LSTM-based Beam Tracking for mmWave Vehicular Networks

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Abstract—The use of millimeter wave (mmWave) frequency bands for transmission can improve data transmission rate with the help of beamforming technology to overcome the high path and penetration losses. However for vehicle, the high mobility of vehicle results in extremely frequent beam alignment and significant overhead. In this paper, a long short-term memory (LSTM)-based beam tracking method was proposed for reducing overhead brought by beam alignment in mmWave Vehicular Networks, by predicting beam angles at next time step through known beam angles at a certain number of consecutive time steps as features. To train this network, a time series array antenna channel data was set up by statistical channel model using time series vehicle information generated from road traffic simulation software named "Simulation of Urban MObility (SUMO)". Simulation results show that proposed LSTM-based method outpreforms Kalman filter based method, and can prevent frequent beam alignment to reduce overhead while ensuring acceptable signal to noise ratio (SNR).

Index Terms—beam tracking, mmWave vehicular networks, LSTM, Kalman filter

I. Introduction

MmWave is one of the important technology for the future development of communications [1], relies on abundant spectrum resources. In order to apply mmWave better, the high path and penetration loss caused by high frequency of mmWave need to be overcome [2]. Thanks to the short wavelength of mmWave, it is possible to integrate array antennas into small chip and use beamforming technique to constrained signal power in specified direction for high gain [3].

Unlike conventional omni-directional antennas, array antennas are directional [4], which means that if high antenna gain needs to be achieved, it is important to ensure that the beams generated by array antennas at the receiver and transmitter are aligned well with each other. Otherwise the misalignment of the beam pairs will result in serious gain reduction, degrading communications performance. A hierarchical codebook design was proposed in [5], which essentially exploit binary search algorithm to reduce overhead by using different widths of beams at different stages to perform beam search. However, this algorithm requires a high quantization level phase shifter and potentially multiple RF chains to implement, which results in high cost. In [6], Zhang et al. proposed a Kalman filter based method for beam tracking using exhaustive search. However, the number of measurements and protocol overhead for exhaustive search required increases with the number of elements in antenna. On the other hand, [7] exploits extended Kalman filter and requires only a single measurement, making it more suitable for beam tracking in fast-changing environments.

Providing reliable beam alignment for high speed vehicles is more challenging than for low speed terminals, due to vehicles move out of the beam coverage faster. To ensure that the vehicles are always covered by high gain beams, high frequency beam alignment is required, and it will further increase system overhead, especially when high resolution code books are used. Fortunately, due to the sparsity of mmWave channel [8] and the correlation between the angle of arrival (AoA) and angle of departure (AoD) of a millimeter-wave channel and the location of the receiver and transmitter [9], the beam angles of a vehicle traveling along a road under LOS condition can be well predicted.

In this paper, a LSTM-based beam tracking method is proposed. By using LSTM network to learn the historical channel state information (CSI) which was obtained from exhaustive beam search, AoA and AoD of current channel between vehicle and BS can be predicted to reduce system overhead. Meanwhile, a road traffic simulation software is use to generate time seires vehicle information (e.g. position, speed, etc.) to set up time seires CSI with a statistical channel model for training the network. Compared to prior work, this paper differs in the following ways: 1) More realistic channel model; 2) Simultaneous prediction of AoA and AoD; 3) The outage judgment is based on the SNR of the receiver instead of the angle difference; 4) Fewer beam measurements required on average in a tracking cycle. 5) In addition, beam search is performed more realistic by using a quantized phase shifter based beamforming.

