

# Instantaneous Relative Positioning of Multiple GNSS Receivers

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A thesis submitted in fulfillment  
of the requirements of the degree of  
Bachelor of Mechatronics (Space) Engineering/Bachelor of Science



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# Declaration

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of the University or other institute of higher learning, except where due acknowledgement has been made in the text.

**Lydia Drabsch**

8 June 2017

# Abstract

Abstract text goes here...

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# Acknowledgements

This thesis would not have been possible without the support of a great many of my work colleagues, family and friends . . .

You could dedicate the thesis to someone, or write something poignant. . .

*Hydrogen, in sufficient quantity and given enough time,  
will end up writing, reviewing and hopefully citing this thesis.*

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# Nomenclature

## List of Symbols

$v$	Variable Name, units
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## List of Acronyms

# Glossary

**L<sup>A</sup>T<sub>E</sub>Xplugin:** A L<sup>A</sup>T<sub>E</sub>Xpackage.

**Term:** This term means stuff.

**Constellation:** A group of satellites in the same network.

**GNSS:** Global Navigation Satellite System.

**GPS:** Global Positioning System, the American GNSS.

# Chapter 1

## Introduction

### 1.1 Background and Motivation

Localisation, outdoors GNSS - accuracy

Localisation is an integral part of the modern world.

- GNSS - talk about different constellations then pick GPS as the topic as it is the most available atm but galileo and gps will soon combine

### 1.2 Problem Summary

- get high drift using dead reckoning in robotics data - for outdoor solutions, gps is the easiest solution as it is already setup - however due to the high error in accuracy, it doesn't really solve the drift problem small scale, other data is required such as laser beacons, it is only used as global drift correction - embedded systems often have minimal computational space/time/processing available - current algorithms that have minimal setup have high computation costs - simple differential algorithms have high setup that require access to a known node, or access to realtime error information from the internet

what is the problem to be solved?

- find a way to have accurate (how accurate?) localisation data with minimal calibration or setup using low cost receivers and low computational requirements - a system that has requirements for high accuracy typically needs it for between parts of the system in which case there will be a receiver on each part, or be accurate to a specific location in the field, not to the global reference frame. Therefore differencing systems

## 1.3 Principle Contributions

this is what i've achieved

- contribution: description

- I carried out the literature survey in order to identify what areas of GNSS can be built on.
- I made the conceptual breakdown of the planar algorithm and the construction of the residuals: It is then solved using the generic solution for an overdetermined, non-homogeneous, linear system using least squares.
- I wrote a simulation in Matlab: The simulation calculates the pseudorange from all visible satellites to all receivers and adds randomised error in a controlled way to mimic different types of error. It then implements the epoch alignment and planar algorithm
- I used existing subfunctions as a part of my simulation that I had previously written: The subfunctions were all written as a part of Assignment 2 of the unit of study AERO4701 Space Engineering 3 in Semester 1 2016. The code that was adapted were; coordinate frame transforms, creation of plots in polar and cartesian frames, GPS constellation data, non-linear least squares solution of absolute position using pseudoranges.
- I carried out the analysis with influence from my supervisor.

## 1.4 Outline

epoch synchronisation followed by two fold optimisation process

need instantaneous relative positioning with minimal calibration or setup of external hardware

Instantaneous Relative displacement/position between GNSS receivers.

Simulation case studies are presented to validate the mathematical models.

The algorithm presented is designed to replace current methods.

With a local reference point with a well known global location, the relative position of the other receivers

# Chapter 2

## Literature Review

Global Navigation Satellite System (GNSS) is a generic term for a satellite navigation system that provides autonomous localisation and tracking on a global scale. Some countries have developed or in the process of developing their own GNSS, see Table 2.1. This thesis focuses on GPS (Global Positioning System) due to information accessibility and extensive usage in literature as it is the oldest constellation.

**Table 2.1** – GNSS Constellations

Country	Name	Number of Satellites	Start Date	End Date
America	GPS	31		
Russia	GLONASS	24	1976	
European Union	Galileo		1998	
China	Beidou			



## 2.1 GPS Operational Components

There are three components to the whole GPS system; the space segment, the user segment, and the ground control segment. The satellite sends radio signals towards the Earth which is received by both users and ground control centers. The ground control tracks the satellite and the data to analyse the satellite parameters and returns periodic updates to the satellite about its status. The satellite then updates its signal to the new parameters. The user receives the signal from the satellite with the updated parameters.

### 2.1.1 Space Segment

- current GNSS: explain GPS, GLONASS, galileo, chinese one constellations and how it works - what orbits are they in and why?: altitude inbetween the two radiation belts?

The GPS constellation consists of 32 operational satellites in six different orbital planes

### 2.1.2 User Segment

The user is a passive participant in the GPS system. - typical accuracy for civilian accessible gps

- military has more precise stuff  
- lower cost receivers have only one frequency band, error in timing

Unfortunately, low cost GNSS receivers rarely provide official access to the GPS raw data. Previous studies have used customised bluetooth headsets or customised android platform mobile phones to investigate algorithms on low-cost GPS receivers. More expensive receivers do allow raw data to be utilised, however they also provide other mechanisms such as dual frequencies and more accurate clocks, rendering the new algorithm \*obscure\*. The mindset of \*crowd-sourcing\*/customising/flexible

technology is changing the way manufactures build GPS receivers. The new Android OS platform Nougat 7.0 provides the developer raw GPS data at the software level.

### 2.1.3 Ground Control Segment

On the ground spread all over the world, are control stations that monitor the satellites. - tracking -> distance and angle - active as it sends back information -Locations of control stations and why they're there

Uses Herrick-Gibbs algorithm to determine the orbit of the satellite. A satellite is tracked over a ground station for a period of time and its position is measured. From three position vectors the velocity vector can be calculated and the ephemeris parameters are estimated. The ephemeris parameters describe the orbit of the satellite and are used to calculate the position of the satellite at any point in time by the user segment. The time parameters and clock corrections of the satellite are also calculated by the ground control station and sent back in the navigation message. The ephemeris data is highly accurate and updated every two hours.

## 2.2 GPS Satellite Signals

There are two frequencies that GPS uses; L1 frequency at 1575.42 MHz and L2 at 1227.60 MHz [?].

There are two sets of signals that are sent from every satellite, the pseudorandom binary sequence (PRN code) and the navigation message.

