1.

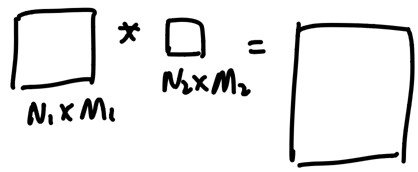
Problem 1 (7 points)

$(N_1+N_2-1) * (M_1+M_2-1)$

Show that forming unweighted local averages, which yields an operation of the form

$$\mathcal{R}_{ij} = \frac{1}{(2k+1)^2} \sum_{u=i-k}^{u=i+k} \sum_{v=j-k}^{v=j+k} \mathcal{F}_{uv}$$

$i+k \sim i-k$



is a convolution. What is the kernel of this convolution?

The mathematical formulation of 2D convolution is given by

$$y[i,j] = \sum_{u=1}^{m} \sum_{v=1}^{n} h[u,v] \cdot x[i-u, j-v]$$

$$\Rightarrow R'_{ij} = \sum_{p=1}^{m} \sum_{l=1}^{n} F(i-p, j-l) \cdot H(p,l)$$

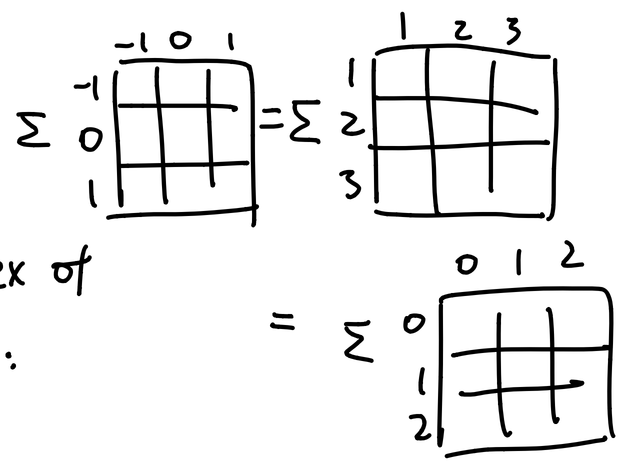
Suppose H is with m rows and n columns, F is with M rows and N columns, R' is with $M-m+1$ rows and $N-n+1$ columns.

Suppose $H_{u,v}$ is a square matrix and all the values are one

then $R'_{ij} = \sum_{p=1}^{m} \sum_{l=1}^{n} F(i-p, j-l) \cdot H(p,l) = \sum_{p=1}^{m} \big(F(-p,0) \cdot H(p,0) + F(-p,-1) \cdot H(p,1) + F(-p,-2) \cdot H(p,2)$ ...

$$= \sum_{p=1}^{m} \sum_{l=1}^{m} F(i-p, j-l)$$

$$= \sum_{u'=i-m}^{i-1} \sum_{v'=j-m}^{j-1} F(u', v')$$



take $2k = m-1$, then just change the index of the matrix similarly to the left hand side:

$$R'_{ij} = \sum_{u=i-k}^{i+k} \sum_{v=j+k}^{j+k} F_{uv} \text{ which means}$$

$$R_{ij} = \frac{1}{(2k+1)^2} \sum_{u=i-k}^{i+k} \sum_{v=j+k}^{j+k} F_{uv} \text{ is a convolution.}$$

H is a matrix $(2k+1) \times (2k+1)$ with all entries being $\frac{1}{(2k+1)^2}$

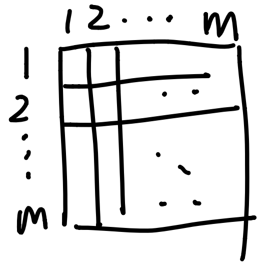
$$\frac{1}{(2k+1)^2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & & \\ \vdots & & \ddots & \\ & & & 1 \end{bmatrix}_{(2k+1) \times (2k+1)}$$

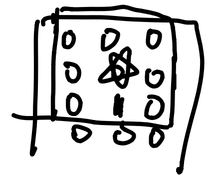
Problem 2 (7 points)

Write $\mathcal{E}_0$ for an image that consists of all zeros with a single one at the center. Show that convolving this image with the kernel

$$H_{ij} = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{((i-k-1)^2+(j-k-1)^2)}{2\sigma^2}\right)$$

(which is a discretised Gaussian) yields a circularly symmetric fuzzy blob.





$$g(i,j)=\sum_{k=-M}^{M}\sum_{l=-M}^{M} f(k+1,l+1)\,h(i-k-1,j-l-1)$$

Suppose H_{ij} forms a $(2m+1)\times(2m+1)$ matrix, F is $(2M+1)\times(2M+1)$ matrix consists of all zeros with a single one at the center. , $f[0,0]=1$, otherwise $f[i,j]=0$, $i\neq 0$, $j\neq 0$.