The following notation will be used in this paper. Matrices, vectors and scalars are denoted by bold uppercase letters (e.g. $\mathbf{A}$), bold lowercase letters (e.g. $\mathbf{a}$) and lowercase letters (e.g. a), respectively. $(\cdot)^T$ denote transpose and $(\cdot)^H$ denote conjugate transpose (Hermitian). $[\mathbf{A}]_{m,:}$, $[\mathbf{A}]_{:,n}$ and $[\mathbf{A}_{m,n}]$ denote the mth row, nth column and the mth row nth column entry of $\mathbf{A}$, respectively. $[\mathbf{a}]_n$ denote the nth entry of $\mathbf{a}$. Besides, $\|\cdot\|_2$ denote ℓ_2-norm of a vector. $\mathbb{C}$ denote the set of complex number and $\mathbb{R}$ denote the set of real number. Gaussian, complex Gaussian, wrapped Gaussian and exponential distribution are denoted by $\mathcal{N}$, $\mathcal{CN}$, $\mathcal{WN}$ and $\mathcal{E}$, respectively.

II. SYSTEM MODEL

A mmWave vehicular networks scenario including a vehicle with M_r antenna as receiver and a base station (BS) with M_t antenna as transmitter are considered. Both of them equips with uniform linear array (ULA) of half wave interval antenna as shown in Fig. 1, and adopted analog beamforming used quantized phase shifter which connect with single analog radio frequency (RF) chain. The array response vector of a uniform linear array with M half wave interval antenna elements is given by:

$$\mathbf{a}(M,\varphi) = \frac{1}{\sqrt{M}}\left[1, e^{j\pi cos(\varphi)}, ..., e^{j\pi(M-1)cos(\varphi)}\right]^T \quad (1)$$

where φ is the arrival angle of the signal. When AoA of the vehicle is ϕ and AoD of the BS is θ, the array response vectors of both are $\mathbf{a}_r(\phi) = \mathbf{a}(M_r, \phi)$ and $\mathbf{a}_t(\theta) = \mathbf{a}(M_t, \theta)$, respectively.

The statistical 28 GHz mmWave channel model in [10] was used, which is modeled by real experimental data collected in New York City. The narrowband time-varying L subpaths channel matrix between the vehicle and BS at nth time step is $\mathbf{H}_n \in \mathbb{C}^{M_r \times M_t}$, which is given as:

$$\mathbf{H}_n = \frac{1}{\sqrt{L}}\sum_{l=1}^{L} g_{ln}\mathbf{a}_r(\phi_{ln})\mathbf{a}_t^H(\theta_{ln}) \quad (2)$$

where $g_{ln} \in \mathbb{C}$ is complex small-scale fading gain on subpath l at nth time step, ϕ_{ln} is AoA of the lth subpath signal received by the vehicle at nth time step, and θ_{ln} is AoD of the lth subpath signal transmitted by the BS at nth time step. $\phi_{ln} = \phi_n + \Delta\phi_{ln}$ and $\theta_{ln} = \theta_n + \Delta\theta_{ln}$, where ϕ_n and θ_n is the AoA and AoD of the cluster center at nth time step, $\Delta\phi_{ln} \sim \mathcal{WN}(\delta_{rn}^2)$, $\Delta\theta_{ln} \sim \mathcal{WN}(\delta_{tn}^2)$, $\delta_{tn} \sim \mathcal{E}(\lambda_t)$ and $\delta_{rn} \sim \mathcal{E}(\lambda_r)$. The complex small-scale fading gain is given by:

$$g_{ln} = \overline{g}_{ln}e^{j2\pi t_n f_{Dn,max}cos(\omega_{ln})}, \overline{g}_{ln} \sim \mathcal{CN}\left(0, 10^{-0.1PL_n}\right) \quad (3)$$

where t_n is the signal transmission time at nth time step, $f_{Dn,max} = v_n/\lambda_c$ is maximum doppler shift at nth time step, v_n is the speed of vehicle at nth time step, λ_c is the carrier wavelength, $\omega_{ln} = \phi_{ln} - \pi/2$ is the AoA of subpath l relative to the direction of vehicle at nth time step, and PL_n is omnidirectional path loss at nth time step, which is defined as:

$$PL_n = 61.4 + 20\log(d_n) + \xi \ [dB] \quad (4)$$

where d_n is distance between the vehicle and BS at nth time step, and $\xi \sim \mathcal{N}(0, \sigma_p^2)$. Note that only the case of single cluster under line-of-sight (LOS) condition is considered in this paper. In addition, the cluster angle depends on the geometric position of the vehicle and base station.