### 2.2.1 Course Acquisition Code

The course acquisition code (C/A)

Low cost GPS receivers only have access to

The PRN code is intentionally complex to stop jamming

L1 frequency of 1575.42 Mhz - code phase Course Acquisition (C/A) code is transmitted on the L1 frequency as 1.023 MHz signal using a bi-phase shift keying modulation technique. Navigation message sent at 50 bits per second

### 2.2.2 Navigation Message

The navigation message is a low

[https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-540-principles-of-geophysics/lecture-notes/MIT12\\_540S12\\_lec7.pdf](https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-540-principles-of-geophysics/lecture-notes/MIT12_540S12_lec7.pdf)

- civilian GNSS using dual frequency, send CDMA, how decryption works
- what is pseudorange?
- what is carrier phase
- clock bias

### 2.2.3 Raw Data

There is a lot of data contained in the total signal, but the following are what is important to this thesis:

- **Time received:** The time in the receivers frame that the sample reading was taken.
- **Pseudorange:** The range calculated by the receiver to the satellite. Depending upon the type of receiver, this measurement may have already been adjusted for some errors that were encoded in the navigation message.
- **Carrier Phase:** The phase of the carrier signal at the receivers point in time.
- **Doppler Shift:** The instantaneous Doppler frequency of the signal.
- **Satellite Epoch:** The time the signal was sent from the satellite, decoded from the navigation message.

- **Ephemeris Data:** The orbital parameters necessary to calculate the position of the satellite.

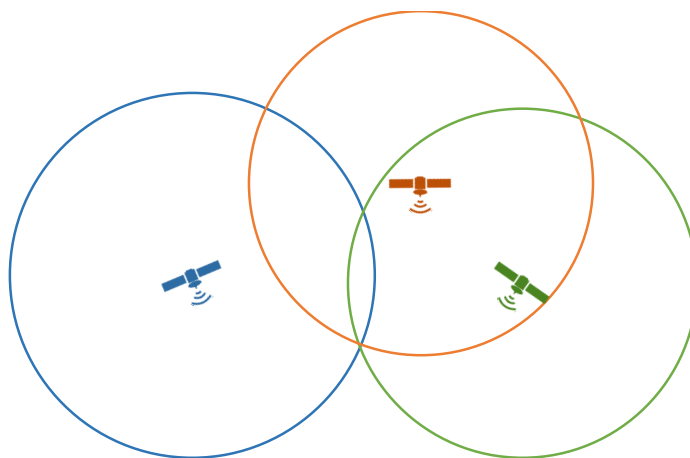
## 2.3 Trilateration

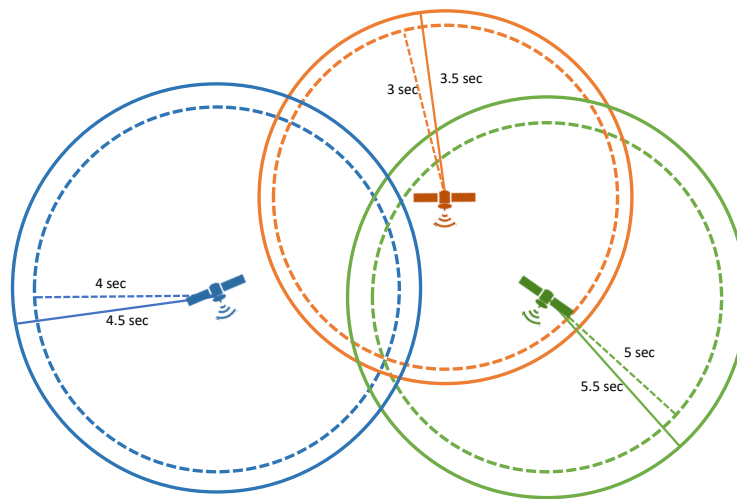
Unlike triangulation, which uses the angles from known points, trilateration uses the distances from known points. This is the base concept behind GNSS. Satellite W sends out a radio signal at time X and it's position Y which the user received at time Z. The time difference is used to calculate the distance from the satellite's position. With this information from multiple satellites, the users position is calculated.

- timing comparison between satellite and receiver to find psudeorange - ECEF frame of reference

- NLLS solve spheres - need 4 satellites minimum -

**Figure 2.1**



**Figure 2.2**

## 2.4 GPS Error Sources

There are many sources of error that plague the GPS data. These errors are categorised into six types; multipath, atmospheric effects, receiver noise, ephemeris error, clock bias and Sagnac effect.

### 2.4.1 Mutlipath Interference

Multipath is the case where the radio signal is reflected off objects before reaching the receiver, which increases the distance travelled. It is especially prevalent in built-up areas with tall buildings.

### 2.4.2 Atmospheric Effects

The distance from a satellite to a user is calculated by the time difference when the radio signal was sent and when it was received multiplied by the speed of light. However, the speed of light is reduced when in the atmosphere compared to that in space. The ionosphere is the upper layer of the atmosphere ranging from 50 to 500 km. It consists of ionized particles that create fluctuating electric fields in the atmosphere that perturb the radio signals that travel through it. This effect can be modelled but is still a significant source of error, approximately 5 m [3]. The troposphere is the lowest part of the atmosphere up to 50 km that varies in temperature and pressure with weather patterns. The radio signals are refracted through the medium, but as it is only for a short period of time this error is significantly less than that of the ionosphere at approximately 0.5 m error [3].

### 2.4.3 Receiver Noise

In any electronic components, especially low cost components, thermal noise introduces error into the system. Thermal noise is caused by the random motion of electrons in conducting materials. The construction of the electrical components are not

identical, which also cause slightly different solutions from receiver to receiver. This can contribute approximately 0.3 m of error [3].

#### 2.4.4 Ephemeris Error

The ephemeris error is the error in the navigation data describing the orbital parameters of the satellite [2]. This information is approximated and updated every 2-4 hours by the control segment to minimise the error. The approximation is based on a prediction model of the orbital parameters, as there are many forces that act on a satellite that can alter the orbit. These include gravitational affects of other masses in the solar system such as the Moon, the Sun, even Jupiter and the outer planets affect the gravitational potential of objects in orbit around Earth. The non-spherical Earth, solar radiation pressure, slight atmospheric pressure are all forces that manipulate the orbit. The satellites also undergo station keeping manoeuvres to manage the orbit, which the requires the ephemeris data to update. This inaccuracy contributes about 2.5 m in error [3].

