Space Engineering 3
Assignment 1
24th March 2016

Orbit Simulation and Determination

Lydia Drabsch
311217591
ldra3557@uni.sydney.edu.au

STUDENT PLAGIARISM: COURSE WORK - POLICY AND PROCEDURE COMPLIANCE STATEMENT

INDIVIDUAL / COLLABORATIVE WORK

I/We certify that:

(1) I/We have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure*;

(2) I/We understand that failure to comply with the *Student Plagiarism: Coursework Policy and Procedure* can lead to the University commencing proceedings against me/us for potential student misconduct under Chapter 8 of the *University of Sydney By-Law 1999* (as amended);

(3) this Work is substantially my/our own, and to the extent that any part of this Work is not my/our own I/we have indicated that it is not my/our own by Acknowledging the Source of that part or those parts of the Work.

Name(s): Lydia Drabsch

Signature(s):
Lydia Drabsch

Date: 24/3/16

Contents

List of Figures

List of Tables

Introduction

Each mainQN.m file has a section called 'User Input' where the animations and state plots can be turned on/off. Also the timestep and the number of days to simulate are defined. The default settings are dt = 100 seconds and days = 1.

1. Simulation of Orbits with Classical Elements

1.1 Introduction

- keplers three laws
- perifocal frame
- The true anomaly θ is the angle taken at the focus of the perifocal frame to the satellite from the perigee. The eccentric anomaly E is the angle taken at the centre of perifocal frame to the satellite from the perigee.
-The mean anomaly M_t is the mean number of orbits per day.
- LEO,MEO
-TLE's

1.2 Methodology

From Kepler's second law, the mean anomaly at time t is calculated using the mean motion n from an epoch time described by $M_0(t_0)$.

$$M_t = M_0 + n(t - t_0) \tag{1}$$

To solve for the eccentric anomaly, newtons method was used

$$E_{i+1} = E_i - \frac{f(E_i)}{f'(E_i)} \tag{2}$$

$$E_{i+1} = E_i - \frac{E - e\sin(E_i) - M_t}{1 - e\cos(E_i)} \tag{3}$$

The true anomaly was calculated using

$$\theta = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)\right) \tag{4}$$

The general equation for the radius of an ellipse

$$r = \frac{p}{1 + e\cos\theta} \tag{5}$$

The two parameters, θ and r can completely define the orbit in the perifocal frame as polar coordinates. The conversion to cartesian coordinates in the perifocal frame are as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{perifocal} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_{perifocal} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}}\sin\theta \\ \sqrt{\frac{\mu}{p}}(e + \cos\theta) \\ 0 \end{bmatrix} \tag{7}$$

(8)

The perifocal parameters were transformed to the ECI frame to animate the 3D model of the satellite orbit around the Earth using the function ***orbit2ECI***. From the ECI coordinates, the ECEF and LLHGD coordinates were calculated for the ground trace.

1.2.1 Orbital Period

The period was calculated analytically and computationally. The period is inversely proportional to the mean motion. From the simulation, the ECI coordinates of the satellite were analysed by auto correlation based on the Wiener-Khinchin Theorem. The data was fast fourier transformed using the built-in matlab function, multiplied with the resultant complex conjugate then transformed back to the time domain by inverse fourier transform.

```
%% L Drabsch 24/3/16
% Find period of orbit with autocorrelation based on Wiener-Khinchin Theorem
% Inputs: X = signal to analyse, eg X_ECI
%         dt = timestep (s)
% Output: P = period (hours)

function P = Period_AC(X,dt)
    Fr = fft(X);
    S = Fr.*conj(Fr);
    R = ifft(S);
    [~,loc] = findpeaks(R);
    P = loc(1)*dt/(60*60);
end
```

1.3 Results/Discussion

1.3.1 Van Allen Probes

The satellites Van Allen Probes, previously known as the Radiation Belt Storm Probes (RBSP A and B), are in a highly eccentric orbit. RBSP-A was modelled based on TLEs from spacetrack.com, see Table 1.1. It was launched on 30th August 2012

The Van Allen Belts are regions of plasma that surround the Earth that are contained by the Earth's magnetic field. The highly energetic particles become trapped in the Earth's magnetic field by travelling along the field lines to a pole and being reflected back and travelling to the opposite pole. This phenomenon is called a magnetic mirror, and is the reason the Van Allen belts have a torus shape about the Earth's magnetic axis, see Figure 1.1. Therefore, the radiation is at the highest intensity about the magnetic equatorial plane, currently at 11.5° inclination. The satellite was put in an orbit close to this inclination at about 10°, with the mission objective to be in an orbit of no greater than 18°. The inclination has fluctuated between 9-11° over the course of it's mission.

