Student ID: ________________________ **Student Name:** ______________________

CS203 Data Structure and Algorithm Analysis **Quiz 1**

Note 1: Write all your solutions in the question paper directly. You can ask additional answer paper if necessary

Note 2: If a question asks you to design an algorithm, full marks will be given if your algorithm runs with optimal time complexity

Note 3: If a question asks you to design an algorithm, you should **first** describe your ideas in general words, **then** write the pseudocode, and **end** with time complexity analysis.

Problem 1 [20 points] Recursive Algorithm.

```
RecursivePrint (int n)
if n ≤ 0 then
    return
if n%2=0 then
    Print(n) // print the integer n into the standard output
    RecursivePrint (n-1)
else
    RecursivePrint (n-1)
    Print(n)
```

(a)[4 points] The time complexity of the algorithm is ____________

(b)[4 points] Let n = 5, write down its output: ______________________________

(c)[4 points] Let n = 8, write down its output: ______________________________

(d)[8 points] Please modify the above pseudocode such that the output is "531246" when n=6. (You only can use the used functions and statements)

Problem 2 [30 points] Given two sorted arrays **A** and **B** (in ascending order), with n and m integers respectively. All (n+m) integers are distinct. Design an algorithm to find the median in all (n+m) integers.

Median definition: If (n+m) is an odd number, median is the (n+m+1)/2-th smallest number. If (n+m) is an even number, median is the (n+m)/2-th smallest number. For example, A = {1,3}, B = {2, 4}. The median in these four integers is “2”.

Problem 3 [20 points] Sorting Algorithm

ShellSort is mainly a variation of Insertion Sort. In insertion sort, we move elements only one position ahead. When an element has to be moved far ahead, many movements are involved. The idea of shellSort is to allow exchange of far items. In shellSort, we make the array h-sorted for a large value of h. We keep reducing the value of h until it becomes 1. An array is said to be h-sorted if all sublists of every h'th element is sorted.

(a) [2 points] Records A[1], A[2], A[3],..., A[N] are said to be h-sorted, if ________
(A) A[i] <= A[i+h] for 1<= i*h <= N
(B) A[h] <= A[i+h] for 1<= i <= N
(C) A[i] <= A[h] for 1<= i <= h
(D) A[i] <= A[i+h] for 1<= i <= N-h

(b) [2 points] An array that is first 7-sorted, then 5-sorted becomes _________
(A) 7-ordered
(B) 5-ordered
(C) both 2-ordered and 5-ordered
(D) both 7-ordered and 5-ordered

(c) [4 points] In the worst case, the quick sort algorithm and shell sort algorithm will degenerate to ________ and ________ sort algorithm, respectively.

(d) [3 points] Shell sort is more efficient than insertion sort if the length of input arrays is small. True or False? Why?

(e) [9 points] Fill the following table to show the running steps of Shell-Sort Algorithm.

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
Input	13	26	18	53	7	17	95	86
4-Sorted								
2-Sorted								
1-Sorted								

Problem 4 [7 points] Proof $5n^3 + 7n^2\sqrt{n} = O(n^3)$.

Problem 5 [5 points] Which of the following function is not $O(n^{2.5})$ ( )

A. $53179546n^2$ B. $n^{2.7}/\log^2 n$ C. $\frac{n^{100}}{2^n}$ D. $(\log_2 n)^{101}$

Problem 6 [5 points] Which of the following functions is $O(n \log \sqrt{n})$ ( )

A. $(1.03)^n$ B. $n \cdot (\log_2 n)^{1.0001}$ C. $358\cdot n\log_2 n$ D. $n^{1.2}/\log^5 n$

Problem 7 [5 points] Suppose you receive n numbers in stream (one by one). Once a number arrives, you need to sort it with all numbers you have received and then output the sorted list. Which sort algorithm you should use ( )

A. Insertion Sort B. Selection Sort C. Merge Sort D. Quick Sort

Problem 8 [8 points] Let S1 be an unsorted array of n integers, and S2 is another sorted array of $\log_2 n$ integers (n is a power of 2, **S2 is in ascending order**). Suppose Bo asked you to find all pairs (x, y) satisfying x ∈ S1, y ∈ S2, and x > y. Please tell him the time complexity of your designed algorithm and explain why. (You do not need to write pseudocode!)