Symmetric: (Suppose when $|i|\le m$, or $|j|\le m$: (otherwise $g(i,j)=g(-i,j)=g(i,-j)$ $=g(-i,-j)=0$)

proof:

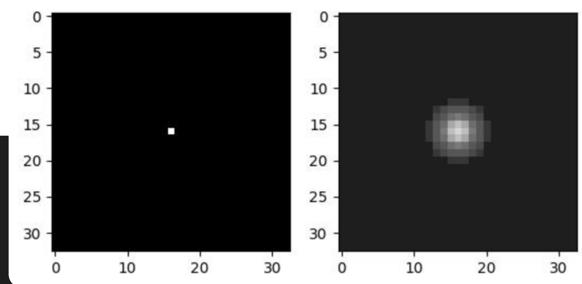
$$g(i,j)= f(0,0)*h(i,j) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{i^2+j^2}{2\sigma^2}\right) \quad (k+1=0,\ l+1=0)$$

$$g(-i,j) = f(0,0)*h(-i,j) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{(-i)^2+j^2}{2\sigma^2}\right)$$

Similarly : $g(i,j)=g(-i,j)=g(i,-j)=g(-i,-j) \Rightarrow$ symmetric

circularly:

Since $g(i,j)=g(-i,-j)$, it is circularly.




```python
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal
from PIL import Image
class GaussianBlur(object):
    def __init__(self, kernel_size=9, sigma=3):
        self.kernel_size = kernel_size
        self.sigma = sigma
        self.kernel = self.gaussian_kernel()

    def gaussian_kernel(self):
        kernel = np.zeros(shape=(self.kernel_size, self.kernel_size), dtype=np.float)
        radius = self.kernel_size//2
        for y in range(-radius, radius + 1):
            for x in range(-radius, radius + 1):
                v = 1.0 / (2 * np.pi * self.sigma ** 2) * np.exp(-1.0 / (2 * self.sigma ** 2) * (x ** 2 + y ** 2))
                kernel[y + radius, x + radius] = v
        kernel2 = kernel / np.sum(kernel)
        return kernel2

    def filter(self):
        a = np.zeros((33,33))
        d = np.array([0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,255,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0])
        a[[16],:]=d
        c = Image.fromarray(a)
        plt.subplot(1, 2, 1)
        plt.imshow(c)
        new_arr = signal.convolve2d(a, self.kernel, mode="same", boundary="symm")
        new_arr = np.array(new_arr, dtype=np.uint8)
        return Image.fromarray(new_arr)

def main():
    img2 = GaussianBlur(sigma=2).filter()

    plt.subplot(1, 2, 2)
    plt.imshow(img2)
```

```python
def main():
    img2 = GaussianBlur(sigma=2).filter()

    plt.subplot(1, 2, 2)
    plt.imshow(img2)

    plt.savefig("222.jpg", dpi=100)
    INPUT_PATH="222.jpg"
    OUPUT_PATH="333.jpg"
    I = Image.open(INPUT_PATH)
    L = I.convert('L')
    L.show()
    L.save(OUPUT_PATH)
    pass

main()
```

Problem 3 (7 points)

Show that convolving a function with a δ function simply reproduces the original function. Now show that convolving a function with a shifted δ function shifts the function.

convolving:

$$R(x,y) = \sum_i \sum_j H(i,j)\,F(x-i,\,y-j)$$

"Reproduces" means not changing the original function

consider $H_1(x,y)$ is $H_1(i,j)=0$ everywhere except $H_1(0,0)=1$,

then $R(x,y) = F(x-0,\,y-0) = F(x,y)$

then δ function is $H_1(x,y)$ that $H_1(i,j)=0$ everywhere except $H_1(0,0)=1$

"shifted": $R(x,y) = F(x+\Delta x,\, y+\Delta y) = \sum_i \sum_j H_2(i,j)\,F(x-i,\,y-j)$

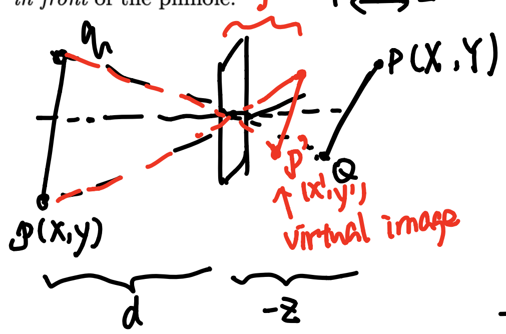
consider $H_2(x,y)$ is $H_2(i,j)=0$ everywhere except $H_2(-\Delta x, -\Delta y)=1$

then $R(x,y) = H_2(-\Delta x, -\Delta y)\cdot F(x+\Delta x,\, y+\Delta y) = F(x+\Delta x,\, y+\Delta y)$

then $H_2(x,y)$ is that $H_2(i,j)=0$ everywhere except $H_2(-\Delta x, -\Delta y)=1$

Problem 4 (7 points)

Derive the perspective equation projections for a virtual image located at a distance f' *in front* of the pinhole. $-f'$ $+ \longleftrightarrow -$



for any point $p(x,y)$ projected by $P(X,Y)$

$$\begin{cases} \dfrac{X}{-z} = \dfrac{x}{d} \\ \dfrac{Y}{-z} = \dfrac{y}{d} \end{cases}$$

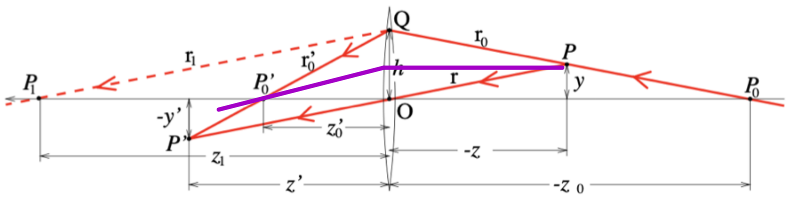
for any point $p'(x',y')$ "projected" by $p(x,y)$