The orbit was highly eccentric in order to cover the entire radiation belt region. The perigee altitude is 500 km, which is below the inner radiation belt and the apogee altitude is 30600 km, which is above the outer edge of the outer radiation belt.

1.3.2 International Space Station

See Figure 4.1 and 4.2 in the appendix. The ISS needs to stay below the Inner Van Allen belt for the astronauts safety, which required a very low orbit and an almost perfect circle.

1.3.3 Orbital Properties

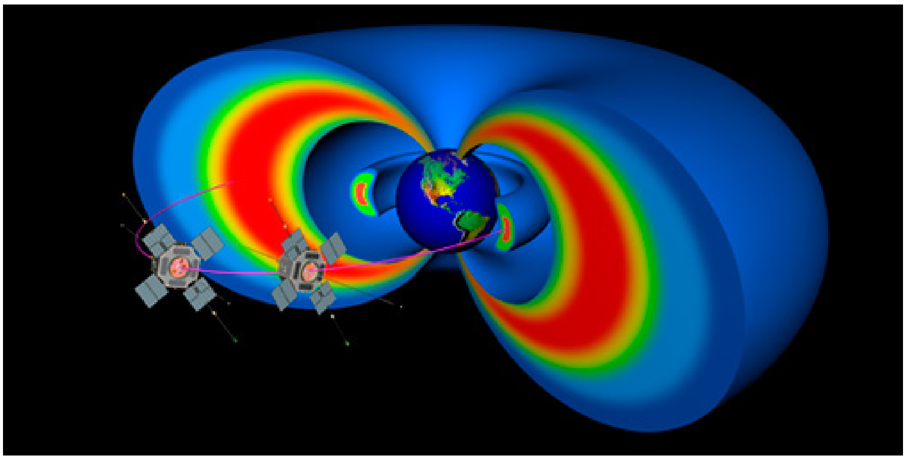


Figure 1.1

Table 1.1: Orbital properties of simulated satellites. (* indicates data taken directly from TLEs)

Orbital Properties	Variable Name	Units	Van Allen Probe A	ISS
NORAD ID				25544
*Julian year/day data was taken	t0	year/day	2016 68.37507029	2016 83.8256713
Semimajor axis	a	m		
*Eccentricity	e	-	0.6813430	0.0002017
*Inclination angle	inc	degrees	10.1687	51.6429
*Right ascension of the node	Rasc	degrees	46.5607	123.7637
*Argument of Perigee	omega	degrees	77.2770	351.0445
*Mean Anomaly	Mt	degrees	346.0491	342.5794
*Mean Motion	MM	orbits per day	2.681033090	15.54235390
Analytical Period	P	hours	8.500	1.5556
Computational Period	Pc	hours	8.9518	1.5442

Figure 1.2: Top View of keplerian orbit simulation

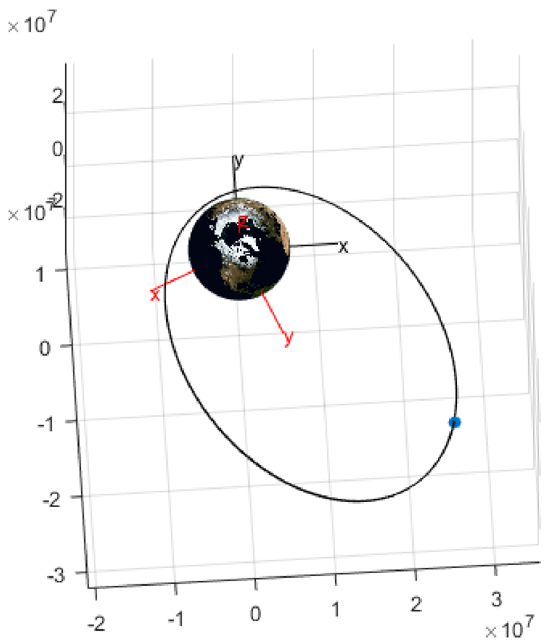
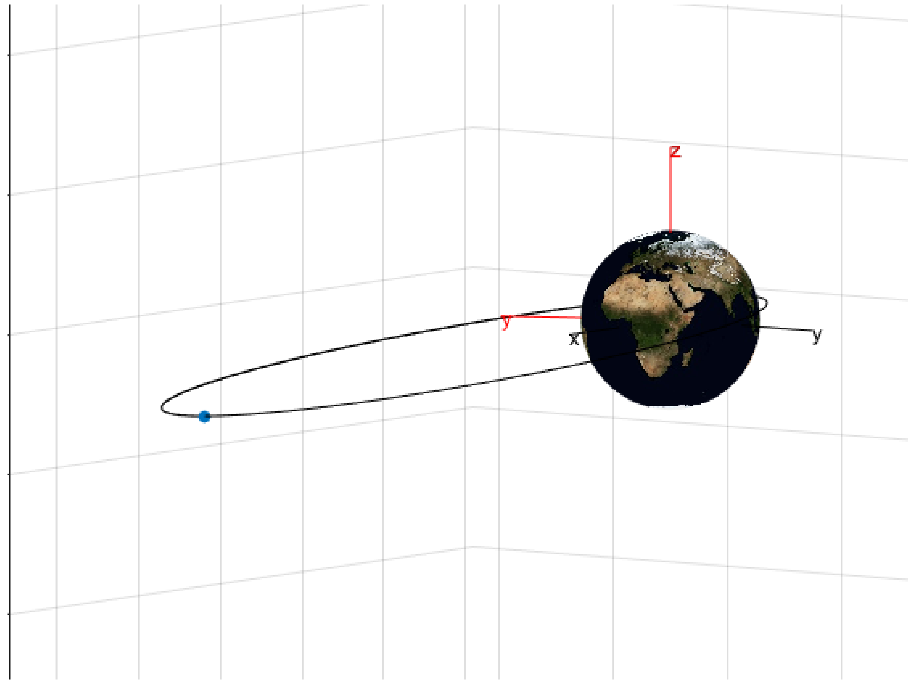


