Precise Estimation of Clock Bias Difference Between Multiple GNSS Receivers for Cooperative Positioning

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***Abstract*—The use of raw-data from multiple GNSS receivers located in a closer vicinity is an emerging concept to enhance the positioning availability in harsh environments, where the number of received measurements by one receiver are not enough to compute a position estimate. Within this modern navigation paradigm, different receivers or users collaborate by exchanging measurements in order to be able to estimate their locations together. However, the cooperation between different receivers at the measurements level needs synchronisation of measurements produced by these different devices. The main issue is that the clock bias and drift of individual receivers with respect to GPS time are not predictable and will produce large errors in measurements and position if the problem of clock difference is not taken into account in the collaborative processing.**

The existing methods either estimate these clock bias values as part of Position, Velocity and Time (PVT) estimation algorithm or eliminate them by double difference. These approaches necessitate the availability of more measurements to estimate the additional clock bias of each receiver or need to have the same satellites visible to the cooperating receivers.

In this paper, we suggest a new approach of clock timing difference compensation when degraded measurements are available from multiple receivers in dense urban environments. These estimates are used to interpolate the raw measurements of multiple receivers to a common reference time. The proposed method is inherently simple, robust and allows us to develop innovative algorithms for cooperative PVT estimation. To illustrate the effectiveness of this method, we present some results by processing real data obtained using low cost commercial GNSS receivers.

***Index Terms*—GNSS, Data Synchronization, Clock Bias, Cooperative Positioning, Adaptive Kalman Filtering.**

I. Introduction

A Global Navigation Satellite System (GNSS) provides a user with the information of location and time. Under normal conditions, a commercially available single frequency receiver can be accurate up to 3m. Higher accuracy and precision can be obtained by various methods which include differential systems and precise positioning with carrier phase measurements.The focus of our project is to obtain better positioning in urban environment using collaborated data from multiple low cost COTS (Commercial Off-The-Shelf) receivers.

The relative and cooperative positioning are two popular techniques which use multiple receivers. In relative positioning, the redundant measurements available from two receivers are used to eliminate the common local correlated errors and get precise positioning. This may not be useful in urban environments where the satellite visibility is limited. To overcome this limitation, in cooperative positioning, receivers share the satellite measurements among them. When more measurements are available, they could be processed to select the best data set to be used according to their quality and the geometric configuration (i.e. DOP) [1] [2]. It is an interesting approach to improve the precision, accuracy and reliability of estimated PVT by increased redundancy in available data.

When more than one receiver is used, it is desirable to take the measurements at all the receivers at common time instants (measurement epochs) for easy combination or comparison of data [3]. Other than PVT estimation, multiple receivers have been used for spoofing detection [4] and multipath estimation [5].

In this work, we use the low cost commercial receivers, Ublox-M8T, which does not have the provision for using external reference clock signal. Each internal oscillator has it's own characteristic clock drift and bias. This is the case for most of the low cost receivers. In some cases of collaborative positioning the receivers are placed far apart and it is difficult to provide a common clock signal. This means that it is not possible to take measurements at the same epoch from multiple receivers, even if they are of same model. The workaround for this problem is to interpolate the measured data to a common reference time.

The interpolation of measurements to the GPS epoch can be done by estimating the clock bias values. Existing methods either estimate these clock bias values as part of PVT estimation algorithm or eliminate them by double difference. The elimination algorithms [6] require redundancy in the measurements to cancel out the common errors, which puts constraints as in relative positioning. The estimation approaches have their own disadvantages and cannot give precise estimates of clock bias. The precision of clock bias estimation is very important [7] as an error of 3×10^{-9} seconds can change PseudoRange (PR) and Carrier Phase (CP) measurements by one meter.

Raw measurements of a GNSS receiver include PR, CP and Doppler values. Our goal is to do precise interpolation of these measurements from multiple receivers to common reference epochs. We propose an approach of preprocessing by which

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the raw measurements of multiple receivers can be interpolated to measurement epoch of one receiver. The output can be used in innovative cooperative PVT estimation [8], spoofing detection and multipath estimation algorithms. This approach is easy to implement and provides robust estimation in dense urban environments characterized by frequent signal outages.

In this paper, we preview the existing methods and their disadvantages in section II. The formulation of proposed method for two receivers is given in section III. The standard PVT estimation and proposed single difference approaches are tested with experimental raw-data collected in open sky environment (static mode) and their performance is compared with data collected in conditions unfavourable for GNSS navigation (kinematic mode in dense urban environment). Some use case scenarios of the proposed interpolation method are also discussed.

II. Using Measurements from Multiple Receivers

In this section, we look into existing methods where the measurements from multiple receivers are combined in PVT estimation algorithms despite of the difference in measurement epochs.

A. Double Differencing Approach

The double difference approach is a well-known method to remove common errors present in measurements of receivers placed with in a short range. The details of implementation can be found in various sources on relative positioning [9]. Apart from RTK positioning, there are implementations of double differencing in collaborative positioning [6] and vehicle attitude determination [10]. The key constraints of using this method are:

- Need a reference satellite with good measurements at both receivers.
- The common multipath effects are also double differenced if the receivers are in close proximity. So it is not useful for multipath estimation and mitigation in such situations.
- Not suitable to perform cooperative positioning in constrained satellite visibility conditions, such as that of dense urban environments.
- Interpolation of raw measurements to common epochs is not direct. The interpolation can only be done after performing PVT estimation to obtain the precise measurement epoch of each receiver [11].

B. Interpolation With Clock Bias Obtained From Standard PVT Estimation

In this approach, the PVT of each receiver is estimated by using standard Extended Kalman Filter (EKF) or Least Squares Estimation (LSE). The clock bias and drift values thus obtained can be used to interpolate the raw measurements to common reference epochs [12]. This is easy to implement and is well studied approach.

