1.A. a. right and above

b. ① both of them are dominant by specular reflection.

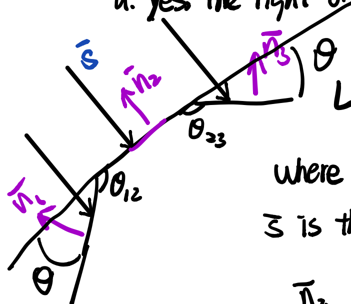
② the brighter one reflects a large scale of illumination to people eyes while the darker one almost can't receive light.

c. the Albedo parameter ρ is small, so most of light is absord by the dark streaks. It caused by lambertian reflectance. the unevenness of the label (shape) lets less light reflect on the surface.

d. No, we could only observe the directly reflected light on the glass.

B.

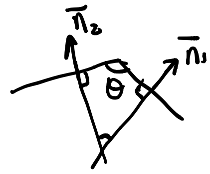
a. yes, the light orientation θ changes



$$L = \frac{\rho_d}{\pi} \cdot \frac{J}{r^2} (\vec{n}_i \cdot \vec{s}) \text{ , where } i = 1, 2, 3$$

where $\vec{n}$ is the surface normal,

$\vec{s}$ is the lighting direction



$$\vec{n}_1 \vec{n}_2 = |n_1| \cdot |n_2| \cdot \cos(\pi - \theta)$$

$$\Rightarrow \theta_{12} = \pi - \arccos \frac{\vec{n}_1 \vec{n}_2}{|n_1| \cdot |n_2|}$$

$$\theta_{23} = \pi - \arccos \frac{\vec{n}_2 \cdot \vec{n}_3}{|\vec{n}_2| \cdot |\vec{n}_3|}$$

$$\frac{L_1}{L_2} = \frac{\vec{n}_1 \cdot \vec{s}}{\vec{n}_2 \cdot \vec{s}} = \frac{|\vec{n}_1| \cdot |\vec{s}| \cos\theta}{|\vec{n}_2| \, |\vec{s}|} = \cos\theta$$

$$= \frac{0.5}{0.9} = \frac{5}{9}$$

then $\theta_{12} = \pi - \arccos \frac{5}{9} = 123.7°$

$\theta_{23} = \pi - \arccos \frac{8}{9} = 152.7°$

2. a. Yes, we need to pre-know the position of the camera, Intrinsics matrix of the camera, and the the transform of the hallway in the real world.

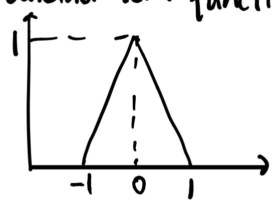
we need to get the rotation matrix of the camera dynamically.

b. Yes, the position of the guard, the direction of the guard.

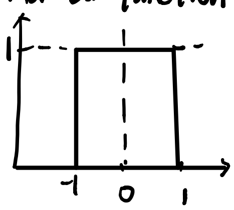
c. No, the projected lines from other views must not vanish at the same point, since the projected plane must not parallel to the hallway to hide the hero.

3. Since $F[f(x) * g(x)] = F[f(x)] \cdot F[g(x)]$

Consider tent function and box function:



$t(x)$

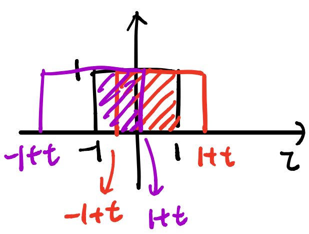


$b(x)$

where $b(x) = \begin{cases} 1 & , |x| \le 1 \\ 0 & , \text{otherwise} \end{cases}$

cosider $b(t) * b(t)$:

$$b(t) * b(t) = \int_{-\infty}^{\infty} b(\tau)\, \underline{b(t-\tau)}\, d\tau$$



when $0 < t < 2$:

$$b(t) * b(t) = 1 - (-1+t) \cdot 1 = 2 - t$$

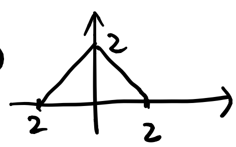
when $-2 < t < 0$:

$$b(t) * b(t) = 1 - t - 1 = 2 + t$$

when $t > 2$ or $t < -2$, $b(t) * b(t) = 0$

therefore: $b(t) * b(t) = \begin{cases} 2 - |t| & , |t| < 2 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow$



is a tent function.

which means if $f(t)$ is tent function,

It can be expressed as $f(t) = g(t) * g(t)$, $g(t)$ is box func.