Figure 1.3: SIde view of keplerian orbit simulation



2. Simulating Perturbations

2.1 Introduction

The basic keperian model only accounted for an ideal two body system with no other effects when in reality, other forces act of the satellite. Compared to the gravitational motion between the Earth and the satellite however, these other effects are small. Therefore, they can be mathematically modelled as perturbations. While it is still an approximation, the error is reduced. Types of perturbations include;

- gravitational effects: such as the oblate Earth, third-body interactions such as the Moon, Sun or other planets
- drag forces: from the Earth's atmosphere and from solar radiation pressure
- unexpected thrusting: from malfunctioning thrusters

2.1.1 Earth oblateness

For the standard simulation model in Question 1, it was assumed that the Earth was spherical. A perfectly spherical mass has an inverse square relation of the gravitational field to the force applied on a body. However, the Earth is slightly oblate, as it is flatter at the poles and wider at the equator than a sphere.

Unlike using the keplerin model of Question 1, the classical orbital parameters change through time.

2.1.2 Van Allen Probes

Due to the highly elliptical orbit of the Van Allen Probes, most of the time the satellite is in MEO away from the Earth's atmosphere. As the inclination of the orbit keeps the satellite close to the equator, the perturbation due to the non-spherical earth

Newtown's second law and his law of gravitation results in the following equation in the ECI frame [1].

$$\ddot{\boldsymbol{r}} + \frac{\mu \boldsymbol{r}}{r^3} = a\frac{3J_2\mu R_e^2}{2R^5}\left(\left(5\frac{z^2}{R^2}-1\right)(x\hat{i}+y\hat{j}) + z\left(5\frac{z^2}{R^2}-3\right)\hat{k}\right) \tag{9}$$

2.2 Methodology

Equinoctial elements were used to remove the possible singularities that affect classical elements. The

2.3 Results/Discussion

For the perturbation model the calculated orbital period was 8.4722 hours, compared to the 8.9518 hrs of the keperlian model. This is because the satellite is pulled further around the Earth as it crosses the equator.