#### 2.4.5 Clock Bias

The clock on the satellites and receivers are not exact with the true GPS time. Low-cost receivers are built with cheap quartz crystal oscillators that keep time to  $1\mu s$  accuracy. The clocks on the satellites however are atomic clocks that have accuracy on the order of  $1ns$ . In addition to this, ground control can measure the satellite clock bias and send back that information to be stored in the navigation message. [4]

#### 2.4.6 Sagnac Effect

The sagnac effect is a more intrinsic source of error. It is due to the rotation of the Earth during the time of the signal transmission. The transmission time is between 0.06-0.08 seconds in which time the Earth has rotated approximately 30 m. If the

**Table 2.2** – Categorisation of Errors

Satellite	Receiver	Uncorrelated
satellite clock bias ionospheric delay tropospheric delay ephemeris error	receiver clock bias receiver noise	multipath Sagnac

ephemeris data was in an inertial frame (ECI) there would not be a problem. However, the data is in the Earth-Centred Earth-Fixed (ECEF) frame which is a frame that rotates with the Earth to allow users to calculate their positions independent of time [1].

### 2.4.7 GPS Error Summary

All of the errors mentioned above can be categorised into three types; satellite correlated, receiver correlated, and uncorrelated errors. That is, a particular type of error may be consistent between all receivers from a particular satellite, or consistent between all satellite measurements for a particular receiver, or neither. See Table 2.2.

## 2.5 Multiple Receivers

- problems arising with multiple receivers

A single sample of pseudoranges from any two receivers will not be taken at the exact same time without a connecting network to implement control. This means that the satellite positions at the time of each signal transmission will be actually different. The primary issue with calculating the change in pseudorange is identifying the transmission time

### 2.5.1 Align Reception time

In [1], they align the reception time of multiple receivers



They precisely align the epoch by accounting for the differing Sagnac effect between two receivers and accounting for the clock biases of the receivers. The Sagnac effect manifests in the multiple receivers case by the signal propagation time for a measurement taken at  $t_2$  would be different than if it was taken at  $t_1$ , the Earth will have rotated by different amounts.

1. The clock bias is calculated by solving for the individual absolute position of a receiver using least squares. This is necessary in order to
- 2.

## 2.6 Current GNSS algorithms

- just reference implementation papers? - algorithms to make it more accurate
  - use for motion tracking
  - performance vs cost trade off
- (<http://ieeexplore.ieee.org.ezproxy1.library.usyd.edu.au/document/7530542/>)

### 2.6.1 Standard Positioning Service

- single frequency and multi frequency to remove atmospheric affects

### 2.6.2 Differential GPS

- explain what it is
- what setup is required
- abs vs rel
- degree of accuracy

**Table 2.3** – Error Components and Potential Improvements for SF-PPP

Error component	Potential Improvement
Ionosphere: Klobuchar model	7 m
Troposphere: Saastamoinen model	2.5 m
Ephemeris data	1 m
Satellite clock drift	1.5 m
Differential code bias	50 cm
Phase windup: rotation of the antenna	dm
Sagnac effect	30 m
ROA: satellite orbit correction	up to 10 cm
Relativistic clock correction	up to 21 m
Moon-Earth interaction	5cm (Hor) and 30 cm (Ver)

### 2.6.3 WAAS DGPS

### 2.6.4 SBAS ?

### 2.6.5 Real Time Kinematic

### 2.6.6 Post Processing Algorithm

### 2.6.7 Single Frequency Precise Point Positioning (SF-PPP)

Rademakers [how to say reference?](#) at University of Delft in the Netherlands developed a solution for finding the absolute position in open areas to a horizontal accuracy of 0.5 m. It uses a single frequency, single antenna low cost GPS receiver by connecting to the internet and using real time information to model all errors. The errors they corrected with the potential improvements are outlined in Table 2.3.

### 2.6.8 Dual-Epoch, Double-Differencing Model

In the paper by the Institute of Software Integrated Systems, Vanderbilt University called *High-Accuracy Differential Tracking of Low-Cost GPS Receivers*, Hedgecock

and party developed a new algorithm for relative motion tracking for multiple receivers. They used low cost GPS receivers with access to raw measurement data to produce centimeter-scale tracking accuracy. Each receiver was shared the whole networks data and ran the localisation algorithm independently to avoid having a single point of failure.

The algorithm uses the change in carrier phase through time of each receiver to estimate the change in relative ranges between a satellite and two receivers. It does not require a reference satellite, a reference node or an integer ambiguity solution. It does require the clock bias for each receiver at each point in time as solved for by non-linear least squares for the absolute position before running the algorithm itself. To reiterate, it does not directly solve for the relative position but the relative motion. However, neither of the initial positions of the receivers need to be precisely known in order for the relative motion to be accurate. Due to the time dependency, consistent satellite locks of at least four satellites are required, otherwise reinitialisation must occur. The calculated change was projected onto the unit direction vector from receiver to satellite. The system of these tracking equations was solved via least squares optimisation.

It uses the assumption that all satellites in the constellation are such a great distance from the surface of the Earth that the unit vector from both receivers are parallel to each individual satellite, as long as it is in the same geographical region. How far apart the receivers can be for this assumption to hold was not stated.

- how many receivers?
- why and how it aligns epoch
- uses difference in time for a single receiver to find change in motion.
- have this one last as it is the most similar
- needs instantaneous relative distance for first point, to speed up processing and make the first few time steps more accurate, also when locking onto new satellites

### 2.6.9 Summary of Algorithms ?

- dynamic tracking (need temporal measurements) vs static measurement - no temporal
- post processing vs pre-processing vs realtime
- ground structure vs free standing
- absolute vs relative
- accuracy (how much)
- computation time/space required
- what error is each method removing
- what piece of data it needs (if raw)
- calibration required
- robustness -> if a satellite goes out of view does it need to re-calibrate? passing information between receivers-> is one a reference? single point of failure

## 2.7 Moving Forward

GPS is currently going through a major upgrade, with the launch of new satellites with alternate structuring of signals that will provide greater accuracy for civil use in the future. The constellation Galileo that is currently being developed and launched, is expected to be finished by XX. Both the new GPS and Galileo are designed to be able to operate in tandem to provide an \*unprecedented\* accuracy with a total of 57 operational joint satellites, while still having the individual capability of global coverage. Some technology today uses both GPS and GLONASS constellations but requires dual receivers as the signals are encoded differently. GPS uses code division multiple access (CMDA) whereas GLONASS uses frequency division multiple access (FMDA). Research is also being conducted on the sources of error in the radio signals. The errors are to be modelled and the appropriate adjustments stored in the encoding of the signal. Through the international cooperation efforts and research, localisation and tracking using baseline GNSS will continue to improve, regardless of alternate

---

data processing algorithms. For now, improved signal accuracy will obviously improve the performance of the different data processing algorithms. However, there will come a time where the extra computational and hardware complexity will not be worth the small gain in accuracy over the default performance of the low-cost civilian GNSS receiver.

