第一部分　选择题（每题 4 分，总共 20 分）

Part One　Select one from the given four options (4 marks each question, in total 20 marks)

$A \cap B = \phi$　$P(AB)=0$

C　1. 设 A、B 为不相容事件，且 $P(A)>0$，$P(B)>0$，下面四个结论中正确的是（ ）：

(A) $P(B \mid A) > 0$

(B) $P(A \mid B) = P(A)$

(C) $P(A \mid B) = 0$　$= \dfrac{P(AB)}{P(B)} = 0$

(D) $P(AB) = P(A)P(B)$

Assume A and B are disjoint events, $P(A)>0$, $P(B)>0$, which conclusion is correct?

(A) $P(B \mid A) > 0$

(B) $P(A \mid B) = P(A)$

(C) $P(A \mid B) = 0$

(D) $P(AB) = P(A)P(B)$

C　2. 设 $F(x,y)$ 是二维随机变量 (X,Y) 的分布函数，下面四个结论中错误的是（ ）：

(A) $F(+\infty,+\infty)=1$ ✓

(B) $F(-\infty,-\infty)=0$ ✓

(C) $F(+\infty,y)=1$ → 边缘

(D) $F(x,-\infty)=0$ ✓

$F(+\infty,+\infty)=1$　$F(-\infty,-\infty)=0$

$F(x,-\infty)=0$

$F(-\infty,y)=0$　☆

Assume $F(x,y)$ is the distribution function of two dimensional r.v. (X,Y), which conclusion is wrong?

(A) $F(+\infty,+\infty)=1$

(B) $F(-\infty,-\infty)=0$

(C) $F(+\infty,y)=1$

(D) $F(x,-\infty)=0$

$1-P_1$　$1-P_2$

C　3. 一种零件的加工由两道工序组成．第一道工序的废品率为 p_1，第二道工序的废品率为 p_2，则该零件加工的成品率为（　）．

$(1-P_1)(1-P_2)$

$=1-P_1-P_2+P_1P_2$

（A） $1-p_1-p_2$； （B） $1-p_1p_2$ ；

（C） $1-p_1-p_2+p_1p_2$； （D） $(1-p_1)+(1-p_2)$.

A component is manufactured through two stages. The rejection rate (the probability of defective products) is p_1 in the first stage, and p_2 in the second stage. Which option ( ) is the finished rate（the probability of qualified products）?

（A） $1-p_1-p_2$； （B） $1-p_1p_2$；

（C） $1-p_1-p_2+p_1p_2$； （D） $(1-p_1)+(1-p_2)$.

单调不减

① $F(+\infty)=1$, $F(-\infty)=0$ ② 右连续 ③ $0\le F\le 1$

A

4. 设 X_1, X_2 是随机变量，其分布函数分别为 $F_1(x)$, $F_2(x)$，为使 $F(x)=aF_1(x)-bF_2(x)$ 是某一随机变量的分布函数，在下列给定的各组数值中应取（ ）.

（A） $a=\frac{3}{5}$, $b=-\frac{2}{5}$； （B） $a=\frac{2}{3}$, $b=\frac{2}{3}$；

（C） $a=-\frac{1}{2}$, $b=\frac{3}{2}$； （D） $a=\frac{1}{2}$, $b=\frac{3}{2}$.

$a-b=1$

$\begin{cases} a>0 \\ b<0 \end{cases}$

Assume random variables X_1, X_2 have their distribution functions $F_1(x)$, $F_2(x)$ respectively. In order to make $F(x)=aF_1(x)-bF_2(x)$ be a distribution function of some random variable, which option ( ) can make it happen?

（A） $a=\frac{3}{5}$, $b=-\frac{2}{5}$； （B） $a=\frac{2}{3}$, $b=\frac{2}{3}$；

（C） $a=-\frac{1}{2}$, $b=\frac{3}{2}$； （D） $a=\frac{1}{2}$, $b=\frac{3}{2}$.

$\frac{X+Y-1}{\sqrt{2}} \sim N(0,1)$

$X+Y\sim N(1,2)$, $X-Y\sim N(-1,2)$

μ 期望

σ^2 方差

$X\sim N(\mu_1,\sigma_1^2)$

$Y\sim N(\mu_2,\sigma_2^2)$

$aX+bY\sim N(a\mu_1+b\mu_2, a^2\sigma_1^2+b^2\sigma_2^2)$

$aX+b\sim N(a\mu_1+b, a^2\sigma_1^2)$

B

5. 设 $X\sim N(0,1)$, $Y\sim N(1,1)$，且 X 与 Y 相互独立，则（ ）.

