Elementos de Probabilidades - Soluções da Folha 1

1. (a) $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$ (**ou** $\{(a_1, a_2) : a_1, a_2 \in \{C_a, C_o\}\}$ **ou** $\{C_a, C_o\}^2$);
   (b) $\Omega = \{(a_1, a_2) : a_1 \in \{C_a, C_o\}, a_2 \in \{1, 2, 3, 4, 5, 6\}\}$ (**ou** $\{C_a, C_o\} \times \{1, 2, 3, 4, 5, 6\}$ **ou** $\{(C_a, 1), (C_a, 2), (C_a, 3), (C_a, 4), (C_a, 5), (C_a, 6), (C_o, 1), (C_o, 2), (C_o, 3), (C_o, 4), (C_o, 5), (C_o, 6)\}$);
   (c) $\Omega = \{(a_1, a_2, a_3) : a_1, a_2, a_3 \in \{1, 2, 3, 4, 5, 6\}\}$ (**ou** $\{1, 2, 3, 4, 5, 6\}^3$)

2. a) $\Omega = \{1, 2, 3, 4, 5, 6\}$ e $P(\{i\}) = \frac{2^{i-1}}{63}$, $i \in \{1, \ldots, 6\}$
   b) Não
   c) $A = \{2, 4, 6\}, P(A) = \frac{42}{63}$; $B = \{3, 6\}, P(B) = \frac{36}{63}$; $\overline{B} = \{1, 2, 4, 5\}, P(\overline{B}) = \frac{27}{63}$;
   $C = \{1, 3, 5\}, P(C) = \frac{21}{63}$; $A \cap B = \{6\}, P(A \cap B) = \frac{32}{63}$;
   $A \cup B = \{2, 3, 4, 6\}, P(A \cup B) = \frac{46}{63}$; $A \backslash B = \{2, 4\}, P(A \backslash B) = \frac{10}{63}$

3. a) $\emptyset$; $\emptyset$; Ω; Ω
   b) $D = A \cap B$; $E = B \cap C$; $F = A \cup B \cup C$; $G = (A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C)$

4. $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$

5. $\frac{260}{630}$; $\frac{120}{630}$; $\frac{470}{630}$; $\frac{160}{630}$; $\frac{380}{630}$

6. 0.68; 0.12; 0.24

7. 0.485; 0.515; 0.025 0.035, 0.295

8. 0.32; 0.68; 0.03; 0.03

9. $\frac{2 \times \binom{13}{5}}{\binom{26}{5}} = \frac{9}{230}$

10. (a) 0.47; 0.55; 0.85; 0.83 (b) —

11. (a) 0.2; (b) 0.575; (c)0.04375

12. Sim; Sim; Não; Não

13. (a) Não (b) Falsa

14. (a) $\Omega = \{C_a, C_o\}^{n-1}$
   (b) i. $P(E_j) = \frac{1}{2}, j \in \{1, \ldots, n\}$; ii. 0 e não são independentes

15. $n > 6.644$ pelo que $n = 7$.

16. Se $n = 2$ não são independentes; se $n = 3$ são independentes.

17. (a) 0.72; (b) $\frac{0.1}{0.28}$; (c) Não

18. a) 0.1; 0.6; 0.3; 0.36; 0.42
   b) $\frac{0.52}{0.64}$

19. (a) —
   (b) 0.719; ; $\frac{0.665}{0.719}$; $\frac{0.246}{0.281}$

1. (a) i. $X : \left\{ \begin{array}{ccc} 0 & 1 & 2 \\ \frac{9}{36} & \frac{18}{36} & \frac{9}{36} \end{array} \right.$, $F_X(c) = \left\{ \begin{array}{ccc} 0 & se & c < 0 \\ 9/36 & se & 0 \leq c < 1 \\ 27/36 & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} \right.$

ii. Resultado igual ao da alínea anterior

iii. $Z : \left\{ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{36} & \frac{3}{36} & \frac{5}{36} & \frac{7}{36} & \frac{9}{36} & \frac{11}{36} \end{array} \right.$, $F_Z(c) = \left\{ \begin{array}{ccc} 0 & se & c < 1 \\ 1/36 & se & 1 \leq c < 2 \\ 4/36 & se & 2 \leq c < 3 \\ 9/36 & se & 3 \leq c < 4 \\ 16/36 & se & 4 \leq c < 5 \\ 25/36 & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} \right.$

