Instantaneous Relative Positioning of Multiple GNSS Receivers

Lydia Drabsch BE (Hons 1)

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Declaration

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of the University or other institute of higher learning, except where due acknowledgement has been made in the text.

Lydia Drabsch

8 June 2017

Abstract

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If you've just opened up this template, you should check out Chapter 1 for a quick introduction. Note also that hyperlinks are rendered in colour for convenience during editing—see ../LaTeX/packages.tex. Note also that there is a simple way to change all of the coloured hyperlinks to black in ../LaTeX/packages.tex—look for `\usepackage[colorlinks`.

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Acknowledgements

Thank you to my supervisor for his patience and understanding.
Thank you to my parents Kathy and Brian, who kept my sanity safe while I wasn't using it.

Science asks: How does the universe work?
Engineering asks: How can I make the universe work for me?

Lydia Drabsch, 2013

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Nomenclature

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Glossary

LaTeXplugin: A LaTeXpackage.

Term: This term means stuff.

Constellation: A group of satellites in the same network.

GNSS: Global Navigation Satellite System.

GPS: Global Positioning System, the American GNSS.

Chapter 1

Introduction

1.1 Background and Motivation

Clearly identifies the problem for investigation and relevant context Clearly sets out the content and aims of the project Localisation, outdoors GNSS - accuracy

Localisation is an integral part of the modern world.

- GNSS - talk about different constellations then pick GPS as the topic as it is the most avalible atm but galelaio and gps will soon combine

1.2 Problem Summary

- get high drift using dead reckoning in robotics data - for outdoor solutions, gps is the easiest solution as it is already setup - however due to the high error in accuracy, it doesn't really solve the drift problem small scale, other data is required such as laser beacons, it is only used as global drift correction - embedded systems often have minimal computational space/time/processing available - current algorithms that have minimal setup have high computation costs - simple differential algorithms have high setup that require access to a known node, or access to realtime error information from the internet what is the problem to be solved?

- find a way to have accurate (how accurate?) localisation data with minimal calibration or setup using low cost receivers and low computational requirements - a system that has requirements for high accuracy typically needs it for between parts of the system in which case there will be a receiver on each part, or be accurate to a specific location in th field, not to the global reference frame. Therefore differencing systems

1.3 Principle Contributions

The following list is an overview of what I have contributed to the field.

- I carried out the literature survey in order to identify what areas of GNSS can be built on.
- I made the conceptual breakdown of the planar algorithm and the construction of the residuals: It is then solved using the generic solution for an overdetermined, non-homogeneous, linear system using least squares.
- I wrote a simulation in Matlab: The simulation calculates the pseudorange from all visible satellites to all receivers and adds randomised error in a controlled way to mimic different types of error. It implements the epoch alignment and planar algorithm with a comparison to
- I used existing subfunctions as a part of my simulation that I had previously written: The subfunctions were all written as a part of Assignment 2 of the unit of study AERO4701 Space Engineering 3 in Semester 1 2016. The code that was adapted were; coordinate frame transforms, creation of plots in polar and cartesian frames, GPS constellation data, non-linear least squares solution of absolute position using pseudoranges.
- I carried out the analysis of the planar algorithm using the simulation as previously mentioned. The conclusions are my own.

1.4 Outline

The rest of the thesis is organised as follows. Chapter 2 is an overview of how GPS works including the operation components and signal structure. The sources of error will be introduced and relevant work in the literature of how the errors can be minimised. Chapter 3 is a discussion of the assumptions and the detailed mathematical methodology of the planar algorithm. Chapter 4 is a full analysis of the planar algorithm and comparison to

epoch synchrosation followed by two fold optimisation process
need instantanous relative positioning with minimal calibration or setup of external hardware

Instantaneous Relative displacement/position between GNSS receivers.
Simulation case studies are presented to validate the mathematical models.

The algorithm presented is designed to replace current methods.

With a local reference point with a well known global location, the relative position of the other receivers

Chapter 2

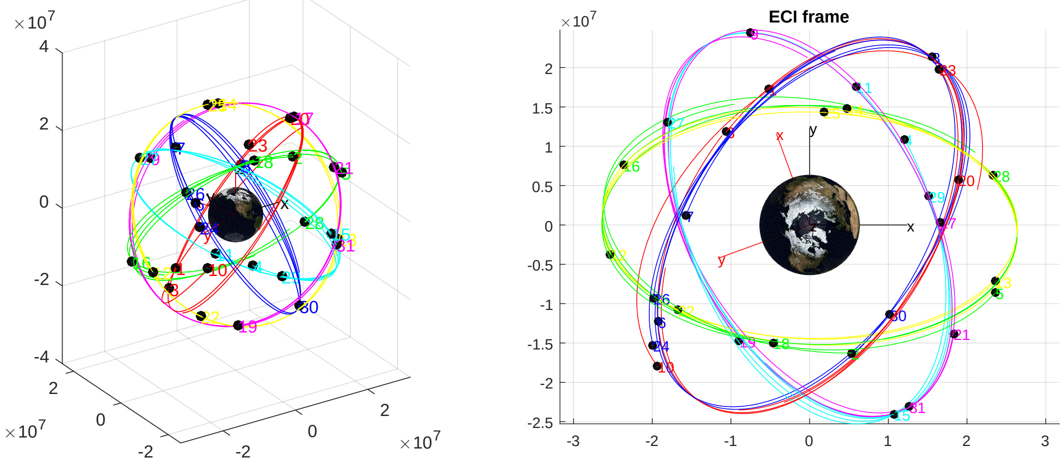
Literature Review

Global Navigation Satellite System (GNSS) is a generic term for a satellite navigation system that provides autonomous localisation and tracking on a global scale. Some countries have developed or in the process of developing their own GNSS. This thesis focuses on the American GPS (Global Positioning System) due to information accessibly and extensive usage in literature as it is the oldest constellation.

Figure 2.1: GPS Constellation in Earth-Centered Inertial frame

(a) Side View

(b) Top View



2.1 GPS Operational Components

There are three components to the whole GPS system; the space segment, the user segment, and the ground control segment. The satellite sends radio signals towards the Earth which is received by both users and ground control centers. The ground control tracks the satellite and the data to analyse the satellite parameters and returns periodic updates to the satellite about its status. The satellite then updates its signal to the new parameters. The user receives the signal from the satellite with the updated parameters.

2.1.1 Space Segment

A set of satellites in the same network is called a constellation. The GPS constellation consists of 30 active and spare NAVSTAR (NAVigation System with Timing and Ranging) satellites [1]. There are 24 satellites in the nominal configuration of 6 orbits with 4 satellites evenly spaced in the orbit with 90° separation, see Figure 2.1. Each orbital plane is inclined at 55° and evenly spaced around the Earth with 60° separation. All of the orbits are almost circular with an eccentricity of 0.02 and an altitude (semi-major axis) of 20 200 km. This type of orbital configuration was chosen so that the period of the satellites are half of a sidereal day, 11 hours 58 minutes 2 seconds. With a masking of 15° from the horizon, this configuration provides coverage of a minimum of four satellites for every point on Earth, consistently [12]. Each satellite sends radio signals towards Earth and receives updates from ground control. The signals are explained more in Section 2.2.

2.1.2 User Segment

The user is a passive participant in the GPS system, it receives the radio signals and processes them to determine position, velocity and timing (PVT). There are millions of receivers around the world covering military and civilian applications by ships, aircraft, ground vehicles and individuals.

WHERE TO PUT THIS?
Unfortunately, low cost GPS receivers rarely provide official access to the GPS raw data. Previous studies have used customised bluetooth headsets or customised android platform mobile phones to investigate algorithms on low-cost GPS receivers. More expensive receivers do allow raw data to be utilised, however they also provide other mechanisms such as duel frequencies and more accurate clocks, rendering the new algorithm *obtuse*. The mindset of *crowd-sourcing*/customising/flexible technology is changing the way manufactures build GPS receivers. The new Android OS platform Nougat 7.0 provides the developer raw GPS data at the software level.

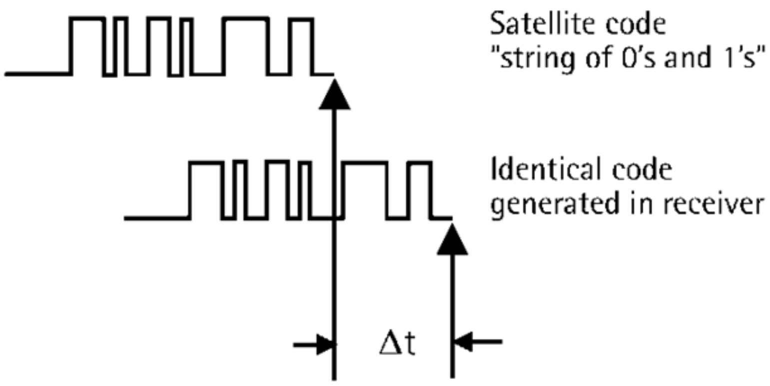
2.1.3 Ground Control Segment

On the ground spread all over the world, are control stations that monitor the satellites. The ground stations use the Herrick-Gibbs algorithm to determine the orbit of the satellite. A satellite is tracked over a ground station for a period of time and its position is measured. From three position vectors the velocity vector can be calculated and the ephemeris parameters are estimated. The ephemeris parameters describe the orbit of the satellite and are used to calculate the position of the satellite at any point in time by the user segment. The time parameters and clock corrections of the satellite are also calculated by the ground control station and sent back in the navigation message. The ephemeris data is highly accurate and updated every two hours.

2.2 GPS Satellite Signals

The are two frequencies that GPS uses; L1 frequency at 1575.42 MHz and L2 at 1227.60 MHz [?]. There are two sets of signals that are sent from every satellite, the pseudorandom binary sequence (PRN code) and the navigation message.