（A） $P(X+Y\le 0)=\frac{1}{2}$； （B） $P(X+Y\le 1)=\frac{1}{2}$；

（C） $P(X-Y\le 0)=\frac{1}{2}$； （D） $P(X-Y\le 1)=\frac{1}{2}$.

Assume $X\sim N(0,1)$, $Y\sim N(1,1)$, and X and Y are independent to each other, which option ( ) is correct?

（A） $P(X+Y\le 0)=\frac{1}{2}$； （B） $P(X+Y\le 1)=\frac{1}{2}$；

（C） $P(X-Y\le 0)=\frac{1}{2}$； （D） $P(X-Y\le 1)=\frac{1}{2}$.

$(A-AB)+(B-AB)$

$\bar{A}B = B - AB$

$A\bar{B} = A - AB$

第二部分 填空题（每空 2 分，总共 20 分）

$\Omega - A\cup B$ $\quad 0.3$

Part Two Fill in the boxes for each Question (2 marks each box, in total 20 marks)

$P(\bar{A}\bar{B}) + P(A\bar{B}) + P(\bar{A}B)$

$P(A\bar{B}+\bar{A}B)=0.3$

$P(A)-P(AB)+P(B)-P(AB)=0.3$

$P(AB)=0.1$

$P(A\cup B)=P(A)+P(B)-P(AB)=0.4$

$1-0.4+0.3=0.9$

1. 已知事件A、B仅发生一个的概率为 0.3（即$P(A\bar{B})+P(\bar{A}B)=0.3$），且$P(A)+P(B)=0.5$，则$A$、$B$至少有一个不发生的概率为 0.9 .
If the probability for exactly one of the events A or B to happen is 0.3 (i.e., $P(A\bar{B})+P(\bar{A}B)=0.3$), and $P(A)+P(B)=0.5$, then the probability for at least one of A and B not to happen is ________.

$P(ABC)=0 \quad P(A)=P(B)=P(C)$

2. 设A,B,C是两两独立且三事件不能同时发生的随机事件，且它们发生的概率相等，则$P(A\cup B\cup C)$的最大值为 $\frac{3}{4}$.
Suppose A,B,C are pairwise independent events while the three events cannot happen at the same time. If each of the events happens with the same probability, then the maximum of $P(A\cup B\cup C)$ is ________.

$P(AB)=P(A)P(B)$

$P(AC)=P(A)P(C)$

$P(BC)=P(B)P(C)$

$P(ABC)=0$

$P(A)=P(B)=P(C)$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(AB)-P(BC)-P(AC)+P(ABC)$

$=3P(A)-3P^2(A)$

$y=3x-3x^2=3x(1-x)$

$x=\frac{1}{2}, \; y_{max}=\frac{3}{4}$

3. 将一枚硬币重复掷五次，则正、反面都至少出现二次的概率为 $\frac{5}{8}$.
A coin is tossed five times, then the probability of getting at least two heads and at least two tails is ________.

$C_5^3(\frac{1}{2})^2(\frac{1}{2})^3 + C_5^2(\frac{1}{2})^2(\frac{1}{2})^3 = \frac{1}{16}\times 10 = \frac{5}{8}$

4. 假设随机变量X与Y相互独立且都服从参数为λ的指数分布$EXP(\lambda)$，即$f_X(x)=\begin{cases}\lambda e^{-\lambda x}, & x>0\\ 0, & x\le 0\end{cases}$，则$\min(X,Y)$服从的分布为 $EXP(2\lambda)$.（写出分布类型及参数）
Suppose the random variables X and Y are independent and they both follow the exponential distribution $EXP(\lambda)$, $f_X(x)=\begin{cases}\lambda e^{-\lambda x}, & x>0\\ 0, & x\le 0\end{cases}$, then the distribution of $\min(X,Y)$ is________. (Please write down the distribution and its parameter.)

$Z=\min(X,Y)$, $F_Z(z)=P\{\min\le z\}$

$=1-[1-F_X(z)][1-F_Y(z)]$

$x>0, y>0$

$=1-[1-(1-e^{-\lambda z})][1-(1-e^{-\lambda z})]$

$=1-e^{-2\lambda z}$

5. 设随机变量(X,Y)的联合频率函数为

$X \backslash Y$	0	1	
0	1/4	1/4	$\frac{1}{2}$
1	0	1/2	$\frac{1}{2}$
	$\frac{1}{4}$	$\frac{3}{4}$	

记(X,Y)的分布函数为$F(x,y)$，则$F\left(\frac{1}{2},1\right)=$ $\frac{1}{2}$.

