## 执行测验: Homework 4

## 测试信息

描述

多次尝试 不允许。此测试只能进行一次。 强制完成 本测试可保存并可稍后继续。

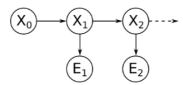
※ 问题完成状态:

问题 1

20分

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Consider the HMM shown below.



The prior probability  $P(X_0)$ , dynamics model  $P(X_{t+1}|X_t)$ , and sensor model  $P(E_t|X_t)$  are as follows:

$X_0$	$P(X_0)$
0	0.35
1	0.65

$X_{t+1}$	$X_t$	$P(X_{t+1} X_t)$
0	0	0.05
1	0	0.95
0	1	0.05
1	1	0.95

$E_t$	$X_t$	$P(E_t X_t)$
а	0	0.15
b	0	0.3
С	0	0.55
а	1	0.7
b	1	0.05
С	1	0.25

We perform a first dynamics update, and fill in the resulting belief distribution  $B'(X_1)$ .

$X_1$	$B'(X_1)$	
0	0.05	
1 0.95		

We incorporate the evidence  $E_1=c$  . We fill in the evidence-weighted distribution  $P(E_1=c|X_1)B'(X_1)$  , and the (normalized) belief distribution  $B(X_1)$ .

$X_1$	$P(E_1 = c X_1)B'(X_1)$
0	0.0275
1	0.2375

$X_1$	$B(X_1)$	
0	0.103773584906	
1	0.896226415094	

You get to perform the second dynamics update. Fill in the resulting belief distribution  $B'(X_2)$ .

X <sub>2</sub>	$B'(\times_2)$
0	0.0500
1	0.9500

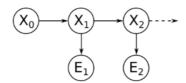
Now incorporate the evidence  $E_2=b$ . Fill in the evidence-weighted distribution  $P(E_2=b|X_2)B^\prime(X_2)$ , and the (normalized) belief distribution  $B(X_2)$ .

$X_2$	$P(E_2 = b \mid X_2) B'(X_2)$
0	0.0150

1	0.0475
$X_2$	$B(\times_2)$
0	0.2400
1	0.7600

问题 2 20 分 已保存

Consider the same HMM.



The prior probability  $P(X_0)$ , dynamics model  $P(X_{t+1}|X_t)$ , and sensor model  $P(E_t|X_t)$  are as follows:

$X_0$	$P(X_0)$
0	0.5
1	0.5

$X_{t+1}$	$X_t$	$P(X_{t+1} X_t)$
0	0	0.5
1	0	0.5
0	1	0.2
1	1	0.8

$E_t$	$X_t$	$P(X_{t+1} X_t)$
а	0	0.5
b	0	0.3
С	0	0.2
а	1	0.8
b	1	0.1
С	1	0.1

In this question we'll assume the sensor is broken and we get no more evidence readings. We are forced to rely on dynamics updates only going forward. In the limit as  $t o\infty$  , our belief about  $X_t$  should converge to a stationary distribution  $ilde{B}(X_\infty)$  defined as follows:

$$ilde{B}(X_{\infty}) := \lim_{t o \infty} P(X_t|E_1,E_2)$$

Recall that the stationary distribution satisfies the equation

$$ilde{B}(X_{\infty}) = \sum_{X_{\infty}} P(X_{t+1}|X_t) ilde{B}(X_{\infty})$$

for all values in the domain of X.

In the case of this problem, we can write these relations as a set of linear equations of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \tilde{B}(X_{\infty} = 0) \\ \tilde{B}(X_{\infty} = 1) \end{bmatrix} = \begin{bmatrix} \tilde{B}(X_{\infty} = 0) \\ \tilde{B}(X_{\infty} = 1) \end{bmatrix}$$

In the spaces below, fill in the coefficients of the linear system. The system you have written has many solutions (consider (0,0), for example), but to get a probability distribution we want the solution that sums to one. Fill in your solution in the table below. (Hint: to check your answer, you can also write some code and run till

Your answers will be evaluated to 4 decimal places.

coefficient	value
а	0.5000
b	0.2000
С	0.5000
d	0.8000

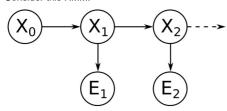
 $B(X_{\infty})$ 

0	0.2857
1	0.7143

问题 3

30分 已保存

Consider this HMM.