Figure 2.2: Pseudorange measurement from the time delay of the PRN code [11]



2.2.1 PRN

Coarse Acquisition Code, or C/A code, is transmitted on the L1 frequency as a 1.023 MHz signal of 1023 bits. Civilians have access to the C/A code and is what is used in receivers. There is another code modulated onto the L1 and L2 frequencies called the P (Precise) code as a 10.23 MHz signal [5]. As it is 10 times faster it is more accurate, and is restricted to military use. Both of these codes are modulated as pseudo random number (PRN) codes and repeat constantly, which have the property of the appearance of random noise but are very precisely defined. It is because it is precisely defined, the GPS receiver can recreate the PRN code at the same time the satellite does. By comparing the incoming signal and the self-generated code the time delay is measured, see Figure 2.2. As the C/A code repeats every millisecond, only a time range of 1 ms needs to be searched [10].

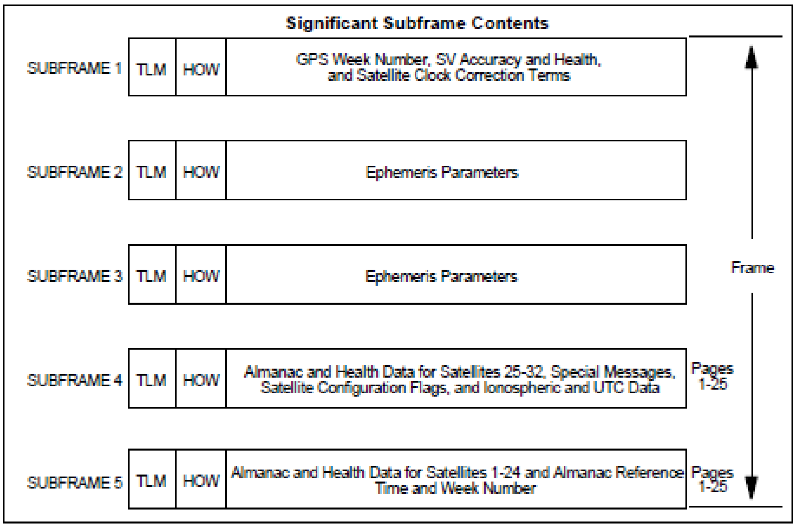
[4]

2.2.2 Navigation Message

The navigation message is a low frequency signal added to the L1 code at a rate of 50 bps. It has three components [7];

1. GPS date and time with satellite status at the time the signal was sent

Figure 2.3: Navigation Message Content and Format Overview [7]



2. Ephemeris data: valid for up to 4 hours

3. Almanac data: valid for 180 days

The almanac data

2.2.3 Raw Data

There is a lot of data contained in the total signal, but the following are what is important to this thesis:

- **Time received**: The time in the receivers frame that the sample reading was taken.
- **Pseudorange**: The range calculated by the receiver to the satellite. Depending upon the type of receiver, this measurement may have already been adjusted for some errors that were encoded in the navigation message.
- **Carrier Phase**: The phase of the carrier signal at the receivers point in time.
- **Doppler Shift**: The instantaneous Doppler frequency of the signal.

- **Satellite Epoch**: The time the signal was sent from the satellite, decoded from the navigation message.
- **Ephemeris Data**: The orbital parameters necessary to calculate the position of the satellite.

2.3 Trilateration

Unlike triangulation, which uses the angles from known points, trilateration uses the distances from known points. This is the base concept behind GNSS. Satellite W sends out a radio signal at time X and it's position Y which the user received at time Z. The time difference is used to calculate the distance from the satellite's position, see Figure 2.4. With this information from multiple satellites, the users position is calculated. In reality, the distance from the satellite to the user has error in it, which will be explained in detail in Section 2.4. The error alters the radius of the circle (or in 3D the sphere), see Figure 2.5. The error is expressed as a clock bias, a change in time at the receiver side in order to find an intersection. As there are four variables to solve for, x, y, z and clock bias b_u, a minimum of four satellites are required to calculate the intersection. The range that has error in it is called the pseudorange.

Figure 2.4: Solve for Position using Trilateration

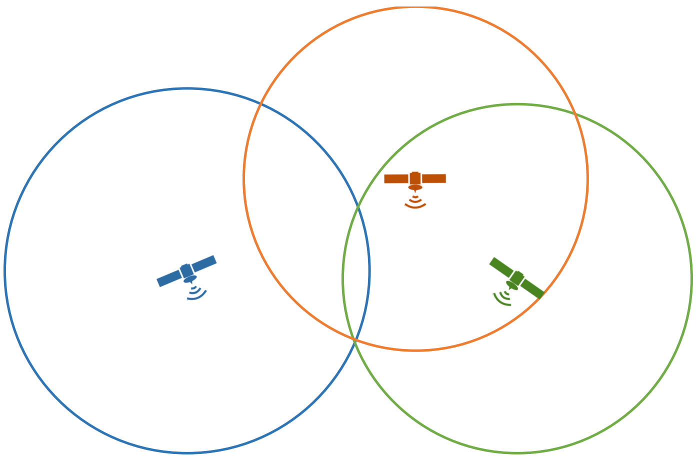
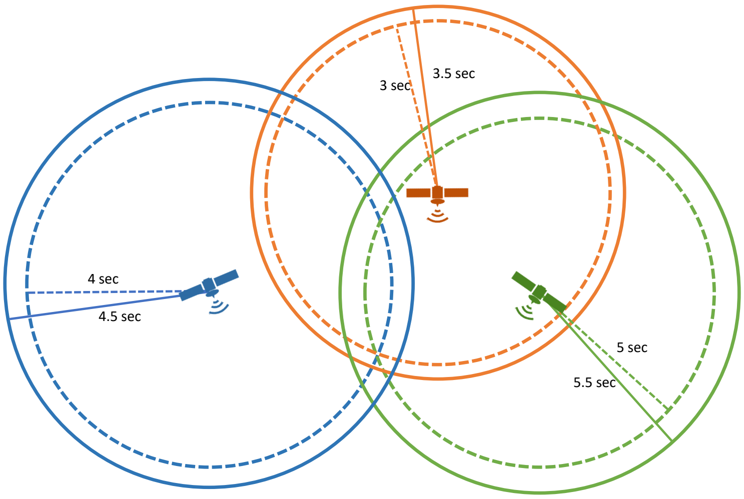


Figure 2.5: Solve for Position and Clock Bias using Trilateration



2.3.1 Nonlinear Least Squares

There may be more than four satellites in view, which creates an overdetermined system. There is not exact solution when the system has error, so it is solved via non-linear least squares (NLLS).

$$\rho_i = \sqrt{(X_{SV_i} - x)^2 + (Y_{SV_i} - y)^2 + (Z_{SV_i} - z)^2} + cb_u + \epsilon \tag{2.1}$$

Where ρ_i is the pseudorange of the receiver to satellite i, $[X_{SV_i}, Y_{SV_i}, Z_{SV_i}]$ is the position of the satellite i, c is the speed of light, and ϵ is some measurement noise. NLLS is a standard algorithm that linearises about an estimate of the state variable

1. Assume an initial estimate of the system state
2. Linearise the system about the estimate by calculating the Jacobian
3. Calculate the residual between the estimated location and the measured ranges
4. Identify the residual due to the linearised system
5. Minimise the quadratic error to calculate the best linear step
6. Update the estimate using the linear step
7. Repeat until convergence

- NLLS solve spheres

- ECEF frame of reference

2.4 GPS Error Sources

There are many sources of error that plague the GPS data. These errors are categorised into six types; multipath, atmospheric effects, receiver noise, ephemeris error, clock bias and Sagnac effect.

2.4.1 Mutlipath Interference

Multipath is the case where the radio signal is reflected off objects before reaching the receiver, which increases the distance travelled. It is especially prevalent in built-up areas with tall buildings.

2.4.2 Atmospheric Effects

The distance from a satellite to a user is calculated by the time difference when the radio signal was sent and when it was received multiplied by the speed of light. However, the speed of light is reduced when in the atmosphere compared to that in space. The ionosphere is the upper layer of the atmosphere ranging from 50 to 500 km. It consists of ionized particles that create fluctuating electric fields in the atmosphere that perturb the radio signals that travel through it. This effect can be modelled but is still a significant source of error, approximately 5 m [9]. The troposphere is the lowest part of the atmosphere up to 50 km that varies in temperature and pressure with weather patterns. The radio signals are refracted through the medium, but as it is only for a short period of time this error is significantly less than that of the ionosphere at approximately 0.5 m error [9].

2.4.3 Receiver Noise

In any electronic components, especially low cost components, thermal noise introduces error into the system. Thermal noise is caused by the random motion of electrons in conducting materials. The construction of the electrical components are not identical, which also cause slightly different solutions from receiver to receiver. This can contribute approximately 0.3 m of error [9].

2.4.4 Ephemeris Error

The ephemeris error is the error in the navigation data describing the orbital parameters of the satellite [6]. This information is approximated and updated every 2-4 hours by the control segment to minimise the error. The approximation is based on a prediction model of the orbital parameters, as there are many forces that act on a satellite that can alter the orbit. These include gravitational affects of other masses in the solar system such as the Moon, the Sun, even Jupiter and the outer planets affect the gravitational potential of objects in orbit around Earth. The non-spherical Earth, solar radiation pressure, slight atmospheric pressure are all forces that manipulate the orbit. The satellites also undergo station keeping manoeuvres to manage the orbit, which the requires the ephemeris data to update. This inaccuracy contributes about 2.5 m in error [9].