The joint probability mass function (PMF) of the random variable (X,Y) is listed as follows:

$X \backslash Y$	0	1
0	1/4	1/4
1	0	1/2

Let $F(x,y)$ be the joint CDF of (X,Y), then $F\left(\frac{1}{2},1\right)=$________.

$X-Y\sim N(-\mu,\sigma_1^2+\sigma_2^2)$ $-\mu=1$ $\mu=-1$

6\. 设随机变量$X\sim N(\mu,\sigma_1{}^2)$，$Y\sim N(2\mu,\sigma_2{}^2)$，$X$与$Y$相互独立．已知$P(X-Y\geq 1)=\frac{1}{2}$，则

$\mu=$ -1 .

Let $X\sim N(\mu,\sigma_1{}^2)$, $Y\sim N(2\mu,\sigma_2{}^2)$, X and Y are independent. If $P(X-Y\geq 1)=\frac{1}{2}$, then

$\mu=$_________.

$P\{X\geq 1\}=\int_1^{+\infty}e^{-x}dx=[-e^{-x}]_1^{+\infty}=e^{-1}$

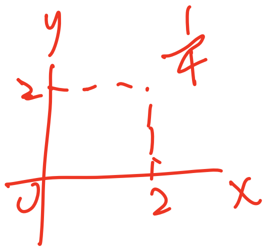
$P\{1\leq X\leq 2\}=\int_1^2 e^{-x}dx=[-e^{-x}]_1^2=e^{-1}-e^{-2}$

$\frac{e^{-1}-e^{-2}}{e^{-1}}=1-e^{-1}$

7\. 设随机变量X的概率密度为$f(x)=\begin{cases}e^{-x}, & x>0\\ 0, & x\leq 0\end{cases}$，则$P(X\leq 2|X\geq 1)=$ $1-e^{-1}$.

The probability density function (PDF) of a random variable X is $f(x)=\begin{cases}e^{-x}, & x>0\\ 0, & x\leq 0\end{cases}$,

then $P(X\leq 2|X\geq 1)=$_________.



$f(x,y)=\begin{cases}\frac{1}{4}, & x\in(0,2), y\in(0,2)\\ 0, & \cancel{其}余\end{cases}$

$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\int_0^2\frac{1}{4}dy=\frac{1}{2}$

8\. 设平面区域 D 由直线$x=2, y=2$及坐标系的x轴和y轴所围成，二维随机变量(X,Y)在区域 D 上服从均匀分布，则(X,Y)关于X的边际密度在$x=1$处的值为 $\frac{1}{2}$.

Suppose a region D is formed by the lines $x=2$, $y=2$ and the x-axis and y-axis of the coordinate system. A two-dimensional random variable (X,Y) follows a uniform distribution in the region D. Then the value of the marginal probability density function (PDF) of X at $x=1$ is _________.

$m+n$ p

9\. 设随机变量X和Y服从二项分布 $X\sim b(n,p), Y\sim b(m,p)$并相互独立，则$X+Y$的分布为：

$X+Y\sim$ $b(m+n,p)$

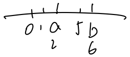
Suppose X and Y follow Binomial Distribution $X\sim b(n,p), Y\sim b(m,p)$ and be independent, then the distribution of $X+Y$ is: $X+Y\sim$_________.

$3a+b=12$ $(b\geq 4)$

$\frac{3-a}{b-a}=\frac{1}{4}$ $\frac{4-a}{b-a}=\frac{1}{2}\Rightarrow a+b=8$ $\Rightarrow\begin{cases}a=2\\ b=6\end{cases}$

10\. 设随机变量X服从均匀分布$X\sim U(a,b)(a>0)$，且$P(0<X<3)=\frac{1}{4}$，$P(X>4)=\frac{1}{2}$，则

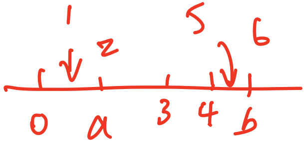
$P(X\leq 4)=\frac{1}{2}$



$P(1<X<5)=$ $\frac{3}{4}$.