1) Raw measurements models: Consider equations (1) and (2) as models of measured PR and Doppler values, respectively [13].

$$\rho_r^s = R_r^s + c(t_r - t^s) + T_r^s + I_r^s + M_r^s + \varepsilon_r, \tag{1}$$

where, R_r^s, is the geometric distance between r-th receiver and s-th satellite, c denotes speed of light, t^s represents the s-th satellite clock error, the atmospheric terms T_r^s and I_r^s denote the tropospheric and ionospheric effects, respectively, M_r^s denotes multipath effects and ε_r is the receiver noise.

$$D_r^s = \frac{\dot{R}_r^s}{\lambda} + f\dot{t_r} + f\dot{t^s} + m_r^s + \epsilon_r, \tag{2}$$

where, D_r^s is the Doppler measured at r-th receiver for s-th satellite signal, the 'dot' represents time derivatives of variables, λ and f are the wavelength and frequency of transmitted satellite signal, respectively, m_r^s is the multipath effect on Doppler shift and ϵ_r is the noise.

2) Interpolation of raw measurements: GNSS receivers making measurements at same rate, will have measurement instants close to common GPS epochs [3]. By design, the Ublox-M8T receivers can have maximum bias of 1 ms from the GPS time. Therefore, two of those receivers making measurements at the same rate can have measurement instants which are 2 ms apart.

Before interpolating the raw measurements from their respective measurement instants to common GPS epoch, the output of receivers are checked for synchronisation by using the following condition:

$$abs(rcvTow_{r1} - rcvTow_{r2}) \leq 2ms, \tag{3}$$

where, $rcvTow_r$ is time of week (in seconds) of r-th receiver at its measurement instant and abs denotes the absolute value function.

The effect of satellite, receiver and earth motions during the different measurement instants is very small compared to clock bias term in pseudorange measurement. Range-rate obtained from the Doppler measurements can be used to compensate for those movements [3]. The change in Doppler during the time between measurement instants of the two receivers is ignored.

If we consider two closely placed receivers making measurements at same rate, then after synchronisation, the interpolation to a common epoch can be done according to the equations (5) and (6).

$$\begin{aligned} D'_r &= D_r - f\hat{\dot{t}}_r \\ &= \frac{\dot{R}_r^s}{\lambda} + f\dot{t^s} + m_r^s + \epsilon_r \end{aligned} \tag{4}$$

$$\begin{aligned} \rho'_{r_1}(GPS_{epoch}) &= \rho_{r_1} - c\hat{t}_{r_1} - \lambda D'_r \hat{t}_{r_1} \\ &= R_r^s - ct^s + T_r^s + I_r^s + M_{r_1}^s + \varepsilon_{r_1} \end{aligned} \tag{5}$$

$$\begin{aligned} \rho'_{r_2}(GPS_{epoch}) &= \rho_{r_2} - c\hat{t}_{r_2} - \lambda D'_r \hat{t}_{r_2} \\ &= R_r^s - ct^s + T_r^s + I_r^s + M_{r_2}^s + \varepsilon_{r_2} \end{aligned} \tag{6}$$

Where, D' is the Doppler frequency adjusted for clock drift, $\rho'_r(GPS_{epoch})$ represents the pseudorange from r-th receiver interpolated to GPS epoch, $\hat{\dot{t}}$ and $\hat{t}$ are the clock drift and bias from standard PVT estimation, respectively. The same model can be extended to n number of receivers.

3) Analysis from measurement campaign 1: The motivation of this measurement set-up is to obtain data for analysis of interpolation method's precision. For that purpose a multipath-free environment is necessary.

In this measurement campaign, two receivers are placed close to each other ($baseline \approx 0.4m$) and raw measurements are logged at a rate of 1 Hz, simultaneously for 15 minutes. The receivers are set to track both GPS and GLONASS satellite vehicles (SV). The data collection is done on roof top of a high rise building in static mode. Therefore, raw measurements from the satellites at high elevation angles should be multipath free.

It is not possible to get a reference value for clock bias of a receiver to compare with estimated values. So for demonstration, we use difference between pseudoranges of two receivers interpolated to GPS time. If the clock biases are estimated correctly the pseudorange measurements interpolated to a common time epoch should be approximately equal (We assume that the baseline is too small and can be considered as noise in code measurements).

As the receivers are not using any external correction parameters, the navigation solution from Ublox receivers should be equivalent to standard positioning solution. The clock bias estimates from Ublox are used for plotting eq.(7) in Fig.1.

$$\begin{aligned} \Delta\rho'_{r_1r_2}(GPS_{epoch}) &= \rho'_{r_1}(GPS_{epoch}) - \rho'_{r_2}(GPS_{epoch}) \\ &= \Delta M^s_{r_1r_2} + \Delta\varepsilon_{r_1r_2} \end{aligned} \tag{7}$$

$$\Delta D'_{r_1r_2} = \Delta m^s_{r_1r_2} + \Delta\epsilon_{r_1r_2} \tag{8}$$

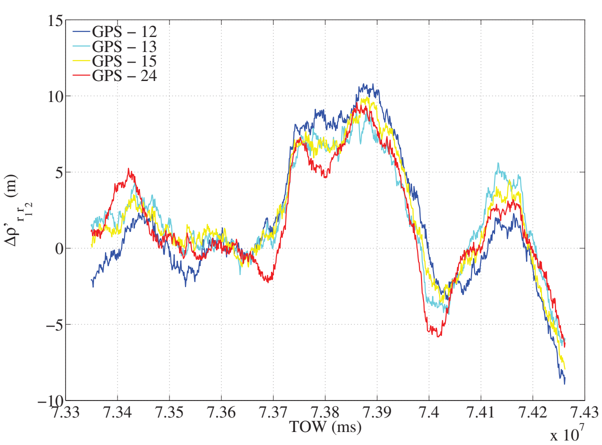

Fig. 1: Difference between interpolated PR of two receivers (m) versus Time of week (ms); Data from measurement campaign 1.