2.4.5 Clock Bias

The clock on the satellites and receivers are not exact with the true GPS time. Low-cost receivers are built with cheap quartz crystal oscillators that keep time to $1\mu s$ accuracy. The clocks on the satellites however are atomic clocks that have accuracy on the order of $1ns$. In addition to this, ground control can measure the satellite clock bias and send back that information to be stored in the navigation message. [13]

2.4.6 Sagnac Effect

The sagnac effect is a more intrinsic source of error. It is due to the rotation of the Earth during the time of the signal transmission. The transmission time is between 0.06-0.08 seconds in which time the Earth has rotated approximately 30 m. If the ephemeris data was in an inertial frame (ECI) there would not be a problem. However, the data is in the Earth-Centred Earth-Fixed (ECEF) frame which is a frame that

Table 2.1: Categorisation of Errors

Satellite	Receiver	Uncorrelated
satellite clock bias ionospheric delay tropospheric delay ephemeris error	receiver clock bias receiver noise	multipath Sagnac

rotates with the Earth to allow users to calculate their positions independent of time [3].

2.4.7 GPS Error Summary

All of the errors mentioned above can be categorised into three types; satellite correlated, receiver correlated, and uncorrelated errors. That is, a particular type of error may be consistent between all receivers from a particular satellite, or consistent between all satellite measurements for a particular receiver, or neither. See Table 2.1.

2.5 Multiple Receivers

- problems arising with multiple receivers

A single sample of pseudoranges from any two receivers will not be taken at the exact same time without a connecting network to implement control. This means that the satellite positions at the time of each signal transmission will be actually different. The primary issue with calculating the change in pseudorange is identifying the transmission time

2.5.1 Align Reception time

In [3], they align the reception time of multiple receivers

They precisely align the epoch by accounting for the differing Sagnac effect between two receivers and accounting for the clock biases of the receivers. The Sagnac effect manifests in the multiple receivers case by the signal propagation time for a measurement taken at t_2 would be different than if it was taken at t_1, the Earth will have rotated by different amounts.

1. The clock bias is calculated by solving for the individual absolute position of a receiver using least squares. This is necessary in order to

2.

2.6 Current GNSS algorithms

- just reference implementation papers? - algorithms to make it more accurate
- use for motion tracking
- performance vs cost trade off
(http://ieeexplore.ieee.org.ezproxy1.library.usyd.edu.au/document/7530542/)

2.6.1 Standard Positioning Service

The Standard Positioning Service (SPS) is the default system that provides PNT signals as described in Section 2.2 for free to civilian, commercial and scientific uses worldwide. It has been operational since 1993 and has minimum performance commitments of 3 meters horizontal accuracy and 5 meters vertical accuracy as of 2007 which was the latest edition of the Performance Standard USA Department of Deference issued [8]. The GPS receivers use trilateration and solve for position and time with NLLS algorithm as described in Section 2.3.1.

- single frequency and multi frequency to remove atmospheric affects

Currently the DOD is code phase

2.6.2 Carrier Phase Solution

2.6.3 Duel Frequency Precise Point Positioning (DF-PPP)

The ionospheric delay is one of the largest sources of error in the system. The electrical interference is diffusive, and is dependent on the frequency of the signal. The two signals are used to remove the ionosphere delay then combined to solve for the carrier phase ambiguities. This system can produce centimeter or decimeter accuracy but requires 20-40 minutes to converge to that accuracy. Also the duel frequency receiver costs more than single frequency receivers [2].

2.6.4 Differential GPS

Differential GPS (DGPS) is a way to correct satellite correlated errors by using a stationary receiver in a well known location. It ties the satellite measurements into a local reference by solving for the reference receiver's position in reverse. That is, it solves for the timing errors in the satellite signal as it knows what position it should be in. The stationary reference receiver then broadcasts the error correction information to any roaming receiver in its vicinity.

- explain what it is
- what setup is required
- abs vs rel
- degree of accuracy

Table 2.2: Error Components and Potential Improvements for SF-PPP

Error component	**Potential Improvement**
Ionosphere: Klobuchar model	7 m
Troposphere: Saastamoinen model	2.5 m
Ephemeris data	1 m
Satellite clock drift	1.5 m
Differential code bias	50 cm
Phase windup: rotation of the antenna	dm
Sagnac effect	30 m
ROA: satellite orbit correction	up to 10 cm
Relativistic clock correction	up to 21 m
Moon-Earth interaction	5cm (Hor) and 30 cm (Ver)

2.6.5 WAAS DGPS

2.6.6 Post Processing Algorithm

2.6.7 Single Frequency Precise Point Positioning (SF-PPP)

Rademakers how to say reference? at University of Delft in the Netherlands developed a solution for finding the absolute position in open areas to a horizontal accuracy of 0.5 m. It uses a single frequency, single antenna low cost GPS receiver by connecting to the internet and using real time information to model all errors. The errors they corrected with the potential improvements are outlined in Table 2.2.

2.6.8 Duel-Epoch, Double-Differencing Model

In the paper by the Institute of Software Integrated Systems, Vanderbilt University called *High-Accuracy Differential Tracking of Low-Cost GPS Receivers*, Hedgecock and party developed a new algorithm for relative motion tracking for multiple receivers. They used low cost GPS receivers with access to raw measurement data to produce centimeter-scale tracking accuracy. Each receiver was shared the whole networks data and ran the localisation algorithm independently to avoid having a single point of failure.

The algorithm uses the change in carrier phase through time of each receiver to estimate the change in relative ranges between a satellite and two receivers. It does not require a reference satellite, a reference node or an integer ambiguity solution. It does require the clock bias for each receiver at each point in time as solved for by non-linear least squares for the absolute position before running the algorithm itself. To reiterate, it does not directly solve for the relative position but the relative motion. However, neither of the initial positions of the receivers need to be precisely known in order for the relative motion to be accurate. Due to the time dependency, consistent satellite locks of at least four satellites are required, otherwise reinitialisation must occur. The calculated change was projected onto the unit direction vector from receiver to satellite. The system of these tracking equations was solved via least squares optimisation.

It uses the assumption that all satellites in the constellation are such a great distance from the surface of the Earth that the unit vector from both receivers are parallel to each individual satellite, as long as it is in the same geographical region. How far apart the receivers can be for this assumption to hold was not stated.

- how many receivers?
- why and how it aligns epoch
- uses difference in time for a single receiver to find change in motion.

- have this one last as it is the most similar
- needs instantaneous relative distance for first point, to speed up processing and make the first few time steps more accurate, also when locking onto new satellites

2.6.9 Summary of Algorithms ?

- dynamic tracking (need temporal measurements) vs static measurement - no temporal
- post processing vs pre-processing vs realtime
- ground structure vs free standing
- absolute vs relative
- accuracy (how much)
- computation time/space required
- what error is each method removing
- what piece of data it needs (if raw)
- calibration required
- robustness -> if a satellite goes out of view does it need to re-calibrate? passing information between receivers-> is one a reference? single point of failure

2.7 Moving Forward

GPS is currently going through a major upgrade, with the launch of new satellites with alternate structuring of signals that will provide greater accuracy for civil use in the future. The constellation Galileo that is currently being developed and launched, is expected to be finished by XX. Both the new GPS and Galileo are designed to be able to operate in tandem to provide an *unprecedented* accuracy with a total of 57 operational joint satellites, while still having the individual capability of global coverage. Some technology today uses both GPS and GLONASS constellations but requires duel receivers as the signals are encoded differently. GPS uses code division multiple access (CMDA) whereas GLONASS uses frequency division multiple access (FMDA). Research is also being conducted on the sources of error in the radio signals. The errors are to be modelled and the appropriate adjustments stored in the encoding of the signal. Through the international cooperation efforts and research, localisation and tracking using baseline GNSS will continue to improve, regardless of alternate data processing algorithms. For now, improved signal accuracy will obviously improve the performance of the different data processing algorithms. However, there will come a time where the extra computational and hardware complexity will not be worth the small gain in accuracy over the default performance of the low-cost civilian GNSS receiver.

Chapter 3

Algorithm Solution

The common theme throughout the literature is to use differencing to minimise errors, both through time and space.

For a single receiver, differencing through time

For a single receiver differencing through space, a second known location is still required. This is set up externally and services a set area.

However for multiple receivers, spacial differencing is available. Even though with multiple receivers there is the added pre-processing with the sending of data between nodes and computation for epoch alignment. When differencing through time, the epoch alignment is required at the receiver end. That is, to manipulate the measurements as if the receivers took readings at the same time. Depending on how precise the alignment required for a system is, the computation time may require a numerical iteration solution.

When double-differencing, that is for both time and space for a single receiver.