vi. $N : \left\{ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{11}{36} & \frac{9}{36} & \frac{7}{36} & \frac{5}{36} & \frac{3}{36} & \frac{1}{36} \end{array} \right.$, $F_N(c) = \left\{ \begin{array}{ccc} 0 & se & c < 1 \\ 11/36 & se & 1 \leq c < 2 \\ 20/36 & se & 2 \leq c < 3 \\ 27/36 & se & 3 \leq c < 4 \\ 32/36 & se & 4 \leq c < 5 \\ 35/36 & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} \right.$

v. $W : \left\{ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \frac{6}{36} & \frac{10}{36} & \frac{8}{36} & \frac{6}{36} & \frac{4}{36} & \frac{2}{36} \end{array} \right.$, $F_W(c) = \left\{ \begin{array}{ccc} 0 & se & c < 0 \\ 6/36 & se & 0 \leq c < 1 \\ 16/36 & se & 1 \leq c < 2 \\ 24/36 & se & 2 \leq c < 3 \\ 30/36 & se & 3 \leq c < 4 \\ 34/36 & se & 4 \leq c < 5 \\ 1 & se & c \geq 5 \end{array} \right.$

vi. $S : \left\{ \begin{array}{cccccccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{5}{36} & \frac{4}{36} & \frac{3}{36} & \frac{2}{36} & \frac{1}{36} \end{array} \right.$, $F_S(c) = \left\{ \begin{array}{ccc} 0 & se & c < 2 \\ 1/36 & se & 2 \leq c < 3 \\ 3/36 & se & 3 \leq c < 4 \\ 6/36 & se & 4 \leq c < 5 \\ 10/36 & se & 5 \leq c < 6 \\ 15/36 & se & 6 \leq c < 7 \\ 21/36 & se & 7 \leq c < 8 \\ 26/36 & se & 8 \leq c < 9 \\ 30/36 & se & 9 \leq c < 10 \\ 33/36 & se & 10 \leq c < 11 \\ 35/36 & se & 11 \leq c < 12 \\ 1 & se & c \geq 12 \end{array} \right.$

(b) i. $\frac{3}{4}$; ii. $\frac{1}{4}$; iii. $\frac{1}{4}$; iv. $\frac{1}{4}$; v. $\frac{1}{6}$; vi. $\frac{5}{6}$; vii. $\frac{1}{6}$

2. (a) –; (b) i. 0.65; ii. 0.5; iii. 0.5

(c) $F_X(c) = \begin{cases} 0 & se \quad c < 0 \\ 0.05 & se \quad 0 \leq c < 1 \\ 0.15 & se \quad 1 \leq c < 2 \\ 0.35 & se \quad 2 \leq c < 3 \\ 0.5 & se \quad 3 \leq c < 4 \\ 0.8 & se \quad 4 \leq c < 5 \\ 1 & se \quad c \geq 5 \end{cases}$; (d) i. $\frac{0.15}{0.8}$; ii. $\frac{0.45}{0.8}$; iii. $\frac{0.3}{0.8}$

3. a) $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$

b) i.

ω	$X(\omega)$	$Y(\omega)$
(Ca, Ca)	2	0
(Ca, Co)	1	1
(Co, Ca)	1	1
(Co, Co)	0	2

ii. Funções massa de probabilidade são iguais a: $\begin{cases} 0 & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{cases}$

Funções de distribuição também são iguais a: $F(c) = \begin{cases} 0 & se & c < 0 \\ 1/4 & se & 0 \leq c < 1 \\ 3/4 & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{cases}$

Comentário: X e Y são duas funções distintas mas, enquanto v.a.'s, têm em comum as funções que as caracterizam (a f.m.p. e a função de distribuição).