The elevation angles for GPS satellite vehicles 12, 13, 15 and 24 are $42°, 37°, 70°$ and $62°$, respectively. These satellites are manually selected upon inspecting for good C/N_0 values on both receivers through out the data logging period.

From eq.(7) we can say that if the bias values are estimated precisely, the difference of pseudorange measurements should not have any significant correlated residuals. But, Fig.1 has a clear correlated residual in curves which can be inferred as imprecise estimation of clock biases.

Disadvantages of this simple interpolation method are summarized as follows:

- At least four or more multipath free satellite measurements are necessary at each receiver to obtain precise bias values.
- The inaccuracies in broadcast atmospheric parameters and ephemeris (usually used in standard PVT estimation) affect the precision of bias estimation.
- The use of measurements from multiple receivers interpolated with imprecise bias values propagates the errors to cooperative positioning solution.

C. PVT Estimation Augmented with Multiple Clock Biases

If multiple receivers are placed in a zero baseline mode, the standard PVT estimation can be extended to accommodate for additional clock biases [8]. The PVT estimation can be formulated as in eq.(9) for two receivers. This formulation can also be applied in scenarios where baseline is known (multiple receivers placed on an aircraft, ship or car), by adjusting the measurements model with the known baseline. In zero/very short baseline arrangement both receivers have same position and therefore line of sight (LOS) vector is common for a particular satellite.

$$\begin{bmatrix} \rho_1^1 \\ \rho_1^2 \\ \vdots \\ \rho_2^1 \\ \rho_2^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} h_x^1 & h_y^1 & h_z^1 & 1 & 0 \\ h_x^2 & h_y^2 & h_z^2 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_x^1 & h_y^1 & h_z^1 & 0 & 1 \\ h_x^2 & h_y^2 & h_z^2 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t_{r_1} \\ t_{r_2} \end{bmatrix} + \epsilon, \tag{9}$$

In this measurements model, ρ_r^s is the pseudorange measurement between r-th receiver and s-th satellite. The LOS unit vectors are denoted by $\{h_x^s, h_y^s, h_z^s\}$. The vector (x, y, z) denotes the common position coordinates of these cooperative receivers.

The individual clock biases are denoted by t_{r_1} and t_{r_2} for r_1-th and r_2-th receivers, respectively. All the remaining errors such as, atmospheric residuals, ephemerides errors, propagation noise, etc., will be represented by the noise term ϵ.

With this formulation, the total number of measurements required to determine the bias values so as to align the measurements is reduced compared to the standard PVT estimation approach described in section II-B, but the remaining disadvantages of standard PVT estimation are still present.

III. Proposed method: Single Differencing Based Estimation of Bias and Drift Differences

A. Measurements Model Using Single Difference

Let us consider equations (1) and (2) as the pseudorange and Doppler measurements models, respectively. Assume that two receivers are placed in zero baseline configuration. Single differencing the measurements will give equations (10) and (11).

$$\Delta\rho^s_{r_1r_2} = c(t_{r_1} - t_{r_2}) + \Delta M^s_{r_1r_2} + \Delta\varepsilon_{r_1r_2} \tag{10}$$

$$\Delta D^s_{r_1r_2} = f(\dot{t_{r_1}} - \dot{t_{r_2}}) + \Delta m^s_{r_1r_2} + \Delta\epsilon_{r_1r_2} \tag{11}$$

Expanding eq.(11) with λ gives eq.(12)

$$\lambda\Delta D^s_{r_1r_2} = c(\dot{t_{r_1}} - \dot{t_{r_2}}) + \lambda\Delta m^s_{r_1r_2} + \lambda\Delta\epsilon_{r_1r_2} \tag{12}$$

Equations (10) and (12) are not dependent on the atmospheric delays or satellite ephemerides. In the absence of multipath and NLOS signals, these derived measurements can give precise estimates of differences of clock biases and drifts. If we consider the receiver noise follows Gaussian distribution among various channels, we can reduce it's effect on the single differenced measurements by averaging them over multiple satellites. The estimation model is given in eq.(15).

$$\Delta\bar{\rho}_{r_1r_2} = mean(\Delta\rho^{s_1}_{r_1r_2}, \Delta\rho^{s_2}_{r_1r_2}, \ldots) \tag{13}$$

$$\lambda\Delta\bar{D}_{r_1r_2} = mean(\lambda_{s_1}\Delta D^{s_1}_{r_1r_2}, \lambda_{s_2}\Delta D^{s_2}_{r_1r_2}, \ldots) \tag{14}$$

$$\begin{aligned} z &= Hx + noise \\ \begin{bmatrix} \Delta\bar{\rho}_{r_1r_2} \\ \lambda\Delta\bar{D}_{r_1r_2} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c(\widehat{t_{r_1} - t_{r_2}}) \\ c(\widehat{\dot{t_{r_1}} - \dot{t_{r_2}}}) \end{bmatrix} + noise, \end{aligned} \tag{15}$$

where, z represents the input measurements, H is the observations model and x is the state estimation vector.

To get measurements from two receivers at common epoch, the estimated difference in their clock biases can be used to interpolate measurements of one receiver to the measurement epoch of the other receiver. Measurements from the latter receiver are already at its own measurement epoch, so no further computations are necessary.

B. Multipath Effect Free Measurements Selection

The selection of satellite signals without multipath effects is important to get precise estimates of bias and drift by the estimation model in eq.(15). To eliminate the multipath effected measurements, we can take advantage of having multiple receivers in close vicinity.