$$F[f(t)] = F[g(t) * g(t)] = F[g(t)]\, F[g(t)]$$

$F\{g(t)\} = \int_{-\infty}^{\infty} g(t) e^{-2\pi j f t} dt = \int_{-T/2}^{T/2} A e^{-2\pi j f t} dt = \cdots = AT[\mathrm{sinc}(fT)]$

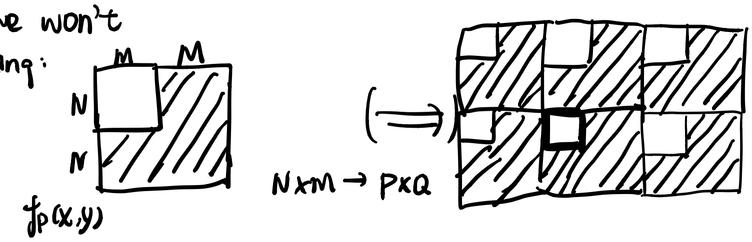
$\Rightarrow F\{f(t)\} = \mathrm{sinc}(f) \cdot \mathrm{sinc}(f)$

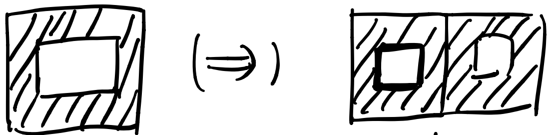
$= \mathrm{sinc}^2(f)$

the fourier transform of a tent function is a squared sinc function.

4. No, we won't

padding:





edges are the same

the spectrum view is also the same. (zeros unchaneg, shape unchanged)

$\Rightarrow$ multipy by $(-1)^{x+y}$ to center the transform

$\Rightarrow$ there will make no difference.

5. $H(u,v) = Ae^{-(u^2+v^2)/2\sigma^2} \Rightarrow$ cts frequency domain

show $h(t,z) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)}$ in spatial domain

$h(t,z) = F^{-1}\{H(u,v)\}$

for 2D-continuous fourier transform:

$F(u,v) = \iint_{-\infty}^{\infty} f(t,z) e^{-j2\pi(ut+vz)} dt dz$

$f(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) \cdot e^{j2\pi(ut+vz)} du dv$

$\Rightarrow h(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdot Ae^{-(u^2+v^2)/2\sigma^2} \cdot e^{j2\pi(ut+vz)} du dv$

since $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \Rightarrow \int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}}$

$h(t,z) = \int_{-\infty}^{\infty} Ae^{-v^2/2\sigma^2} \cdot e^{j2\pi vz} \int_{-\infty}^{\infty} e^{-u^2/2\sigma^2} \cdot e^{j2\pi ut} du dv$

$= \int_{-\infty}^{\infty} Ae^{-v^2/2\sigma^2} \cdot e^{j2\pi v^2} \int_{-\infty}^{\infty} e^{-(\frac{u^2}{2\sigma^2} - j2\pi t u)} du dv$ $\quad \nearrow x = \frac{u}{\sqrt{2}\sigma} \Rightarrow \frac{du}{dx} = \sqrt{2}\sigma$

$= \int_{-\infty}^{\infty} Ae^{-v^2/2\sigma^2} \cdot e^{j2\pi vz} \int_{-\infty}^{\infty} e^{-(\frac{u^2}{2\sigma^2} - j2\pi t u + 2j^2\pi^2 t^2\sigma^2) - 2\pi^2 t^2\sigma^2} du dv$ $\quad a = 1$

$= Ae^{-2\pi^2 t^2 \sigma^2} \cdot \sqrt{2}\sigma \cdot \sqrt{\pi} \cdot \int_{-\infty}^{\infty} Ae^{-v^2/2\sigma^2} \cdot e^{j2\pi vz} dv$, similarly,

$= Ae^{-2\pi^2\sigma^2(t^2+z^2)} \cdot 2 \cdot \sigma^2 \cdot \pi = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)}$

6.