When differencing through space between the receivers, the epoch alignment is required at the satellite end. The time the satellite sent the - make some ideas for when the receivers are moving - your giving the time of the positions between the receivers as different

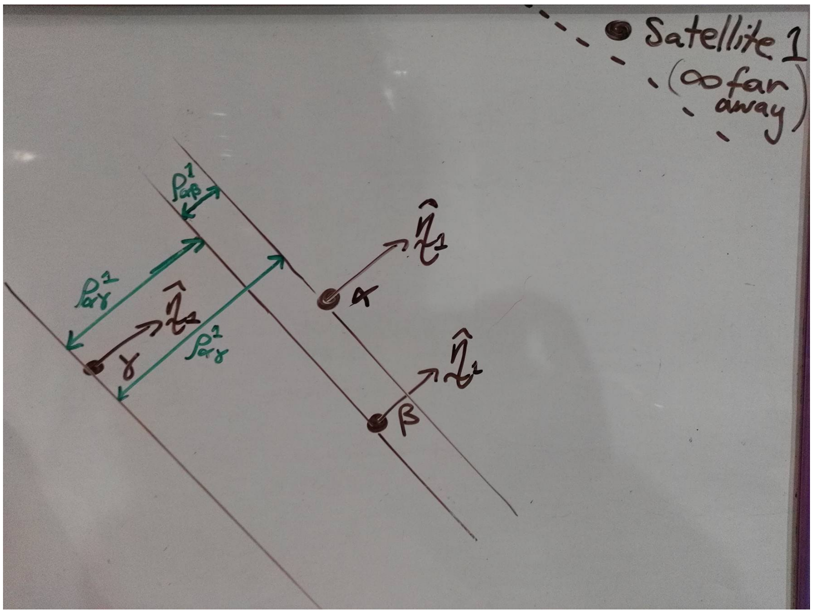
3.1 Proposed Planar Intersection Algorithm

- Fit better in the next chapter, put in references to literature - these are assumptions, heres why, rationale, these ppl also did it

Following the literature, the two main options for increasing the accuracy is extensive modelling of the errors, or some type of differencing algorithm. Error modelling requires external hardware, internet connection and extra computation time which increases the budget requirements. Whereas

This new algorithm is derived from taking the difference in pseudorange between multiple receivers from one satellite and expressing the distances as planes.

Figure 3.1: 2D representation

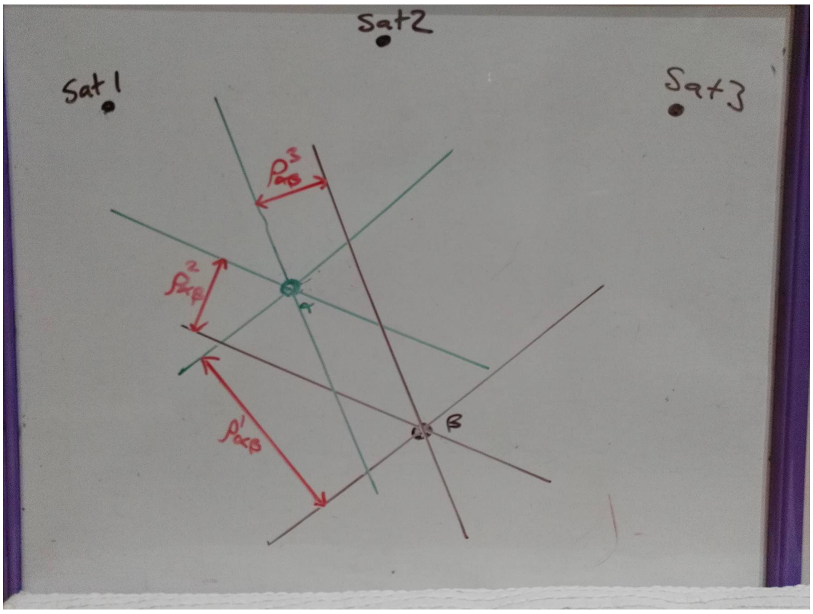


With multiple satellites in view, the intersection of planes for a particular receiver is the position of the receiver.

With this strategy, some of the errors that plague the absolute position are negated for the relative position. Eq(3.1) describes the measured pseudorange for a single receiver ω from a single satellite s.

$$\rho_\omega^s = R_\omega^s - cb_\omega + c(T_s + I_s + \nu_s + b_s) + \epsilon \tag{3.1}$$

Figure 3.2



Where R^s_ω is the real range with the following sources of error; b_ω is the receiver clock bias, T_s is the tropospheric error, I_s is the ionospheric error, ν_s is the relativistic error, b_s is the satellite clock bias and ϵ is random noise. The satellite correlated effects are removed with the individual receiver clock biases, random error and true range difference, Eq(3.3).

$$D^s_{\omega_i\omega_j} = \rho^s_{\omega_i} - \rho^s_{\omega_j} \tag{3.2}$$

$$D^s_{\omega_i\omega_j} = R^s_{\omega_i} - R^s_{\omega_j} - c(b_{\omega_i} - b_{\omega_j}) + \epsilon \tag{3.3}$$

This chapter explains the assumptions and details the mathematics behind the planar algorithm.

3.2 Assumptions

All the assumptions are based on each other so need a pre-statement

3.2.1 Approximate Global Location

An approximate location is required for where the system is on the Earth within 1 km of all of the receivers. This is to calculate the normal direction vectors to each of the satellites. A simple solution to this is to set the reference receiver to calculate it's absolute position first using NLLS. This will give the system a reference of within 20 m at worst, well within the acceptable parameters.

3.2.2 Static Receivers

All receivers are assumed to be static for the time in between all receivers to get a sample reading. This makes for an easier transform to align the satellite positions to a common epoch. It also ignores the problem of how the pseudorange from each receiver would be sent to either all of the receivers or to a central device for computation and the time delay associated with that.

For the dynamic case, it is likely that the data would be a part of a system containing other signals that describe the motion such as dead reckoning. In that case, the location of the receiver at the common epoch time can be back calculated to minimise the dynamic error. The incorporation of moving receivers is an area to explore for future work on the algorithm. However, there are existing algorithms in the literature XX where the relative motion is tracked with sub meter accuracy that may be more appropriate for complex dynamic systems.

3.2.3 Parallel plane assumption

It as assumed for the plane equations that all receivers point to a satellite along the same vector. This is valid for a dispersion of receivers for 10km for an error of XX. This is synonymous to if the satellites were at infinity and all the receiver vectors are parallel to a satellite

$$
\begin{aligned}
\delta &= \tan^{-1}\left(\frac{d}{a}\right) && (3.4)\\
e &= 2d\tan\delta && (3.5)\\
(3.4)\&(3.5) \Rightarrow e &= \frac{2d^2}{a} && (3.6)
\end{aligned}
$$

Where a is the altitude, d is the distance between two receivers and e is the error in the plane created. The worst configuration for error in the vector normal to the plane is if the satellite is directly above the receivers at the smallest distance from the Earth in orbit, $a > 20000\ km$. For d=5 km the perpendicular error is 2.5 m

3.3 Epoch Alignment

A single sample of pseudoranges from any two receivers will not be taken at the exact same time without a connecting network to implement control. This increases the setup requirements which in turn increases the cost of the system and reduces the ease of compatibility with different receivers. The earliest time between all the receivers will be used as the time reference point called common epoch. The satellite position in the future time steps were backcalculated to find the difference in the pseudorange. The time between receivers would be a maximum of one second, as the slowest sampling time is typically 1 Hz. This extra distance is only in the vacuum of space and is not affected by potential nonlinear affects such as ionosphere and troposphere errors that affect the speed of light.

3.3.1 Satellite Time

The known variables from the raw GPS data is the time the sample measurement was taken t_ω and the pseudorange to each visible satellite ρ^1_ω for each receiver ω.

The error in the normal vector due to the time difference is

By the distributive law

$$\hat{\eta} \cdot s + \hat{\eta} \cdot v = |\eta| \tag{3.7}$$

$$\begin{aligned}
\Delta\rho &= |\eta| - |v| && (3.8)\\
&= \hat{\eta} \cdot s + \hat{\eta} \cdot v - |v| && (3.9)\\
&= \hat{\eta} \cdot s + |\hat{\eta}||v| \cos(\theta) - |v| && (3.10)
\end{aligned}$$

As |v| is >20 000 km and s is <4 km, $\theta \approx 0$ which means $\cos\theta \approx 1$. The magnitude of a unit vector is one therefore (3.10) simplifies to

$$\Delta\rho = \hat{\eta} \cdot s \tag{3.11}$$

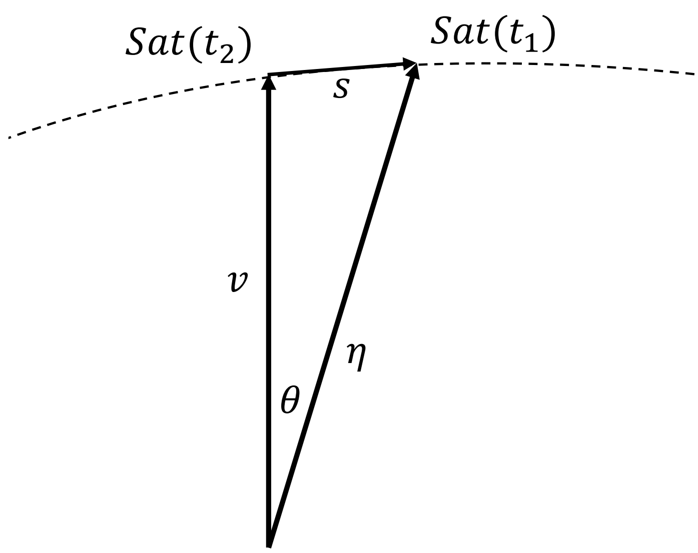
The vector s is the transform of the satellite from time t_2 to the common epoch time t_1. It is calculated using the

$t_1 = distance/cforalphareceiver$ $t_2 = t_{sampleoffset} - |v|/c$

The vector v is the measured pseudorange of a receiver from a particular visible satellite at t_2 with an unknown direction vector. The direction vector is unknown because the position of the receiver is what is being solved for. The vector η describes the known direction vector from the single approximate location of the geographic region to the common epoch of a particular visible satellite. It is the magnitude of η that describes what the pseudorange would have been if the measurement was taken at the common epoch. The difference between the $|\eta| - |v|$

Because of the asynchronous time, some of the satellite correlated errors would not correlate as well as if it was from the same point in time. This is because the path through the ionosphere and troposphere is not exactly the same.