Absolute values of differenced interpolated PRs (eq.(16)) and absolute values of differenced C/N_0 values from two receivers are plotted in Fig.2.

$$\Delta\rho'_{r_1r_2}(Rx2_{epoch}) = \Delta M^s_{r_1r_2} + \Delta\varepsilon_{r_1r_2}, \tag{16}$$

where, $Rx2_{epoch}$ denotes the measurement epoch of r_2-th receiver and the interpolation of r_1-th receiver measurements is done by the proposed method.

The significant residual in eq.(16) is differenced multipath error observed at two receivers. If signals arriving at both

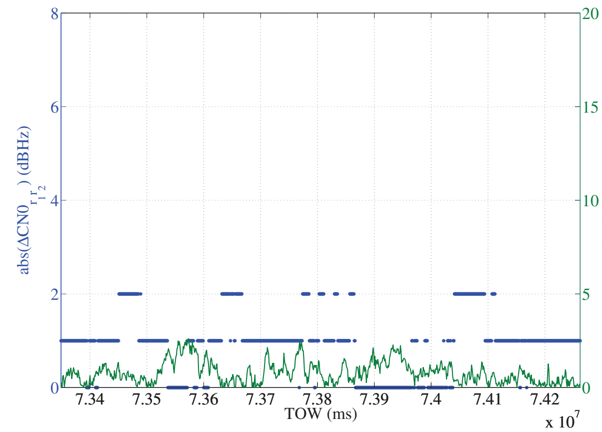

Fig. 2: C/N_0 difference as multipath effect indicator; Data from measurement campaign 1; GPS SV - 15 (Elevation - 70°).

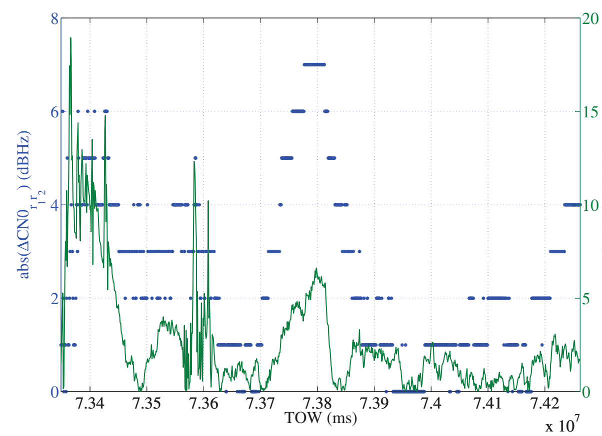

Fig. 3: C/N_0 difference as multipath effect indicator; Data from measurement campaign 1; GPS SV - 18 (Elevation - 28°).

receivers have undergone no or same multipath error, then the absolute values of differenced interpolated PRs will be minimum. This residual shows a strong correlation to differenced C/N_0 values. So, along with elevation and C/N_0 masks, difference in C/N_0 values observed at two receivers can be used as an indicator to select multipath free satellite measurements for clock bias estimation.

Same analysis is performed on data from measurement campaign 2.

Measurement campaign 2: The purpose of this campaign is to get data in very harsh environment for GNSS signals. The data obtained is used to determine whether the assumptions and observations made from analysis of data in measurement campaign 1 hold good under severe conditions also.

The data is logged by placing two Ublox receivers on roof of a car ($baseline \approx 0.4m$) and driven in the down town area of Toulouse, France. The receivers are set to track GPS and GLONASS SVs and measurement rate is set at 5 Hz. The route has narrow streets with tall buildings on either side. The measurements obtained have very high noise components. Traffic is congested for most of the time during this campaign but, there was a five minute duration when we were able to accelerate and reach speeds of 60 km/h. The total duration of campaign is around 25 minutes.

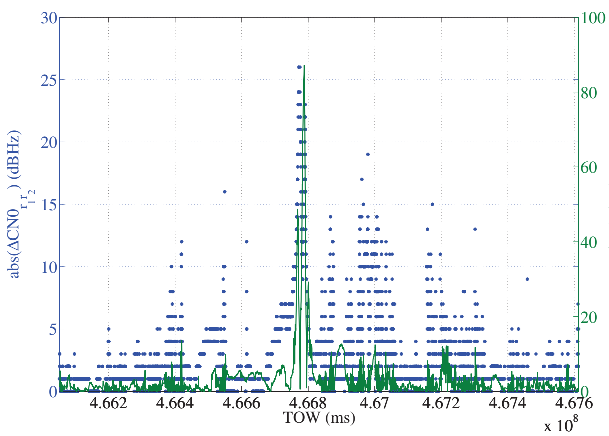

Fig. 4: C/N_0 difference as multipath effect indicator. Data from measurement campaign 2; GPS SV - 3; Elevation - 69°.

Even in harsh environment the differenced C/N_0 values have same trend as that of $\Delta\rho'$ residual (Fig.4). By fixing a threshold on maximum C/N_0 difference allowed, we can limit the amount of errors present in the input of bias estimation model of eq.(15).

Cut-off values: Ideally, the standard deviation of $\Delta\rho'$ is less than twice of standard deviation of code measurements (Δ makes standard deviation twice and averaging lowers the standard deviation). Considering that the standard deviation of multipath-free PR measurements noise is 3 meters, any $\Delta\rho' > 4m$ is identified as multipath affected data.

The C/N_0 values from Ublox receivers are rounded and output as unsigned integers. Therefore, the standard deviation of C/N_0 measurement error is approximately 1 dB-Hz. So, 2 dB-Hz is taken as threshold for $\Delta C/N_0$.

