CS208 · Algorithm Design

Assignment 1&2

刘乐奇 / 12011327

March 14, 2022

1 Chapter 1

1.1 Exercise 1

False.
Consider the instance as below:
Suppose men list = {m, m′}, women list = {w, w′}, and

m prefers w to w′;
m′ prefers w′ to w;
w prefers m′ to m;
w′ prefers m to m′;

There are no pairs where each ranks the other first.

1.2 Exercise 2

True.
Suppose that there are two pairs (m, w′) and (m′, w) where neither w′ nor m′ ranks first at the list of m or w, respectively.
Since m prefers w than any others and w prefers m than any others, this matching is not stable.

1.3 Exercise 3

There is not always a stable pair of schedules.
To simplify the question, suppose we have network $\mathcal{A}$ with 2 shows $\{a_1, a_2\}$, network $\mathcal{B}$ with 2 shows $\{b_1, b_2\}$, where ratings are ranked like $a_1 > b_1 > a_2 > b_2$. If $S = \{a_1, a_2\}$ and $T = \{b_1, b_2\}$, then $\mathcal{B}$ would want to change T to be $T' = \{b_2, b_1\}$ to obtain one slot instead of none. But at this time, $\mathcal{A}$ would want to change S to be $S' = \{a_2, a_1\}$ for two slots instead of one.
For another perspective, the same networks and the same shows but with different ratings $a_1 > b_1 > b_2 > a_2$. At this time, if $S = \{a_1, a_2\}$ and $T = \{b_1, b_2\}$, the pair of schedules (S, T) is stable.

1.4 Optional: Exercise 8

Suppose we have man list $\{m_1, m_2, m_3\}$ and woman list $\{w_1, w_2, w_3\}$. Consider the preference list as below:

Man's Appearance No.	1st	2nd	3rd
m_1	w_3	w_1	w_2
m_2	w_1	w_3	w_2
m_3	w_3	w_1	w_2

Woman's Appearance No.	1st	2nd	3rd
w_1	w_1	w_2	w_3
w_2	w_1	w_2	w_3
w_3	w_2	w_1	w_3

For the preference list, after GS algorithm, we have

m_1	w_3		
m_2		w_1	
m_3			$(w_3)(w_1)w_2$

m_1 proposes to w_3, m_2 proposes to w_1, m_3 proposes to w_2 after failing to propose w_3 and w_1. Finally we have the pairs $(m_1, w_3), (m_2, w_1), (m_3, w_2)$.
If we consider there is a false preference list of w_3 as $\{m_2, m_3, m_1\}$, then we have

m_1	w_3		break up	w_1		
m_2		w_1		break up	w_3	
m_3			w_3		break up	$(w_1)w_2$

After GS algorithm the pairs will be $(m_1, w_1), (m_2, w_3), (m_3, w_2)$, making the liar w_3 get a preferable partner. So we can conclude that a switch on preference list will improve the partner of a woman.

2 Chapter 2

2.1 Exercise 1

1. When the input size is doubled, the algorithms will get slower by
   (a) $\frac{(2n)^2}{n^2} = 4$
   (b) $\frac{(2n)^3}{n^3} = 8$
   (c) $\frac{100(2n)^2}{100n^2} = 4$
   (d) $\frac{2nlog2n}{nlogn} = 2\frac{log2+logn}{logn} = 2 + 2\frac{log2}{logn}$
   (e) $\frac{2^{2n}}{2^n} = 2^n$
2. When the input size is increased by an additive one, the algorithms will get slower by
   (a) $\frac{(n+1)^2}{n^2} = 1 + \frac{2}{n} + \frac{1}{n^2}$
   (b) $\frac{(n+1)^3}{n^3} = 1 + \frac{1}{n^3} + \frac{2}{n} + \frac{2}{n^2}$
   (c) $\frac{100(n+1)^2}{100n^2} = 1 + \frac{2}{n} + \frac{1}{n^2}$
   (d) $\frac{(n+1)log(n+1)}{nlogn}$
   (e) $\frac{2^{n+1}}{2^n} = 2$

2.2 Exercise 5

1. False.
   Since $f(n) = O(g(n))$, then $\exists c(\forall n \geq n_0, f(n) \leq cg(n))$. So $log_2 f(n) \leq log_2 c + log_2 g(n)$. If $\forall n \geq n_0, g(n) \geq 2$, then $log_2 c + log_2 g(n) \leq (log_2 c)(log_2 g(n))$; else, the inequality does not hold.
   Let $f(n) = 2$ and $g(n) = 1$ which satisfies $f(n) = O(g(n))$. However, $log_2 f(n) = 1$ while $log_2 g(n) = 0$. There will never exist $log_2 f(n) \leq clog_2 g(n)$.
2. False.
   Let $f(n) = 2n$ and $g(n) = n$ which satisfies $f(n) = O(g(n))$, i.e., $2n = O(n)$. However, $2^{f(n)} = 4^n \neq O(2^{g(n)}) = O(2^n)$.
3. True.
   Since $f(n) = O(g(n))$, then $\exists c(\forall n \geq n_0, f(n) \leq cg(n))$. So $f(n)^2 \leq c^2 g(n)^2$, i.e., $f(n)^2 = O(g(n)^2)$