$\frac{1}{4}\times 3$

Suppose the random variable X follow Uniform distribution $X\sim U(a,b)(a>0)$, and $P(0<X<3)=\frac{1}{4}$, $P(X>4)=\frac{1}{2}$, then $P(1<X<5)=$_________.



$P(0<X<3)=\frac{3-a}{b-a}=\frac{1}{4}\Rightarrow 3a+b=12$

$P(X>4)=\frac{b-4}{b-a}=\frac{1}{2}\Rightarrow b+a=8$

$\begin{cases}a=2\\ b=6\end{cases}$

第三部分 大题（每题 10 分，总共 60 分）

Part Three Questions and Answers (10 marks each question, in total 60 marks)

设：A_i={选到i级选手}, B={通过选拔}.

$P(B)=\sum_{i=1}^{4}P(B|A_i)P(A_i)=\frac15\times0.9+\frac25\times0.7+\frac{7}{20}\times0.5+\frac{1}{20}\times0.2=\frac{129}{200}$

1. 某射击小组有 20 名射手，其中一级射手 4 人，二级 8 人，三级 7 人，四级 1 人。各级射手能通过选拔进入比赛的概率依次为 0.9、0.7、0.5、0.2。求任选一名射手能通过选拔进入比赛的概率。

   A shooting team has 20 shooters, of whom 4 are in the first level, 8 are in the second level, 7 are in the third level, and 1 is in the fourth level. The probability of each level of the shooters entering the competition through selection is 0.9, 0.7, 0.5, 0.2. Compute the probability that a randomly selected shooter could enter the competition.

2. 设猎人在猎物 100m 处对猎物打第一枪，命中猎物的概率为 0.5。若第一枪未命中，则猎人继续打第二枪，此时猎物与猎人已相距 150m。若第二枪仍未命中，则猎人继续打第三枪，此时猎物与猎人已相距 200m。若第三枪还未命中，则猎物逃逸。假如该猎人命中猎物的概率与距离成反比 $P(X=x)=\frac{k}{x}$（x 是距离，k是待求的常数），试求该猎物被击中的概率。

   A hunter shoots at the first time in 100m from the prey, and the probability of hitting the prey is 0.5. If the first shooting misses, the hunter continues to shoot at the second time, and now the prey is in 150m away from the hunter. If the second shooting still misses, the hunter continues to shoot at the third time, and right now the prey is in 200m away from the hunter. If the third shooting has not hit, the prey escapes. If the probability of the hunter hitting the prey is inversely proportional to the distance $P(X=x)=\frac{k}{x}$ (x is the distance and k is a constant), find the probability of the prey having been hit.

$P(X=100)=\frac{k}{100}=0.5\Rightarrow k=50$

$P(X=150)=\frac{50}{150}=\frac13$, $P(X=200)=\frac{50}{200}=\frac14$

A={在100m}, B={在150m}, C={在200m}

D={击中}

$P(D)=P(A)+P(\bar{A}B)+P(\bar{A}\bar{B}C)$

$=0.5+0.5\times\frac13+0.5\times\frac23\times\frac14$

$=\frac12+\frac16+\frac1{12}=\frac34$

3. 设随机变量X的密度函数满足：

$$f(x)=\begin{cases}kx(1-x), & 0<x<1,\\ 0, & 其他\end{cases}$$

   (1) 求常数k;

   (2) 求$Y=-3X+3$的取值范围和密度函数.

   Suppose a random variable X has the density function:

$$f(x)=\begin{cases}kx(1-x), & 0<x<1,\\ 0, & other\end{cases}$$

   (1) Compute the constant k;

   (2) Find the value range and the density function of $Y=-3X+3$.

(1) $\int_{-\infty}^{+\infty}f(x)dx=\int_0^1 k(1-x)x\,dx=k[\frac12x^2-\frac13x^3]_0^1=\frac{k}{6}=1\Rightarrow k=6$

(2) $y=-3x+3\Rightarrow x=\frac13(3-y)\Rightarrow x'=-\frac13$

$x\in(0,1)$, $y\in(0,3)$, $f_Y(y)=|x'|f(x)=\frac13\cdot k\frac13(3-y)[1-\frac13(3-y)]$

$=\frac29(3-y)y$

故 $f_Y(y)=\begin{cases}\frac29y(3-y), & 0<y<3\\ 0, & 其他.\end{cases}$

$X\to f_X(x)$

$Y=g(X)\Rightarrow x=g^{-1}(y)$

$x'=[g^{-1}(y)]'$

$f_Y(y)=\sum|g^{-1}(y)'|f_X(g^{-1}(y))$

4. 假设随机变量 X，Y 服从泊松分布$X\sim P(\lambda_1), Y\sim P(\lambda_2)$，进一步地，假设$X$和$Y$独立.