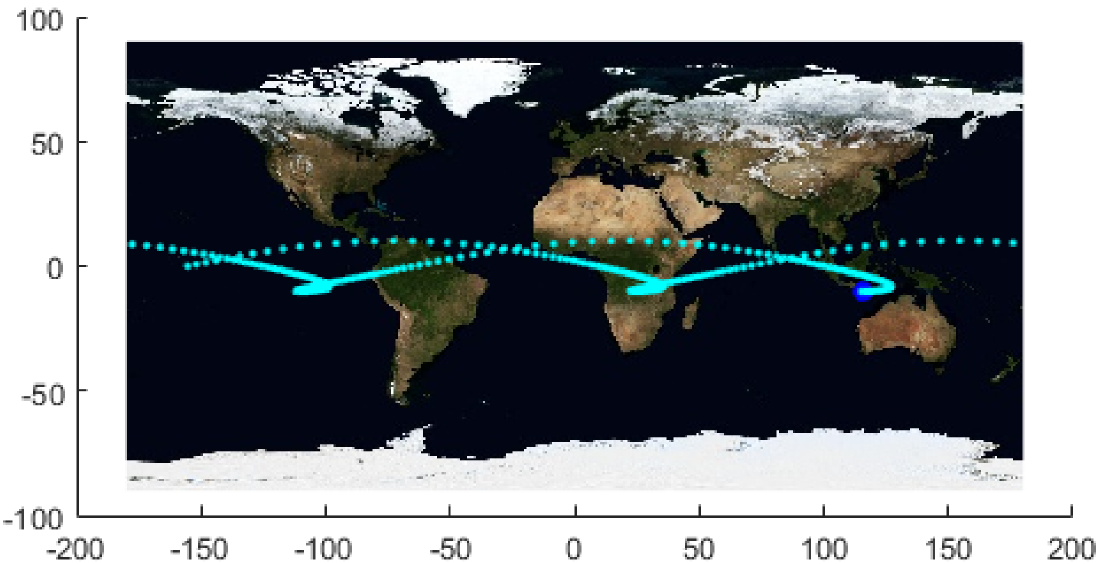


Figure 2.1: Ground trace of perturbation model

The J2 perturbation model was implemented for the satellite Van Allen Probe A and integrated over twelve days and compared to the TLEs in the same time frame. Figure 2.2 shows the progression of the classical elements. The right ascending node and argument of perigee agreed very well with the perturbation model, indicating that the oblate Earth effect is strong on the satellite. The general trends of the inclination, semi-major axis and eccentricity indicate that there is another effect occurring on the satellite, such as atmospheric drag effects as it has a very low perigee. However, the percentage of the change in the parameter is very small ¡ 0.1 %. Figure 2.3 displays the equinoctial elements, with strong agreement for all parameters except for the semilatus rectum (p) however it is again ¡0.6 % difference and exists for the same reasons as prior. The parameter θ was calculated from the TLEs using the revolution number plus the calculated true anomaly from eq(4).

In the keplerian model, the classical elements are not variables, but are constants used the in the equations for position and velocity in the perifocal frame. The blue circle marker on Figure 2.2 shows the Keplerian value, and the initial value used in the perturbation model.

