Cálculo

Funções importantes

(Omitem-se os domínios das funções.)

$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$$1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$$

$$1 + \operatorname{cotg}^2 x = \frac{1}{\operatorname{sen}^2 x}$$

$\operatorname{sen}(-x) = -\operatorname{sen} x$ (a função é ímpar)

$\cos(-x) = \cos x$ (a função é par)

$$\operatorname{sen}(\tfrac{\pi}{2} - x) = \cos x$$

$$\cos(\tfrac{\pi}{2} - x) = \operatorname{sen} x$$

$$\cos(x+y) = \cos x \cos y - \operatorname{sen} y \operatorname{sen} x$$

$$\operatorname{sen}(x+y) = \operatorname{sen} x \cos y + \operatorname{sen} y \cos x$$

$$\cos x - \cos y = -2 \operatorname{sen} \tfrac{x-y}{2} \operatorname{sen} \tfrac{x+y}{2}$$

$$\operatorname{sen} x - \operatorname{sen} y = 2 \operatorname{sen} \tfrac{x-y}{2} \cos \tfrac{x+y}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

$$\operatorname{ch} x + \operatorname{sh} x = e^x$$

$$\operatorname{th}^2 x + \frac{1}{\operatorname{ch}^2 x} = 1$$

$$\operatorname{coth}^2 x - \frac{1}{\operatorname{sh}^2 x} = 1$$

$\operatorname{sh}(-x) = -\operatorname{sh} x$ (a função é ímpar)

$\operatorname{ch}(-x) = \operatorname{ch} x$ (a função é par)

$$\operatorname{sh}(x+y) = \operatorname{sh} x \operatorname{ch} y + \operatorname{sh} y \operatorname{ch} x$$

$$\operatorname{ch}(x+y) = \operatorname{ch} x \operatorname{ch} y + \operatorname{sh} y \operatorname{sh} x$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\operatorname{sen} x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$$\operatorname{sen}(\arccos x) = \sqrt{1-x^2}$$

$$\operatorname{tg}(\arccos x) = \frac{\sqrt{1-x^2}}{x}$$

$$\cos(\operatorname{arcsen} x) = \sqrt{1-x^2}$$

$$\operatorname{tg}(\operatorname{arcsen} x) = \frac{x}{\sqrt{1-x^2}}$$

Regras de derivação

(Omitem-se os domínios das funções e considera-se a uma constante apropriada.)

$(f \pm g)'(x) = f'(x) \pm g'(x)$

$(f\,g)'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

$\left(\dfrac{f}{g}\right)'(x) = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{g^2(x)}$