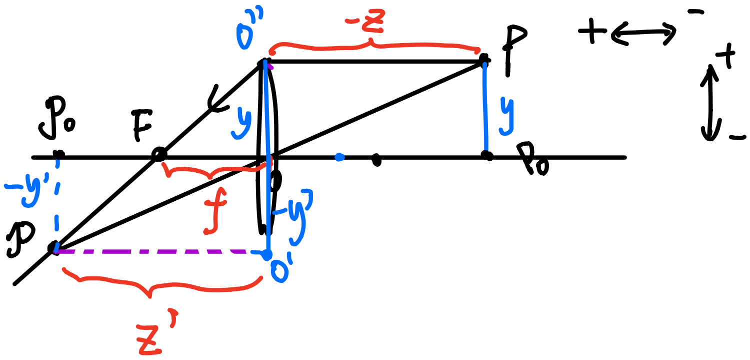
$$\begin{cases} \dfrac{x}{d} = -\dfrac{x'}{f'} \\ \dfrac{y}{d} = -\dfrac{y'}{f'} \end{cases}$$

$$\Rightarrow \begin{cases} x' = f'\dfrac{X}{z} \\ y' = f'\dfrac{Y}{z} \end{cases}$$ where z is the distance to the pinpole, which is positive.

Problem 5 (7 points)

Derive the thin lens equation.
Hint: consider a ray r_0 passing through the point P and construct the rays r_1 and r_2 obtained respectively by the refraction of r_0 by the right boundary of the lens and the refraction of r_1 by its left boundary.





$\triangle POP_0$ and $\triangle pOp_0$ are similar

$$\frac{y}{-y'} = \frac{-z}{z'} \Rightarrow \frac{y}{y'} = \frac{z}{z'}$$

$\triangle p_0pF$ and $\triangle OO''F$ are similar

$$\frac{y}{-y'} = \frac{f}{z'-f}$$

$$\Rightarrow \frac{-z}{z'} = \frac{f}{z'-f}$$

$$\Rightarrow z'f = -zz' + zf \qquad / z'zf$$

$$\Rightarrow \frac{1}{z} = -\frac{1}{f} + \frac{1}{z'}$$

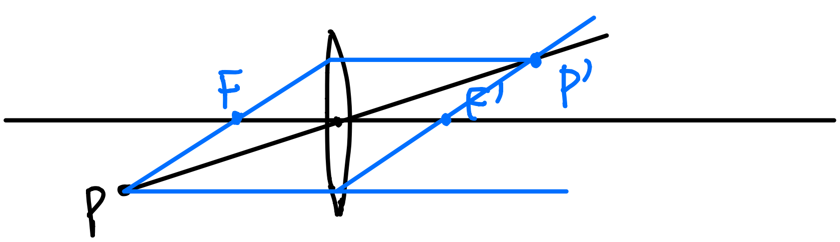
$$\Rightarrow \frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

P located at (negative) depth z off the optical axis.

Problem 6 (7 points)

Give a geometric construction of the image P' of a point P given the two focal points F and F' of a thin lens. ?

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

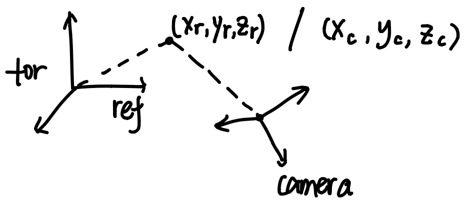


s is the object distance
s' is the image distance
f is the focal length of lens

Problem 7 (7 points)

Let $\boldsymbol{O}$ denote the *homogeneous* coordinate vector of the optical center of a camera in some reference frame, and let $\mathcal{M}$ denote the corresponding perspective projection matrix. Show that $\mathcal{M}\boldsymbol{O} = \mathbf{0}$.

we model the pose of the camera using 3x1 translation vector $\vec{t}$ and a 3x3 rotation matrix R

then 

where $\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix}$ is a random 3D point coordinate vector in some reference frame

$\begin{bmatrix} x_c \\ x_c \\ z_c \end{bmatrix}$ is its coordinate vector in camera frame

we have $\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = R\left(\begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} - t\right) = R\begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} - Rt$

for homogeneous coordinates :

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R & -Rt \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \quad \cdots \text{①}$$

we know that, for the optical center of the camera, its coordinate vector under "reference frame" is its traslation vector $\vec{t}$.

therefore $O = [t]$, homogeneous coordinates : $\begin{bmatrix} t \\ 1 \end{bmatrix}$

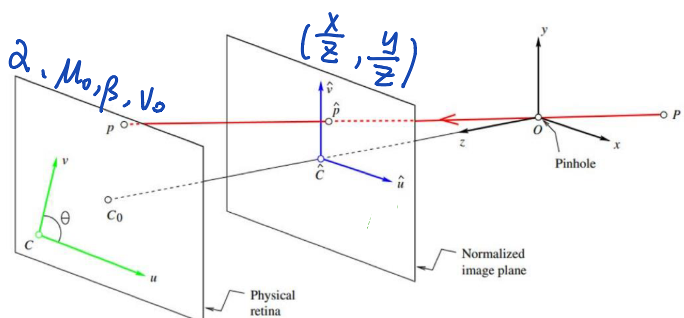
$$\overset{①}{\Longrightarrow} \begin{bmatrix} R & -Rt \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix} = \begin{bmatrix} Rt - Rt \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \vec{0} \text{ in Cartesian Coordinates.}$$

