南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

2018-2019 年春季学期

期末考试试卷

课程名称： 概率论与数理统计	Course name: Probability & Statistics
课程代码： MA212	Course code: MA212
开课单位： 数学系	Course run by: Mathematics department
考试时长： 120 分钟	Test duration: 120 minutes

你的姓名: ____________________ 学号: ____________________

Your Name: ________________ **Your ID:** ________________

第一部分 选择题（每题 4 分，总共 20 分）

Part One Select one from the given four options (4 marks each question, in total 20 marks):

B

1. 设随机变量$X \sim N(\mu_1, {\sigma_1}^2), Y \sim N(\mu_2, {\sigma_2}^2)$， 且有$P(|X-\mu_1|<1) > P(|Y-\mu_2|<1)$，则 ______.

（A）$\sigma_1 > \sigma_2$, （B）$\sigma_1 < \sigma_2$, （C）$\mu_1 < \mu_2$, （D）$\mu_1 > \mu_2$.

Assume there are two random variables$X \sim N(\mu_1, {\sigma_1}^2), Y \sim N(\mu_2, {\sigma_2}^2)$. If $P(|X-\mu_1|<1) > P(|Y-\mu_2|<1)$, then ______.

（A）$\sigma_1 > \sigma_2$, （B）$\sigma_1 < \sigma_2$, （C）$\mu_1 < \mu_2$, （D）$\mu_1 > \mu_2$.

A

2. 在区间$[0,1]$中随机取两个数X, Y，则$P(|X-Y| < \frac{1}{2}) =$ ______.

（A）$\frac{3}{4}$, （B）$\frac{1}{2}$,
（C）$\frac{1}{4}$, （D）$\frac{1}{3}$.

Select two values X, Y in the range of $[0,1]$ randomly, then $P(|X-Y| < \frac{1}{2}) =$______.

（A）$\frac{3}{4}$, （B）$\frac{1}{2}$,
（C）$\frac{1}{4}$, （D）$\frac{1}{3}$.

C

3. 设$X_1, X_2, \cdots, X_n$是来自总体$X \sim N(\mu, \sigma^2)$的独立样本，$\bar{X}$是样本均值, 则_____.

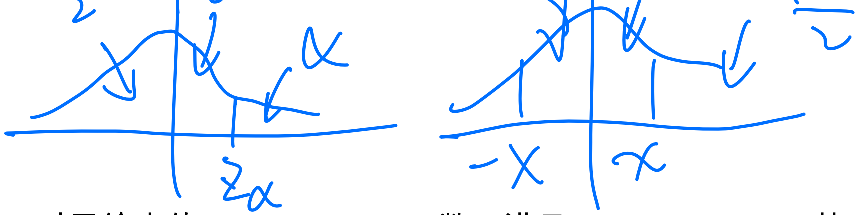
(A) $E(\bar{X}) = 0, D(\bar{X}) = 1$, (B) $E(\bar{X}) = 0, D(\bar{X}) = \sigma^2$.

(C) $E(\bar{X}) = \mu, D(\bar{X}) = \frac{\sigma^2}{n}$, (D) $E(\bar{X}) = \mu, D(\bar{X}) = \sigma^2$.

Assume$X_1, X_2, \cdots, X_n$are independent samples from the population $X \sim N(\mu, \sigma^2)$, $\bar{X}$ is the sample mean, then _____.

(A) $E(\bar{X}) = 0, D(\bar{X}) = 1$, (B) $E(\bar{X}) = 0, D(\bar{X}) = \sigma^2$.

(C) $E(\bar{X}) = \mu, D(\bar{X}) = \frac{\sigma^2}{n}$, (D) $E(\bar{X}) = \mu, D(\bar{X}) = \sigma^2$.



4. 设随机变量$X\sim N(0,1)$，对于给定的$\alpha\ (0<\alpha<1)$，数z_α满足$P(X>z_\alpha)=\alpha$，若$P(|X|<x)=\alpha$，则$x=$___ .

