hSSALT Rpackage: simple heterogeneous SSALT with exponential life distribution based on CE assumption

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Contents

1 Description

This package offers a comprehensive framework for analyzing a simple heterogeneous step stress accelerated life testing (hSSALT) model, assuming the cumulative exposure (CE) model and heterogeneity with two subgroups under the second stress level for both continuous and interval monitoring. The supported life time distribution is the exponential distribution. For continuous monitoring, the package supports both Type-I and Type-II censoring, while for interval monitoring, it supports only Type-I censoring with equally spaced inspections. Additionally, the package can examine whether a dataset from an SSALT experiment justifies the implementation of an hSSALT model by a homogeneity test. However, only continuous monitoring under the exponential distribution is supported. The package includes the following four core functions and one example dataset:

- rhSSALT(): Generates a random sample from a simple hSSALT model;
- MLEhSSALT(): Computes the maximum likelihood estimates (MLEs) for a simple hSSALT model;
- CIhSSALT(): Constructs different confidence intervals (CIs) for a simple hSSALT model;
- HomohSSALT(): Performs a homogeneity test under the second stress level of a simple hSSALT model;
- hSSALTdata: A simulated simple hSSALT dataset based on [1].

2 Functions and an example dataset

2.1 rhSSALT

Simulate a simple hSSALT random dataset with exponential (continuous) or geometric (interval) distribution.

rhSSALT(n, censoring = 1, tau, r = NULL, monitoring= "continuous", delta= NULL, theta1, theta21, theta22, p):

- n: sample size, an integer.
- censoring: 1 for Type-I censoring or 2 for Type-II censoring. Default value is 1.
- tau: If censoring type is 1, tau is a vector with length 2; if censoring type is 2, tau is a positive numeric value.
- r: If censoring type is 2, r provides the pre-specified number of failures, a positive integer.
- monitoring: continuous or interval, default value is continuous. For interval monitoring, only equally spaced inspection is supported.
- delta: if interval monitoring, interval length, a positive numeric value. Default value is NULL.
- theta1: mean lifetime parameter in the exponential distribution under s_1, a numeric value.
- theta21: mean lifetime parameter in the exponential distribution of the first group under s_2, a positive numeric value.
- theta22: mean lifetime parameter in the exponential distribution of the second group under s_2, a positive numeric value.
- p: mixture proportion, a numeric value between 0 and 1.

The output is a list consisting of four sub-lists: censored sample, the observed number of censored failures under s_1 and s_2, complete sample, the observed number of failures under s_1 and s_2.

2.2 MLEhSSALT

Provide point estimation of a simple hSSALT model with exponential (continuous) or geometric (interval) distribution.

MLEhSSALT(data, n, censoring, tau, r= NULL, monitoring= "continuous", delta= NULL, theta21, theta22, p, maxit = 1000, tol = 1e-8, language = "CPP", parallel = FALSE, ncores):

- data: sample, a vector. The given data should be a censored vector with observations less than or equal to n. When censoring type is 2, the length of data should be r.
- n: sample size, a positive integer.
- censoring: 1 for Type-I censoring or 2 for Type-II censoring. Default value is 1.
- tau: If censoring type is 1, tau is a vector with length 2; if censoring type is 2, tau is a positive numeric value.
- r: If censoring type is 2, r provides the pre-specified number of failures, a positive integer.
- monitoring: "continuous" or "interval". Default value is "continuous". For interval monitoring, only equally spaced inspection is supported.
- delta: if interval monitoring, interval length, a positive numeric value. Default value is NULL.
- theta21: initial value of theta21 for the EM algorithm, can be both a numeric value or a vector of values. For an initial-value vector, the (ultimate) value with the largest log-likelihood is returned as the MLE.
- theta22: initial value of theta22 for the EM algorithm, can be both a numeric value or a vector of values.
- p: initial value of mixture proportion p, can be both a numeric value or a vector of values.
- maxit: The maximum number of iterations allowed, an integer. Default value is 1000.
- tol: Tolerance limit for declaring algorithm convergence based on the change between two consecutive iterations. Default value is 1e-8.

- language: "R" or "CPP". Only for bootstrap methods. Default value is "CPP".
- parallel: support parallel computation for multiple initial values, a logical value. Default value is FALSE.
- ncores: the number of cores that are used in parallelization, a positive integer.

The output is an *MLEhSSALT* object that summarizes all relevant information, including: the observed number of censored failures under s_1 and s_2 (denoted n_1 and n_2), the MLEs of the four model parameters, the log-likelihood value, the number of iterations in the adapted EM algorithm, convergence status of the algorithm, the sample's censoring rate, and the posterior probabilities from the final iteration of the adapted EM algorithm.