For a transmitted signal x_n ($|x_n| = 1$) from the BS, received signal y_n in the vehicle at nth time step is:

$$y_n = \mathbf{w}_n^H\mathbf{H}_n\mathbf{f}_n x_n + \mathbf{w}_n^H\mathbf{v}_n \quad (5)$$

where $\mathbf{w}_n$ and $\mathbf{f}_n$ are combining vector and beamforming vector at nth time step, respectively. $\mathbf{v}_n$ is Gaussian noise at time step n and $\mathbf{v}_n \sim \mathcal{CN}(0, \sigma_v^2\mathbf{I}_{M_r})$.
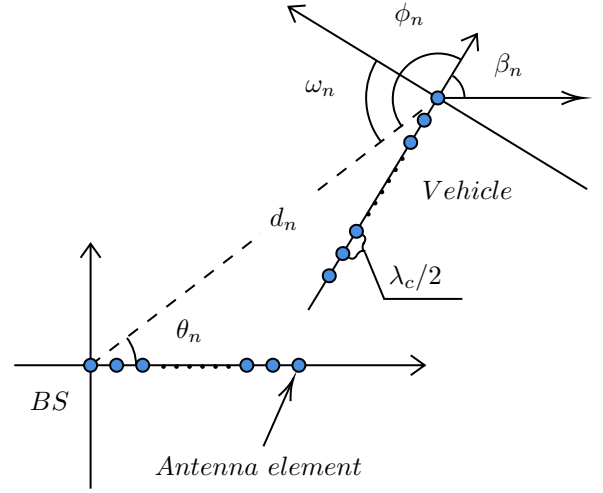

Fig. 1. Channel geometry

III. LSTM-BASED BEAM TRACKING SOLUTION

A. The LSTM Network

The purpose of beam tracking is to obtain beam angles (ϕ_{ln} and θ_{ln}) with as few beam measurements as possible for reducing overhead brought by beam search. Since the vehicle must travel in a particular direction on the road and the mmWave beams are correlated with the position of vehicles, the problem of obtaining beam angles can be formulated as a time series regression problem after the historical CSI of vehicles is collected.

An artificial neural network named LSTM network was used to solve this regression problem. It has been implemented successfully in several fields such as machine translation, image captioning, speech recognition, and even stock prices prediction in economy. As one of the recurrent neural networks (RNNs), LSTM are generally considered to be more robust to long time series than simpler vanilla RNN implementations [11]. It is powerful for handling time series data, beacase relying on three gates which are forget, input and output gate composed of neural networks (NNs) in LSTM cells, it can learn long-term relations between features.

Fig. 2 show the structure of the network, which is a sequential model comprises LSTM layer with 50 hidden units, a dropout layer with 0.2 probability, a LSTM layer with 50 hidden units, a dropout layer with 0.2 probability and a fully connected layer with 2 nodes. The output of the first LSTM layer is a sequence of length n_{LSTM}, which is used as the input of the second LSTM layer. And two dropout layers are set to prevent overfitting.

