

A decorative graphic on the left side of the page, consisting of a network of blue lines and circles. The lines are of varying thickness and connect to circles of different sizes, creating a circuit-like or orbital pattern that extends from the top to the bottom of the page.

SPACE ENGINEERING 3  
Assignment 1  
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**ORBIT SIMULATION AND  
DETERMINATION**

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**STUDENT PLAGIARISM: COURSE WORK – POLICY AND PROCEDURE****MTRX 2700 COMPLIANCE STATEMENT****INDIVIDUAL / COLLABORATIVE WORK**

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## INTRODUCTION

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## 1. SIMULATION OF ORBITS WITH CLASSICAL ELEMENTS

### 1.1 Introduction

- keplers three laws
- perifocal frame
- The true anomaly  $\theta$  is the angle taken at the focus of the perifocal frame to the satellite from the perigee. The eccentric anomaly  $E$  is the angle taken at the centre of perifocal frame to the satellite from the perigee.
- The mean anomaly  $M_t$  is the mean number of orbits per day.
- LEO,MEO
- TLE's

### 1.2 Methodology

From Kepler's second law, the mean anomaly at time  $t$  is calculated using the mean motion  $n$  from an epoch time described by  $M_0(t_0)$ .

$$M_t = M_0 + n(t - t_0) \quad (1)$$

To solve for the eccentric anomaly, newtons method was used

$$E_{i+1} = E_i - \frac{f(E_i)}{f'(E_i)} \quad (2)$$

$$E_{i+1} = E_i - \frac{E - e \sin(E_i) - M_t}{1 - e \cos(E_i)} \quad (3)$$

The true anomaly was calculated using

$$\theta = 2 \tan^{-1} \left( \sqrt{\frac{1+e}{1-e}} \tan \left( \frac{E}{2} \right) \right) \quad (4)$$

The general equation for the radius of an ellipse

$$r = \frac{p}{1 + e \cos \theta} \quad (5)$$

The two parameters,  $\theta$  and  $r$  can completely define the orbit in the perifocal frame as polar coordinates. The conversion to cartesian coordinates in the perifocal frame are as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{perifocal} = \begin{bmatrix} r \cos \theta \\ r \sin \theta \\ 0 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_{perifocal} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}} \sin \theta \\ \sqrt{\frac{\mu}{p}} (e + \cos \theta) \\ 0 \end{bmatrix} \quad (7)$$

$$(8)$$

The perifocal parameters were transformed to the ECI frame to animate the 3D model of the satellite orbit around the Earth using the function **orbit2ECI**. From the ECI coordinates, the ECEF and LLHGD coordinates were calculated for the ground trace.

## 1.3 Results/Discussion

### 1.3.1 Van Allen Probes

The satellite Van Allen Probes, previously known as the Radiation Belt Storm Probes (RBSP-A), is in a highly eccentric orbit. RBSP-A has a perigee in LEO at an altitude of 596 km and an apogee in MEO at an altitude of 30421 km assuming a spherical Earth.

### 1.3.2 Orbital Properties

Table 1.1: Orbital Properties - maybe put in classical parameters

Orbital Properties	Van Allen Probe	Other sat
Period		
Altitude at Perigee		
Altitude at Apogee		

## 2. SIMULATING PERTURBATIONS

### 2.1 Introduction

Different perturbing sources acting on Van Allen Probes - earth oblateness, gravity harmonics, solar/lunar gravity forces, aerodynamic drag, solar radiation

#### 2.1.1 Earth oblateness

For the standard simulation model in Question 1, it was assumed that the Earth was spherical. A perfectly spherical mass has an inverse square relation of the gravitational field to the force applied on a body. However, the Earth is slightly oblate, as it is flatter at the poles and wider at the equator than a sphere. As the difference in the force is small compared to the total force, it can be mathematically treated as a perturbation. While this method is only an approximation it works well.

Unlike using the keplerin model of Question 1, the classical orbital parameters change through time.

Newtown's second law and his law of gravitation results in

$$\ddot{\mathbf{r}} + \frac{\mu \mathbf{r}}{r^3} = \mathbf{a}_p \quad (9)$$

### 2.2 Methodology

Equinoctial elements were used to remove the possible singularities that affect classical elements. The

### 2.3 Results/Discussion

### **3. ORBITAL DETERMINATION**

#### **3.1 Introduction**

#### **3.2 Methodology**

#### **3.3 Results/Discussion**

### **4. CONCLUSIONS**

## REFERENCES



## **5. APPENDIX**