$f(x,y) = A\sin(u_0 x + v_0 y) \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2}$

$f(x,y) = A \cdot \frac{e^{j(u_0 x + v_0 y)} - e^{-j(u_0 x + v_0 y)}}{2j}$

$F(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A \frac{e^{j(u_0 u + v_0 v)} - e^{-j(u_0 u + v_0 v)}}{2j} \cdot e^{-j2\pi(ut+vz)} du dv$

$$= \frac{A}{2j}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(u_0u+v_0v)-j2\lambda(ut+vz)} - e^{-j(u_0u+v_0v)-j2\lambda(ut+vz)}\,du\,dv$$

consider $\iint_{-\infty}^{\infty} e^{j(u_0u+v_0v)}\cdot e^{-j2\lambda(tu+zv)}\,du\,dv$

$$= \iint_{-\infty}^{\infty}\left[e^{j(u_0-2\lambda t)u}\cdot e^{j(v_0-2\lambda z)v}\right]du\,dv \Rightarrow \text{Since } \mathfrak{F}\{e^{j\frac{2\lambda n}{\Delta T}t}\} = \int_{-\infty}^{\infty} e^{-j2\lambda(\mu-\frac{n}{\Delta T})t}\,dt$$

$$= \iint_{-\infty}^{\infty}\left[e^{-j2\lambda(\frac{-u_0}{2\lambda}+t)u}\cdot e^{-j2\lambda(-\frac{v_0}{2\lambda}+z)}\right]du\,dv \qquad = \delta(\mu-\frac{n}{\Delta T})$$

$$= \delta(t-\frac{u_0}{2\lambda}, z-\frac{v_0}{2\lambda})$$

Similarly: $\iint_{-\infty}^{\infty} e^{-j(u_0u+v_0v)}\cdot e^{-j2\lambda(ut+zv)}\,du\,dv$

$$= \iint_{-\infty}^{\infty} e^{-j(u_0+2\lambda t)u}\cdot e^{-j(v_0+2\lambda z)v}\,du\,dv$$

$$= \iint_{-\infty}^{\infty} e^{-j2\lambda(\frac{u_0}{2\lambda}+t)u}\cdot e^{-j2\lambda(\frac{v_0}{2\lambda}+z)v}\,du\,dv$$

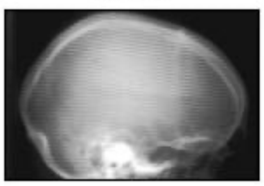
$$= \delta(t+\frac{u_0}{2\lambda}, z+\frac{v_0}{2\lambda})$$

then $F(t,z) = \frac{A}{2j}\left[\delta(t-\frac{u_0}{2\lambda}, z-\frac{v_0}{2\lambda}) - \delta(t+\frac{u_0}{2\lambda}, z+\frac{v_0}{2\lambda})\right]$

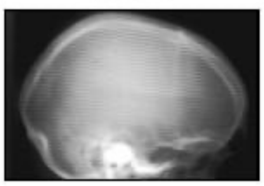
$F(u,v) = -j\frac{A}{2}\left[\delta(u-\frac{u_0}{2\lambda}, v-\frac{v_0}{2\lambda}) - \delta(u+\frac{u_0}{2\lambda}, v+\frac{v_0}{2\lambda})\right]$

Problem 1, (a)