Figure 3.3: Vector Epoch Synchronisation



3.4 Planar Intersection Algorithm

https://www.e-education.psu.edu/geog862/node/1759 - errors in pseduorange

http://www.insidegnss.com/node/2898 - how to get pseudorange from raw data

Select reference receiver The receiver α is used as the reference location and common epoch time. The reference location is calculated by performing NLLS on a receiver to calculate the absolute position for one point in time. Any receiver can be used as the reference point. For a real world implementation, all of the raw data can be sent to each node and it uses itself as the reference point.

Collect data of one timestep from all receivers The raw data as well as the estimated absolute location and clock bias (what frame of reference is this?) from non-linear least squares optimisation is collected from all GPS receivers.

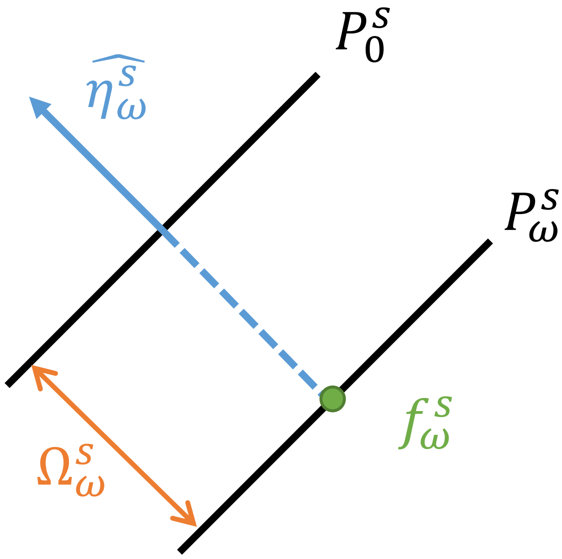
Align to reference Epoch time

Average normal Vector Find the average normal vector pointing to each satellite $\hat{\eta}_s$ from the receivers. The normal vector is calculated by using the position all of the satellites in view at the common time t_α as previously transformed in 3.4 and the estimated absolute position of all receivers. The average for each satellite is calculated by taking the mean across all receivers.

Difference in Pseudorange The differences in pseudorange are calculated Ω_ω^s in the ECEF frame where s is the satellite, ω is the receiver.

Create Planes Sets of planes are created for each receiver ω. The equation of a plane is $Ax + By + Cz + D = 0$ where the coefficients [A,B,C] describe the normal vector of the plane and the coefficient D sets the plane in 3D space along the vector,

Figure 3.4: Construction of Planes



Eq(3.12). As the normal vector is already calculated for each satellite, only the D coefficient must be solved for each receiver and satellite pair.

$$P_\omega^s \quad : \quad (\mathbf{i} \cdot \hat{\eta_s})x + (\mathbf{j} \cdot \hat{\eta_s})y + (\mathbf{k} \cdot \hat{\eta_s})z + D_\omega^s = 0 \tag{3.12}$$

Solve for D The coefficient D is calculated by finding a point on the plane f_ω^s, then substituting it into Eq(3.12) for x,y,z; Eq(3.14). The point of the plane is calculated by moving along the normal vector by the difference in pseudorange from the reference point Eq(3.13), see Figure 3.4.

$$f_\omega^s \quad = \quad -\Omega_\omega^s \hat{\eta_s} \tag{3.13}$$

As the algorithm is solved in 3D space relative to the reference receiver (the origin is the reference receiver), the coefficient D is actually the measured difference in pseudorange Ω_ω^s. The following is the proof.

$$\begin{aligned}
\hat{\eta_s} \cdot f^s_\omega + D^s_\omega &= 0 &(3.14)\\
D^s_\omega &= -\hat{\eta_s} \cdot f^s_\omega\\
D^s_\omega &= -\hat{\eta_s} \cdot (-\Omega^s_\omega \hat{\eta_s})\\
D^s_\omega &= \Omega^s_\omega ||\hat{\eta_s}||^2\\
as\ ||\hat{\eta_s}|| &= 1\\
D^s_\omega &= \Omega^s_\omega &(3.15)
\end{aligned}$$

Solve for Intersection Four variables for each receiver must be solved for $X_\omega = [x, y, z, \tau]^T$. The vector X_ω describes the position of receiver ω in NED coordinates and τ_ω describes a final receiver clock bias that alters the displacement of all the planes in the set P_ω by the same parameter.

In order to solve all of the receivers with the least amount of error in the whole system, all of the position vectors X_ω are solved at the same time. The reference planes of α must be included as a constraint on the system. All of the clock biases are also constrained with the clock bias from τ_α, see (3.16). The receiver clock bias only affects the equation of the planes by altering the constant as a change in the pseudorange has no affect over the angle of the plane. Each receiver clock bias alters all the planes associated with that receiver proportionally.

$$(\mathbf{i} \cdot \hat{\eta_s})(x - x_0) + (\mathbf{j} \cdot \hat{\eta_s})(y - y_0) + (\mathbf{k} \cdot \hat{\eta_s})(z - z_0) + (\tau_\omega - \tau_0) = -\Omega^s_\omega \quad (3.16)$$

In matrix form, one

$$[\hat{\eta_s}, 1] \times [X_\omega - X_0] = -\Omega^s_\omega \quad (3.17)$$

The matrix N (4 x n) is describes the set of normal vectors to all of the satellites

$s \in \{1...n\}$

$$N = \begin{bmatrix} \hat{\eta}_1 & 1 \\ \hat{\eta}_2 & 1 \\ \vdots & \vdots \\ \hat{\eta}_n & 1 \end{bmatrix} \tag{3.18}$$

G_ω is a n x 1 matrix that contains all of the difference in pseudoranges between the reference receiver and the receiver ω for all satellites $s \in \{1...n\}$

$$G_\omega = \begin{bmatrix} \Omega_\omega^1 \\ \Omega_\omega^2 \\ \vdots \\ \Omega_\omega^n \end{bmatrix} \tag{3.19}$$

There are m+1 receivers in the system, therefore the total set of distances encompasses G_ω for $\omega \in \{1...m\}$ where G_0 is the empty set that constrains the position of the reference receiver and constrains the system to the relative frame.

$$\begin{bmatrix} N & \{0\} & \dots & \{0\} \\ -N & N & \{0\} & \dots \\ \vdots & \{0\} & \ddots & \{0\} \\ -N & \{0\} & \dots & N \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_m \end{bmatrix} = \begin{bmatrix} G_0 \\ G_1 \\ \vdots \\ G_m \end{bmatrix} \tag{3.20}$$

$$\Phi\chi = \Gamma \tag{3.21}$$

As the number of satellites each receiver can see can be 4 or more, the system is overdetermined. An overdetermined system is solved by least squares. That is, there is a solution that minimises the residuals $||\Phi\chi - \Gamma||^2$ by using the pseudo-inverse of Φ:

$$\chi = (\Phi^T\Phi)^{-1}\Phi^T\Gamma \tag{3.22}$$

Limitations The pseudo-inverse requires that the matrix equations be linearly independent. Therefore the satellite configuration cannot have two satellites at the same location. Also, all of the satellites cannot have the exact same elevation or azimuth as the inverse approaches singularity which introduces error based on the mathematical instability. In reality it is extremely unlikely that these configurations would occur, but it does effect the satellite configurations analysis in the next Chapter.

Chapter 4

Experiments That Prove Things

- properties of the simulation then show results

4.1 Simulation Creation

4.1.1 Creating a Simulation

Matlab was chosen as the platform to simulate and evaluate the program due to the ease of matrix manipulation and graphical interaction.

Simulate Satellite Locations

Using real ephemeris data for the GPS constellation, the location of all of the satellites were calculated, see Table 4.1. The approximate reference location of the network of receivers was inputed into the program as longitude, latitude, height geocentric coordinate system. The satellite positions were transformed to the local tangent plane of the reference location in polar coordinates. The satellites that had an elevation of above 12deg were selected as potential visible satellites. This lower elevation limit was selected to minimised multipath effects that would likely occur at ground level REF.

Table 4.1: Ephemeris data for GPS constellation

Orbital parameters	Value

True Location of Receivers

The dispersion of receivers from α is a variable to the program. The actual displacement is calculated by multiplying the dispersion magnitude by a uniformly distributed random vector ranging from [0,1] in three dimensions for all remaining receivers in North-East-Down (NED) frame localised at α. The positions were then transformed to the global ECEF frame.

Calculation of Pseudorange

In ECEF frame, the instantaneous distance between each visible satellite and each receiver was calculated. The errors were simulated by adding random distance proportional to error models in the literature to each individual satellite, see Table 4.2. The errors followed the structure in Eq(4.1)

For the error analysis, the errors associated with the pseudorange were divided into three sections; satellite correlated errors, receiver correlated errors and random errors. For the satellite correlated errors, the same amount was added to the true range for each receiver from a single satellite. For the receiver correlated errors, the same amount was added to the true range for each satellite from a single receiver. All errors were an addition to the pseudorange, so that how each type of error affected the algorithms

$$Error(seconds) = magnitude + 10 \times magnitude \times rand(elements) \tag{4.1}$$

Where *rand* is an inbuilt matlab function that returns a value on the domain [0,1] in a standard uniform distribution.

