天线第三次作业

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1 2.1-2

解：最大相位误差为 $\dfrac{\pi}{4}$，即要求波程差为 $\dfrac{\lambda}{8}$。
由远区场和辐射近区的边界知，

$$\frac{L^2}{8r} \geq \frac{\lambda}{8} \quad \Longrightarrow \quad r \leq \frac{L^2}{\lambda}$$

由辐射近区和感应近区的边界知，

$$\frac{L^3}{24\sqrt{3}r^2} \leq \frac{\lambda}{8} \quad \Longrightarrow \quad r^2 \geq \frac{L^3}{3\sqrt{3}\lambda} \quad \Longrightarrow \quad r \geq 0.44\sqrt{\frac{L^3}{\lambda}}$$

综上，

$$0.44\sqrt{\frac{L^3}{\lambda}} \leq r \leq \frac{L^2}{\lambda}$$

当 $L = 10\,m$，$\lambda = 7.6\,cm = 7.6 \times 10^{-2}\,m$，时，代入上式得，

$$0.44\sqrt{\frac{10^3}{7.6 \times 10^{-2}}} \leq r \leq \frac{10^2}{7.6 \times 10^{-2}} \quad \Longrightarrow \quad 50.47\,m \leq r \leq 1315.79\,m$$

2 2.1-4

解：本题主要利用远区场的条件来解决。$r \gg L, \quad r \gg \dfrac{\lambda}{2\pi}, \quad r \geq \dfrac{2L^2}{\lambda}$。
1) $L = 1\,m,\, f = 1\,MHz$

易知，$\lambda = \dfrac{v}{f} = \dfrac{3 \times 10^8}{10^6} = 300\,m, \quad \dfrac{\lambda}{2\pi} = \dfrac{300}{2\pi} = 47.75\,m,$

$\dfrac{2L^2}{\lambda} = \dfrac{2}{300} = 0.0067\,m$

综上，$r \geq 47.75\,m$，即获得天线方向图最小距离为 $47.75\,m$。

2) $L = 0.8\,m,\, f = 1.8\,GHz$

易知，$\lambda = \dfrac{v}{f} = \dfrac{3 \times 10^8}{1.8 \times 10^9} = \dfrac{1}{6}m, \quad \dfrac{\lambda}{2\pi} = \dfrac{0.8}{2\pi} = 0.127\,m,$

$\dfrac{2L^2}{\lambda} = \dfrac{2 \times 0.8^2}{1/6} = 7.68\,m$

综上，$r \geq 7.68\,m$，即获得天线方向图最小距离为 $7.68\,m$。

3) $\quad L = 13\,m,\ f = 6\,GHz$

易知，$\lambda = \frac{v}{f} = \frac{3\times10^8}{6\times10^9} = \frac{1}{20}m, \quad \frac{\lambda}{2\pi} = \frac{13}{2\pi} = 2.07\,m,$

$\frac{2L^2}{\lambda} = \frac{2\times13^2}{1/20} = 6760\,m$

综上，$r \geq 6760\,m$，即获得天线方向图最小距离为 $6760\,m$。

3 2.1-8

解：长为 L 的对称振子的辐射电阻为

$R_r = 30[2(C + ln\,kL - Ci\,kL) + (sin\,kL)(Si\,2kL - 2Si\,kL) + (cos\,kL)(C + ln\,\frac{1}{2}kL + Ci\,2kL - 2Ci\,kL)$

其中，$C = 0.5772$，Si 为正弦积分，Ci 为余弦积分。

1) $\quad L = 2l = \frac{\lambda}{4}$

此时，$kL = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$，查表 2.1-2 知，$Si\,\frac{\pi}{2} = 1.371,\ Ci\,\frac{\pi}{2} = 0.472,\ Si\,\pi = 1.852,\ Ci\,\pi = 0.074$。

$R_r = 30[2(0.5772 + ln\,\frac{\pi}{2} - 0.472) + sin\,\frac{\pi}{2} \times (1.852 - 2\times1.371) + cos\,\frac{\pi}{2} \times (0.5772 + ln\,\frac{\pi}{4} + 0.074 - 2\times0.472)] = 6.71\,\Omega$

2) $\quad L = 2l = \lambda$

此时，$kL = \frac{2\pi}{\lambda}\lambda = 2\pi$，查表 2.1-2 知，$Si\,2\pi = 1.418,\ Ci\,2\pi = -0.0227,\ Si\,4\pi = 1.492,\ Ci\,4\pi = -0.006$。

$R_r = 30[2\times(0.5772 + ln\,2\pi + 0.0227) + cos2\pi \times (0.5772 + ln\,\pi - 0.006 - 2\times(-0.0027))] = 197.9\,\Omega$

4 2.2-1

解：已知辐射功率 $P_r = 10\,W$，方向系数 $D = 200$，距离 $r = 37590\,km = 3.759\times10^7\,m$。

到达北京的平均功率为 $P_1 = \frac{P_r D}{4\pi r^2} = \frac{10\times200}{4\pi\times(3.759\times10^7)^2} = 1.126\times10^{-13}\,W/m^2$