B. Loss Function

Mean Square Error (MSE) is the most commonly used regression loss function for quantifying the error of prediction. In [12], considering the problem of angle ambiguity due to the periodicity of the beam angle results in unnecessary loss for large difference of beam angle when used MSE, D. Burghal

$$\begin{bmatrix} \phi_k & \cdots & \phi_{k+n_{LSTM}-1} \\ \theta_k & \cdots & \theta_{k+n_{LSTM}-1} \end{bmatrix}$$

LSTM layer with n_{LSTM} cells

Dropout layer with 0.2 probability

LSTM layer with n_{LSTM} cells

Dropout layer with 0.2 probability

Fully connected layer with 2 nodes
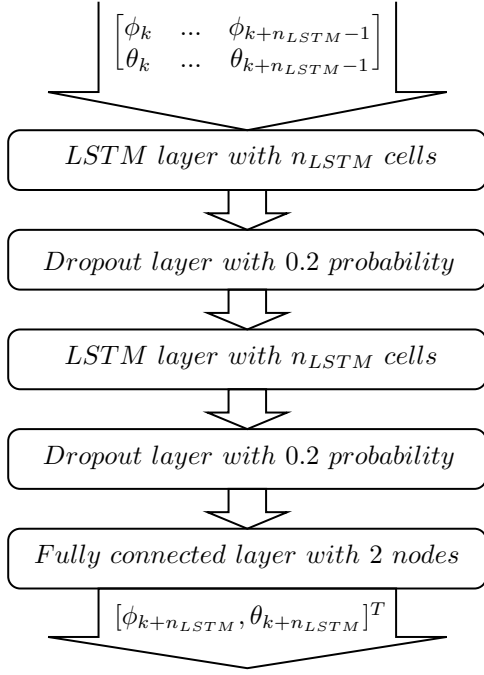
$[\phi_{k+n_{LSTM}}, \theta_{k+n_{LSTM}}]^T$

Fig. 2. Structure of network

et al. proposed to use a shifted cosine function as the loss function. However, when using this formula as a loss function, the training progress may be unstable due to the presence of multiple global minima, and the responses may not converge in a reasonable interval.

In addition, another problem is that the codewords whose beam angle near 0 or π have close gains at two different angles, resulting in unnecessary loss in some cases. For instance, in the case of the codeword whose beam angle is π, there are same gains at 0 and π. The loss should be small, even zero, if estimated angle is 0 and predicted angle is π, although the absolute difference of them is π. A straightforward idea is taking the negative of the dot product of the array response vector of estimated angle and predicted angle plus a positive constant as the loss function. Calculating this loss, however, requires array response vectors of estimated angle and predicted angle at the corresponding time step, which will increase the computational cost when training network. On the other hand, considering this function contains complex numbers, it will be difficlut to perform backpropagation (BP).

Hence, to avoid the problem of angle ambiguity and close gains, the AoAs and AoDs are limited in $(0, \pi]$ by simple conversion before training. Considering complexity of computation and training performance, a modified MSE is used as loss function, which is given as:

$$Loss = \frac{1}{N} \sum_{i=1}^{N} \left[(\phi_i^p - \phi_i^e)^2 + (\theta_i^p - \theta_i^e)^2 \right] \quad (6)$$

where N is the number of training samples, ϕ_i^e, θ_i^e, ϕ_i^p, θ_i^p, are the estimated AoA, estimated AoD, predicted AoA, and

predicted AoD, respectively, for training sample i.

C. Procedure of Beam Tracking

Before the beam tracking, the BS demands to collect historical CSI, which are estimated by the vehicles using exhaustive beam search, for training the LSTM network. For time step k, the network takes the AoAs and AoDs from time step $(k - n_{LSTM})$ to $(k - 1)$ as features and the AoA and AoD at time step k as responses. The dataset is consisted of time series AoAs and AoDs of the vehicles from initial access to disconnection.

After the LSTM network has been trained, at the beginning of a round of beam tracking, a vehicle takes n_{LSTM} consecutive exhaustive beam search to obtain initial estimated AoAs and AoDs, and the network will take the estimated AoAs and AoDs to predict AoA and AoD at next time step. Then, the predicted AoA and AoD at kth time steps will be used to compose new feature which from $(k - n_{LSTM} + 1)$th to kth time steps to predict the AoA and AoD at $(k+1)$ time step. For every time step, SNR under current predicted AoA and AoD will be calculated and compares with a threshold SNR_T. The network will repeat to use predicted result to compose new features to predict AoA and AoD, when the calculated SNR is greater than SNR_T. Otherwise, a new round of estimation and prediction will be repeated.