Table 4.2: Magnitude of simulated errors

Error Source	meters

Planar Algorithm

The normal vector was

- what data is it using from receiver? psudorange, time

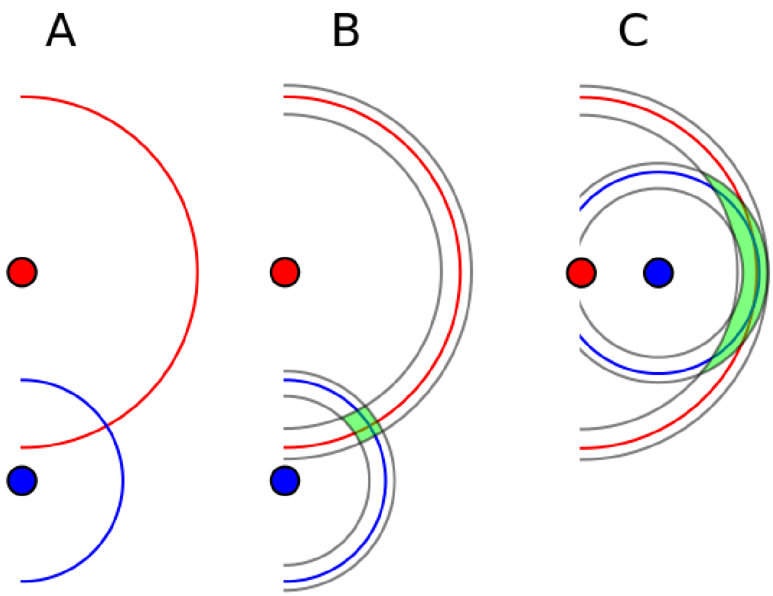
- what errors to include and how to incorporate into the simulation.
- how to include the different(asynchronous) time received for all receivers-> is for the one receiver
- how extra receivers affects computational time/ accuracy
- how number of sats affect comp time/accuracy
- configuration of sats
- large multipath affects
- no receiver sees the same sat? - does it just output the relative difference between abs values? -> incorrect? just have it fail? not actually implementing, can control the environment
- distance of receivers apart
- configuration of receivers

- what data received and how to simulate misaligned timing between receivers
- what magnitude are the errors and how to simulate them
- simulate the errors individually (to see how each type affects the sim - convergence time and accuracy) and/or all errors at once

4.1.2 Evaluation

The expected error due to the assumptions were outlined in Section XX. To evaluate the parallel plane assumption, two receivers were a set distance apart.

Figure 4.1: Geometric Dilution of Precision



Dilution of Precision The dilution of precision (DOP) is a measure of how the geometry of the satellites affect the position measurement precision. It is used in GNSS to evaluate how much error there may be in a measurement. Figure 4.1 shows two different configurations of satellites in the 2D circular solution case. With some error bounds as shown in B and C, the solution can lie anywhere in the green area. Due to the geometry configuration of the satellites in C, the area is considerably larger even though the error bounds on the signals are the same.

- fake gps data
- how to simulate noise - what level SNR
- to calculate your own GPS location using the normal algorithm? - space 3
- use real GPS locations? (and through time) -space 3

How to evaluate?: - accuracy in relative space

- compare to just taking differences in absolute position
- between individual receivers and the total error in the whole system
- markov? error analysis -> cannot do precision without statistical analysis but isolate

errors in x,y,z. how much worse is z than horizontal?
- computational time-> how does more receivers/satellites affect the comp time -> what time and space complexity?

4.2 Evaluation

Base Evaluation In NED frame, have two receivers - vary the distance by 0.1m, 1m, 10m ... 100km - set it as x, y, z for each distance and satellite configuration - Use 'fake' satellite positions? Sat configs: - only sats in one orbit - 1D - cluster directly above - cluster at the edges - sats in the same plane as the difference in receivers -

- how close the 'approximate location' setting is to the alpha receiver

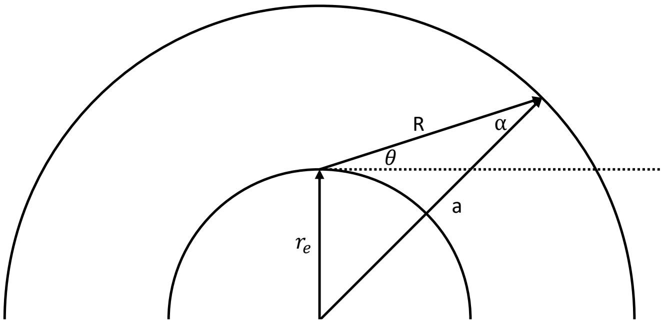
- what affect random errors have on the accuracy - use a time scale

- vary number of satellites in view
- vary GDOP (good GDOP and bad)
- when receivers don't see the exact same satellites
- vary number of receivers
- simulate a multipath error and how does it account for it or how much error does it introduce

Fake Satellite Positions

It is assumed that the orbits are circular, this is valid as the eccentricity (e) of the GPS satellites are all $e < 10^{-3}$. The eccentricity is a measure of how elliptical an orbit is; where e = 0 is a perfect circle and e = 1 is a parabola. It is also assumed that the Earth is circular and has a radius of $r_e = 6.378137 \times 10^6 m$.

Figure 4.2: text



$$\sin\alpha = \frac{r_e \sin(\pi/2+\theta)}{a} \tag{4.2}$$

$$R = \frac{a\sin(\pi-(\pi/2+\theta)-\alpha)}{\sin\theta} \tag{4.3}$$

$$R = \frac{a\cos\left(\theta+\sin^{-1}\left(\frac{r_e\cos(\theta)}{a}\right)\right)}{\cos(\theta)} \tag{4.4}$$

Some configurations will not reflect what can be achieved in our current reality but may be able to in the future. The European GNSS constellation *Galileo* will have 30 satellites operating in tandem with GPS. There are also the constellations GLONASS and Beibo the Russian and Chinese constellations that there may be international cooperation in the future to have all the GNSS constellations operating together in a high density system.

4.3 Planar Assumption Analysis

4.3.1 Setup

These results had no error induced in the measurements in order to analyse the effectiveness and robustness of the algorithm. No epoch alignment and no correlated or uncorrelated errors. Only two receivers were used with the reference receiver exactly at the approximate location. The second receiver was set at varied configurations of a set distance north, east and down of the reference receiver. The set distance between the receivers was varied from 1 mm to 100 km. As there are no randomly induced errors, there is no statistical analysis. The algorithm was also adjusted to not solve for a clock bias variable to see how the least squares solution adjusted the distance between planes without error in the system.

The satellites were configured in a small cluster to analyse how the position of the satellites relative to the configuration of the receivers affected the plane assumption. The total error and the individual component error were calculated to identify what component produces the most error for the configuration.

This analysis was not required to be compared to NLLS as there was no error and it would solve the system perfectly to mathematical precision.

4.3.2 Discussion

See Figure 4.3, two receiver configuration with one receiver at the approximate location and one receiver at varying distance along the north direction. The cluster of satellites were set perpendicular to the receiver configuration as it was predicted to be a configuration with less error. This isolated the error due to the plane assumption. The algorithm was compared to solving the same plane intersections but without the clock bias variable. In Figure 4.3, for distances < 1 meter, the planar algorithm solution has a consistent error two orders of magnitude larger than the solution without the clock bias. This indicates that without errors, the least squares matrix adjusts the distance between planes proportionally but will introduce errors. These errors however are on the order of micrometers (10^{-6}) and will be below the noise floor with any error in the system.

The results show that the least squares setup by constraining to the reference receiver is successful as the error is not proportional to distance below 10 m. Above 10 m the error increases for both the solutions with and without the clock bias, indicating that the error is due to the planar assumption. The slope has a gradient of $m = \log(10^4)/\log(10^2) = 2$ which means the error is proportional to the square of the distance. This was to be expected as in reality, the distance from the satellites are spheres, as stated in Eq(3.6). However the magnitude of error was 1 m at 10 km distance, which for localised systems is acceptable.

The individual component error shows where the error has come from. In Figure 4.3, it the error in the north component is much smaller than the error due to the east or down component. This is based on the satellite configuration relative to the receiver configuration, however in this case it can be misleading. As the satellite cluster is directly east of the reference receiver at a high elevation, the error due to the plane assumption bends towards the east and up. This is also evident in Figure 4.4 where the satellite configuration is directly north of the system. The east direction has the least amount of error for the same reason. A full analysis of the effect of the satellite configuration was conducted for each two receiver configuration.

The total error based on satellite configuration shown in Figure 4.5 for Northerly displaced receivers, Figure 4.6 for East and Figure 4.7 for Down. Note that the minimum for the Down configuration was 2 m whereas the minimum for the horizontal was 0.5 m. The down vector will inherently will have more error as the sphere curves up away from the plane assumption, however it is still in the same magnitude. It is due to the nature of the system, that the satellites are above the Earth, that PNT has less accuracy in the vertical plane than the horizontal plane as stated by the SPS Performance Standard [8]. The north and east configurations are the same but 90° out of phase. A full breakdown of which component causes the error is shown in Figure 4.8. High elevation causes the most error for both configurations, due to the down component, see (e) and (f) of Figure 4.8. The large error along the axis of

Figure 4.3: Error of Plane Assumption vs Distance along North vector. **Top Left**: Receiver configuration in NED of the approximate location. One receiver at the center and one along the north vector. **Bottom Left**: Satellite configuration. Cluster of 5 satellites around elevation 60° and azimuth 90°. **Top Right**: Magnitude of total error vs the distance between receivers with and without solving for the clock bias. **Bottom Right**: North/East/Down error components vs the distance between receivers.

