2025 Assignment Dynamics of Structures under Moving Loads – Part A

In this part of the assignment, your task is to solve small problems related to the Canonical Problems 1-4 and interpret the results you obtain. In doing so, you get hands-on experience, and you gain a deeper understanding of the structural responses to moving loads.

<u>Problem 1 (about Canonical Problem 1)</u>
Derive and plot (for a fixed moment in time) the steady-state response of a semi-infinite beam subject to an oscillatory load at the boundary with a relatively small excitation frequency as well as with a relatively large excitation frequency (as compared to the cut-off frequency). Interpret the result.

<u>Problem 2 (about Canonical Problem 1)</u>
Consider an infinite beam with continuous visco-elastic foundation, subject to a constant moving load. The equation of motion reads as follows (η denotes the distributed-dashpot constant; other symbols have the same meaning as in the lecture slides of Canonical Problem 1):

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + \chi w + \eta \frac{\partial w}{\partial t} = Q_0 \delta(x - Vt)$$

Derive and compute the steady-state response in the moving reference system (i.e., $w(\xi)$, where $\xi = x - Vt$) for a sub-critical velocity (note that the critical velocity is approximately the same as that of the undamped system) and plot the result for the following parameter values:

$$\rho A = 268.3 \text{ kg/m}$$
$$EI = 6.42 \cdot 10^6 \text{ Nm}^2$$
$$\chi = 7.3 \cdot 10^6 \text{ N/m}^2$$
$$\eta = 1 \cdot 10^2 \text{ Ns/m}^2$$
$$Q_0 = 80 \cdot 10^3 \text{ N}$$

It is advised to take the integral over frequency numerically, for example using the Trapezium rule (which is most easy). In doing so, make sure you respect the sampling theorem related to the discrete Fourier transform. The use of Maple to conduct the numerical integration is in this case not advised, as the result is very sensitive to the precise settings (accuracy, discretization, etc.).

<u>Problem 3 (about Canonical Problem 1)</u>
For the same problem considered in the previous question, derive and compute the equivalent stiffness at the loading/contact point and plot it versus velocity. Consider in the plot only the sub-critical velocity range (up to the 99% of the critical velocity) and explain what you observe.

(other problems will be added soon)