
  - □ Volume must be between 1 and 5 liters
  - □ Pressure must be between 200 and 400 milliliters
  - □ Temperature must be between 100 and 200 degrees centigrade
  - □ Mixture must be made up entirely of A, B, and C
  - □ For the drug to perform properly, only half the mixture at most can be product A.

11

Ex. 1: Solution continued

- Mathematically, these constraints can be expressed:

$V \leq 5$	$A \geq 0$
$V \geq 1$	$B \geq 0$
$P \leq 400$	$C \geq 0$
$P \geq 200$	$A + B + C = 1.0$
$T \leq 200$	$A \leq 0.5$
$T \geq 100$	

12

Ex. 1: Solution continued

■ The Complete Daisy Optimization Model

□ Letting z represent the value of the objection function (the yield), the entire optimization model may be written as:

Maximize $z = 300 + 0.8V + 0.01P + 0.06T + 0.001T*P - 0.01T^2$

$\qquad - 0.001P^2 + 11.7A + 9.4B + 16.4C + 19A*B + 11.4A*C - 9.6B*C$

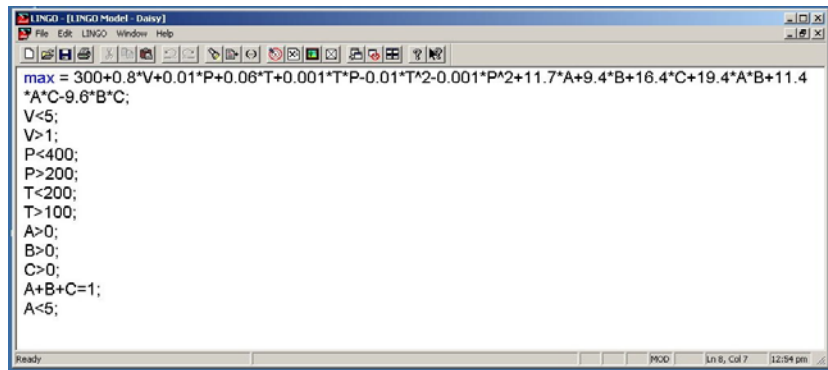| Subject to (s.t.)	V ≤ 5	T ≤ 200	A ≥ 0
	V ≥ 1	T ≥ 100	B ≥ 0
	P ≤ 400	A + B + C = 1.0	C ≥ 0
	P ≥ 200	A ≤ 0.5	

Ex. 1: Solution continued

■ Any specification of the decision variables that satisfies all the model's constraints is said to be in the **feasible region**.

□ For example, V = 2, P = 300, T = 150, A = 0.4, B = 0.3 and C = 0.3 is in the feasible region.

■ An **optimal solution** to an optimization model any point in the feasible region that optimizes (in this case maximizes) the objective function.

□ Using LINGO, it can be determined that the optimal solution to its model is V = 5, P = 200, T = 100, A = 0.294, B = 0, C = 0.706, and z = 209.384.

Ex. 1: Solution continued

- Daisy problem formulation in LINGO 7.0

```
LINGO - [LINGO Model - Daisy]
File  Edit  LINGO  Window  Help
max = 300+0.8*V+0.01*P+0.06*T+0.001*T*P-0.01*T^2-0.001*P^2+11.7*A+9.4*B+16.4*C+19.4*A*B+11.4
*A*C-9.6*B*C;
V<5;
V>1;
P<400;
P>200;
T<200;
T>100;
A>0;
B>0;
C>0;
A+B+C=1;
A<5;

Ready                                          MOD    Ln 8, Col 7    12:54 pm
```
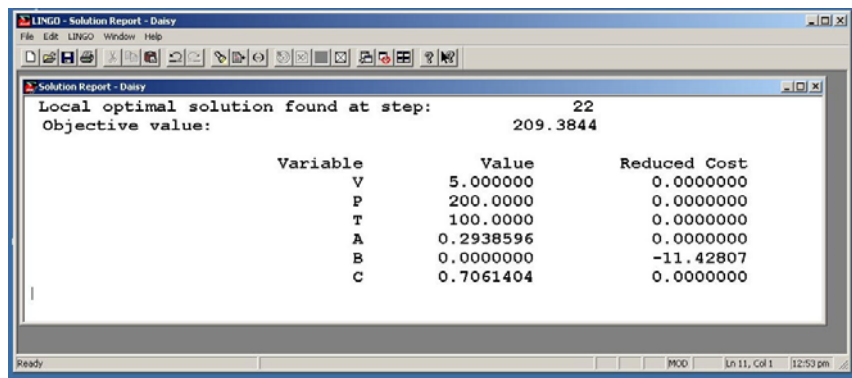