Problem 8 (7 points)

偏斜

Show that when the camera coordinate system is skewed and the angle θ between the two image axes is not equal to 90 degrees, then Eq. (2.11) transforms into Eq. (2.12).

$$\begin{cases} u = \alpha \dfrac{x}{z} - \alpha \cot\theta \dfrac{y}{z} + u_0 \\ v = \dfrac{\beta}{\sin\theta} \dfrac{y}{z} + v_0 \end{cases} \qquad (2.12)$$



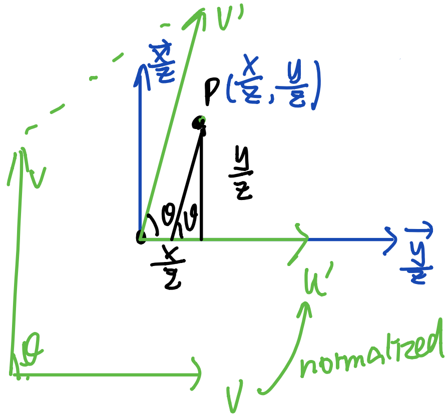
$$\begin{cases} u = \alpha \frac{x}{z} + u_0 \\ v = \beta \frac{y}{z} + v_0 \end{cases} \qquad (2.11)$$

→ the pixel coordinates of the projection of a point in the scene onto the retina.

$\vec{\frac{x}{z}}, \vec{\frac{y}{z}}$ are the coordinates of the projection of a point in the real scene onto the normalized image plane

α, β are the pixel magnification factors along the v and u axes

u_0, v_0 are then offsets of the image center from the origin of the camera coordinate system.



$P\left(\frac{x}{z}, \frac{y}{z}\right)$ after changing into skewed coordinate system:

$$\begin{cases} x' = \frac{x}{z} - \frac{y}{z\tan\theta} = \frac{x}{z} - \frac{y\cot\theta}{z} \\ y' = \frac{y}{z\sin\theta} \end{cases}$$

$\Rightarrow$ after changing the normalized coordinate plane into pixel coordinates of the projection in the scene onto retina:

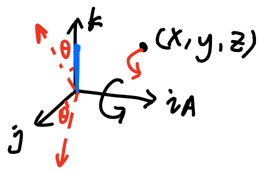
$$u = \alpha\left(\frac{x}{z} - \frac{y\cot\theta}{z}\right) + u_0 \qquad \Rightarrow \begin{cases} u = \alpha\frac{x}{z} - \alpha\cot\theta\frac{y}{z} + u_0 \\ v = \frac{\beta}{\sin\theta}\frac{y}{z} + v_0 \end{cases}$$

$$v = \beta\left(\frac{y}{z\sin\theta}\right) + v_0$$

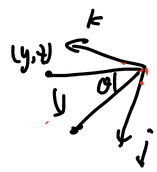
Problem 9 (7 points)

Write formulas for the matrices ${}^A_B\mathcal{R}$ when (B) is deduced from (A) via a rotation of angle θ about the axes $\boldsymbol{i}_A$, $\boldsymbol{j}_A$, and $\boldsymbol{k}_A$ respectively.

rotate about i_A:



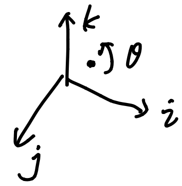
then $\begin{cases} x_i = x \\ y_i = \cos\theta y - \sin\theta z \\ z_i = \sin\theta y + \cos\theta z \end{cases}$



$$\begin{cases} y = r\cos\alpha \\ z = r\sin\alpha \end{cases}$$

$$\Rightarrow \begin{cases} y' = r\cos(\alpha+\theta) = r\cos\alpha\cos\theta - r\sin\alpha\sin\theta = y\cos\theta - z\sin\theta \\ z' = r\sin(\alpha+\theta) = r\sin\alpha\cos\theta + r\cos\alpha\sin\theta = z\cos\theta + y\sin\theta \end{cases}$$

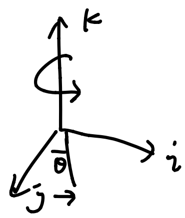
then $R_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$



rotate about j_A:

$$\begin{cases} x_j = x_i\cos\theta + z_i\sin\theta \\ y_j = y_i \\ z_j = z_i\cos\theta - x_i\sin\theta \end{cases}$$

then $R_j = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$



rotate about k_A:

$$\begin{cases} x_k = x_j\cos\theta - y_j\sin\theta \\ y_k = y_j\cos\theta + x_j\sin\theta \\ z_k = z_j \end{cases}$$

then $R_k = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow {}^A_B R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin^2\theta & \sin\theta\cos\theta \\ 0 & \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta\sin\theta & \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \sin^2\theta\cos\theta - \sin\theta\cos\theta & \sin\theta\cos^2\theta + \sin^2\theta \\ \sin\theta\cos\theta & \sin^3\theta + \cos^2\theta & \sin^2\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta & \cos\theta\sin\theta & \cos^2\theta \end{bmatrix}$$

Problem 10 (7 points)

Show that rotation matrices are $\boxed{\text{characterized}}$ by the following properties: (a) the inverse of a rotation matrix is its transpose and (b) its determinant is 1.