# Chapter 3

## Algorithm Solution

The common theme throughout the literature is to use differencing to minimise errors, both through time and space.

For a single receiver, differencing through time

For a single receiver differencing through space, a second known location is still required. This is set up externally and services a set area.

However for multiple receivers, spacial differencing is available. Even though with multiple receivers there is the added pre-processing with the sending of data between nodes and computation for epoch alignment. When differencing through time, the epoch alignment is required at the receiver end. That is, to manipulate the measurements as if the receivers took readings at the same time. Depending on how precise the alignment required for a system is, the computation time may require a numerical iteration solution.

When double-differencing, that is for both time and space for a single receiver.

When differencing through space between the receivers, the epoch alignment is required at the satellite end. The time the satellite sent the - make some ideas for when the receivers are moving - your giving the time of the positions between the receivers as different

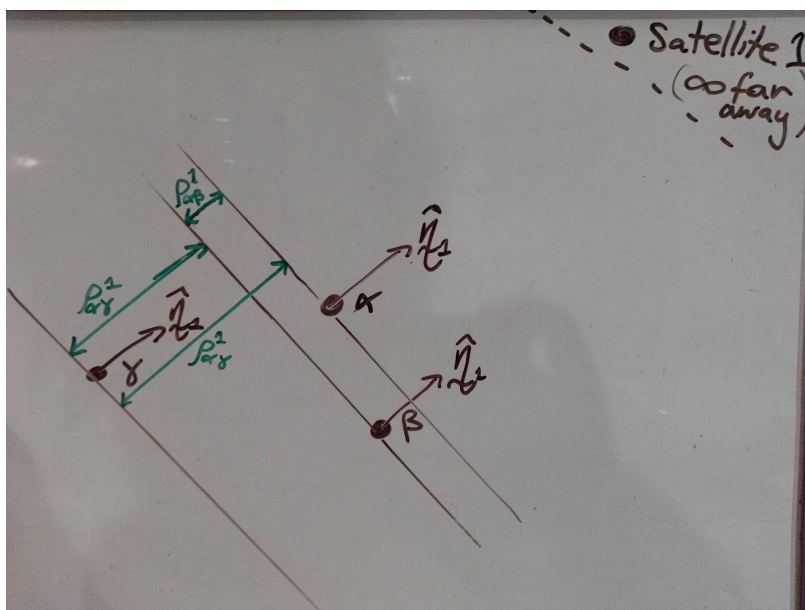
### 3.1 Proposed Planar Intersection Algorithm

- Fit better in the next chapter, put in references to literature - these are assumptions, heres why, rationale, these ppl also did it

Following the literature, the two main options for increasing the accuracy is extensive modelling of the errors, or some type of differencing algorithm. Error modelling requires external hardware, internet connection and extra computation time which increases the budget requirements. Whereas

This new algorithm is derived from taking the difference in pseudorange between multiple receivers from one satellite and expressing the distances as planes.

**Figure 3.1** – 2D representation

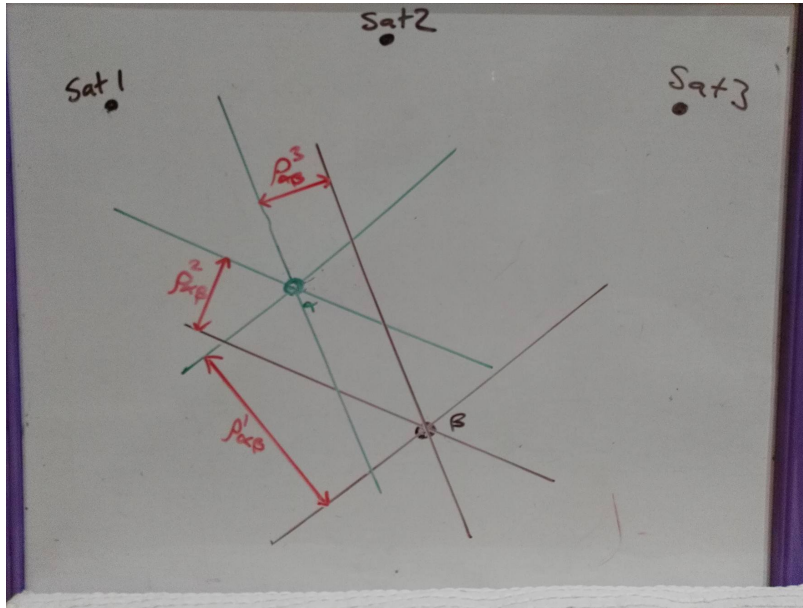


With multiple satellites in view, the intersection of planes for a particular receiver is the position of the receiver.

With this strategy, some of the errors that plague the absolute position are negated for the relative position.

$$\rho_i = \rho_n - cb_\omega + c(T_s + I_s + \nu_s + b_s) \quad (3.1)$$

Figure 3.2



where  $\rho_n$  is the real range with the following sources of error;  $b_w$  is the receiver clock bias,  $T_s$  is the tropospheric error,  $I_s$  is the ionospheric error,  $\nu_s$  is the relativistic error and  $b_s$  is the satellite clock bias.

## 3.2 Assumptions

All the assumptions are based on each other so need a pre-statement

### 3.2.1 Approximate Location

### 3.2.2 Static Receivers

All receivers are assumed to be static for the time in between all receivers to get a sample reading. This makes for an easier transform to align the satellite positions to a common epoch. It also ignores the problem of how the pseudorange from each receiver would be sent to either all of the receivers or to a central device for compu-



tation and the time delay associated with that.