Figure 2.2: Classical Elements over 12 days

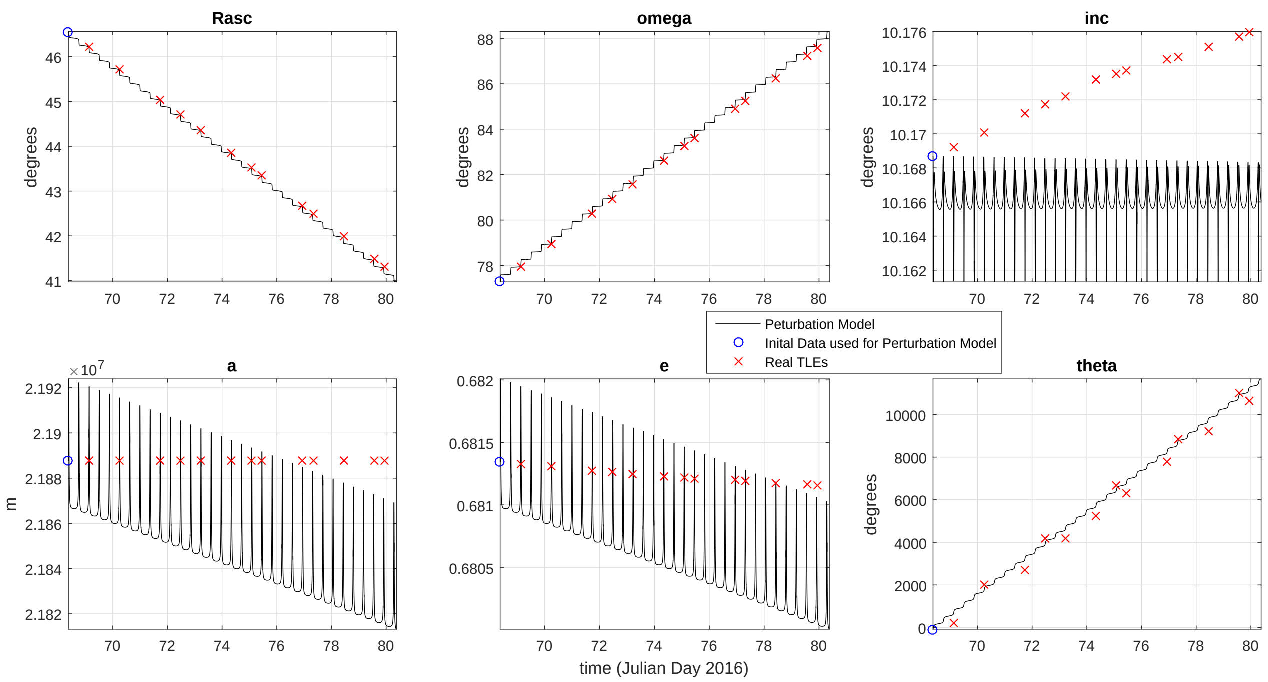
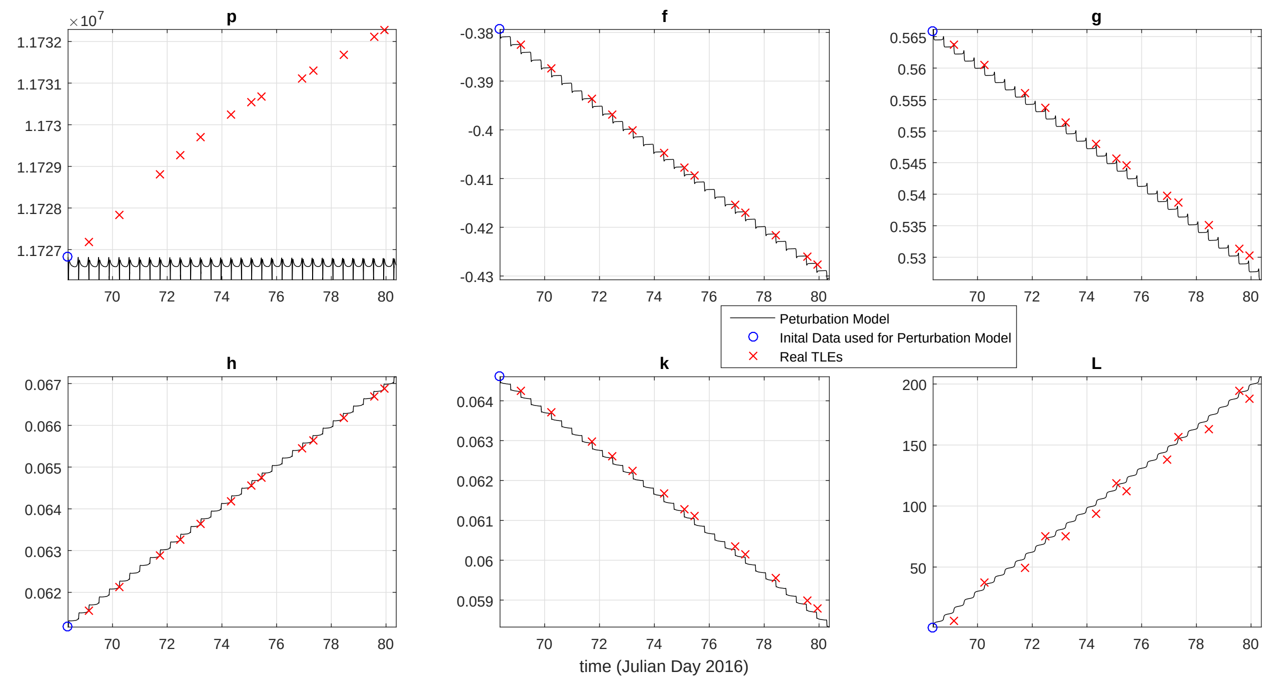