4. (a) $F_X(c) = \begin{cases} 0 & se & c < 0 \\ \frac{1}{8}c & se & 0 \leq c < 4 \\ \frac{1}{2} + \frac{1}{4}(c-4) & se & 4 \leq c < 6 \\ 1 & se & c \geq 6 \end{cases}$; (b) i. $\frac{3}{16}$; ii. $\frac{13}{16}$; iii. igual a ii.; iv. $\frac{3}{8}$

(c) $\frac{13}{16}$; $\frac{11}{13}$

5. (a) $k = \frac{1}{4}$; $F(c) = \begin{cases} 0 & se & c < 1 \\ \frac{1}{8}(c^2 - 1) & se & 1 \leq c < 3 \\ 1 & se & c \geq 3 \end{cases}$; (b) i. $\frac{27}{32}$; ii. igual a i.; iii. $\frac{39}{128}$; c) $\frac{20}{27}$

6. (a) $F_T(c) = \begin{cases} 0 & se & c < 0 \\ 1 - e^{-\lambda c} & se & c \geq 0 \end{cases}$; (b) $\frac{3e^{-4}}{e^{-2} + 3e^{-4}}$

7. (a) $F_X(c) = \begin{cases} \frac{1}{2}e^c & se & c < 0 \\ 1 - \frac{1}{2}e^{-c} & se & c \geq 0 \end{cases}$; (b) $\frac{1}{2}$; , $\frac{1}{2}$; $\frac{1}{2} - \frac{1}{2}e^{-1}$; ; $1 - e^{-1}$

(c) Função de distribuição de Y: $F_Y(c) = \begin{cases} 0 & se & c < 0 \\ 1 - e^{-c} & se & c \geq 0 \end{cases}$. Logo $Y \sim Exp(1)$.

8. $P(Y = 0) = 1 - e^{-\lambda a}$, $F_Y(c) = \begin{cases} 0 & se & c < 0 \\ 1 - e^{-\lambda(c+2a)} & se & c \geq 0 \end{cases}$

1. (a)

	$E[X]$	$Var[X]$	σ_X	$\chi_{0.25}$	$\chi_{0.5}$	$\chi_{0.75}$
1(a) i.	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	0	1	1
1(a) ii.	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	0	1	1
1(a) iii.	$\frac{161}{36}$	$\frac{2555}{1296}$	$\sqrt{\frac{2555}{1296}}$	3	5	5
1(a) iv.	$\frac{91}{36}$	$\frac{2555}{1296}$	$\sqrt{\frac{2555}{1296}}$	1	2	3
1(a) v.	$\frac{70}{36}$	$\frac{2660}{1296}$	$\sqrt{\frac{2660}{1296}}$	1	2	3
1(a) vi.	7	$\frac{210}{36}$	$\sqrt{\frac{210}{36}}$	5	7	9
2.	3.15	2.1275	$\sqrt{2.1275}$	2	3	4

(b)

	$E[X]$	$Var[X]$	σ_X	$\chi_{0.25}$	$\chi_{0.5}$	$\chi_{0.75}$
4.	$\frac{7}{2}$	$\frac{37}{12}$	$\sqrt{\frac{37}{12}}$	2	4	5
5.	$\frac{13}{6}$	$\frac{11}{36}$	$\frac{\sqrt{11}}{6}$	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{7}$
7.	0	2	$\sqrt{2}$	$-\log 2$	0	$\log 2$

Decis (Ex 1.(a) vi.): 4, 5, 6, 6, 7, 8, 8, 9, 10

2. (a) $F_X(c) = \begin{cases} 0 & se \quad c < 0 \\ c^3(4-3c) & se \quad 0 \le c \le 1 \\ 1 & se \quad c \ge 1 \end{cases}$; $E[X] = \frac{3}{5}$; $Var[X] = \frac{1}{25}$

   (b) i. $L : \begin{cases} 8 & 13 & 18 \\ \frac{3}{27} & \frac{13}{27} & \frac{11}{27} \end{cases}$; ii. $\frac{391}{27}$