The prior probability  $P(X_0)$ , dynamics model  $P(X_{t+1} | X_t)$ , and sensor model  $P(E_t | X_t)$  are

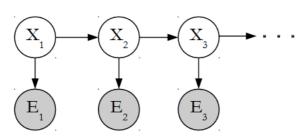
$X_0$	$P(X_0)$
0	0.5
1	0.5

$X_{t+1}$	$X_t$	$P(X_{t+1} X_t)$
0	0	0.5
1	0	0.5
0	1	0.2
1	1	0.8

$E_t$	$X_t$	$P(E_t X_t)$
а	0	0.5
b	0	0.3
С	0	0.2
а	1	0.8
b	1	0.1
С	1	0.1

If the  $E_1=a$ ,  $E_2=b$ ,  $E_3=c$ , what is the most likely explanation  $X_{1:3}^*=argmaxP(X_{1:3}\mid E_{1:3})$ 

问题 4



The Viterbi algorithm finds the most probable sequence of hidden states  $X_{1:T}$ , given a sequence of observations  $e_{1:T}$ . For the HMM structure above, which of the following probabilities are maximized by the sequence of states returned by the Viterbi algorithm? Select all correct option(s).

□ A. P(X₁. →)

10分

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B.

$$P(X_T | e_T)$$

' V'1:1/

$$\mathbb{Z}$$
 E.
$$P(X_1)P(e_1|X_1)\prod_{t=2}^{T}P(e_t|X_t)P(X_t|X_{t-1})$$

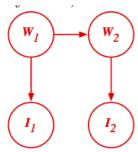
G. None of above

问题 5

30分

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Consider the following graph, where  $W_1$  and  $W_2$  can be either be R or S, and  $I_1$  and  $I_2$  can be either be T or F:



The conditional probability distributions are given by:

$$\begin{array}{c|c} W_1 = \mathtt{S} & W_1 = \mathtt{R} \\ \hline .6 & .4 \\ \hline & (\mathtt{a}) \ P(W_1) \end{array}$$

Now we want to try approximate inference through sampling. Applying likelihood weighting, suppose we generate the following six samples given the evidence  $I_1 = T$  and  $I_2 = F$ :

$$(W_1,I_1,W_2,I_2) = \Big\{ \langle \mathtt{S},\mathtt{T},\mathtt{R},\mathtt{F} \rangle, \langle \mathtt{R},\mathtt{T},\mathtt{R},\mathtt{F} \rangle, \langle \mathtt{S},\mathtt{T},\mathtt{R},\mathtt{F} \rangle, \langle \mathtt{S},\mathtt{T},\mathtt{S},\mathtt{F} \rangle, \langle \mathtt{S},\mathtt{T},\mathtt{S},\mathtt{F} \rangle, \langle \mathtt{R},\mathtt{T},\mathtt{S},\mathtt{F} \rangle \Big\}$$

Then the weight of the first sample (S, T, R, F) is 0.72

The result from likelihood weighting is:

$\widehat{P}(W_2 = R   I_1 = T, I_2 = F)$	0.8889
$\widehat{P}(W_2 = S   I_1 = T, I_2 = F)$	0.1111

问题 6

20 分

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After observing step of particle filtering, the particles and its weight are as follow:

Particles	Weight
A	0.3
В	0.4
С	0.9
D	0.5
Α	0.3
С	0.9
Α	0.3
D	0.5
D	0.5
Α	0.3

	imal places.			
P'(A) =	0.2449			
P'(B) =	0.0816			
P'(C) =	0.3673			
P'(D) =	0.3061			
		保存所有答案。		