For the dynamic case, it is likely that the data would be a part of a system containing other signals that describe the motion such as dead reckoning. In that case, the location of the receiver at the common epoch time can be back calculated to minimise the dynamic error. The incorporation of moving receivers is an area to explore for future work on the algorithm. However, there are existing algorithms in the literature XX where the relative motion is tracked with sub meter accuracy that may be more appropriate for complex dynamic systems.

### 3.2.3 Parallel plane assumption

It is assumed for the plane equations that all receivers point to a satellite along the same vector. This is valid for a dispersion of receivers for 10km for an error of XX. This is synonymous to if the satellites were at infinity and all the receiver vectors are parallel to a satellite

$$\delta = \tan^{-1} \left( \frac{d}{a} \right) \quad (3.2)$$

$$e = 2d \tan \delta \quad (3.3)$$

$$(3.2) \& (3.3) \Rightarrow e = \frac{2d^2}{a} \quad (3.4)$$

Where  $a$  is the altitude,  $d$  is the distance between two receivers and  $e$  is the error in the plane created. The worst configuration for error in the vector normal to the plane is if the satellite is directly above the receivers at the smallest distance from the Earth in orbit,  $a > 20000 \text{ km}$ . For  $d=5 \text{ km}$  the perpendicular error is 2.5 m

### 3.3 Epoch Alignment

A single sample of pseudoranges from any two receivers will not be taken at the exact same time without a connecting network to implement control. This increases the setup requirements which in turn increases the cost of the system and reduces the ease of compatibility with different receivers. The earliest time between all the receivers will be used as the time reference point called common epoch. The satellite position in the future time steps were backcalculated to find the difference in the pseudorange. The time between receivers would be a maximum of one second, as the slowest sampling time is typically 1 Hz. This extra distance is only in the vacuum of space and is not affected by potential nonlinear affects such as ionosphere and troposphere errors that affect the speed of light.

#### 3.3.1 Satellite Time

The known variables from the raw GPS data is the time the sample measurement was taken  $t_\omega$  and the pseudorange to each visible satellite  $\rho_\omega^1$  for each receiver  $\omega$ .

The error in the normal vector due to the time difference is

By the distributive law

$$\hat{\eta} \cdot s + \hat{\eta} \cdot v = |\eta| \quad (3.5)$$

$$\Delta\rho = |\eta| - |v| \quad (3.6)$$

$$= \hat{\eta} \cdot s + \hat{\eta} \cdot v - |v| \quad (3.7)$$

$$= \hat{\eta} \cdot s + |\hat{\eta}||v| \cos(\theta) - |v| \quad (3.8)$$

$$(3.9)$$

As  $|v|$  is  $>20\,000$  km and  $s$  is  $<4$  km,  $\theta \approx 0$  which means  $\cos \theta \approx 1$ . The magnitude

of a unit vector is one therefore (3.8) simplifies to

$$\Delta\rho = \hat{\eta} \cdot s \quad (3.10)$$

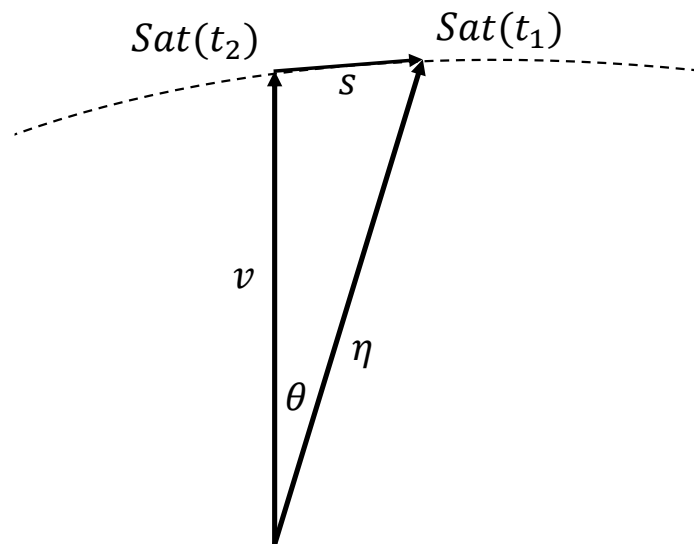
The vector  $s$  is the transform of the satellite from time  $t_2$  to the common epoch time  $t_1$ . It is calculated using the

$$t_1 = distance/c \text{ for a receiver } t_2 = t_{sampleoffset} - |v|/c$$

The vector  $v$  is the measured pseudorange of a receiver from a particular visible satellite at  $t_2$  with an unknown direction vector. The direction vector is unknown because the position of the receiver is what is being solved for. The vector  $\eta$  describes the known direction vector from the single approximate location of the geographic region to the common epoch of a particular visible satellite. It is the magnitude of  $\eta$  that describes what the pseudorange would have been if the measurement was taken at the common epoch. The difference between the  $|\eta| - |v|$

Because of the asynchronous time, some of the satellite correlated errors would not correlate as well as if it was from the same point in time. This is because the path through the ionosphere and troposphere is not exactly the same, however the

Error as the clock bias is unknown at this point. In the literature, an approximate clock bias is calculated through NLLS first then the epoch is transformed to process an updated pseudorange. The updated pseudorange is then used again in NLLS and the process is repeated until convergence is reached.

**Figure 3.3** – Vector Epoch Synchronisation

## 3.4 Planar Intersection Algorithm

- assume satellite is at infinity for comparing difference in pseudorange for a particular reference satellite.
  - use all satellites as reference satellite - no single point of failure, also not all satellites might be in view for all receivers
  - get the normal vector between all receivers and each sat.
  - Calculate the average normal vector.
  - get the difference in pseudorange between all receivers along each normal vector
  - create a plane with the normal vector with that distance
  - solve via optimization (least squares)
  - use clock adjustment from abs gps? or have as another optimisation variable
  - antenna problems? misalignment?
  - share clock bias's between solving for different reference sets? - do it one by one or all together?
  - need to align the time of signal sent to the receivers before calculating average normal vector
- 
- have weighted planes based on ? have weighted area on the planes?
  - if one plane intercepts far away from the others then ignore it (multipath). hyper-dimensional surface to minimise

how to send data between receivers? do it offline on a different platform?

<https://www.e-education.psu.edu/geog862/node/1759> - errors in pseudorange

<http://www.insidegnss.com/node/2898> - how to get pseudorange from raw data

### 3.4.1 Algorithm

#### Pre-Processing

**Select reference receiver  $\alpha$**  The receiver  $\alpha$  is used as the reference location and common time in the NED frame.