It makes more sense to use the SNR of received signal than to use the absolute difference of the angles as a threshold, because there is no way to obtain an accurate beam angle to calculate the threshold until it is accurately measured or predicted.

IV. DATA GENERATION

A. Channel Matrix

A road traffic simulation software named "SUMO" was used to generate time series information of the vehicle in the scene mentioned in Section II, which including a BS and a vehicle driving on curve road as shown in Fig. 3. The vehicle departs from $(0, 100)$ at first time step, and arrives $(100, 0)$ at final time step. Note that the speed of vehicle is different at each time step, that means the variation of vehicle position and beam angles is non-linear.

The generated vehicle time series information is composed of position, v_n and directional angle β_n of the vehicle at each time step. For nth time step, the position of vehicle is given by (x_{rn}, y_{rn}), v_n is used to determine maximum doppler shift in (3), and β_n is used to determine ϕ_{ln} and ω_{ln}.

Once the position of the vehicle and the BS are obtained, θ_n and ϕ_n can be calculated by simple geometric methods as following:

$$\begin{cases} \theta_n = \arg(\Gamma_{xn} + j\Gamma_{yn}) \\ \phi_n = \arg(\Gamma_{xn} + j\Gamma_{yn}) - \beta_n \end{cases} \quad (7)$$

where $\Gamma_{xn} = x_{rn} - x_t$, $\Gamma_{yn} = y_{rn} - y_t$, (x_t, y_t) are coordinates of the BS, β_n is directional angle of the vehicle. Furthermore, the signal transmission time $t_n = d_n/c$ can be calculated, where the distance between the BS and the vehicle is $d_n =$
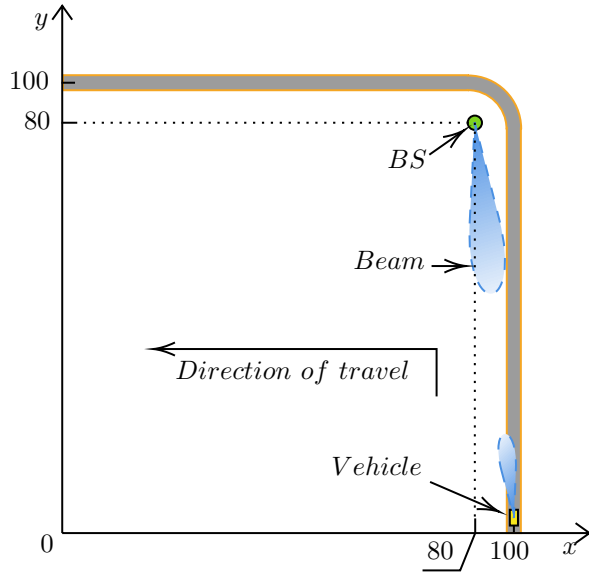
Fig. 3. Scene of mmWave vehicular networks for data generation

$\sqrt{\Gamma_{xn}^2 + \Gamma_{yn}^2}$, and c is light speed. Now the channel matrix at each time step can be obtained from (2), (3), (4) and (7).

B. Beam Search

In this paper, Downlink (DL) configuration scheme is considered for beam management. The beam management procedure is as follows. The reference signals are sent to the vehicle from the BS, the vehicle measures and determines the optimal beam for communication, then transmits beam reporting back to the BS.