（A）$z_{\frac{\alpha}{2}}$,　　（B）$z_{1-\frac{\alpha}{2}}$,　　（C）$z_{\frac{1-\alpha}{2}}$,　　（D）$z_{1-\alpha}$,

Assume the random variable $X\sim N(0,1)$. Given $\alpha\ (0<\alpha<1)$, the number z_α has $P(X>z_\alpha)=\alpha$. If $P(|X|<x)=\alpha$, then $x=$_____.

（A）$z_{\frac{\alpha}{2}}$,　　（B）$z_{1-\frac{\alpha}{2}}$,　　（C）$z_{\frac{1-\alpha}{2}}$,　　（D）$z_{1-\alpha}$,

C

5. 随机变量X与Y相互独立，且都服从标准正态分布 $N(0,1)$， 则下面结论不正确的________

(A) $Z_1=X^2+Y^2\sim\chi^2(2)$,　　(B) $Z_2=X+Y\sim N(0,2)$,

(C) $Z_3=\frac{X}{\sqrt{\frac{Z_1}{2}}}\sim t(2)$,　　(D) $Z_4=\frac{X^2}{Y^2}\sim F(1,1)$

The two random variables X, Y are independent to each other, both follow standard normal distribution $N(0,1)$, which statement as follows is not correct________

(A) $Z_1=X^2+Y^2\sim\chi^2(2)$,　　(B) $Z_2=X+Y\sim N(0,2)$,

(C) $Z_3=\frac{X}{\sqrt{\frac{Z_1}{2}}}\sim t(2)$,　　(D) $Z_4=\frac{X^2}{Y^2}\sim F(1,1)$.

第二部分 填空题（每空 2 分，总共 20 分）

Part Two Fill in the boxes for each Question (2 marks each box, in total 20 marks)

$D(XY)=E(XY)^2-E^2(XY)=E(X^2)E(Y^2)-[E(X)E(Y)]^2$
$=2\cdot 2-1=3$

1. 设两个独立的随机变量X和Y服从正态分布$N(1,1)$，则D(XY)=__3__.

Suppose two independent random variables X and Y follow normal distribution $N(1,1)$, then D(XY)=______.

2. 设样本$X_1,X_2,\cdots,X_n$为来自总体$X\sim N\,(0,1^2)$的独立样本，则$\sum_{i=1}^{n}X_i^2$服从__$\chi^2(n)$__分布，其期望为__n__。

Assume $X_1,X_2,\cdots,X_n$ are samples from the population $X\sim N\,(0,1^2)$, and then $\sum_{i=1}^{n}X_i^2$ follows _______ distribution, and its expected value is ______.

$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\sim N(0,1)$

3. 设$x_1,x_2,\cdots,x_n$为来自总体$X{\sim}N(\mu,\sigma^2)$的样本值，其均值为$\bar{x}=9.0$。若参数μ的置信度为0.9的双侧置信区间的下限为7.6，则其双侧置信上限为__10.4__.

$\bar{X}-\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}=7.6\Rightarrow\bar{X}+\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}=9+1.4=10.4$

Assume $x_1,x_2,\cdots,x_n$ are sample values from the population $X{\sim}N(\mu,\sigma^2)$ with the average $\bar{x}=9.0$. Set the confidence level to be 0.9. If the two sides confidence interval for μ has the lower bound 7.6, then the upper bound is ______.

4. ....

1. 设事件A_i={乘第i种交通工具}, B={迟到}.
$P(A_1)=0.3, P(A_2)=0.2, P(A_3)=0.1, P(A_4)=0.4$, $P(B|A_1)=\frac{1}{4}, P(B|A_2)=\frac{1}{3}, P(B|A_3)=\frac{1}{12}$, $P(B|A_4)=0$,

$$P(A_1|B)=\frac{P(B|A_1)P(A_1)}{\sum_{i=1}^{4}P(B|A_i)P(A_i)}=\frac{(\frac{3}{40})}{\frac{3}{40}+\frac{2}{30}+\frac{1}{120}}=\frac{9}{9+8+1}=\frac{1}{2}$$

第三部分 问答题（每题 10 分，总共 60 分）

Part Three Questions and Answers (10 marks each, in total 60 marks)

1. 有朋友自远方来，他乘火车、轮船、汽车、飞机来的概率分别是 0.3, 0.2, 0.1, 0.4。如果他乘火车、轮船、汽车，则迟到的概率分别是$\frac{1}{4},\frac{1}{3},\frac{1}{12}$，而乘飞机不会迟到。可他迟到了，问他是乘火车来的概率为多少?