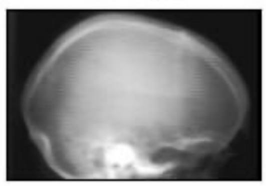
N = 3



N = 7



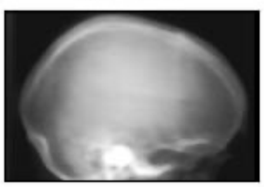
N = 11



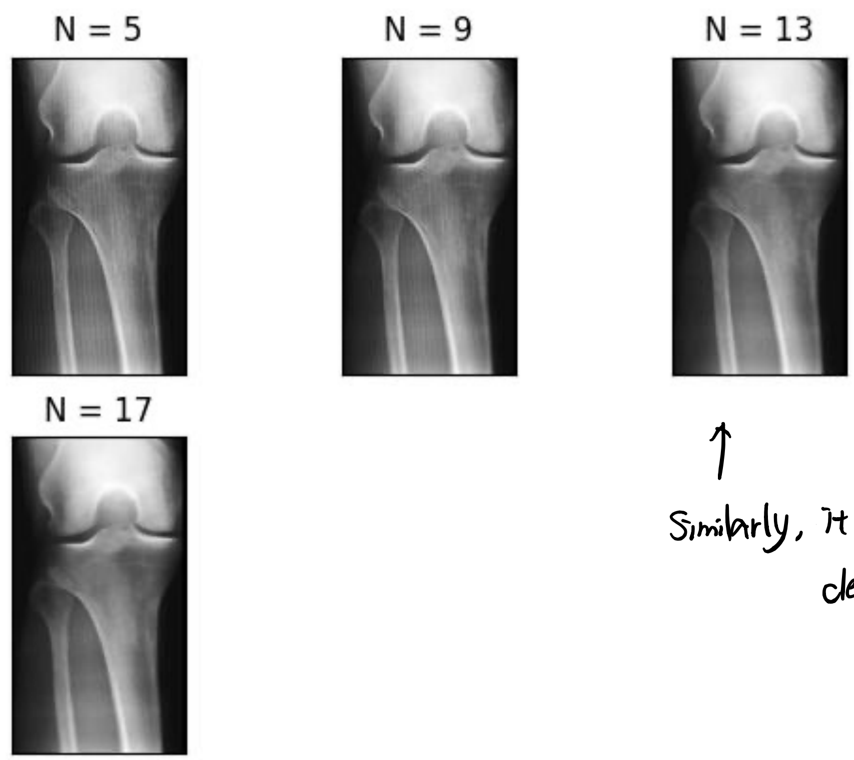
the moire is obvious

better

N = 15

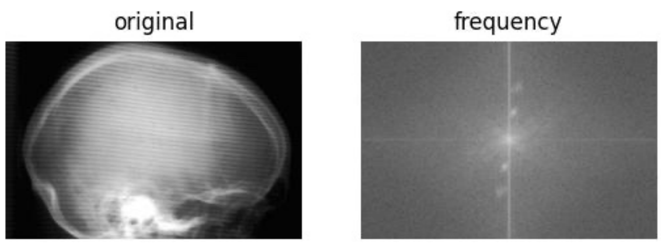


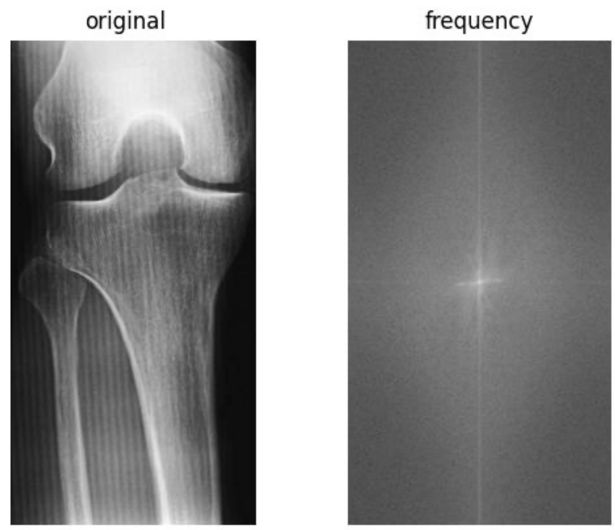
the edge is not clear

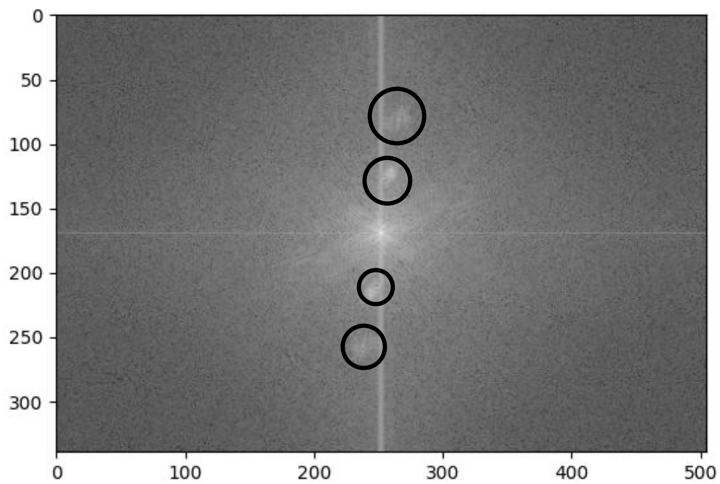


Similarly, it has less pattern and clearer edges.

(b)





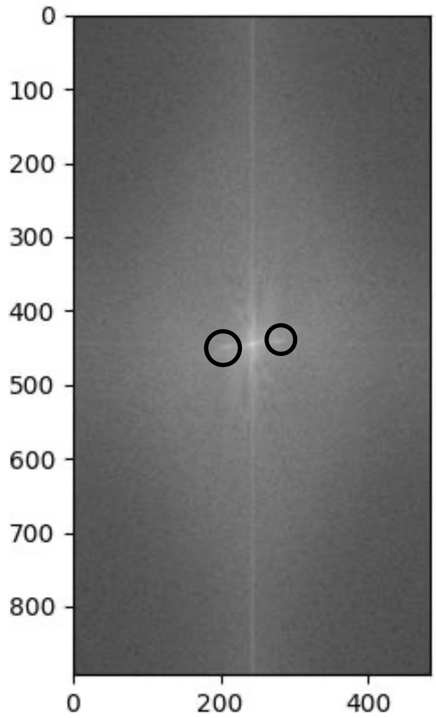


x=234. y=259.
[188.]