2.3 CIhSSALT

Provide interval estimation of a simple hSSALT model with exponential (continuous) or geometric (interval) distribution.

CIhSSALT(data, n, MLEhSSALT, censoring = 1, tau, r = NULL, monitoring= "continuous", delta = NULL, CImethod = "asymptotic", alpha = 0.05, B = 1000, maxit = 1000, tol = 1e-8, language = "CPP", parallel = FALSE, ncores)

- data: sample, a vector. The given data should be a censored vector with observations less than or equal to n. When censoring type is 2, the length of data should be r.
- n: sample size, a positive integer.
- MLEhSSALT: an *MLEhSSALT* object, returned by MLEhSSALT().
- censoring: 1 for Type-I censoring or 2 for Type-II censoring. Default value is 1.
- tau: If censoring type is 1, tau is a vector with length 2; if censoring type is 2, tau is a positive numeric value.
- r: If censoring type is 2, r provides the pre-specified number of failures, a positive integer.
- monitoring: "continuous" or "interval". Default value is "continuous". For interval monitoring, only equally spaced inspection is supported.
- delta: if interval monitoring, interval length, a positive numeric value. Default value is NULL.
- CImethod: "asymptotic", "percentile" or "bca" for asymptotic CIs, bootstrap percentile CIs and bootstrap bias-corrected and accelerated (BCa) bootstrap intervals. Default value is "asymptotic".
- alpha: significance level. Default value is 0.05.
- B: number of bootstrap repetitions, a positive integer, default value is 1000.
- maxit: The maximum number of iterations allowed, a positive integer. Only for bootstrap methods. Default value is 1000.
- tol: Tolerance limit for declaring algorithm convergence based on the change between two consecutive iterations. Only for bootstrap methods. Default value is 1e-8.
- language: "R" or "CPP". Only for bootstrap methods. Default value is "CPP".
- parallel: support parallel computation, a logical value. Only for bootstrap methods. Default value is FALSE.
- ncores: the number of cores that are used in parallelization, a positive integer.

The output is a *CIhSSALT* object that includes the type of returned CIs and the CIs for four parameters at a given significance level.

2.4 HomohSSALT

Perform a homogeneity test under the second stress level s_2 of a simple hSSALT model with exponential (continuous) distribution.

HomohSSALT(data, n, censoring = 1, tau, r, alpha = 0.05, M = 10000)

- data: sample, a vector. The given data should be a censored vector with observations less than or equal to n. When censoring type is 2, the length of data should be r.
- n: sample size, a positive integer.
- censoring: 1 for Type-I censoring or 2 for Type-II censoring. Default value is 1.
- tau: If censoring type is 1, tau is a vector with length 2; if censoring type is 2, tau is a positive numeric value.
- r: If censoring type is 2, r provides the pre-specified number of failures, a positive integer.
- alpha: significance level. Default value is 0.05.
- M: number of simulations used to generate critical values, a positive integer. Default value is 10000.

The output is an *hSSALTtest* object containing a hypothesis test table that reports the test statistic, the simulated critical value at the given significance level, the alternative hypothesis, and the test decision.

2.5 hSSALTdata

The complete sample is generated using the simulated dataset from [1], where $n = 35$, $\tau_1 = 8$, $\theta_1 = e^{3.5} = 33.12$ and $\theta_2 = e^{2.0} = 7.39$. Particularly, θ_2 in the homogeneous SSALT corresponds to θ_{22} in our setup. Additionally, we set $p = 0.4$ as the mixture proportion for the component with a smaller mean, θ_{21}. To create a heterogeneous sample, we randomly select 60% of the original data from stress level s_2. The remaining 40% of the original data set in s_2 is then replaced by an exponential distribution with $\theta_{21} = e^{-0.2} = 0.82$, representing another component. The resulting combined sample is presented in Table 1.

Table 1: Simulated sample based on [1]

Stress level	Mean failure time	Times-to-failure							
s_1	$\theta_1 = e^{3.5}$	1.46	2.22	3.92	4.24	5.47	5.60	6.12	6.56
s_2	$\theta_{21} = e^{-0.2}$ $(p = 0.4)$	8.01	8.10	8.22	8.59	8.77	8.80	8.80	8.84
		8.90	8.97	9.62					
	$\theta_{22} = e^{2.0}$	8.30	8.98	9.43	9.87	11.14	11.85	12.14	13.49
		14.19	14.33	15.28	16.58	17.80	21.09	26.34	28.66

References

[1] N. Balakrishnan, Q. Xie., D. Kundu. Exact inference for a simple step-stress model from the exponential distribution under time constraint. *Annals of the Institute of Statistical Mathematics*, **61**: 251–274, 2009.