(1) 求X和Y的联合频率函数；

(2) 求条件概率$P(X=k|X+Y=n)$，其中$n\ge k$是非负整数.

[提示：$P(X+Y=n)=\sum_{k=0}^{n}P(X=k,Y=n-k)$.]

Suppose random variables X, Y follow Poison distribution $X\sim P(\lambda_1), Y\sim P(\lambda_2)$. Furthermore X and Y are independent.

(1) Find the joint frequency function of X and Y;

(2) Find the conditional probability $P(X=k|X+Y=n)$, where $n\ge k$ is a non-negative integer.

[Hint: $P(X+Y=n)=\sum_{k=0}^{n}P(X=k,Y=n-k)$]

(1) $X\sim P(\lambda_1)$, $P\{X=k\}=\frac{\lambda_1^k}{k!}e^{-\lambda_1}$

$Y\sim P(\lambda_2)$, $P\{Y=k\}=\frac{\lambda_2^k}{k!}e^{-\lambda_2}$

$P_{ij}=P\{X=x_i, Y=y_j\}$

$=P\{X=x_i\}P\{Y=y_j\}$

$P_{xy}=\frac{\lambda_1^x}{x!}e^{-\lambda_1}\cdot\frac{\lambda_2^y}{y!}e^{-\lambda_2}$

$P_{xy}=\frac{\lambda_1^x\lambda_2^y}{x!\,y!}e^{-(\lambda_1+\lambda_2)}$

5. 设二维随机变量(X,Y)的联合密度函数为

$$f(x,y)=\begin{cases}2(x+y), & 0<x<1,\ 0<y<x\\ 0, & 其他\end{cases},$$

(1) 求边际分布函数$F_Y(y)$；

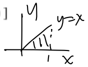
(2) 判断X与Y的独立性，并给出理由.

Suppose the two-dimensional random variable (X, Y) has the joint density function

$$f(x,y)=\begin{cases}2(x+y), & 0<x<1,\ 0<y<x\\ 0, & other\end{cases},$$

(1) Find the marginal density function $F_Y(y)$;

(2) Justify the independency of X and Y, and give the explanation.



(1) $f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\int_y^1 2(x+y)dx=1+2y-3y^3$

(2) $f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\int_0^x 2(x+y)dy=3x^2$

$\because f_X(x)f_Y(y)\ne f(x,y)$ $\therefore$ 不独立

可分 → 独立

形式　　区间

$f(x,y)=xy$　　$0<x<1$, $0<y<1$

$0<x<1$, $0<y<x$

6. 设随机变量$X\sim U(0,1)$，当给定$X=x$时，随机变量Y的条件密度函数为

$$f_{Y|X}(y|x)=\begin{cases}x, & 0<y<x,\\ 0, & 其他.\end{cases}$$

(1) 求X和Y的联合密度函数 $f(x,y)$；

(2) 求边际密度函数 $f_Y(y)$；

(3) 求$P\{X\le Y\}$的值.

Suppose the random variable $X\sim U(0,1)$. Given $X=x$, the random variable Y has the conditional density function

$$f_{Y|X}(y|x)=\begin{cases}x, & 0<y<x,\\ 0, & other\end{cases}$$

(1) Find the joint density function $f(x,y)$;

(2) Find the marginal density function $f_Y(y)$;

(3) Compute $P\{X\le Y\}$.

(1) $f_X(x)=\begin{cases}1, & x\in(0,1)\\ 0, & 其他\end{cases}$

$f(x,y)=f_{Y|X}(y|x)f_X(x)=\begin{cases}x, & x\in(0,1),\ y\in(0,x)\\ 0, & 其他\end{cases}$

(2) $f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\begin{cases}\int_y^1 x\,dx=\frac{1}{2}-\frac{1}{2}y^2, & y\in(0,1)\\ 0, & 其他\end{cases}$

(3) $P\{X\le Y\}=\iint_{x\le y}f(x,y)\,dx\,dy=\int_0^1\int_0^y x\,dx\,dy=\int_0^1\frac{1}{2}y^2\,dy=\frac{1}{6}$