x=245. y=213.
[231.]

x=261. y=123.
[232.]

x=269. y=80.
[152.]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100
200
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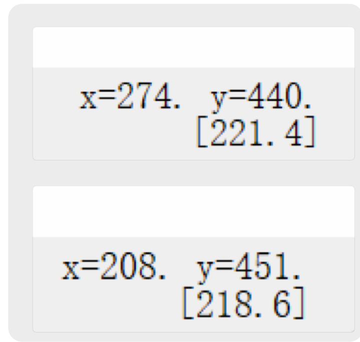
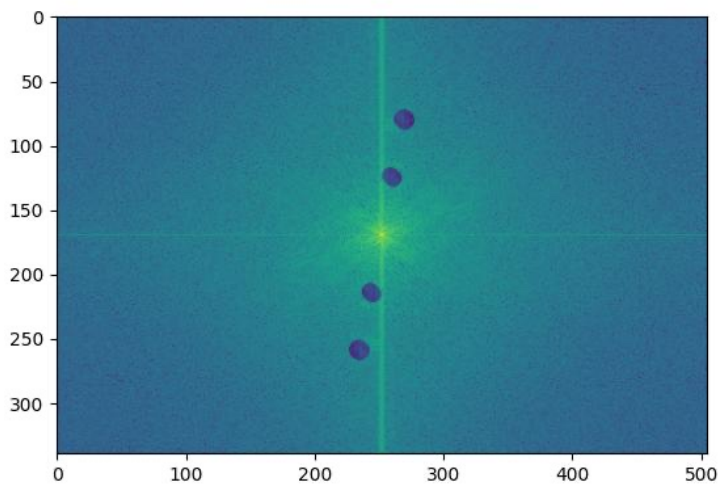
400
500
600
700
800
0
200
400


x=274. y=440.
[221. 4]
x=208. y=451.
[218. 6]

(c)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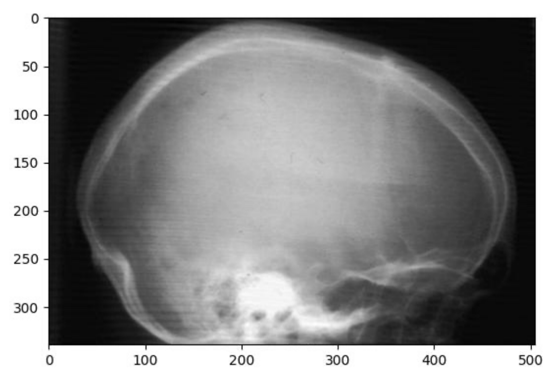
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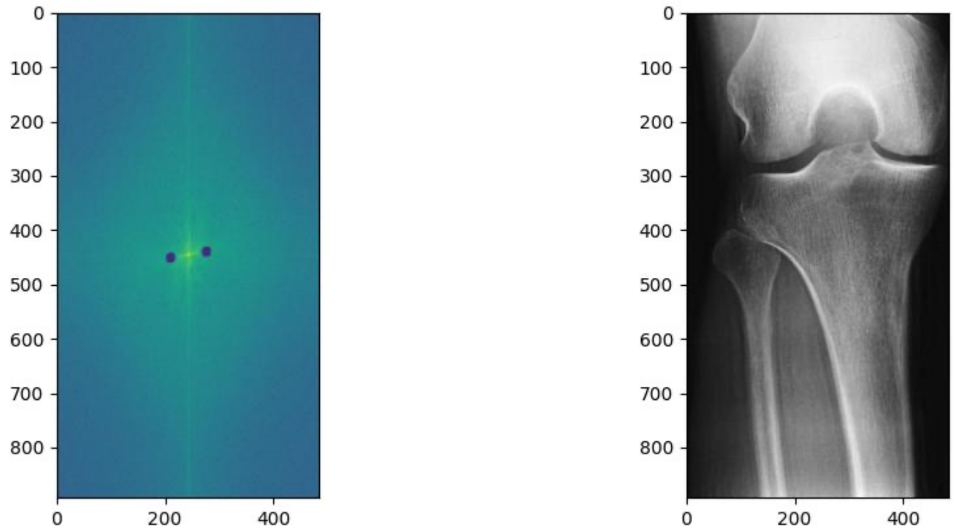
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