Figure 2.3: Equinoctial Elements over 12 days



3. Orbital Determination

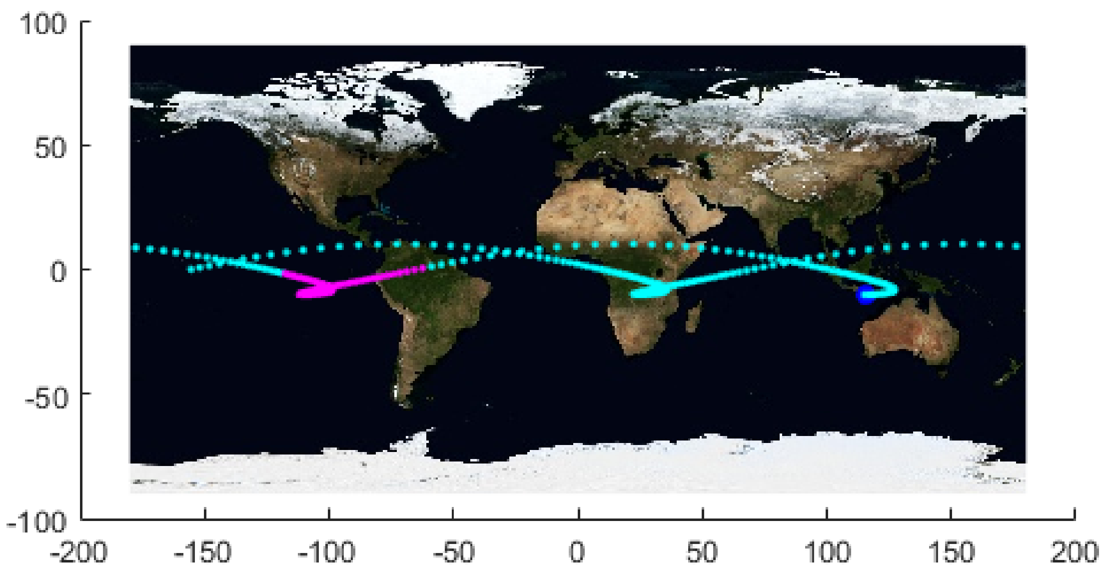
3.1 Introduction

Three networks are used to communicate with the Van Allen Probes [3]. The main communications ground station is at the Applied Physics Laboratory in Washington, USA. The LLH coordinates are 39.10N 76.53W 140 m above mean seal level. [4]
For backup, the satellite communicates with the United Space Network using the Near Earth Network with ground stations in Hawaii(19.0N 155.6W 367m) and Australia. The data rates that can be achieved with this network is high enough to gather scientific data from the probe. Also as a backup, the NASA program Space Network is used for monitoring data, which relays through geosynchronous Tracking and Data Relay Satellites (TDRS). The TLEs are collected at approximately the same point in the orbit, at a mean anomaly of 345°. The Van Allen Probes have two S-band RF antennas and use RF doppler data acquired for ground contacts.
For the noise analysis, Nominal uplink 20dB, minimum downlink margin 3dB

3.2 Methodology

The perturbation model from Question 2 was used to simulate the orbit.

Figure 3.1



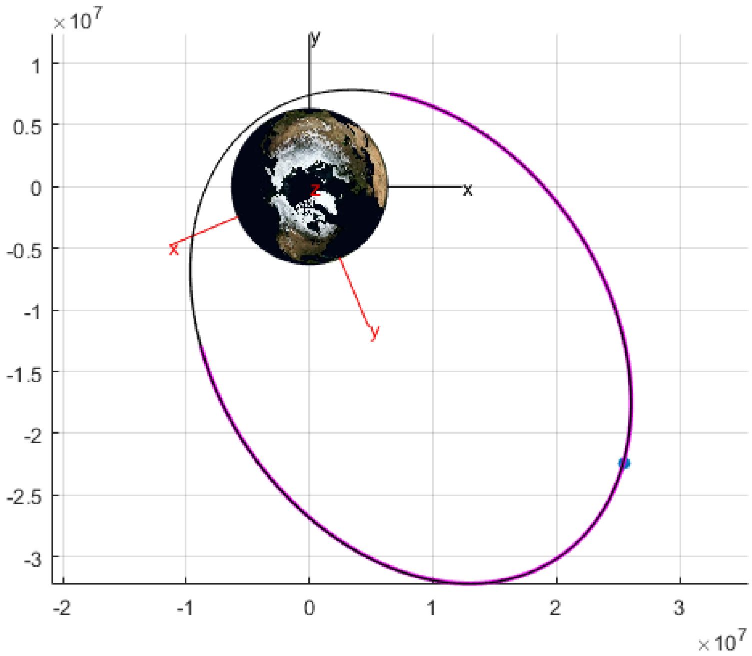


Figure 3.2: Pink highlights when the satellite was in view of the ground station.

3.3 Results/Discussion

The main ground station in Washington USA was simulated. Therefore the ground station location was 39.10N 76.53W 140 m and a minimum elevation of 10 degrees from the horizon was implemented. This is so that any trees or small terrain fluctuations are accounted for.