**Collect data of one timestep from all receivers** The raw data as well as the estimated absolute location and clock bias (what frame of reference is this?) from non-linear least squares optimisation is collected from all GNSS receivers.

#### Align to reference Epoch time

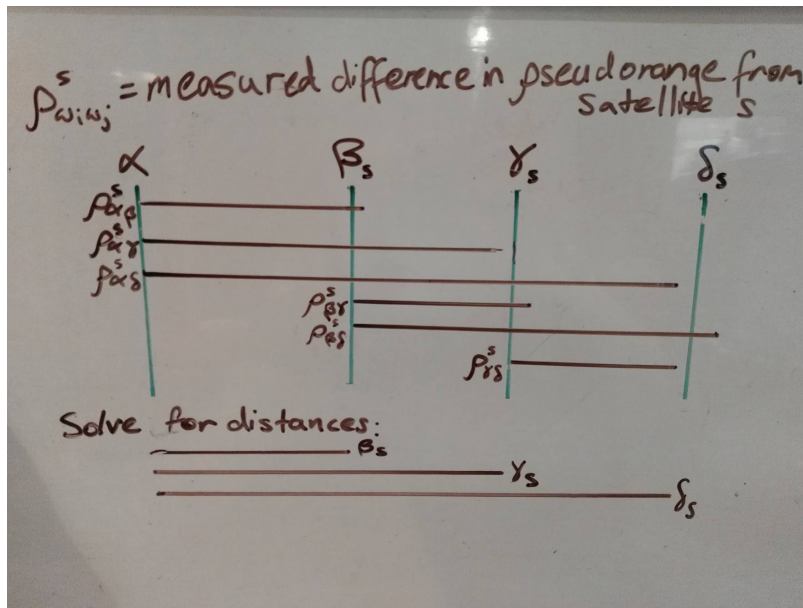
#### Distance Optimisation

By optimising the distance between each pair of receivers, the error in the whole system is minimised. This means that the position receivers are not only relative to the reference receiver  $\alpha$  but between all receivers just with the reference frame origin located at  $\alpha$ . It is because of this step a receiver does not need to have all the same satellites in view as all other receivers, including the designated  $\alpha$ .

**Average normal Vector** Find the average normal vector pointing to each satellite  $\hat{\eta}_s$  from the receivers. The normal vector is calculated by using the position all of the satellites in view at the common time  $t_\alpha$  as previously transformed in 3.4.1 and the estimated absolute position of all receivers. The average for each satellite is calculated by taking the mean across all receivers.

**Difference in Pseudorange** The differences in pseudorange are calculated  $\Delta\rho_{\omega_i\omega_j}^s$  where  $s$  is the satellite,  $\omega_i$  and  $\omega_j$  are receivers (*for*  $i < j, i \neq j$ ).

Figure 3.4 – text



**Optimise Pseudorange** The pseudorange between each pair of receivers along each normal vector  $\hat{\eta}_s$  creates an overdetermined linear system that is solved via least squares.

$$\Phi = \begin{bmatrix} 0 & -1 & 0 & \dots \\ 0 & 0 & -1 & \dots \\ \dots & \dots & \dots & \dots \\ 1 & -1 & 0 & \dots \\ 1 & 0 & -1 & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 1 & -1 & \dots \end{bmatrix} \quad (3.11)$$

$$\Omega_s = \begin{bmatrix} \beta_s \\ \gamma_s \\ \delta_s \\ \vdots \end{bmatrix} \quad (3.12)$$

$$\rho_s = \begin{bmatrix} \rho_{\alpha\omega_1} \\ \rho_{\alpha\omega_2} \\ \dots \\ \rho_{\omega_1\omega_2} \\ \rho_{\omega_1\omega_3} \\ \vdots \end{bmatrix} \quad (3.13)$$

$$\Phi \times \Omega_s = \rho_s \quad (3.14)$$

Solve by linear least squares for an overdetermined system by the pseudo inverse matrix

$$\Omega_s = (\Phi^T \Phi)^{-1} \Phi^T \rho_s \quad (3.15)$$

### Point Optimisation

**Create Planes** Create sets of planes for each receiver  $\omega$  from the normal vectors  $\hat{\eta}_s$  and the set of distances from the reference point  $\alpha$  to receiver  $\omega$  along each of the normal vectors denoted  $\Omega_\omega$ .

The equation of a plane is  $Ax + By + Cz + D = 0$  where the coefficients [A,B,C] describe the normal vector of the plane and the coefficient D sets the plane in 3D space along the vector. As the normal vector is already calculated for each satellite, only the D coefficient must be solved for each receiver and satellite pair.

$$P_\omega^s = (i \cdot \hat{\eta}_s)x + (j \cdot \hat{\eta}_s)y + (k \cdot \hat{\eta}_s)z + D_\omega^s \quad (3.16)$$

$$P_\omega^s = I \cdot H + D_\omega \quad (3.17)$$

$$(3.18)$$

Where  $I = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$  is the \*identity\* vector and H is a matrix of normal vectors



to each satellite:

$$H = \begin{bmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \\ \vdots \\ \hat{\eta}_n \end{bmatrix} \quad (3.19)$$

The coefficient D can be calculated by finding a point on the plane  $f_\omega^s$ , then substituting it into (3.16) for x,y,z. The point of the plane is calculated by moving along the normal vector by the optimised pseudo distance from the reference point (3.20).