Assume a friend drop at your place via train, ship, sedan or plane with the probability of each being 0.3, 0.2, 0.1, 0.4. The probability that your friend was late of each method is $\frac{1}{4},\frac{1}{3},\frac{1}{12}$ and 0 (would not be late if take the plane). Now your friend is late, compute the probability that he took the train.

2. (1) $F_Y(y)=P\{Y\le y\}=P\{X^2\le y\}$
① 当$y<0$时, $F_Y(y)=0$
② 当$0\le y<1$时, $F_Y(y)=P\{-\sqrt{y}\le X\le\sqrt{y}\}=\int_{-\sqrt{y}}^{0}\frac{1}{2}dx+\int_{0}^{\sqrt{y}}\frac{1}{4}dx=\frac{3}{4}\sqrt{y}$
③ 当$1\le y<4$时, $F_Y(y)=P\{-\sqrt{y}\le X\le\sqrt{y}\}=\int_{-1}^{0}\frac{1}{2}dx+\int_{0}^{\sqrt{y}}\frac{1}{4}dx=\frac{1}{2}+\frac{1}{4}\sqrt{y}$
④ 当$y\ge 4$时, $F_Y(y)=P\{-\sqrt{y}\le X\le\sqrt{y}\}=1$
综上: $F_Y(y)=\begin{cases}0, & y<0\\ \frac{3}{4}\sqrt{y}, & 0\le y<1\\ \frac{1}{2}+\frac{1}{4}\sqrt{y}, & 1\le y<4\\ 1, & y\ge 4\end{cases}$
即 $f_Y(y)=\begin{cases}\frac{3}{8\sqrt{y}}, & 0\le y<1\\ \frac{1}{8\sqrt{y}}, & 1\le y<4\\ 0, & 其他.\end{cases}$

(2) $Cov(X,Y)=Cov(X,X^2)$
$=E(X^3)-E(X)E(X^2)$
$=\int_{-1}^{0}\frac{1}{2}x^3dx+\int_0^2\frac{1}{4}x^3dx-(\int_{-1}^{0}\frac{1}{2}xdx+\int_0^2\frac{1}{4}xdx)(\int_{-1}^{0}\frac{1}{2}x^2dx+\int_0^2\frac{1}{4}x^2dx)$
$=-\frac{1}{8}+1-(-\frac{1}{4}+\frac{1}{2})(\frac{1}{6}+\frac{2}{3})$
$=\frac{7}{8}-\frac{1}{4}\times\frac{5}{6}$
$=\frac{16}{24}=\frac{2}{3}$

(3) $F(-\frac{1}{2},4)=P\{X\le-\frac{1}{2},Y\le 4\}=P\{X\le-\frac{1}{2},X^2\le 4\}$
$=P\{-2\le X\le-\frac{1}{2}\}=\int_{-1}^{-\frac{1}{2}}\frac{1}{2}dx=\frac{1}{4}$

2. 设随机变量X的概率密度为

$$f_X(x)=\begin{cases}\frac{1}{2}, & -1<x<0,\\ \frac{1}{4}, & 0\le x<2,\\ 0, & 其它.\end{cases}$$

令$Y=X^2$，$F(x,y)$为二维随机变量(X,Y)的分布函数, 求:

(a) Y的概率密度$f_Y(y)$;

(b) $Cov(X,Y)$;

(c) $F\left(-\frac{1}{2},4\right)$.

Let the density function of X to be

$$f_X(x)=\begin{cases}\frac{1}{2}, & -1<x<0,\\ \frac{1}{4}, & 0\le x<2,\\ 0, & otherwise.\end{cases}$$

Set $Y=X^2$ and $F(x,y)$ is the cumulative distribution function for (X,Y).

(a) Find the density function $f_Y(y)$ for Y;

(b) compute $Cov(X,Y)$;

(c) find $F\left(-\frac{1}{2},4\right)$.