3.3.1 HG - no noise

```
The maximum elevation was 28.04 deg
and the percentage of time the satellite was in view was 33 percent
For no noise:
Semimajor Axis a:          True: 2.189e+07     Observed: 2.187e+07
Inclination i:             True: 1.775e-01     Observed: 1.775e-01
Eccentricity e:            True: 6.813e-01     Observed: 6.809e-01
Right ascention node Omega:    True: 8.126e-01     Observed: 8.073e-01
Argument of Perigee omega:     True: 1.349e+00     Observed: 1.360e+00
True Anomaly theta:        True: -1.341e+00    Observed: 1.974e+00
>>
```

Figure 3.3: HG analysis no noise

3.3.2 Noise Analysis

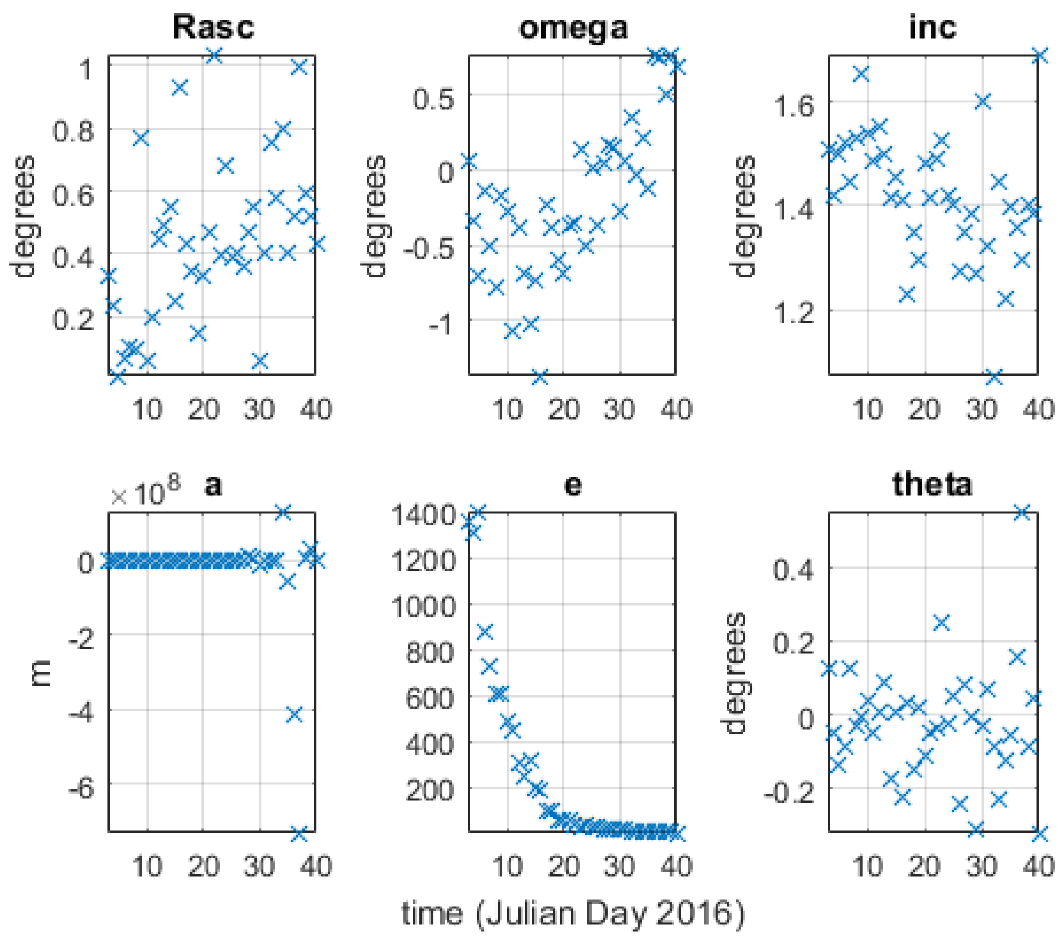


Figure 3.4: Noise analysis *Note x axis label wrong 'Sound to Noise ratio dB'

[1] Kaplan M.Modern Spacecraft Dynamics and Control. New York: Wiley; 1976. 415p
[2] Van Allen Probes, http://vanallenprobes.jhuapl.edu/mission/conversation/overview/index.php
[3] Space Network, Applied Physics Lab http://www.scf.jhuapl.edu/