(a) the rotation matrices are in the form

of $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ or $R_1(\theta) = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$

$R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ $\qquad R_1^T = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$

$R^{-1}R = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} a\cos\theta + b\sin\theta = 1 \\ c\cos\theta + d\sin\theta = 0 \\ -a\sin\theta + b\cos\theta = 0 \\ -c\sin\theta + d\cos\theta = 1 \end{cases} \Rightarrow \begin{cases} a = \cos\theta \\ b = \sin\theta \\ c = -\sin\theta \\ d = \cos\theta \end{cases}$$

$$\Rightarrow R^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = R^T$$

$R_1^{-1}R_1 = I$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$\Rightarrow R_1^{-1} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} = R_1^T$$

(b) $\det(R) = \cos^2\theta + \sin^2\theta = 1$ $\qquad \det(R_1) = 0 + \cos^2\theta + \sin^2\theta + 0 = 1$

Project Problem

(a) for any two point (x_1, y_1) on a line $ax + by + c = 0$:

$$\begin{cases} ax_1 + by_1 + c_1 = 0 \\ ax_2 + by_2 + c_2 = 0 \end{cases} \Rightarrow \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

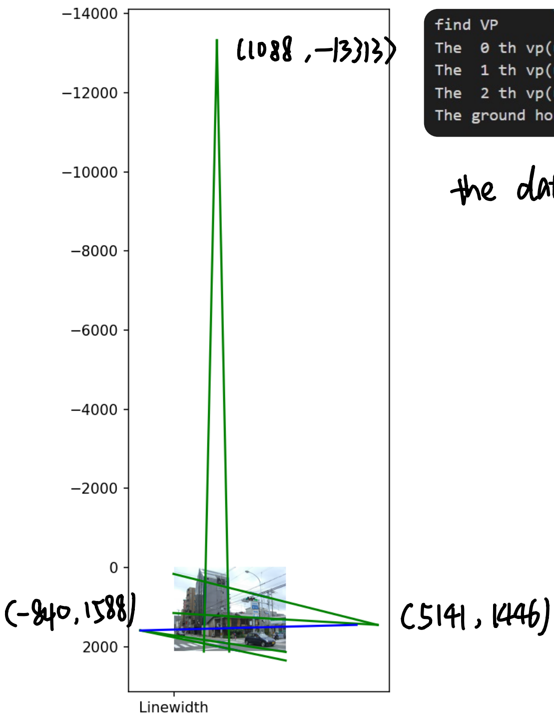
$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \text{null} \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} = [x_1\ y_1\ 1]^T \times [x_2\ y_2\ 1]^T$$

for two lines forming a vanishing point (u, v):

$$\begin{cases} a_1u + b_1v + c_1 = 0 \\ a_2u + b_2v + c_2 = 0 \end{cases} \Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = null \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} = [a_1 \ b_1 \ c_1]^T \times [a_2 \ b_2 \ c_2]^T$$

the result is :




```
find VP
The  0 th vp(u,v) is:  ( -840.415 1588.552 )
The  1 th vp(u,v) is:  ( 5141.626 1446.612 )
The  2 th vp(u,v) is:  ( 1088.093 -13313.063 )
The ground horizon line is in the form:  0.024 *u +  1.0 *v +  -1237285975832496.2 = 0
```

the data keep three decimal places.

(b) Since $x = K(R|t)X$ where x is the point in the image, X is the point in real world.

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} (R|t) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = K(R|t) \begin{bmatrix} 1 \\ y/x \\ z/x \\ 1/x \end{bmatrix}$$

Since every vanish point represent the infinite distance from origin along certain axis

for x axis: $x \to \infty$: $\lambda_x \begin{bmatrix} u_x \\ v_x \\ 1 \end{bmatrix} = K(R|t) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Similarly, for the homo coor vanish point in y and z axis $V_y = \lambda_y \begin{bmatrix} u_y \\ u_y \\ 1 \end{bmatrix}$ $V_z = \lambda_z \begin{bmatrix} u_z \\ u_z \\ 1 \end{bmatrix}$

$$\Rightarrow [V_x \ V_y \ V_z] = K(R|t) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since $(R|t)^T = (R|t)^{-1}$

$K^{-1}K = I$

$$\Rightarrow (R|t)^T K^{-1} [V_x \ V_y \ V_z] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = [l_x \ l_y \ l_z]$$ then by definition

$l_x^T l_y = l_x^T l_z = l_y^T l_z = 0$

$$\Rightarrow \left((R|t)^T K^{-1} V_x\right)^T \cdot \left((R|t)^T K^{-1} V_y\right) = 0$$

$$(K^{-1}V_x)^T \cdot (R|t) \cdot (R|t)^T (K^{-1} V_y) = (K^{-1}V_x)^T \cdot (R|t) \cdot (R|t)^{-1} (K^{-1} V_y) = (K^{-1}V_x)^T (K^{-1} V_y) = 0$$

