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§7.2

1、令 $Y = X_{i+1} - X_i \sim N(0, 2\sigma^2)$

$E(Y^2) = E[(X_{i+1}-X_i)^2] = 2\sigma^2$

无偏估计、

$\sigma^2 = E\left[\frac{1}{k}\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right] = \frac{1}{k}\sum_{i=1}^{n-1}E[(X_{i+1}-X_i)^2] = \frac{n-1}{k}2\sigma^2$

$\Rightarrow k = 2(n-1)$

2、$E(Y) = aE(\bar{X}_1) + bE(\bar{X}_2) = (a+b)\mu$

$D(Y) = a^2D(\bar{X}_1) + b^2D(\bar{X}_2) = \left(\frac{a^2}{n_1} + \frac{b^2}{n_2}\right)\sigma^2$

$\because a+b=1 \quad \therefore E(Y)=\mu,\ D(Y) = \left[\left(\frac{1}{n_1}+\frac{1}{n_2}\right)a^2 - \frac{2}{n_2}a + \frac{1}{n_2}\right]\sigma^2$

当 $a = \frac{\left(\frac{1}{n_2}\right)}{\frac{1}{n_1}+\frac{1}{n_2}} = \frac{n_1}{n_1+n_2}$，$b = \frac{n_2}{n_1+n_2}$ 时，$D(Y)$ 最小为 $\frac{2\sigma^2}{n_1+n_2}$

3、(1) $E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n}E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\left(\int_0^{+\infty}\frac{x}{\theta}e^{-\frac{x}{\theta}}dx\right) = \frac{1}{n}\sum_{i=1}^{n}\theta = \theta$

$P\{X_{(1)} > t\} = \prod_{i=1}^{n}P(X_i > t) = e^{-\frac{n}{\theta}t}\ (t>0) \Rightarrow X_{(1)} \sim EXP\left(\frac{n}{\theta}\right)$

$E(X_{(1)}) = \frac{\theta}{n} \Rightarrow E(nX_{(1)}) = \theta$

故 $\bar{X}$ 和 $n\cdot\min\{X_1 \dots X_n\}$ 都是 $\theta$ 的无偏估计.

(2) $D(\bar{X}) = \frac{1}{n^2}\sum_{i=1}^{n}D(X_i) = \frac{\theta^2}{n}$

$D(n\cdot\min\{X_1 \dots X_n\}) = n^2D(X_{(1)}) = n^2\cdot\frac{\theta^2}{n^2} = \theta^2 > \frac{\theta^2}{n} = D(\bar{X})$

故估计量 $\bar{X}$ 更有效.

§7.3

1、$\bar{X} = \frac{1}{9}\sum_{i=1}^{9}X_i = \frac{1}{9}(6+5.7+5.8+6.5+7+6.3+5.6+6.1+5) = 6$，$n=9$，$\alpha = 1-0.95 = 0.05$

$S = \sqrt{\frac{1}{9}\sum_{i=1}^{9}(X_i-\bar{X})^2} = 0.574$

(1) $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$\mu = 0.95$ 的置信区间为 $\left(\bar{X} - \frac{\sigma}{\sqrt{n}}\mu_{1-\frac{\alpha}{2}}, \bar{X} + \frac{\sigma}{\sqrt{n}}\mu_{1-\frac{\alpha}{2}}\right)$

代入得：$(5.608, 6.392)$

(2) $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1) = t(8)$

$\mu = 0.95$ 的置信区间为 $\left(\bar{X} - \frac{S}{\sqrt{n}}t_{1-\frac{\alpha}{2}}(8), \bar{X} + \frac{S}{\sqrt{n}}t_{1-\frac{\alpha}{2}}(8)\right)$

代入得：$(5.559, 6.441)$

2. $n=16$, $\bar{X}=\frac{1}{16}\sum_{i=1}^{16}X_i=503.75$, $S^2=\frac{1}{16}\sum_{i=1}^{16}(X_i-\bar{X})^2=38.47$, $S=6.2$, $\alpha=1-0.95=0.05$

(1) $\frac{\bar{X}-\mu}{S/\sqrt{n}}\sim t(n-1)=t(15)$

$\mu$ =0.95 的置信区间为 $\left(\bar{X}-\frac{S}{\sqrt{n}}t_{1-\frac{\alpha}{2}}(15), \bar{X}+\frac{S}{\sqrt{n}}t_{1-\frac{\alpha}{2}}(15)\right)$

代入得：$(500.45, 507.05)$

(2) $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)=\chi^2(15)$

$\sigma^2$ =0.95 的置信区间为 $\left(\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right)$

代入得：$(20.99, 92.18)$

3. $\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\sim t(n_1+n_2-2)$, $S_w=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}=1.08$, $\alpha=1-0.95=0.05$

$P\left(\frac{|(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)|}{S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\leq t_{1-\frac{\alpha}{2}}(n_1+n_2-2)\right)=1-\alpha$

$\Rightarrow \mu_1-\mu_2$ =0.95 的置信区间为 $(3.143, 4.857)$

4. $\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}=\frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2}\sim F(n_1-1, n_2-1)$, $\alpha=1-0.90=0.1$

$P\left(F_{\frac{\alpha}{2}}(n_1-1,n_2-1)<\frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2}<F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right)=1-\alpha$

即 $\frac{\sigma_1^2}{\sigma_2^2}$ 的0.90置信区间为 $\left(\frac{S_1^2/S_2^2}{F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)}, \frac{S_1^2/S_2^2}{F_{\frac{\alpha}{2}}(n_1-1,n_2-1)}\right)$

即 $(0.454, 2.791)$