Ex. 1: Solution continued

- Solution extract of Daisy example (shown without slack, surplus, or dual prices) using LINGO 7.0

```
LINGO - Solution Report - Daisy
File  Edit  LINGO  Window  Help

Solution Report - Daisy
Local optimal solution found at step:                22
Objective value:                            209.3844

                   Variable           Value        Reduced Cost
                          V        5.000000           0.0000000
                          P        200.0000           0.0000000
                          T        100.0000           0.0000000
                          A       0.2938596           0.0000000
                          B       0.0000000          -11.42807
                          C       0.7061404           0.0000000

Ready                                          MOD    Ln 11, Col 1    12:53 pm
```

- A **static model** is one in which the decision variables do not involve sequences of decisions over multiple periods.

- A **dynamic model** is a model in which the decision variables do involve sequences of decisions over multiple periods.

- A **linear model** is one in which decision variables appear in the objective function and in the constraints of an optimization model.

- The Daisy example is a **nonlinear model**. In general, nonlinear models are much harder to solve.

- If one or more of the decision variables must be integer, then we say that an optimization model is an **integer model**.

- If all the decision variables are free to assume fractional values, then an optimization model is a **noninteger model**.

  □ The Daisy example is a noninteger example since volume, pressure, temperature, and percentage composition are all decision variables which may assume fractional values.

- Integer models are much harder to solve then noninteger models.

■ A **deterministic model** is a model in which for any value of the decision variables the value of the objective function and whether or not the constraints are satisfied is known with certainty. If this is not the case, then we have a **stochastic model**.

□ If we view the Daisy example as a deterministic model, then we are making the assumption that for given values of V, P, T, A, B, and C the process yield will always be the same.

□ Since this is unlikely, the objective function can be viewed as the average yield of the process for given decision variable values.

1.2 The Seven-Step Model-Building Process

1. Formulate the Problem

□ Define the problem.

□ Specify objectives.

□ Determine parts of the organization to be studied.

2. Observe the System

□ Determines parameters affecting the problem.

□ Collect data to estimate values of the parameters.

3. Formulate a Mathematical Model of the Problem

4. Verify the Model and Use the Model for Prediction

☐ Does the model yield results for values of decision variables **not** used to develop the model?

☐ What eventualities might cause the model to become invalid?

5. Select a Suitable Alternative

☐ Given a model and a set of alternative solutions, determine which solution best meets the organizations objectives.

6. Present the Results and Conclusion(s) of the Study to the Organization

☐ Present the results to the decision maker(s)

- If necessary, prepare several alternative solutions and permit the organization to chose the one that best meets their needs.

☐ Any non-approval of the study's recommendations may have stemmed from an incorrect problem definition or failure to involve the decision maker(s) from the start of the project.

- In such a case, return to step 1, 2, or 3.

7. Implement and Evaluate Recommendations

- Assist in implementing the recommendations.
- Monitor and dynamically update the system as the environment and parameters change to ensure that recommendations enable the organization to meet its goals.

1.3 CITGO Petroleum

- Klingman et. Al. (1987) applied a variety management-science techniques to CITGO Petroleum. Their work saved the company an estimated $70 per year.

- Focus on two aspects of the CITGO's team's work:
    1. A mathematical model to optimize the operation of CITGO's refineries.
    2. A mathematical model – supply, distribution and marketing (SDM) system – used to develop an 11-week supply, distribution and marketing plan for the entire business.

■ Optimizing Refinery Operations

Step 1 (Formulate the Problem) Klingman et. Al. wanted to minimize the cost of CITGO's refineries.

Step 2 (Observe the System) The Lake Charles, Louisiana, refinery was closely observed in an attempt to estimate key relationships such as:

1. How the cost of producing each of CITGO's products (motor fuel, no. 2 fuel oil, turbine fuel, naptha, and several blended motor fuels) depends upon the inputs used to produce each product.

2. The amount of energy needed to produce each product (requiring the installation of a new metering system).

25

3. The yield associated with each input – output combination. For example, if 1 gallon of crude oil would yield 0.52 gallons of motor fuel, then the yield would be 52%.

4. To reduce maintenance costs, data was collected on parts inventories and equipment breakdowns. Obtaining accurate data required the installation of a new data based - management system and integrated maintenance information system. Additionally, a process control system was also installed to accurately monitor the inputs and resources used to manufacture each product.

26

Step 3 (Formulate a Mathematical Model of the Problem) Using linear programming (LP), a model was developed to optimize refinery operations.

- The model:
  - □ Determines the cost-minimizing method for mixing or blending together inputs to obtain desired results.
  - □ Contains constraints that ensure that inputs are blended to produce desired results.
  - □ Includes constraints that ensure inputs are blended so that each output is of the desired quality.
  - □ Ensures that plant capacities are not exceeded and allow for inventory for each end product.