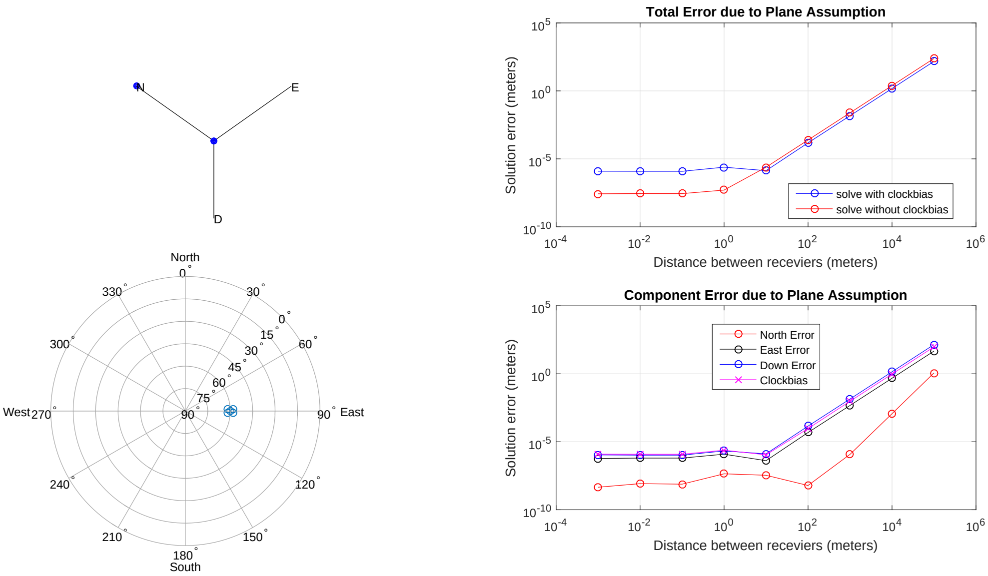
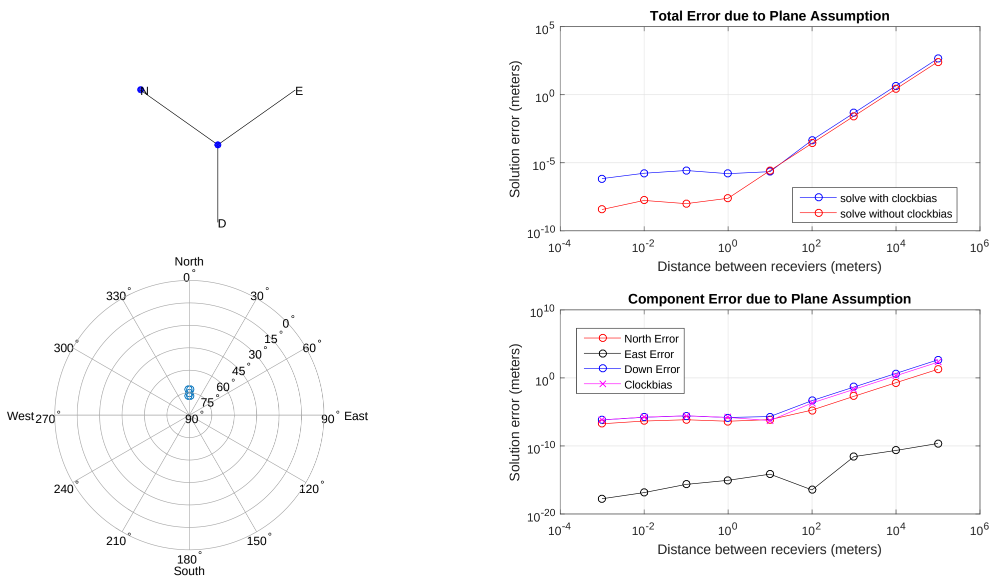


Figure 4.4



the horizontal displacement, that is north for north and east for east, was due to the same direction, see (a). The orthogonal direction contributed significantly less error.

Based on this information, a satellite configuration was created to minimise the error due to the plane assumption in order to explore other affects.

Figure 4.5: NORTH: Total Error based on satellite configuration for two receivers 10 km apart along North vector

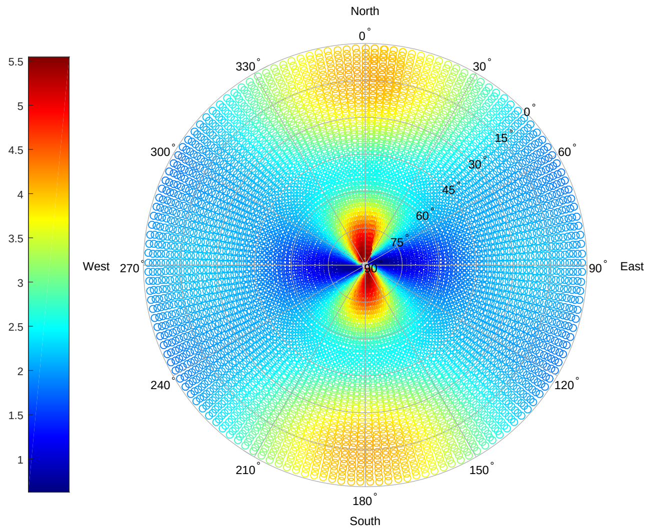


Figure 4.6: EAST: Total Error based on satellite configuration for two receivers 10 km apart along East vector

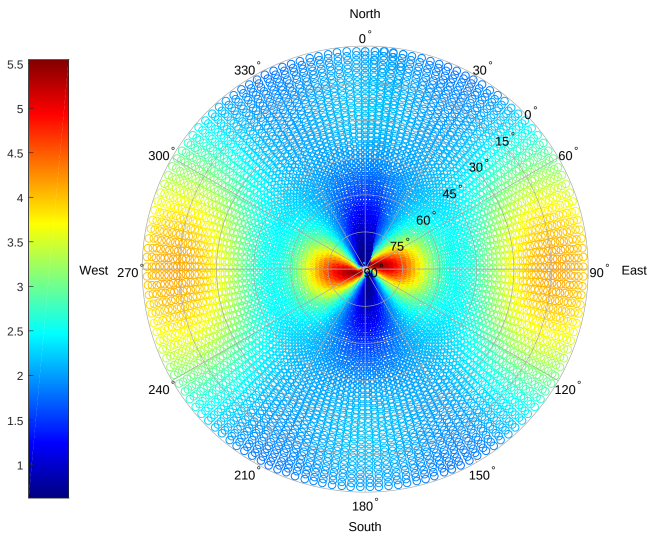


Figure 4.7: DOWN: Total Error based on satellite configuration for two receivers 10 km apart along Down vector

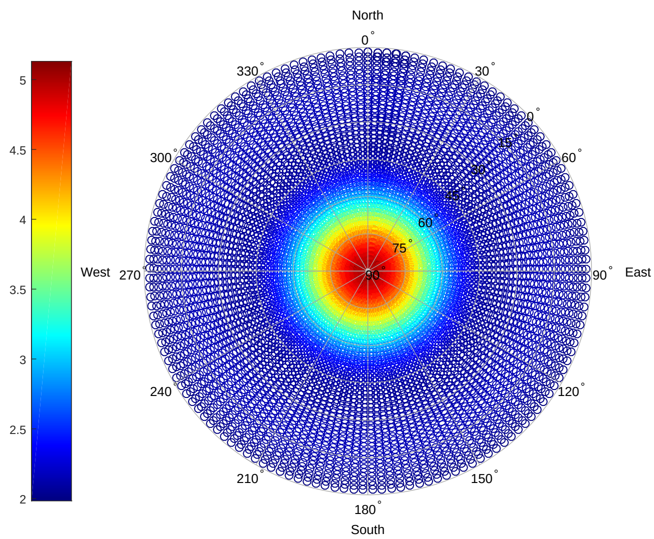
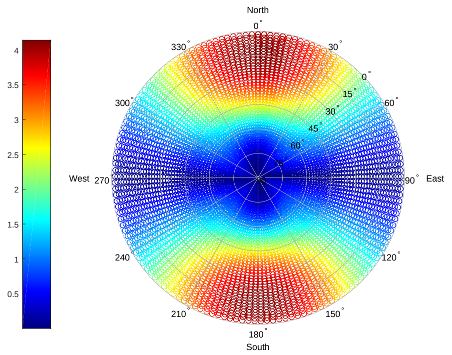
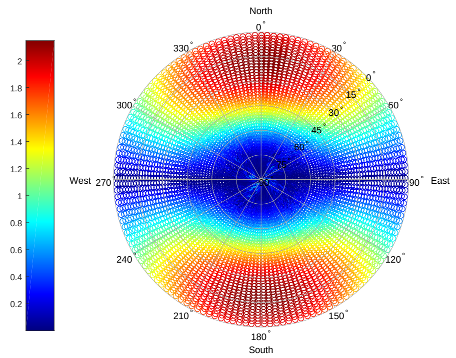


Figure 4.8: Full breakdown of component error based on satellite configuration due to plane assumption (Note that not all the scales are the same but all are in meters)