Similarly, $(K^{-1}v_x)^T(K^{-1}v_y) = (K^{-1}v_x)^T(K^{-1}v_z) = (K^{-1}v_y)^T(K^{-1}v_z) = 0$

$$\Rightarrow v_x^T\left(K^{-T}K^{-1}\right)v_y = v_x^T\left(K^{-T}K^{-1}\right)v_z = v_y^T\left(K^{-T}K^{-1}\right)v_z = 0$$

consider $v_x^T(K^{-T}K^{-1})v_y = 0$

Since $K=\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1\end{bmatrix}$ $\quad K^T=\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ u_0 & v_0 & 1\end{bmatrix}$ $\quad K^{-T}=\begin{bmatrix} \frac{1}{f} & 0 & 0 \\ 0 & \frac{1}{f} & 0 \\ -\frac{u_0}{f} & -\frac{v_0}{f} & 1\end{bmatrix}$ $\quad K^{-1}=\begin{bmatrix} \frac{1}{f} & 0 & -\frac{u_0}{f} \\ 0 & \frac{1}{f} & -\frac{v_0}{f} \\ 0 & 0 & 1\end{bmatrix}$

$$\Rightarrow K^{-T}K^{-1} = \begin{bmatrix} \frac{1}{f} & 0 & 0 \\ 0 & \frac{1}{f} & 0 \\ -\frac{u_0}{f} & -\frac{v_0}{f} & 1\end{bmatrix}\begin{bmatrix} \frac{1}{f} & 0 & -\frac{u_0}{f} \\ 0 & \frac{1}{f} & -\frac{v_0}{f} \\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix} \frac{1}{f^2} & 0 & -\frac{u_0}{f^2} \\ 0 & \frac{1}{f^2} & -\frac{v_0}{f^2} \\ -\frac{u_0}{f^2} & -\frac{v_0}{f^2} & \frac{u_0^2}{f^2}+\frac{v_0^2}{f^2}+1\end{bmatrix}$$

consider $v_x^T(K^{-T}K^{-1})v_y = 0$

$$(u_x\ v_x\ 1)\begin{bmatrix} \frac{1}{f^2} & 0 & -\frac{u_0}{f^2} \\ 0 & \frac{1}{f^2} & -\frac{v_0}{f^2} \\ -\frac{u_0}{f^2} & -\frac{v_0}{f^2} & \frac{u_0^2}{f^2}+\frac{v_0^2}{f^2}+1\end{bmatrix}\underbrace{\begin{pmatrix} u_y \\ v_y \\ 1\end{pmatrix}}_{\lambda \text{ canceled}} = (u_x\ v_x\ 1)\begin{bmatrix} \frac{u_y}{f^2}-\frac{u_0}{f^2} \\ \frac{v_y}{f^2}-\frac{v_0}{f^2} \\ -\frac{u_0u_y}{f^2}-\frac{v_0v_y}{f^2}+\frac{u_0^2}{f^2}+\frac{v_0^2}{f^2}+1\end{bmatrix}$$

$$= \frac{u_xu_y}{f^2} - \frac{u_xu_0}{f^2} + \frac{v_xv_y}{f^2} - \frac{v_xv_0}{f^2} - \frac{u_0u_y}{f^2} - \frac{v_0v_y}{f^2} + \frac{u_0^2}{f^2} + \frac{v_0^2}{f^2} + 1$$

where u_x, u_y, v_x, v_y is known, $\frac{1}{f^2}, -\frac{u_0}{f^2}, -\frac{v_0}{f^2}, \frac{u_0^2}{f^2}+\frac{v_0^2}{f^2}+1$ is the unknown

$$= \frac{1}{f^2}(u_xu_y+v_xv_y) + \left(-\frac{u_0}{f^2}\right)(u_x+u_y) + \left(-\frac{v_0}{f^2}\right)(v_x+v_y) + \left(\frac{u_0^2}{f^2}+\frac{v_0^2}{f^2}+1\right)\cdot(1)$$

then:
$$\begin{bmatrix} u_xu_y+v_xv_y & u_x+u_y & v_x+v_y & 1 \\ u_xu_z+v_xv_z & u_x+u_z & v_x+v_z & 1 \\ u_yu_z+v_yv_z & u_y+u_z & v_y+v_z & 1\end{bmatrix}\begin{bmatrix} \frac{1}{f^2} \\ -\frac{u_0}{f^2} \\ -\frac{v_0}{f^2} \\ \frac{u_0^2}{f^2}+\frac{v_0^2}{f^2}+1\end{bmatrix} = 0 \qquad \begin{bmatrix} 1 \\ -u_0 \\ -v_0 \\ u_0^2+v_0^2+f^2\end{bmatrix}$$

find $\text{null}\begin{bmatrix} u_xu_y+v_xv_y & u_x+u_y & v_x+v_y & 1 \\ u_xu_z+v_xv_z & u_x+u_z & v_x+v_z & 1 \\ u_yu_z+v_yv_z & u_y+u_z & v_y+v_z & 1\end{bmatrix}$ we can solve f, u_0, v_0.

```
click second point
find VP
The  0 th vp(u,v) is:  [ 6985.44 , 1494.298 ]
The  1 th vp(u,v) is:  [ -1092.422 , 1655.975 ]
The  2 th vp(u,v) is:  [ 1093.171 , -16588.128 ]
The ground horizon line is in the form:  0.020010735703347632 *u +  0.9997997651813141 *v +  -517814749447464.3 = 0
[u0,v0] =  [ 1441.7941672951795 , 830.1850797529943 ]
f =  3674.2858852295767
The rotation matrix R is:
 [[-22362.21715365  -1825.47122187    262.38084386]
 [  -211.7941314     154.28065381  13569.6878856 ]
 [-14821.50655703   2646.69759947  -2765.34266427]]
```

