CS208 · Algorithm Design

Assignment 4

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1 Chapter 4

1.1 Exercise 2

1. True.
   If we replace each edge cost c_e by c_e^2, it will sort them in the same order as before replacement. Since Kruskal's algorithm is a kind of greedy algorithm, the time when we put every edge into the edge set of MST is also the same as origin.
2. False.
   Suppose we have a directed graph G with edges $(u, v, 2)$, $(v, w, 2)$, and $(u, w, 3)$, which means (source, destination, weight). We leave from vertex u and for vertex w. The shortest path is $u \to w$ with total weight 3. But after replacement, for $(u, v, 4)$, $(v, w, 4)$, and $(u, w, 9)$, the shortest path will be $u \to v \to w$ with total weight 8.

1.2 Exercise 8

Suppose we have two distinct MST T and T' on graph G. Since they contain the same number of nodes (both of $|V|$, where V is the vertex set of G) and hence they have the same number of edges (both of $|V| - 1$) though the edges are not totally equal.
If edge e is in T but not in T' (obviously this edge is in G), add it into T, which will make a cycle C. The cycle C is also appear in graph G. By cycle property, the most cost edge f in C is not in MST. Therefore, one of T or T' is not an MST, leading a contradiction.

1.3 Exercise 22

We cannot draw this conclusion.
Consider an undirected graph with vertexes $V = \{v_1, v_2, v_3\}$ and edges with cost $E = \{(v_1, v_2, 2), (v_2, v_3, 2), (v_1, v_3, 1)\}$.
The counterexample is listed as below.

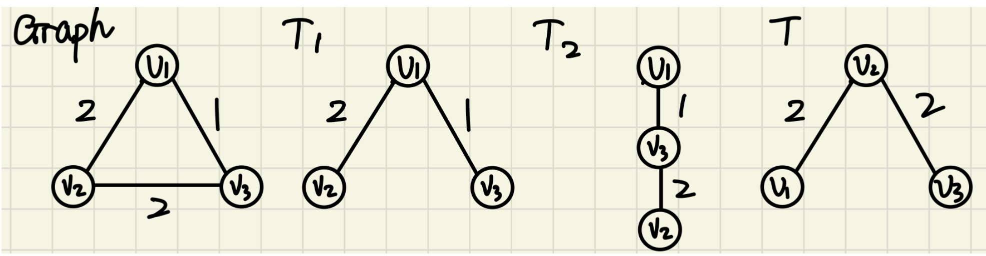

Figure 1: Counterexample of Exercise 22

Both T_1 and T_2 are minimum-cost spanning tree and the edges of T are contained in T_1 or T_2. However, T is not a minimum-cost spanning tree of graph G.