4. Appendix

ISS plots

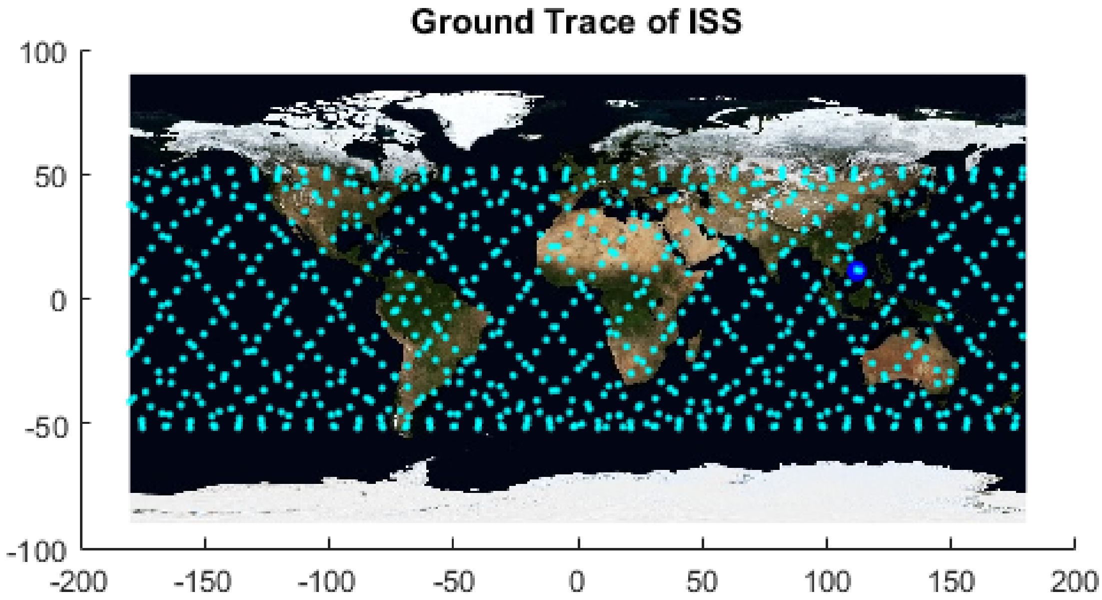


Figure 4.1: ISS ground trace

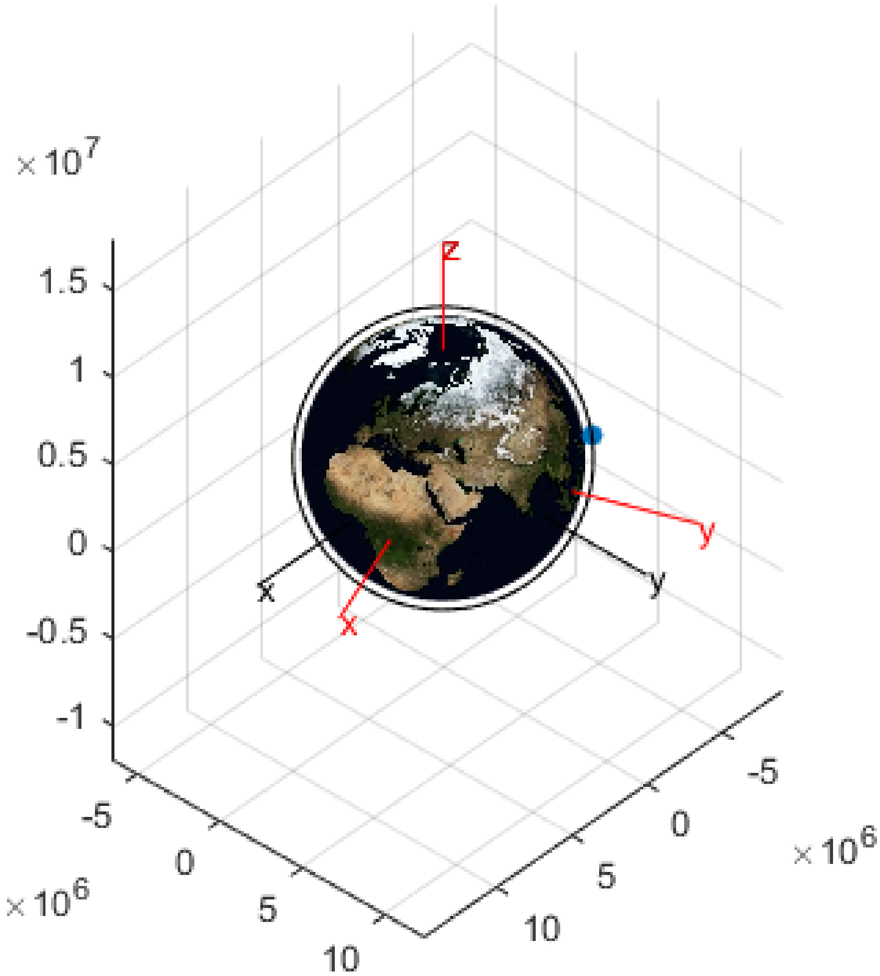


Figure 4.2: ISS Sim