(c) Since

$$[v_1 \; v_2 \; v_3] = K[R|t]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

taking into account the effect of multiplication of the homo points a inifity with the translation vector, we obtain:

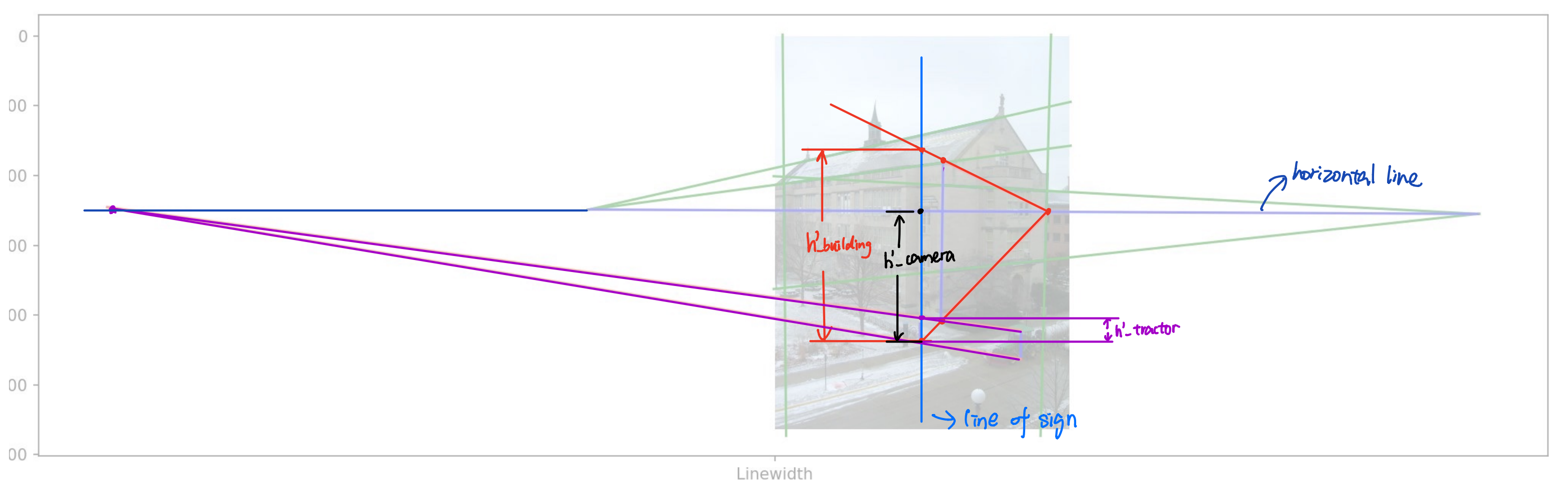
$$[v_1 \; v_2 \; v_3] = KR$$

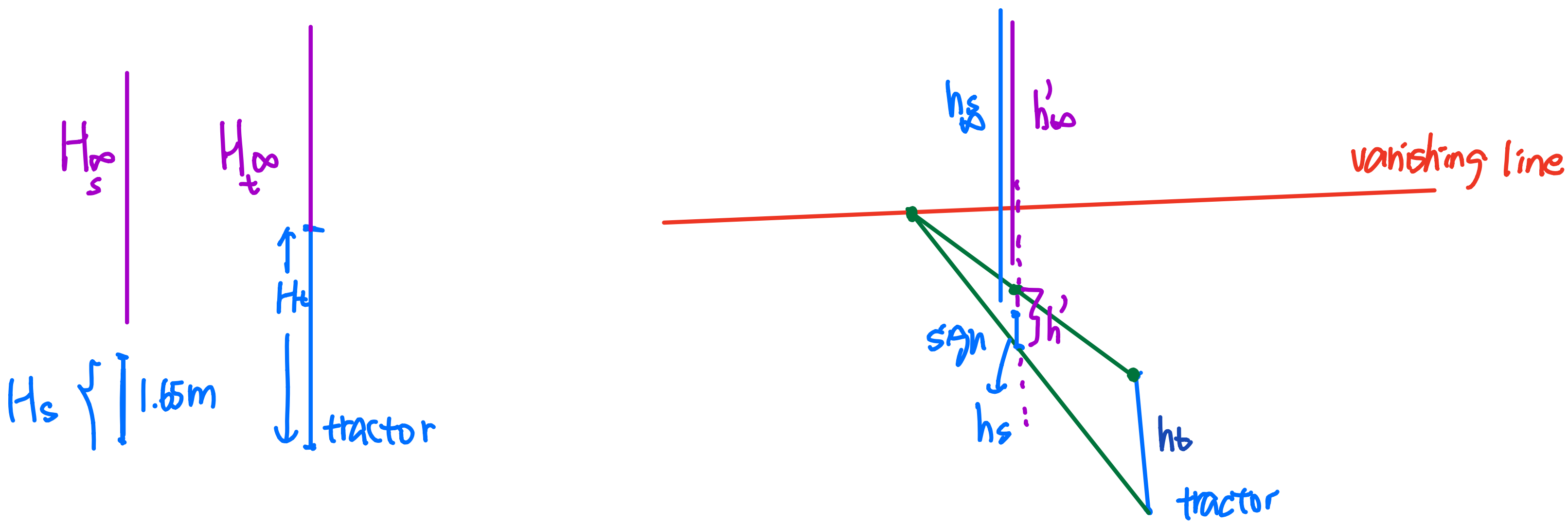
$$K^{-1}[vp_1 \; vp_2 \; vp_3] = K^{-1}KR = R$$