$$f_\omega^s = \Delta_\omega^s \hat{\eta}_s \quad (3.20)$$

$$P_\omega^s = \hat{\eta}_s \cdot f_\omega^s + D_\omega^s = 0 \quad (3.21)$$

$$D_\omega^s = -\hat{\eta}_s \cdot f_\omega^s \quad (3.22)$$

$$D_\omega^s = -\Delta_\omega^s ||\hat{\eta}_s|| \quad (3.23)$$

$$||\hat{\eta}_s|| = 1 \quad (3.24)$$

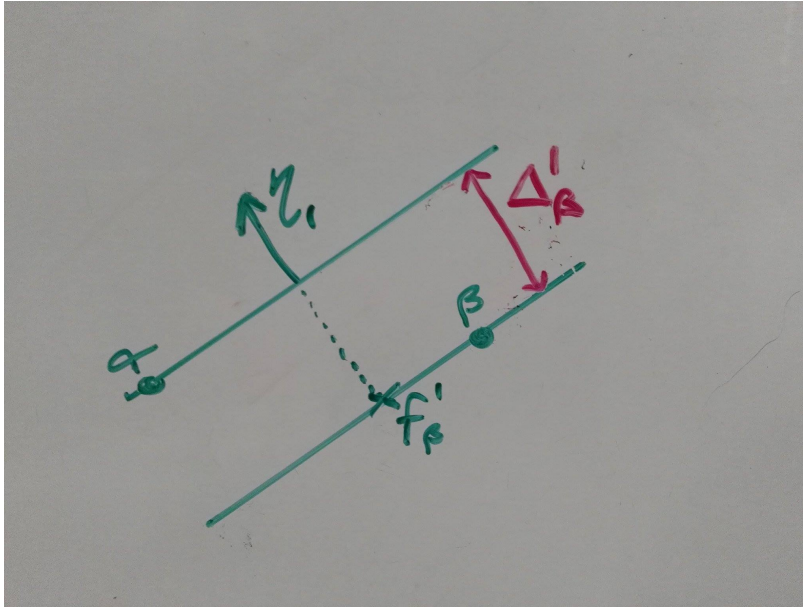
$$D_\omega^s = -\Delta_\omega^s \quad (3.25)$$

$$\Rightarrow P_\omega^s = I \cdot H - \Omega_\omega \quad (3.26)$$

$\Omega_\omega$  is a vector of optimised pseudo-distances from reference  $\alpha$  to receiver  $\omega$  for all satellites  $s \in 1, 2 \dots n$

$$\Omega_\omega = \begin{bmatrix} \Delta_\omega^1 \\ \Delta_\omega^2 \\ \vdots \\ \Delta_\omega^n \end{bmatrix} \quad (3.27)$$

Where  $\Omega_s$  is the vector of optimised pseudo-distances from  $\alpha$  to each receiver  $\omega \in$

**Figure 3.5** – Find position  $f_\omega^s$  on the plane

1, 2... $m$  for a single satellite  $s$ :

$$\Omega_s = \begin{bmatrix} \Delta_{\omega_1}^s \\ \Delta_{\omega_2}^s \\ \vdots \\ \Delta_{\omega_m}^s \end{bmatrix} \quad (3.28)$$

**Solve for Intersection** As the system of homogeneous linear equations is overdetermined, it can be solved using singular value decomposition to find a point that has the minimum residuals from all of the planes in its set  $P_\omega$ . Each set of planes for a particular receiver is independent to all other receivers. The vector  $X_\omega$  describes the position of receiver  $\omega$  in NED coordinates and  $\tau_\omega$  describes a final receiver clock bias that alters the displacement of all the planes in the set  $P_\omega$  by the same parameter.

$$X_\omega = \begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \\ \tau_\omega \end{bmatrix} \quad (3.29)$$

$$P_\omega X_\omega = D_\omega \quad (3.30)$$

$$(3.31)$$

In order to solve all of the receivers with the least amount of error in the whole system, all of the position vectors  $X_\omega$  are solved at the same time. The reference planes of  $\alpha$  must be included as a constraint on the system. All of the clock biases are also constrained with the clock bias from  $\tau_\alpha$ , see (3.32). The receiver clock bias only affects the equation of the planes by altering the constant as a change in the pseudorange has no affect over the angle of the plane. Each receiver clock bias alters all the planes associated with that receiver proportionally.

$$P_\omega^s = (i \cdot \hat{\eta}_s)x + (j \cdot \hat{\eta}_s)y + (k \cdot \hat{\eta}_s)z + D_\omega^s + (\tau_\omega - \tau_\alpha) \quad (3.32)$$

$$(3.33)$$

## 3.5 Summary

?? and Chapter 4 take things to a more advanced level.

# Chapter 4

## Experiments That Prove Things

- properties of the simulation then show results

### 4.1 Simulation Creation

#### 4.1.1 Creating a Simulation

Matlab was chosen as the platform to simulate and evaluate the program due to the ease of matrix manipulation and graphical interaction.

#### **Simulate Satellite Locations**

Using real ephemeris data for the GPS constellation, the location of all of the satellites were calculated, see Table 4.1. The approximate reference location of the network of receivers was inputted into the program as longitude, latitude, height geocentric coordinate system. The satellite positions were transformed to the local tangent plane of the reference location in polar coordinates. The satellites that had an elevation of above 12deg were selected as potential visible satellites. This lower elevation limit was selected to minimised multipath effects that would likely occur at ground level **REF.**

**Table 4.1** – Ephemeris data for GPS constellation

Orbital parameters	Value
--------------------	-------

### True Location of Receivers

The dispersion of receivers from  $\alpha$  is a variable to the program. The actual displacement is calculated by multiplying the dispersion magnitude by a uniformly distributed random vector ranging from  $[0,1]$  in three dimensions for all remaining receivers in North-East-Down (NED) frame localised at  $\alpha$ . The positions were then transformed to the global ECEF frame.

### Calculation of Pseudorange

In ECEF frame, the instantaneous distance between each visible satellite and each receiver was calculated. The errors were simulated by adding random distance proportional to error models in the literature to each individual satellite, see Table 4.2. The errors followed the structure in Eq(4.1)

For the error analysis, the errors associated with the pseudorange were divided into three sections; satellite correlated errors, receiver correlated errors and random errors. For the satellite correlated errors, the same amount was added to the true range for each receiver from a single satellite. For the receiver correlated errors, the same amount was added to the true range for each satellite from a single receiver. All errors were an addition to the pseudorange, so that how each type of error affected the algorithms

$$Error(seconds) = magnitude + 10 \times magnitude \times rand(elements) \quad (4.1)$$

Where *rand* is an inbuilt matlab function that returns a value on the domain  $[0,1]$  in a standard uniform distribution.