$(x^a)' = a\,x^{a-1}$

$(a^x)' = a^x \ln a$

$(\log_a x)' = \dfrac{1}{x \ln a}$

$(e^x)' = e^x$

$(\ln x)' = \dfrac{1}{x}$

$\operatorname{sen}' x = \cos x$

$\cos' x = -\operatorname{sen} x$

$\operatorname{tg}' x = \dfrac{1}{\cos^2 x}$

$\operatorname{cotg}' x = -\dfrac{1}{\operatorname{sen}^2 x}$

$\sec' x = \sec x \operatorname{tg} x$

$\operatorname{cosec}' x = -\operatorname{cosec} x \operatorname{cotg} x$

$\operatorname{sh}' x = \operatorname{ch} x$

$\operatorname{ch}' x = \operatorname{sh} x$

$\operatorname{th}' x = \dfrac{1}{\operatorname{ch}^2 x}$

$\operatorname{coth}' x = -\dfrac{1}{\operatorname{sh}^2 x}$

$\operatorname{sech}' x = -\operatorname{sech} x \operatorname{th} x$

$\operatorname{cosech}' x = -\operatorname{cosech} x \operatorname{coth} x$

$\operatorname{arcsen}' x = \dfrac{1}{\sqrt{1-x^2}}$

$\arccos' x = \dfrac{-1}{\sqrt{1-x^2}}$

$\operatorname{arctg}' x = \dfrac{1}{1+x^2}$

$\operatorname{arccotg}' x = \dfrac{-1}{1+x^2}$

$\operatorname{arcsec}' x = \dfrac{1}{x\sqrt{x^2-1}}$

$\operatorname{arccosec}' x = \dfrac{-1}{x\sqrt{x^2-1}}$

$\operatorname{argsh}' x = \dfrac{1}{\sqrt{1+x^2}}$

$\operatorname{argch}' x = \dfrac{1}{\sqrt{x^2-1}}$

$\operatorname{argth}' x = \dfrac{1}{1-x^2}$

$\operatorname{argcoth}' x = \dfrac{1}{1-x^2}$

$\operatorname{argsech}' x = \dfrac{-1}{x\sqrt{1-x^2}}$

$\operatorname{argcosech}' x = \dfrac{-1}{x\sqrt{1+x^2}}$

$(f \circ u)'(x) = f'(u(x))\,u'(x)$

$(f^{-1})'(y) = \dfrac{1}{f'(f^{-1}(y))}$

Primitivas imediatas

($u\colon I \longrightarrow \mathbb{R}$ é uma função derivável num intervalo I e $\mathcal{C}$ denota uma constante real arbitrária)

$\displaystyle\int a\,dx = ax + \mathcal{C}$

$\displaystyle\int u'\,u^\alpha\;dx = \frac{u^{\alpha+1}}{\alpha+1} + \mathcal{C}\;\;(\alpha \neq -1)$

$\displaystyle\int \frac{u'}{u}\,dx = \ln|u| + \mathcal{C}$

$\displaystyle\int u'\,a^u\;dx = \frac{a^u}{\ln a} + \mathcal{C}\;\;(a \in \mathbb{R}^+ \setminus \{1\})$

$\displaystyle\int u'\,\cos u\;dx = \operatorname{sen} u + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{sen} u\;dx = -\cos u + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{tg} u\;dx = -\ln|\cos u| + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{cotg} u\;dx = \ln|\operatorname{sen} u| + \mathcal{C}$

$\displaystyle\int \frac{u'}{\cos^2 u}\,dx = \operatorname{tg} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{\operatorname{sen}^2 u}\,dx = -\operatorname{cotg} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{\cos u}\,dx = \ln\left|\frac{1}{\cos u} + \operatorname{tg} u\right| + \mathcal{C}$

$\displaystyle\int \frac{u'}{\operatorname{sen} u}\,dx = \ln\left|\frac{1}{\operatorname{sen} u} - \operatorname{cotg} u\right| + \mathcal{C}$

$\displaystyle\int \frac{u'}{\sqrt{1-u^2}}\,dx = \operatorname{arcsen} u + \mathcal{C}$

$\displaystyle\int \frac{-u'}{\sqrt{1-u^2}}\,dx = \arccos u + \mathcal{C}$

$\displaystyle\int \frac{u'}{1+u^2}\,dx = \operatorname{arctg} u + \mathcal{C}$

$\displaystyle\int \frac{-u'}{1+u^2}\,dx = \operatorname{arccotg} u + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{ch} u\;dx = \operatorname{sh} u + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{sh} u\;dx = \operatorname{ch} u + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{th} u\;dx = \ln(\operatorname{ch} u) + \mathcal{C}$

$\displaystyle\int u'\,\operatorname{coth} u\;dx = \ln|\operatorname{sh} u| + \mathcal{C}$

$\displaystyle\int \frac{u'}{\operatorname{ch}^2 u}\,dx = \operatorname{th} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{\operatorname{sh}^2 u}\,dx = -\operatorname{coth} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{\sqrt{u^2+1}}\,dx = \operatorname{argsh} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{\sqrt{u^2-1}}\,dx = \operatorname{argch} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{1-u^2}\,dx = \operatorname{argth} u + \mathcal{C}$

$\displaystyle\int \frac{u'}{1-u^2}\,dx = \operatorname{argcoth} u + \mathcal{C}$