$$\begin{bmatrix} \frac{1}{f} & 0 & -\frac{u_0}{f} \\ 0 & \frac{1}{f} & -\frac{v_0}{f} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = R \Rightarrow R = \begin{bmatrix} \frac{u_1 - z_1 u_0}{f} & \frac{u_2 - z_2 u_0}{f} & \frac{u_3 - z_3 u_0}{f} \\ \frac{v_1 - z_1 v_0}{f} & \frac{v_2 - z_2 v_0}{f} & \frac{v_3 - z_3 v_0}{f} \\ z_1 & z_2 & z_3 \end{bmatrix}$$

origin u_1, v_1 (haven't been normolized)

(d)



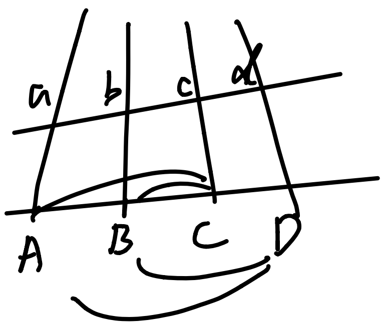


Since h' is the height that the tractor projected on the sign, then:

$$\frac{h_s}{h'} = \frac{H_s}{H_t}$$

then consider the infinite point in real life, and the vanishing point in z axis:

from:



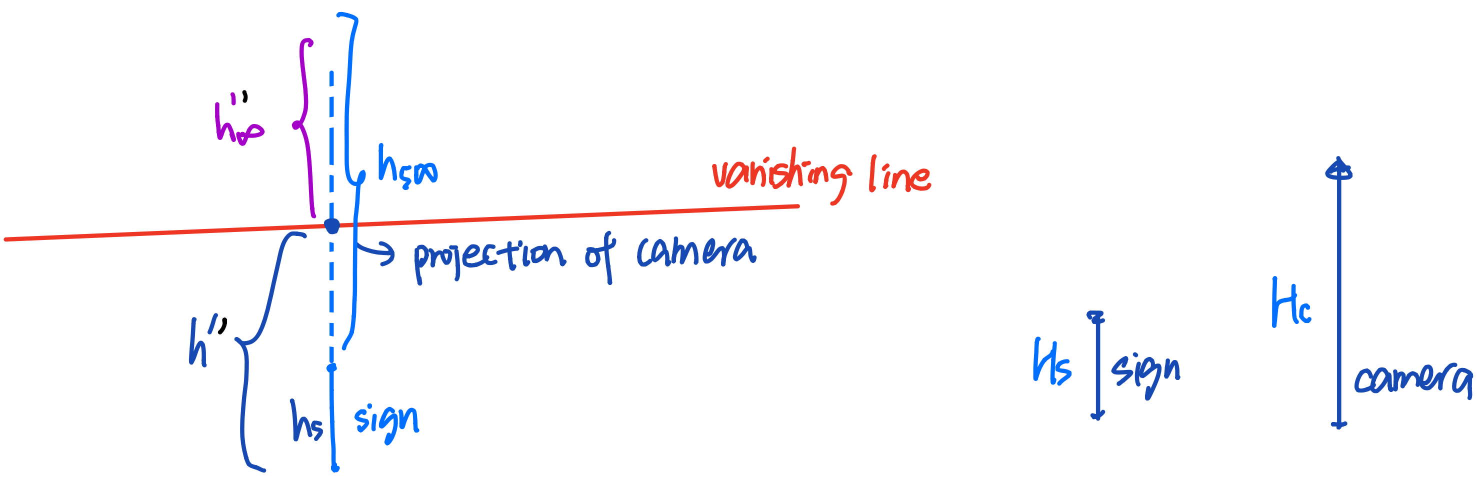
$$\frac{AC}{BC}\cdot\frac{BD}{AD} = \frac{ac}{bc}\cdot\frac{bd}{ad}$$

we have:

$$\frac{h_s}{h'}\cdot\frac{h'_\infty}{h_{\infty s}+h_s} = \frac{H_s}{H_t}\cdot\frac{H_{t\infty}}{H_{s\infty}+H_s} = \frac{H_s}{H_t}\cdot\frac{\infty}{\infty} = \frac{H_s}{H_t}$$

$$\Rightarrow H_t = \frac{H_s h'(h_{\infty s}+h_s)}{h_s h'_\infty}$$

Similarly for H_b



$$\Rightarrow H_c = \frac{H_s h''(h_{\infty s}+h_s)}{h_s h''_\infty}$$

the result is:

```
The height of the tractor is:  2.3239708464558926
The height of the building is:  16.817007375731777
The height of the camera is:  11.657892064741914
```