AoAs and AoDs measured from exhaustive beam search method is used to train the network, rather than using the generated CSI directly. Although AoAs and AoDs are in range $[0, 2\pi]$, only the angles in range $[0, \pi]$ are considered, since the cosine function of the angles is symmetric around π. To reduce the overhead of beamforming, a discrete resolution b-bit codebook is adopted to perform beam search, where b is the number of bits for quantized phase shifters. For a b-bit codebook, it is consisted of 2^b beam patterns. In this paper, b is assumed to be $2log_2 M$ for a M-element antenna terminal. For a terminal with M-element antenna, the candidate beam angles vector $\mathbf{b}_M$ is given by:

$$\mathbf{b}_M = \left[\frac{1}{2M}\pi, \frac{2}{2M}\pi, ..., \pi \right] \tag{8}$$

So, there are $2M_r$ beam patterns for the vehicle, and $2M_t$ beam patterns for the BS. Note that another codebook which achieves uniform maximum gain in all direction is not used for this solution, beacause its low spatial resolution near 0 and π resulting in network underfitting. The beam codebooks of the vehicle and the BS are:

$$\mathbf{W} = [\mathbf{a}_r([\mathbf{b}_{M_r}]_1), \mathbf{a}_r([\mathbf{b}_{M_r}]_2), ..., \mathbf{a}_r([\mathbf{b}_{M_r}]_{2M_r})]$$
$$\mathbf{F} = [\mathbf{a}_t([\mathbf{b}_{M_t}]_1), \mathbf{a}_t([\mathbf{b}_{M_t}]_2), ..., \mathbf{a}_t([\mathbf{b}_{M_t}]_{2M_t})] \tag{9}$$

where $\mathbf{W} \in \mathbb{C}^{M_r \times 2M_r}$ is the codebook matrix of vehicle, and $\mathbf{F} \in \mathbb{C}^{M_t \times 2M_t}$ is the codebook matrix of BS. Each column of the codebook matrix represents a beam pattern, each entry in the column is phase rotation for corresponding antenna element to generate directional beam.

Assume $x_n = 1$ is sent as refernece signal, the observation matrix comprises all measurement of exhaustive search now can be given by:

$$\mathbf{Y}_n = \mathbf{W}^H \mathbf{H}_n \mathbf{F} + \mathbf{V}_n \tag{10}$$

where $\mathbf{V}_n \in \mathbb{C}^{2M_r \times 2M_t}$ is Guassian noise matrix composed of independent identically distribution (i.i.d.) elements, which are the noise part of (5). $\mathbf{Y}_n \in \mathbb{C}^{2M_r \times 2M_t}$, $[\mathbf{Y}_n]_{i,j}$ is the received signal of using $[\mathbf{W}]_{:,i}$ as combining vector and $[\mathbf{F}]_{:,j}$ as beamforming vector.

For each time step, both of the vehicle and the BS use their beam codebook to perform exhaustive beam search to find the optimal beam pair. The indexs of optimal beam pair can be obtained by solving a optimization problems as following:

$$\begin{aligned} (i_n^*, j_n^*) = \ &\underset{i_n, j_n}{\operatorname{argmax}} \| [\mathbf{Y}_n]_{i_n, j_n} \|_2 \\ &s.t. \ i_n \in [1, 2M_r], \\ &j_n \in [1, 2M_t] \end{aligned} \tag{11}$$

A simple nested loop algorithm can be expolited to solve this problem. Then, at nth time step, the optimal combining vector $\mathbf{w}_n^*$ and the optimal beamforming vector $\mathbf{f}_n^*$ are $\mathbf{w}_n^* = [\mathbf{W}]_{:,i_n^*}$ and $\mathbf{f}_n^* = [\mathbf{F}]_{:,j_n^*}$, respectively. And the estimated AoA ϕ_n^e and AoD θ_n^e used to training network are given by:

$$\begin{cases} \phi_n^e = \frac{i_n^* \pi}{2M_r} \\ \theta_n^e = \frac{j_n^* \pi}{2M_t} \end{cases} \tag{12}$$

The advantage of training with radian instead of the codeword indices is that it allows the proposed beam tracking solution to be generalized under terminals with different antenna configuration and beamforming techniques, without additional computation and transimtted data.