A qualitative analysis with $\Delta C/N_0$ mask of 2 dB-Hz, elevation mask of 25° and C/N_0 mask of 35 dB-Hz as multipath indicators is presented in table I. Where, false negative is an epoch when, $\Delta\rho' > 4m$ is allowed in the input and false positive is an epoch when, $\Delta\rho' < 4m$ is rejected as multipath affected measurements.

TABLE I: Qualitative analysis of multipath indicators

Measurement campaign	False negatives	False positives
1	1%	15%
2	4%	19%

C. Innovation-based Adaptive Estimation / Adaptive Fading Kalman Filter

To improve further the accuracy of clock bias difference estimation, we use an IAE/AFKF to estimate the unknowns. This choice allows us to dynamically adjust the covariance matrices of the Kalman Filter to get best estimates without the knowledge of measurement noise model (R). This tracks the error covariance parameters (P) more precisely and avoids Kalman Filter (KF) divergence problem [14].

The main parameters of the KF are as follows:

1) State transition matrix, F:

$$F = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \tag{17}$$

Where, ΔT is the time elapsed between two consecutive measurement epochs.

2) Process noise covariance matrix, Q: The Ublox has an internal Temperature Controlled Crystal Oscillator (TCXO). The TCXO system noise model is well studied [15]. This is modified to accommodate for our case of two receivers. We assume that there is no correlation between the clock signals of the two receivers. As the state vector elements are difference of two clocks, the system noise will be twice the variance of single TCXO clock, according to the eq.(18).

$$Var(\Delta\epsilon_{12}) = Var(\epsilon_1) + Var(\epsilon_1) = 2\sigma_r^2 \tag{18}$$

3) Adaptive measurement noise covariance update, R: The R matrix is updated every epoch based on innovation values as in the following equations:

$$v_k = z_k - \hat{z}_k^-, \tag{19}$$

$$\hat{C}_{v_k} = \frac{1}{N_R}\sum_{j=j_0}^{k} v_j v_j^T, \qquad j_0 = \begin{cases} 0, & if(k < N_R) \\ k - N_R, & otherwise \end{cases} \tag{20}$$

$$C_{v_k} = H_k P_k^- H_k^T + R_k, \tag{21}$$

$$\lambda_R = \frac{tr(\hat{C}_{v_k})}{tr(C_k)}, \tag{22}$$

$$R_k = \lambda_R * R_k, \tag{23}$$

where, k denotes current iteration number, v is the innovation, z is actual input measurement, $\hat{z}$ is estimated input, $\hat{C}_v$ is estimated innovation covariance, N_R is number of epochs over which innovation covariance is estimated, T denotes transpose of the matrix, C_v is calculated innovation covariance and tr denotes trace of the matrix.

4) Adaptive error covariance update, P: To enhance the tracking capability of the KF, P is updated as follows:

$$\hat{C}_{v_k} = \frac{1}{N_P}\sum_{j=j_0}^{k} v_j v_j^T, \qquad j_0 = \begin{cases} 0, & if(k < N_P) \\ k - N_P, & otherwise \end{cases} \tag{24}$$

$$C_k = H_k P_k^- H_k^T + R_k, \tag{25}$$

$$\lambda_P = \frac{tr(\hat{C}_{v_k})}{tr(C_k)}, \tag{26}$$

$$P_{k+1}^{-} = \lambda_P(F_k P_k F_k' + Q_k) \qquad ;\lambda_P = max\{1, \lambda_P\}, \quad (27)$$

where, N_P is number of epochs over which innovation covariance is estimated.

The use of λ_P, the time-varying suboptimal scaling factor, increases the tracking capability of the filter.

D. Interpolation of Raw Measurements

Using the estimated values from the above filter, the raw measurements of one receiver can be interpolated to the measurement epoch of another receiver. Interpolation is performed as per eq.(28).

$$\rho'_{r_1}(Rx2_{epoch}) = \rho_{r_1} - c(t_{r_1} - t_{r_2}) - \lambda D_r(\widehat{t_{r_1} - t_{r_2}}), \quad (28)$$

$$D'_{r_1} = D_{r_1} - \frac{c}{\lambda}(\widehat{\dot{t}_{r_1} - \dot{t}_{r_2}}), \quad (29)$$

where, $\rho'_{r_1}(Rx2_{epoch})$ is r_1-th receiver pseudorange interpolated to the measurement epoch of r_2-th receiver, D_r is Doppler frequency not adjusted for clock drift error, $(\widehat{t_{r_1} - t_{r_2}})$ is estimated clock bias difference, D'_{r_1} is Doppler of r_1-th receiver shifted to drift error of r_2-th receiver and $(\widehat{\dot{t}_{r_1} - \dot{t}_{r_2}})$ is estimated clock drift difference.

The clock drift error in Doppler term of eq.(28) is neglected in the equation. In general, if drift estimation from Ublox navigation solution is available, it is used to obtain a better Doppler value.

The same interpolation method can be used in situations with more than two receivers by selecting a reference receiver and interpolating raw measurements of all the other receivers to its measurement epochs. Other techniques such as Lagrange polynomial interpolation can be used instead of direct subtraction, for interpolating raw measurements [12].