又有 $P_1 = \frac{1}{2}\frac{|E|^2}{120\pi}$,

可得，到达北京的场强为 $|E| = \sqrt{240\pi P_1} = \sqrt{1.126\times10^{-13}\times240\pi} = 9.22\times10^{-6}\,v/m = 9.22\,\mu v$

要保证场强不变，无向天线的功率为 $P_r' = P_r D = 10\times200 = 2000\,W$

5 2.2-4

解：方向函数 $F(\theta,\phi) = sin\theta sin^2\phi \quad (0\le\theta\le\pi,\,0\le\phi\le\pi)$
1) 方向系数

$$D = \frac{4\pi}{\int_0^\pi\int_0^\pi F^2(\theta,\phi)sin\theta d\theta d\phi} = \frac{4\pi}{\int_0^\pi\int_0^\pi \sin^4\phi sin^3\theta d\theta d\phi} = \frac{4\pi}{\frac{4}{3}\times\frac{3}{8}\pi} = 8$$

2) 在 $\theta = \frac{\pi}{2}$ 平面，由 $F^2(\frac{\pi}{2},\phi_{0.5}) = sin^4\phi = 0.5$ 知，$\phi_{0.5} = 57.23°$，故半功率宽度为 $HP_\phi = 2\phi_{0.5} = 114.46°$。
在 $\phi = \frac{\pi}{2}$ 的平面，由 $F^2(\theta,\frac{\pi}{2}) = sin^2\theta = 0.5$ 知，$\theta_{0.5} = 45°$，故半功率宽度为 $HP_\theta = 2\theta_{0.5} = 90° HP_\theta = 2\theta_{0.5} = 90°$ 3) 由克劳斯近似式知，方向系数为 $D \approx \frac{41253}{HP_\phi\cdot HP_\theta} = \frac{41253}{114.46\times90} = 4$

6 2.2-7

解：波长为 $\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^6} = 30\,m$
表面电阻为 $R_s = \sqrt{\frac{\pi f\mu}{\sigma}} = \sqrt{\frac{\pi\times10\times10^6\times4\pi\times10^{-7}}{5.8\times10^7}} = 8.25\times10^{-4}\,\Omega$
方向系数为 $D = \frac{120f_M^2}{R_r} = \frac{120(1-coskl)^2}{R_r}$
1) 当 $2l = \frac{\lambda}{4}$ 时，由 $2.1-8$ 知，辐射电阻 $R_r = 6.71\,\Omega$。

方向系数为 $D = \frac{120\times(1-cos\frac{\pi}{4})}{6.71} = 1.534$
损耗电阻为 $R_\sigma = \frac{2P_\sigma}{I_M^2} = \frac{1}{I_M^2}\frac{R_s}{2\pi a}\int_{-l}^{l}|I(z)|^2dz = \frac{R_s}{\pi a}\int_0^l sin^2k(l-z)dz = \frac{R_s}{\pi a}[\frac{1}{2}z + \frac{1}{4k}sin2k(l-z)]_0^l = \frac{R_s}{\pi a}(\frac{l}{2} - \frac{1}{4k}) = \frac{8.25\times10^{-4}}{\pi\times10^{-3}\times30}(\frac{1}{2}\frac{30}{8} - \frac{30}{4\times2\pi} =$

$0.006\,\Omega$

天线效率为 $e_r = \dfrac{R_r}{R_r + R_\sigma} = \dfrac{6.71}{6.71 + 0.006} = 99.9\,\%$

天线增益为 $G = De_r = 1.534 \times 0.999 = 1.532$，即 $G = 10lg\,1.532 = 1.85\,dB$

2) 当 $2l = \lambda$ 时，由 $2.1 - 8$ 知，辐射电阻 $R_r = 197.9\,\Omega$。

方向系数为 $D = \dfrac{120 \times (1 - cos\,\pi)^2}{197.9} = 2.425$

损耗电阻为 $R_\sigma = \dfrac{2P_\sigma}{I_M^2} = \dfrac{1}{I_M^2}\dfrac{R_s}{2\pi a}\int_{-l}^{l}|I(z)|^2 dz = \dfrac{R_s}{\pi a}\int_0^l sin^2 k(l-z)dz = \dfrac{R_s}{\pi a}[\dfrac{1}{2}z + \dfrac{1}{4k}sin 2k(l-z)]_0^l = \dfrac{R_s}{\pi a}\dfrac{l}{2} = \dfrac{8.25 \times 10^{-4}}{\pi \times 10^{-3} \times 30} \times \dfrac{1}{2} \times \dfrac{30}{2} = 0.0657\,\Omega$

天线效率为 $e_r = \dfrac{R_r}{R_r + R_\sigma} = \dfrac{197.9}{197.9 + 0.0657} = 99.97\,\%$

天线增益为 $G = De_r = 2.425 \times 0.9997 = 2.424$，即 $G = 10lg\,2.424 = 3.85\,dB$