27

Step 4 (Verify the Model and Use the Model for Prediction) To validate the model, inputs and outputs from the refinery were collected for one month.

- Given actual inputs used at the refinery during that month, actual outputs were compared to those predicted by the model.
- After extensive changes, the model's predicted outputs were close to actual outputs.

Step 5 (Select Suitable Alternative Solutions) Running the LP yielded a daily strategy for running the refinery.

- For instance, the model might say, produce 400,000 gallons of turbine fuel using 300,000 gallons of crude 1 and 200,000 of crude 2.

28

Step 6 (Present the Results and Conclusions) and **Step 7** (Implement and Evaluate Recommendations) Once the data base and process control were in place, the model was used to guide day-to-day operations.

- CITGO estimated that the overall benefits of the refinery system exceeded $50 million annually.

- The Supply Distribution Marketing (SDM) System

Step 1 (Formulate the Problem) CITGO wanted a mathematical model that could be used to make supply, distribution, and marketing decisions such as:

- Where should the crude oil be purchased?
- Where should products be sold?
- What price should be charged for products?
- How much of each product should be held in inventory?

The goal was to maximize profitability associated with these decisions.

Step 2 (Observe the System) A database that kept track of sales, inventory, trades, and exchanges of all refined goods was installed. Also regression analysis was used to develop forecasts of for wholesale prices and wholesale demand for each CITGO product.

Step 3 (Formulate a Mathematical Model of the Problem) and **Step 5** (Select Suitable Alterative Solutions) A minimum-cost network flow model (MCNFM) is used a determine an 11-week supply, marketing, and distribution strategy.

- The model makes all decisions discussed in Step 1.

- A typical model run involved 3,000 equations and 15,000 decision variables required only 30 seconds on an IBM 4381.

31

Step 4 (Verify the Model and Use the Model for Prediction)

- The forecasting models are continuously evaluated to ensure that they continue to give accurate forecasts.

Step 6 (Present the Results and Conclusions) and **Step 7** (Implement and Evaluate Recommendations) Implementing the SDM required several organizational changes.

- A new vice-president was appointed to coordinate the operation of the SDM and refinery LP model.

- The product supply and product scheduling departments were combined to improve communications and information flow.

32

1.4 San Francisco Police Department Scheduling

- Taylor and Huxley (1989) developed a police patrol scheduling system (PPSS).
  - All San Francisco (SF) police precincts use the PPSS to schedule their officers.
  - It is estimated that the PSS saves the SF police more than $5 million annually.
  - Other cities have also adopted PPSS.
- Following the seven-step model here is a description of PPSS.

- **Step 1** The SFPD wanted a method to schedule patrol officers in each precinct that would:
  - Quickly produce a schedule and graphically display it.
  - Minimize the sum of hourly shortages and surpluses
  - Would allow precinct captains to easily fine-tune to produce the optimal schedule.
- **Step 2** The SPFD had a sophisticated computer-aided dispatch (CAD) system to keep track of calls for police help, police travel time, police response time, and so on.

- **Step 3** An LP model was formulated
  - □ Primary objective was to minimize the sum of hourly shortages and surpluses.
  - □ The constraints in the PPSS model reflected the limited number of officers available and the relationship of the number of officers working each hour to the shortages and surpluses for that hour.
  - □ PPSS would produce a schedule that would tell the precinct captains how many officers should start work at each possible shift time.
    - ■ The fact that this number was required to be an integer made it far more difficult to find an optimal solution.

35

- **Step 4** Before implementing PPSS, the SFPD tested the PPSS schedule against manually created schedules.
  - □ It produced approximately a 50% reduction in both shortages and surpluses.
- **Step 5** PPSS can be used to minimize the shortages and surpluses as well as being used to experiment with shift times and work rules.
  - □ Officers had wanted to switch to a 4/10 schedule versus a 5/8 schedule for years but were not permitted to because city felt it would reduce productivity.
  - □ PPSS showed that it would not reduce productivity but improve productivity.

36

- **Step 6** and **Step 7**
  - ☐ Estimated that PPSS created an extra 170,000 productive hours per year, thereby saving the city of San Francisco $5.2 million dollars per year.
  - ☐ 96% of workers preferred PPSS generated schedules
  - ☐ PPSS enabled SFPD tot make strategic changes which made officers happier and increased productivity.
  - ☐ Response times to calls improved 20% after PPSS was adopted.
  - ☐ Precinct captains could fine-tune the computer-generated schedule and obtain a new schedule in 1 minute.

37