Note that due to the limited resolution of the beam codebook and the noise received at the vehicle, there will be some errors for the channel estimation, which is reflected in fluctuations in the estimated time series AoAs and AoDs. To reduce the fluctuations, a first order Savitzky-Golay filter [13] with frame length 11 was used to smooth the estimated AoAs and AoDs for training the LSTM network better.

V. SIMULATION RESULTS

In this section, the performance of proposed LSTM-based beam tracking solution is presented by analyzing the SNR of received signal, number of measurements and outage probability. The outage probability for a simulation is given by:

$$P_o = \frac{\sum_{i=1}^N \mathbf{I}(SNR_i \leqslant SNR_T)}{N} \tag{13}$$

where SNR_i is the SNR of received signal for ith sample and $\mathbf{I}$ is an indicator function.

Parameters	Values
Carrier frequency f_c	28×10^9 Hz
Coordinates of BS (x_t, y_t)	$(80, 80)$
Number of antenna elements for BS M_t	16
Number of antenna elements for Vehicle M_r	4
Number of subpaths L	20
Expectation of BS rms angular spread λ_t	$10.2°$
Expectation of Vehicle rms angular spread λ_r	$15.5°$
Path loss standard deviation σ_p	5.8 dB
Noise variance of received signal σ_v^2	-140 dBW
Sequence length of LSTM n_{LSTM}	5
Threshold of SNR SNR_T	5 dB
Data sample rate	20 Hz

In this paper, the sample rate of series channel data is 20 Hz (20 samples per second) and there are total 613 samples for training and 401 samples for testing. Table I show the common simulation parameters. For each simulation, parameters default to the values in Table I, except for those mentioned specifically.

Fig. 4 show the performances of algorithms versus noise variance of recevied signal. It illustrates that for both exhaustive search and LSTM-based method, the average SNR of received signal is decreasing and the probability of outages is increasing with the increasing of σ_v^2. The difference in performance between the LSTM-based method and the exhaustive search method is small, up to about 6 dB. When $\sigma_v^2 = -140$ dBW, the average SNR of recevied signal in exhaustive search is 54.67 dB and in LSTM-based method is 52.32 dB, there is only 2.35 dB loss in SNR, and about 4.3% loss in spectral efficiency.

It is observed that the loss in spectral efficiency is acceptable, compared with the reduction of measurement overhead.
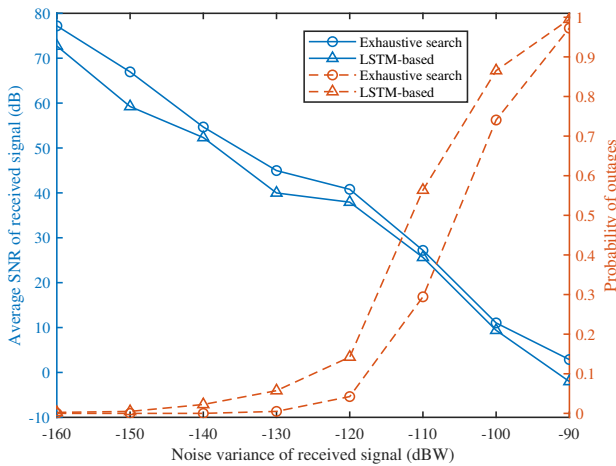
Since the SNR performance of exhaustive search is considered to be the best, there is a high probability that if there are outages when using exhaustive search, there will be outages when using other methods at the same time. Fig. 4 show that there is not any outages when the noise variance of received signal is lower than -140 dBW. Therefore, the default noise variance of received signal is defined as -140 dBW for following simulations to ensure that there are not any outages when using exhaustive search to acquire AoAs and AoDs.

VI. CONCLUSIONS

ACKNOWLEDGMENT

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Fig. 4. Insert caption