E. Analysis

1) Measurement campaign 1, static mode with clear view of sky: The single difference of interpolated PRs (eq.(30)) are plotted in Fig.5. The ΔPR values after interpolation are significantly smaller compared to the standard PVT estimation approach (Fig.1) and there is no visible correlation between different satellite measurements which can be attributed to more precise interpolation.

$$\begin{aligned} \Delta\rho'_{r_1 r_2}(Rx2_{epoch}) &= \rho'_{r_1}(Rx2_{epoch}) - \rho_{r_2} \\ &= \Delta M^s_{r_1 r_2} + \Delta\varepsilon_{r_1 r_2} \end{aligned} \quad (30)$$

The same can be inferred about clock drift estimation. The single differences of Doppler adjusted for drift error by two approaches are plotted in Fig.6 (eq.(7) versus eq.(31)). After initial convergence of covariance matrices, the drift estimates are very precise (Fig.6).

$$\begin{aligned} \Delta D'_{r_1 r_2} &= D'_{r_1} - D_{r_2} \\ &= \Delta m^s_{r_1 r_2} + \Delta\epsilon_{r_1 r_2} \end{aligned} \quad (31)$$

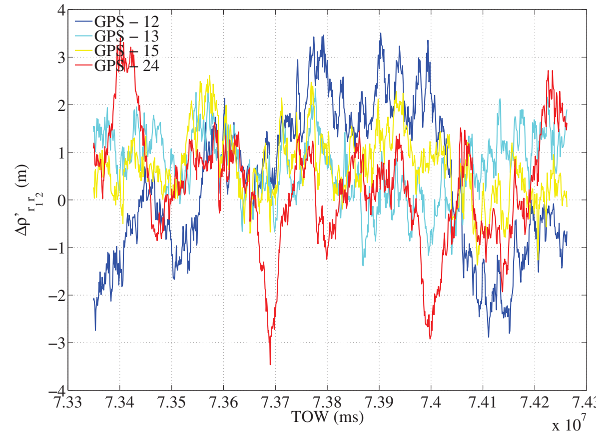

Fig. 5: Difference between interpolated PR of two receivers (m) versus Time of week (ms); Data from measurement campaign 1.

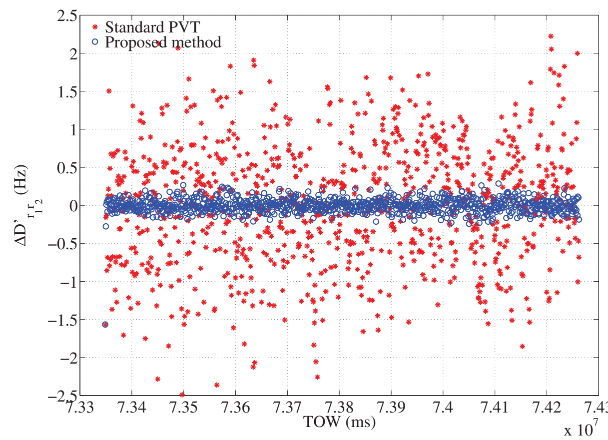

Fig. 6: Difference between adjusted Doppler of two receivers (Hz) versus Time of week (ms); Data from measurement campaign 1; GPS SV - 15; Elevation - 70°.

2) Measurement campaign 2, kinematic mode in a dense urban environment: The proposed method outperforms existing methods even in highly dense urban environment, where there might not even be a single measurement without multipath error. Fig.7 shows the residual of interpolated ΔPR, which can be used for better multipath estimation and elimination. The effect of improved clock bias drift estimation can be seen in Fig.8.

3) Robustness during short term signal outages: Fig.9 plots the clock drift difference of two receivers estimated using standard PVT approach and proposed single difference approach. It can be observed that the proposed interpolation technique outperforms the output of standard PVT method during signal outages.

Especially, around 5600-th epoch, the requirement of less number of satellites helps to get better estimates where the

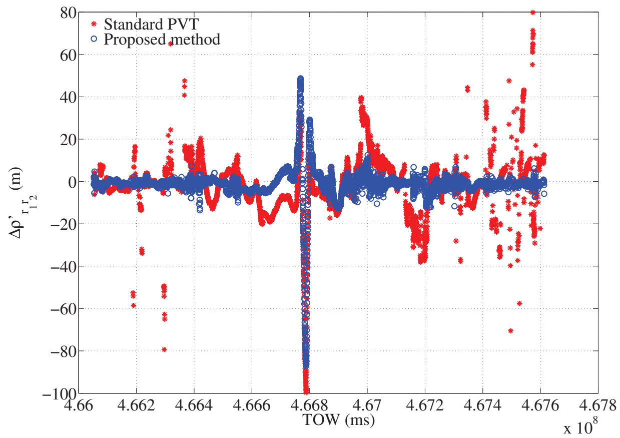

Fig. 7: Difference between interpolated PR of two receivers (m) versus Time of week (ms); Data from measurement campaign 2; GPS SV - 3; Elevation - 69°.

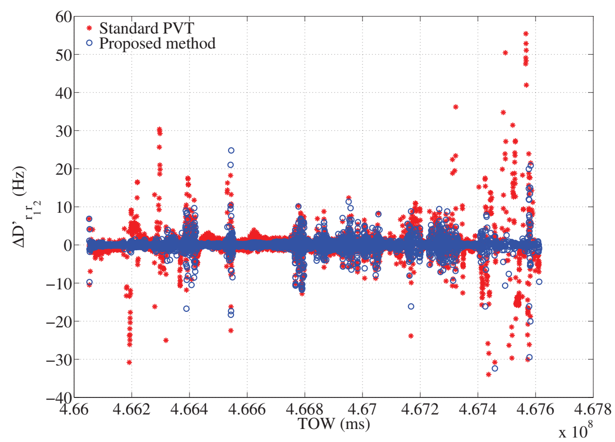

Fig. 8: Difference between adjusted Doppler of two receivers (m) versus Time of week (ms); Data from measurement campaign 2; GPS SV - 3; Elevation - 69°.