**Table 4.2** – Magnitude of simulated errors

Error Source	meters
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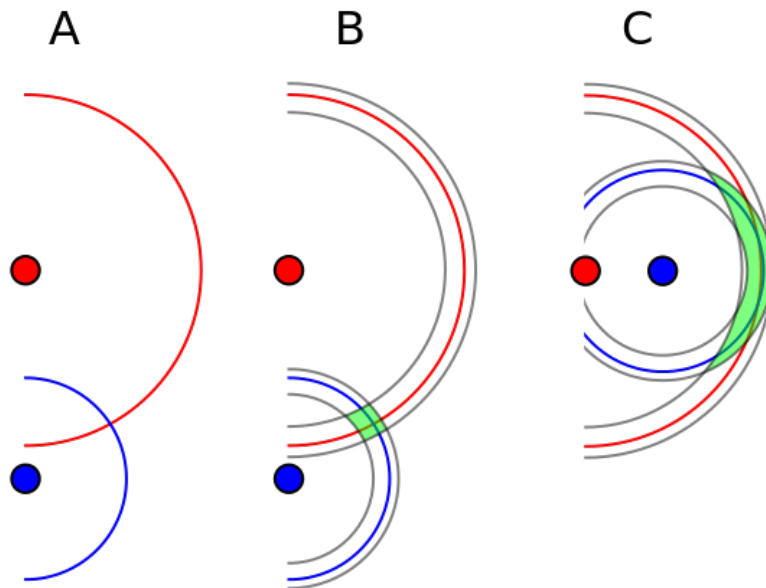
### Planar Algorithm

The normal vector was

- what data is it using from receiver? pseudorange, time
- what errors to include and how to incorporate into the simulation.
- how to include the different(asynchronous ) time received for all receivers-> is for the one receiver
- how extra receivers affects computational time/ accuracy
- how number of sats affect comp time/accuracy
- configuration of sats
- large multipath affects
- no receiver sees the same sat? - does it just output the relative difference between abs values? -> incorrect? just have it fail? not actually implementing, can control the environment
- distance of receivers apart
- configuration of receivers
- what data received and how to simulate misaligned timing between receivers
- what magnitude are the errors and how to simulate them
- simulate the errors individually (to see how each type affects the sim - convergence time and accuracy) and/or all errors at once

#### 4.1.2 Evaluation

The expected error due to the assumptions were outlined in Section XX. To evaluate the parallel plane assumption, two receivers were a set distance apart.

**Figure 4.1** – Geometric Dilution of Precision

**Dilution of Precision** The dilution of precision (DOP) is a measure of how the geometry of the satellites affect the position measurement precision. It is used in GNSS to evaluate how much error there may be in a measurement. Figure 4.1 shows two different configurations of satellites in the 2D circular solution case. With some error bounds as shown in B and C, the solution can lie anywhere in the green area. Due to the geometry configuration of the satellites in C, the area is considerably larger even though the error bounds on the signals are the same.

- fake gps data
- how to simulate noise - what level SNR
- to calculate your own GPS location using the normal algorithm? - space 3
- use real GPS locations? (and through time) -space 3

How to evaluate?: - accuracy in relative space

- compare to just taking differences in absolute position
- between individual receivers and the total error in the whole system
- markov? error analysis -> cannot do precision without statistical analysis but isolate

errors in x,y,z. how much worse is z than horizontal?

- computational time-> how does more receivers/satellites affect the comp time -> what time and space complexity?

## 4.2 Evaluation

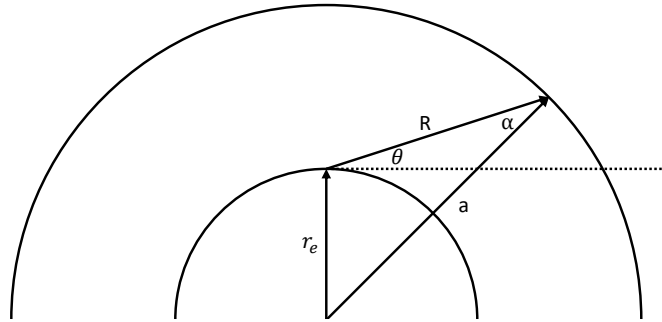
Base Evaluation In NED frame, have two receivers - vary the distance by 0.1m, 1m, 10m ... 100km - set it as x, y, z for each distance and satellite configuration - Use 'fake' satellite positions? Sat configs: - only sats in one orbit - 1D - cluster directly above - cluster at the edges - sats in the same plane as the difference in receivers -

- how close the 'approximate location' setting is to the alpha receiver
- what affect random errors have on the accuracy - use a time scale
- vary number of satellites in view
- vary GDOP (good GDOP and bad)
- when receivers don't see the exact same satellites
- vary number of receivers
- simulate a multipath error and how does it account for it or how much error does it introduce

### Fake Satellite Positions

It is assumed that the orbits are circular, this is valid as the eccentricity ( $e$ ) of the GPS satellites are all  $e < 10^{-3}$ . The eccentricity is a measure of how elliptical an orbit is; where  $e = 0$  is a perfect circle and  $e = 1$  is a parabola. It is also assumed that the Earth is circular and has a radius of  $r_e = 6.378137 \times 10^6 m$ .



**Figure 4.2** – text

$$\sin \alpha = \frac{r_e \sin(\pi/2 + \theta)}{a} \quad (4.2)$$

$$R = \frac{a \sin(\pi - (\pi/2 + \theta) - \alpha)}{\sin \theta} \quad (4.3)$$

$$R = \frac{a \cos \left( \theta + \sin^{-1} \left( \frac{r_e \cos(\theta)}{a} \right) \right)}{\cos(\theta)} \quad (4.4)$$

Some configurations will not reflect what can be achieved in our current reality but may be able to in the future. The European GNSS constellation *Galileo* will have 30 satellites operating in tandem with GPS. There are also the constellations GLONASS and Beibo the Russian and Chinese constellations that there may be international cooperation in the future to have all the GNSS constellations operating together in a high density system.

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# Chapter 5

## Conclusion

Future work: Explain how to do it with real systems: The algorithm is designed with maximum compatibility in mind. - any type of receiver that has access to GPS L1 frequency -

# List of References

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- [2] E. Kaplan, *Understanding GPS - Principles and applications*, 2nd ed. Artech House, 2005.
- [3] Trimble, “Trimble gps tutorial- error correction,” 2017. [Online]. Available: [http://www.trimble.com/gps\\_tutorial/howgps-error2.aspx](http://www.trimble.com/gps_tutorial/howgps-error2.aspx)
- [4] N. Zinas, “Satellite and receiver clock errors,” 2015. [Online]. Available: <http://www.tekmon.eu/1-3-2-satellite-and-receiver-clock-errors/>

# Appendix A

## An Example Appendix

As an appendix, this should contain some content that's not really required for the argument in the main body of the thesis, but is clearly relevant and supports the work.