A9 Mini-TeX Project

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1 Introduction

Prove that if $n \in \mathbb{Z}^+$ and when $\sin \neq 0$,

$$(cos(\theta))(cos(2\theta)(cos(4\theta))(cos(8\theta)))...(cos(2^{n-1}(\theta))) = \frac{sin2^n(\theta)}{2^n sin(\theta)}$$

2 Proof

Base Case: P(1) must be true:

$$(cos(2^0(\theta))) = \frac{sin(2^1(\theta)}{2^1 sin(\theta)}$$

$$\boxed{\textbf{Recall the Ptoley's Indentities: } sin(\theta+\theta) = sin(2\theta) = 2(sin(\theta)cos(\theta))}$$

$$cos(\theta) = \frac{2(sin(\theta)cos(\theta))}{2sin(\theta)}$$

$$cos(\theta) = cos(\theta)$$

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Hypothesis:

$$(cos(\theta))(cos(2\theta)(cos(4\theta))(cos(8\theta)))...(cos(2^{k-1}(\theta))) = \frac{sin2^k(\theta)}{2^k sin(\theta)}$$

Proof(k+1) =

$$(cos(\theta))(cos(2\theta)(cos(4\theta))(cos(8\theta)))...(cos(2^{k-1}(\theta))cos(2^k(\theta)) = \frac{sin2^{k+1}(\theta)}{2^{k+1} sin(\theta)}$$

$$= \frac{sin2^k(\theta)}{2^k sin(\theta)}(cos2^k(\theta))$$

$$\boxed{\textbf{Recall the Double Angle Formula: } 2sin(\alpha)cos(\alpha) = sin2(\alpha)}$$

$$= \frac{sin(2 \cdot 2^k(\theta))}{2 \cdot 2^k sin(\theta)}$$

$$= \frac{sin(2^{k+1}(\theta)}{2^{k+1} sin(\theta)}$$

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Using Proof by Induction, I was able to prove that there exists a positive number that supports the proof question.