3. (a) $L(\theta)=\prod_{i=1}^{n} f(x_i;\theta)=\prod_{i=1}^{n}\left(2e^{-2(x_i-\theta)}\right) = 2^n e^{-2\sum_{i=1}^{n}(x_i-\theta)} = 2^n e^{2n\theta-2\sum_{i=1}^{n}x_i}$ $(x>\theta)$

$\ln L(\theta) = n\ln 2 + 2n\theta - 2\sum_{i=1}^{n}x_i$, $\frac{d\ln L(\theta)}{d\theta} = 2n>0$, 单增.

$\because x_i>\theta$ $\therefore$当n取x_i中最小值时，$L(\theta)$取最大值.

$\therefore \hat{\theta} = \min_{1\le i\le n}\{x_i\}$.

3. (a)设某种原件的使用寿命X的概率密度为

$$f(x;\theta)=\begin{cases}2e^{-2(x-\theta)}, & x>\theta,\\ 0, & x\le\theta,\end{cases}$$

(b) $E(X)=\int_0^\theta \frac{2}{\theta^2}x(\theta-x)dx = \frac{2}{\theta^2}\left[\theta\int_0^\theta x\,dx - \int_0^\theta x^2 dx\right] = \frac{2}{\theta^2}\left[\frac{\theta^3}{2}-\frac{\theta^3}{3}\right] = \frac{\theta}{3}$, $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

令$E(X)=\bar{X}$, 则$\hat{\theta}=\frac{3}{n}\sum_{i=1}^{n}X_i$

其中θ为未知参数。设$X_1, X_2, \ldots, X_n$ 是来自总体X的样本，求参数θ的最大似然估计量。

(b) 设总体X的密度函数为

$$f(x;\theta)=\begin{cases}\frac{2}{\theta^2}\cdot(\theta-x) & , \quad 0<x<\theta\\ 0 & , \quad \text{其他}\end{cases},$$

其中$\theta>0$为未知参数。$X_1, X_2, \cdots, X_n$为来自总体X的样本，求未知参数θ的矩估计量。

(a) Suppose the life X of a kind of product has the density function

$$f(x;\theta)=\begin{cases}2e^{-2(x-\theta)}, & x>\theta,\\ 0, & x\le\theta,\end{cases}$$

θ is the unknown parameter. Let $X_1, X_2, \ldots, X_n$ be independent samples from the population X. Find the maximal likelihood estimate (MLE) for θ.

(b) Let the population X has the density function

$$f(x;\theta)=\begin{cases}\frac{2}{\theta^2}\cdot(\theta-x) & , \quad 0<x<\theta\\ 0 & , \quad otherwise\end{cases}$$

Here $\theta>0$ is the unkonw parameter. Let $X_1, X_2, \cdots, X_n$ be independent samples from X, find the method of moments estimate (MME) of θ.

4.(a) $X\sim N(\mu,\sigma^2)$, $\frac{\bar{X}-\mu}{S/\sqrt{n}}\sim N(0,1)$

μ的置信水平为$1-\alpha$的单侧置信下限为$\bar{X}-\frac{S}{\sqrt{n}}u_\alpha$

4. 设总体$X\sim N(\mu,\sigma^2)$,μ,σ^2均未知，$X_1, X_2, \cdots, X_n$为X的样本，$\bar{X}, S^2$分别为样本均值、样本方差。给定置信水平$1-\alpha$,试导出：

(a) μ 的置信水平为$1-\alpha$的单侧置信下限；

(b) σ^2的置信水平为$1-\alpha$的单侧置信上限。

(b) $X\sim N(\mu,\sigma^2)$, $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$

σ^2的置信水平为$1-\alpha$的单侧置信上限为$\frac{(n-1)S^2}{\chi^2_{1-\alpha}(n-1)}$

Assume the population $X\sim N(\mu,\sigma^2)$, both μ,σ^2 are unknown. $X_1, X_2, \cdots, X_n$ are independent samples from the population X, $\bar{X}, S^2$ are the sample mean and the sample variance. Given the confidence level $(1-\alpha)$, what is:

(a) the one-sided confidence lower limit of the unknown parameter μ?