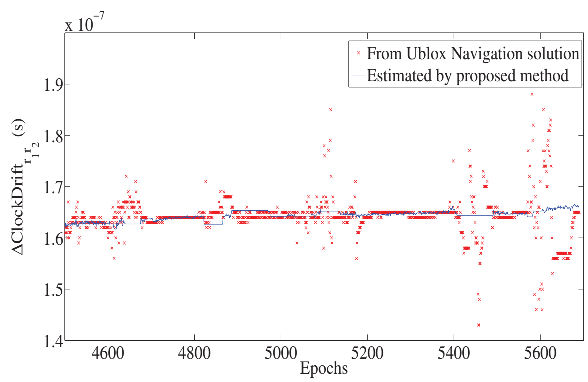

Fig. 9: Drift estimation comparison; Kinematic mode in dense urban environment; Data from measurement campaign 2.

standard method fails. During short signal outages, the drift is maintained constant and bias is estimated by time update steps of KF. Due to the high tracking capability of IAE/AFKF the estimated bias values will converge quickly without much fluctuations or divergence once the signals are reacquired.

TABLE II: Comparison of interpolation methods

Approach	**Minimum number of common satellites required per receiver**	**Minimum total number of good satellite measurements required for precise interpolation (n is number of receivers)**	**Precison level**
Double Differencing	4	n*4	high
Standard PVT	0	n*4	low
Augmented PVT	0	n+3	low
Single Differencing	1	n	high

4) Advantages of the proposed method include the following:

- Very precise estimates with just one satellite in view. The number of satellites required by each approach are tabulated in table II. It is an important criteria in constrained satellite visibility conditions where, the cooperative positioning should be more advantageous.
- Usage of Doppler measurements and IAE/AFKF in estimation process increases the robustness.
- Simple to implement and Low computational complexity.
- Flexibility with the choice of PVT estimation algorithms in later stages (No dependence on double differencing).
- Works well even in short signal outages as clock drift is almost constant over short durations..

F. Utility of interpolating raw measurements

1) Cooperative receivers: If two receivers have visibility of different sets of satellites, the satellite raw measurements from one receiver can be used at the other receiver (which does not have visibility of particular satellites) through interpolation.

This cooperation method can be used between any two receivers with known base line. Examples of such cases include multiple receivers mounted on aeroplanes, ships, cars, side walls of buildings etc.

a) Data for cooperative receivers scenario: To see whether the proposed method of interpolation is good enough in the scenarios mentioned above, a raw measurements data set is formed with a mix of original and interpolated measurements. It is used to obtain PVT solution and is compared to that of a single receiver with visibility of all the available satellites. A simple multi-constellation PVT estimation algorithm developed in Matlab is used for this purpose.

Two sets of raw-data are prepared with data from measurement campaigns.

Data set 1: Raw measurements logged by r_2-th Ublox receiver.

Data set 2: This is prepared to simulate a condition where two closely placed receivers have the visibility of different sets of satellites. Raw measurements of GPS SVs from 17 to 32 and GLONASS SVs from 13 to 24 satellites are interpolated from r_1-th receiver to the measurement epoch of r_2-th receiver. Measurements belonging to rest of the satellites are taken from r_2-th receiver and are combined with that of interpolated.

b) Measurement campaign 1: The positioning solutions from the two sets of data prepared as explained section III-F1a are plotted in Fig.10.

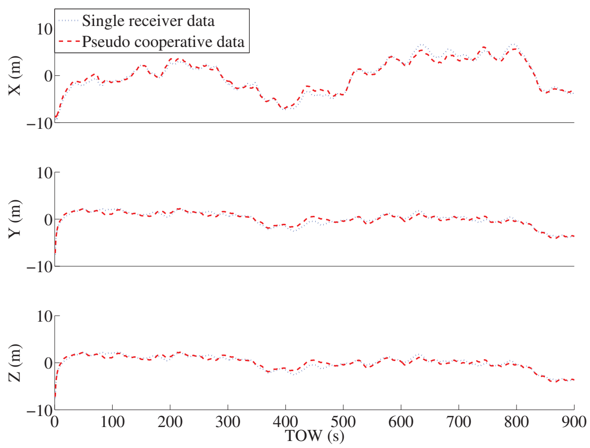

Fig. 10: Comparison of cooperative receivers' positioning error with that of r_2-th receiver; Data from Measurement campaign 1.

The difference between standard deviations of two solutions is given in table III. This difference is accepted in code based single frequency GNSS positioning.

TABLE III: Statistics of Fig.10

ECEF	Standard Deviation (m)	
	Single Receiver	Cooperative Receivers
X	3.66	3.32
Y	1.53	1.46
Z	1.91	1.51

c) Measurement campaign 3: A third measurement campaign is conducted to show that the proposed method can be used for cooperative positioning in kinematic conditions.

Two Ublox-M8T receivers with patch antennas are used to log data which is used for cooperative positioning. A Septentrio AsteRx3 dual frequency multi-constellation receiver with Novatel geodetic grade antenna is used to get the reference trajectory.

The antennas' positions on car roof can be seen in Fig.11. In the picture left most bump is the Novatel antenna and the other two are patch antennas supplied with Ublox-M8T EVK.

Fig. 11: Measurement campaign 3: Antennas' position on car roof

The antennas are placed in a straight line with an approximate distance of 20cm between them.

All the receivers are set to track GPS and GLONASS SVs and measurement rate is set at 1 Hz. The duration of the campaign is 20 minutes. The route taken includes all typical environments encountered in a suburban area such as open sky, trees and tall buildings on either side.

Measurements of Septentrio receiver are post processed using NRCAN PPP tool to get precise reference trajectory. Data from Ublox receiver is used to form data sets of single and cooperative receivers as explained in section III-F1a.

The output of this campaign is explained with Fig.12. The r_2-th and cooperative receivers' solutions trace the same path even in difficult environment (with buildings and trees as in the picture). This shows that the proposed interpolation method is good for cooperative positioning even in kinematic mode.

Fig. 12: Measurement campaign 3: Reference receiver (blue line), r_2-th receiver (white) and cooperative receiver (red).

Error of the two solutions with respect to the reference trajectory is given in Fig.13.