3. (a) $E[Y] = 10$; $Var[Y] = 4.4$; (b) 10.1

4. (a) 2.5 (valor médio) e 28.75 (variância); (b) $N = 11$ é o menor

5. (a) $X : \begin{cases} 1 & 2 & 3 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{cases}$; $Y : \begin{cases} 2 & 3 \\ \frac{1}{3} & \frac{2}{3} \end{cases}$; $E[X] = 2$; $Var[X] = \frac{2}{3}$; $E[Y] = \frac{8}{3}$; $Var[Y] = \frac{2}{9}$

   (b) $\frac{1}{4}$ e 0; X e Y não são independentes

   (c) i. $S : \begin{cases} 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{2}{6} & \frac{1}{6} & \frac{2}{6} \end{cases}$; $T : \begin{cases} 0 & 1 & 2 \\ \frac{3}{6} & \frac{2}{6} & \frac{1}{6} \end{cases}$;

   (c) ii. $E[S] = \frac{14}{3}$; $E[T] = \frac{2}{3}$; $Var[S] = \frac{11}{19}$; $Var[T] = \frac{5}{9}$

1. (a) – [Sug.: $(M \le c) \Leftrightarrow (X_1 \le c, X_2 \le c, \ldots, X_n \le c)$ e $(N > c) \Leftrightarrow (X_1 > c, X_2 > c, \ldots, X_n > c)$]
   (b) $N \sim Exp(n\lambda)$

2. (a) Sim (b) Sim

3. (a) 4×0.6^9; 4.6×0.6^9; 0.4^{10}
   (b) 45×0.5^{10}, $1 - 11 \times 0.5^{10}$
   (c) igual a (b)
   (d) $\left(\frac{3}{5}\right)^4$; $\left(\frac{2}{5}\right)^4$; 0 no caso de extracção sem reposição

4. (a) $\lambda = 1$; (b) i. $\frac{1}{2}e^{-1}$; ii. $\frac{5}{2}e^{-1}$; iii. $1 - 2e^{-1}$

5. (a) $e^{-0.6}$; (b) $e^{-3.6}$; (c), (d) – [Sug.: Usar TPT com a partição $(X = k), k \in \mathbb{N}_0$.];
(e) i. 60 (valor médio) e 6000 (variância); ii. 300 (valor médio) e 30000 (variância)

6. (a) $X \sim U([2, 12])$; (b) i. 0.6; ii. $10 \times 0.4^3 \times 0.6^2 + 5 \times 0.4^4 \times 0.6 + 0.4^5$

7. (a) $\frac{5}{20}$; (b) igual a (a); (c) $\frac{16}{20}$; (d) $\chi_p = 20p + 340$, $p \in]0, 1[$

8. (a) $\lambda = \frac{1}{10}$; (b) $1 - e^{-0.8}$; (c) e^{-1}

9. e^{-1}

Elementos de Probabilidades - Soluções da Folha 5

---

1. (d)

2. (a) 1.96 (b) -1.96 (c) 1.645 (d) 1.645

3. (a) $N(270, 67)$ (b) 0.0336

4. $\overline{X}_{10} \sim N\left(3.2, \frac{1.8^2}{10}\right)$; $P(\overline{X}_{10} > 3.5) = 0.2981$

5. $\simeq 0.5$

6. $\simeq 0.8577$

7. $\simeq 0.9525$

8. 4 e $\frac{1.2}{\sqrt{70}}$ são, respetivamente, a média e o desvio-padrão de $\overline{X}_{70}$; $P\left(|\overline{X}_{70} - 4| \leq \frac{1.2}{12}\right) \simeq 0.516$

9. i) $E[Y] = 350$; $Var[Y] = \frac{875}{3}$; $P(Y > 300) \simeq 0.9983$;
ii) $X \sim Bin(100, \frac{1}{6})$; $E[X] = \frac{100}{6}$; $Var[X] = \frac{500}{36}$; $P(X \leq 30) \simeq 0.9998$
iii) Não são independentes

10. (a) 0.0241;
(b) $S \sim Bin(100, 0.0228)$; $E[S] = 2.28$;
$P(S < 6) \simeq 0.9936$ (aproximado); $P(S < 6) = \sum_{k=0}^{5} \binom{100}{k} 0.0228^k \times 0.9772^{100-k}$ (exacto)

11. $\simeq 0.1423$

12. $\simeq 0.8816$