5. $X\sim N(3.4,6^2)$, $\frac{\bar{X}-3.4}{6/\sqrt{n}} \sim N(0,1)$

∴ $P(1.4<\bar{X}<5.4) = P\left(\frac{1.4-3.4}{6/\sqrt{n}} < \frac{\bar{X}-3.4}{6/\sqrt{n}} < \frac{5.4-3.4}{6/\sqrt{n}}\right) = P\left(-\frac{\sqrt{n}}{3} < \frac{\bar{X}-3.4}{6/\sqrt{n}} < \frac{\sqrt{n}}{3}\right) = \Phi\left(\frac{\sqrt{n}}{3}\right) - \Phi\left(-\frac{\sqrt{n}}{3}\right)$

$= 2\Phi\left(\frac{\sqrt{n}}{3}\right) - 1 \geq 0.95 \Rightarrow \Phi\left(\frac{\sqrt{n}}{3}\right) \geq 0.975 = \Phi(1.96)$

(b) the one-sided confidence upper limit of the unknown parameter σ^2?

∴ $\frac{\sqrt{n}}{3} \geq 1.96 \Rightarrow \sqrt{n} \geq 5.88 \Rightarrow n \geq 34.57$ ∴ n至少 35.

5. 从正态总体$N(3.4,6^2)$中抽取容量为n的样本。如果要求其样本均值位于区间$(1.4,5.4)$内的概率不小于 0.95，问样本容量n至少应取多大？（注：标准正态分布函数值$\Phi(1.96) = 0.975, \Phi(1.645) = 0.95$）

Take a sample of capacity n from the population $X\sim N(3.4,6^2)$. To guarantee that the sample average lies into the interval $(1.4,5.4)$ with the probability no less than 0.95, how many samples at least should be taken? (Remark: standard normal distribution $\Phi(1.96) = 0.975, \Phi(1.645) = 0.95$).

6. $X\sim N(\mu, 6^2)$. $n=36$, $\bar{X}=66.5$, $S=15$. $\alpha=0.05$.

检验假设 $H_0: \mu=\mu_0=70$, $H_1: \mu\neq\mu_0$.

取 $T=\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$.

6. 设某次考试的考生成绩服从正态分布$X\sim N(\mu,\sigma^2)$，从中随机地抽取 36 位考生的成绩，算得他们的平均成绩为 66.5 分，标准差为 15 分。问在显著性水平 0.05 下，是否可以认为这次考试全体考生的平均成绩为 70 分？并给出具体检验过程。

H_0的拒绝域为 $R=\left\{\left|\frac{\bar{X}-\mu}{S/\sqrt{n}}\right| \geq t_{1-\frac{\alpha}{2}}(n-1)\right\}$

（注：标准正态分布函数值$\Phi(1.96) = 0.975, \Phi(1.645) = 0.95$，t 分布表 $P\{t(n) \leq t_\alpha(n)\} = \alpha. t_{0.95}(35) = 1.6896$，$t_{0.975}(35) = 2.0301$，$t_{0.95}(36) = 1.6883$，$t_{0.975}(36) = 2.0281$．）

∴ $\left|\frac{\bar{X}-\mu}{S/\sqrt{n}}\right| = \left|\frac{66.5-70}{15/\sqrt{36}}\right| = 1.4 < t_{1-\frac{\alpha}{2}}(n-1) = t_{0.975}(35) = 2.0301$ ∴应接受H_0 可以认为.

Suppose the scores from one exam follow normal distribution $X\sim N(\mu,\sigma^2)$. Take a sample of 36 students, the average score of them is 66.5 and the sample standard derivation is 15. Set the significance level to be 0.05, can we conclude that the average score of the whole population is 70? Please give the detailed process of your test.

(Remark: standard normal distribution $\Phi(1.96) = 0.975, \Phi(1.645) = 0.95$, t-distribution: $P\{t(n) \leq t_\alpha(n)\} = \alpha. t_{0.95}(35) = 1.6896$，$t_{0.975}(35) = 2.0301$，$t_{0.95}(36) = 1.6883$，$t_{0.975}(36) = 2.0281$．)