It is worthy to note that in harsh environments, like deep urban canyons, the single r_2-th receiver may not have enough measurements to produce a PVT solution while the cooperative solution will combine satellites from two or more receivers to come up with a positioning solution (i.e. the red trajectory of Fig.12).

2) Virtual receivers: When satellites measurements available at multiple receivers are redundant, this method can be used to generate virtual raw measurements. Examples include using C/N_0 values as weighting parameters and deriving better raw measurements by combining that of multiple receivers' to form a virtual receiver. This will have the benefit of improvement in positioning solution due to multiple sources.

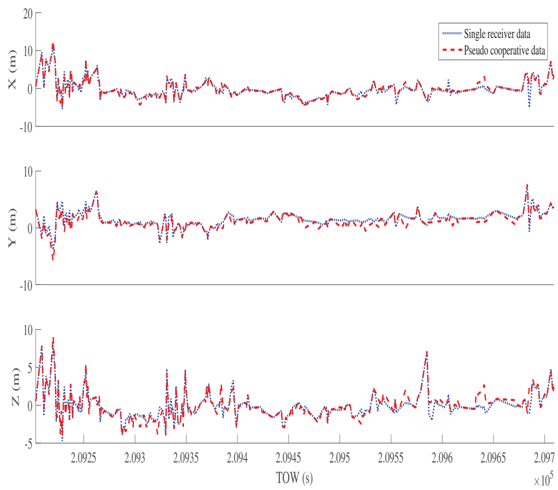

Fig. 13: Comparison of cooperative receivers' positioning error (with respect to reference trajectory) with that of r_2-th receiver; Data from Measurement campaign 3.

IV. Conclusion

This paper has provided the overview of existing methods to use measurements from multiple GNSS receivers for cooperative positioning. A new method is proposed to precisely interpolate raw measurements from multiple receivers with individual clocks to common epochs. The robustness of this method in dense urban environment is demonstrated. The use of this interpolated data in cooperative positioning and multipath estimation are analysed with experimental data. We showed how this method can be exploited to develop innovative PVT estimation algorithms. Further study will be conducted with increased number of receivers and new PVT estimation algorithms will be implemented to use the combined data of multiple receivers for better positioning.

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References

[1] S. Quebe, J. Campbell, S. DeVilbiss, and C. Taylor, "Cooperative gps navigation," in *Position Location and Navigation Symposium (PLANS), 2010 IEEE/ION*, May 2010, pp. 834–837.

[2] B. Huang, Z. Yao, X. Cui, and M. Lu, "Dilution of precision analysis for gnss collaborative positioning," *IEEE Transactions on Vehicular Technology*, vol. PP, no. 99, pp. 1–1, 2015.

[3] F. de Ponte Müller, A. Steingass, and T. Strang, "Zero-Baseline Measurements for Relative Positioning in Vehicular Environments," in *Sixth European Workshop on GNSS Signals and Signal Processing*, Munich, Germany, Dec 2013.

[4] D. Borio and C. Gioia, "A dual-antenna spoofing detection system using gnss commercial receivers," *Proceedings of the 28th International Technical Meeting of The Satellite Division of the Institute of Navigation*, pp. 325–330, sep 2015.

[5] O. Le Marchand, P. Bonnifait, J. Ibanez-Guzmán, D. Betaille, and F. Peyret, "Characterization of gps multipath for passenger vehicles across urban environments," *ATTI dell'Istituto Italiano di Navigazione*, no. 189, pp. 77–88, 2009.

[6] A. Kealy, G. Retscher, C. Toth, and D. Brzezinska, "Collaborative positioning–concepts and approaches for more robust positioning," in *Proceedings of the XXV International FIG Congress*, 2014, pp. 16–21.

[7] P. Defraigne and J. M. Sleewaegen, "Correction for code-phase clock bias in ppp," in *Frequency Control Symposium the European Frequency and Time Forum (FCS), 2015 Joint Conference of the IEEE International*, April 2015, pp. 662–666.

[8] D. W. Waters, T. Pande, and J. Balakrishnan, "Cooperative gnss positioning & navigation," in *Proceedings of the 24th International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS 2011)*, 2001, pp. 3945–3951.

[9] T. Takasu and A. Yasuda, "Development of the low-cost rtk-gps receiver with an open source program package rtklib," in *international symposium on GPS/GNSS*. International Convention Centre, Jeju, Korea, 2009, pp. 4–6.

[10] Z. Dai, S. Knedlik, and O. Loffeld, "Toolbox for gps-based attitude determination: An implementation aspect," *SCIYO. COM*, p. 389, 2010.

[11] J. García, P. Mercader, and C. Muravchik, "Use of gps carrier phase double differences," *Latin American applied research*, vol. 35, no. 2, pp. 115–120, 2005.

[12] S. Caldera, A. Gatti, L. Pertusini, E. Realini, M. Reguzzoni, and D. Sampietro, "Using gogps software with a multi-antenna gps system to improve navigation with low-cost receivers," in *EGU General Assembly Conference Abstracts*, vol. 15, 2013, p. 11911.

[13] P. Misra and P. Enge, *Global Positioning System: Signals, Measurements and Performance Second Edition*. Lincoln, MA: Ganga-Jamuna Press, 2006.

[14] F.-C. C. Dah-Jing Jwo and T.-P. Weng, *Adaptive Kalman Filter for Navigation Sensor Fusion, Sensor Fusion and its Applications*, C. Thomas, Ed. InTech, 2010.

[15] R. G. Brown and P. Y. Hwang, "Introduction to random signals and applied kalman filtering: with matlab exercises and solutions," *Introduction to random signals and applied Kalman filtering: with MATLAB exercises and solutions, by Brown, Robert Grover.; Hwang, Patrick YC New York: Wiley, c1997.*, vol. 1, 1997.