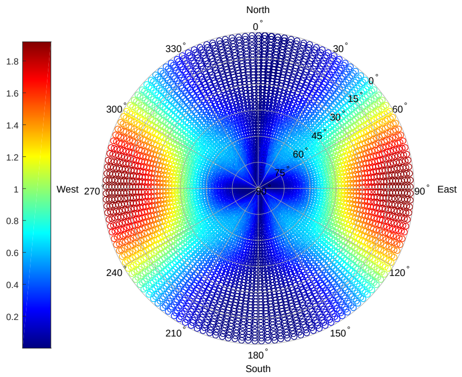
(a) Config:North, Error:North



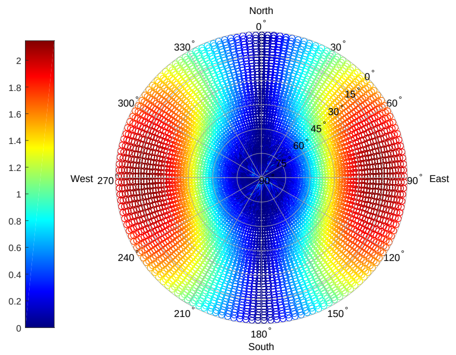
(b) Config:Down, Error:North



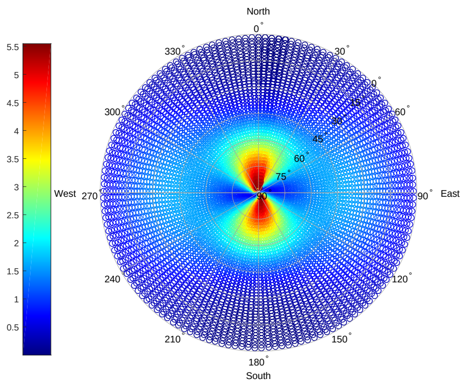
(c) Config:North, Error:East



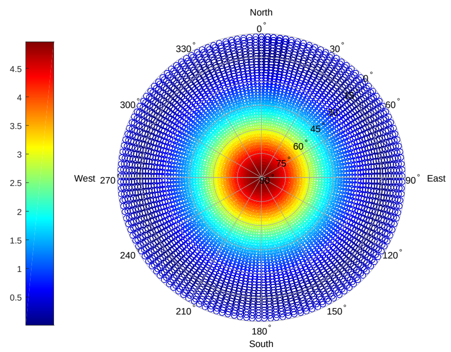
(d) Config:Down, Error:East



(e) Config:North, Error:Down



(f) Config:Down, Error:Down



4.4 Incorporating Error

The error was systematically added in a randomly uniform fashion in three types of ways; satellite correlated error, receiver correlated error and random error. A statistically significant sample size of 100 iterations was conducted with the simulation using error created in Eq(4.1). The planar algorithm was compared to NLLS using the same errors for each iteration. The mean and standard deviation were calculated. The satellite configuration for this section was Figure 4.9.

The planar algorithm (PA) worked very well for satellite correlated errors, confirming the differential method of removing correlated errors, see Figure 4.10. The error for PA was consistent at 10^{-6} m, this lower limit was observed previously and is a product of the clock bias solution. Relative NLLS error increased linearly with the satellite correlated errors until it reached numerical instability.

PA also worked well at removing receiver correlated errors with an error of 10^{-6} m, however NLLS had better performance, see Figure 4.11. Large errors of $> 10^{-3}$ seconds, which corresponds to a pseudorange error of $10^{-3}c = 298\ km$ might have the planar assumption effecting the solution.

Uncorrelated errors had the worst affect on the system, see Figure 4.12. But PA performed just as well as NLLS with error proportional to $c \times t$. This was expected as PA is based on removing errors by linear difference.

Figure 4.9: An Ideal Satellite Configuration for Northerly Displaced Receivers

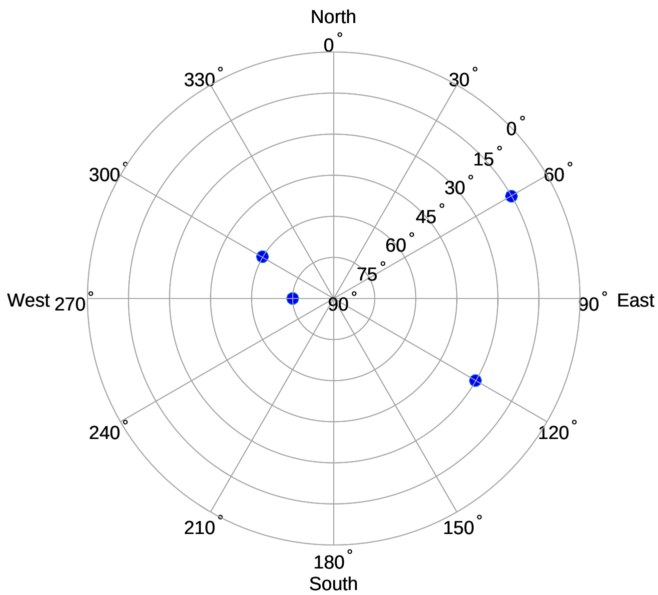


Figure 4.10

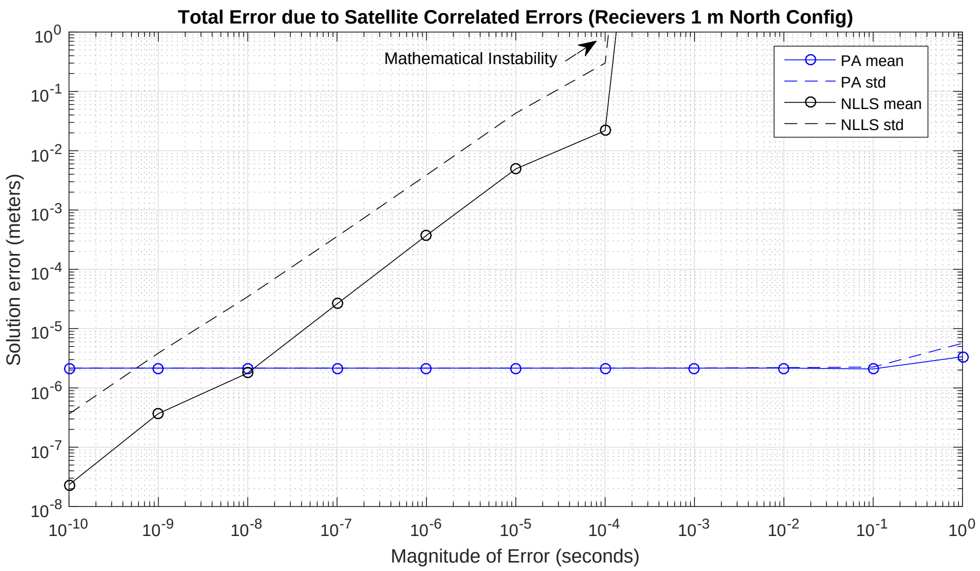


Figure 4.11

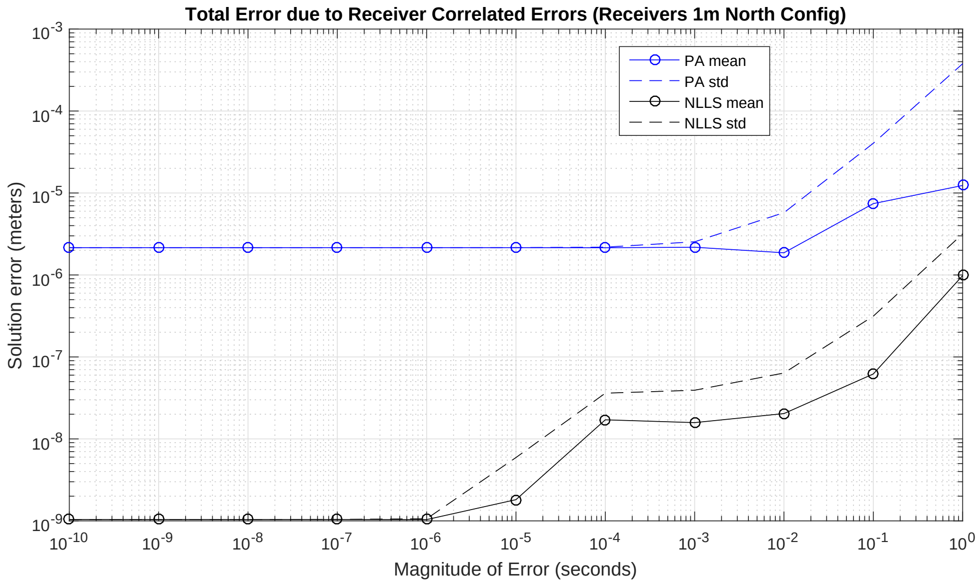
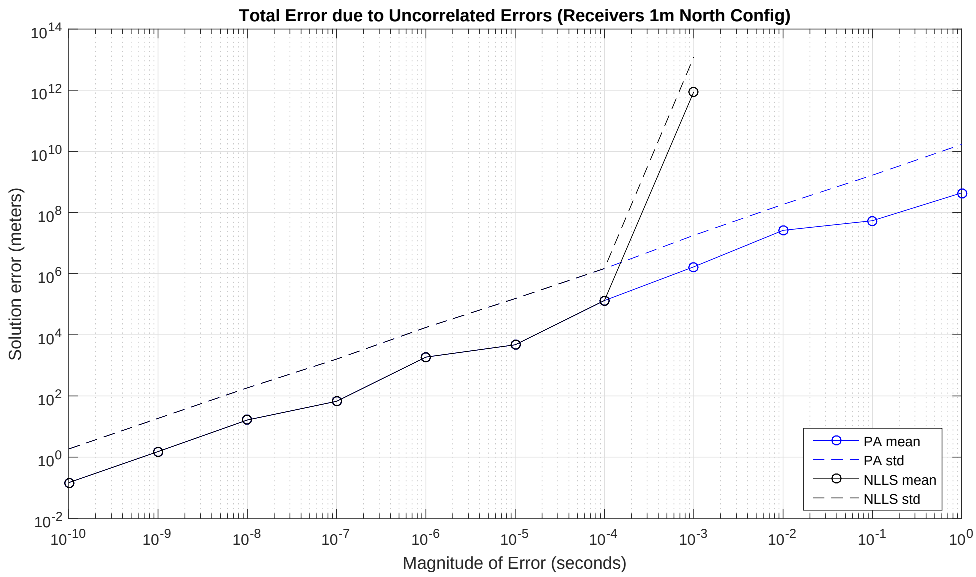


Figure 4.12



Chapter 5

Conclusion

Future work: Explain how to do it with real systems: The algorithm is designed with maximum compatibility in mind. - any type of receiver that has access to GPS L1 frequency -

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Appendix A

An Example Appendix

As an appendix, this should contain some content that's not really required for the argument in the main body of the thesis, but is clearly relevant and supports the work.