第一部分 选择题（每题 4 分，总共 20 分）

Part One Select one from the given four options (4 marks each question, in total 20 marks):

B

1. 若随机事件 ***A*** 与 ***B*** 相互独立，则 $P(A\cup B)=$

   A. $P(A)+P(B)$
   B. $P(A)+P(B)-P(A)P(B)$
   C. $P(A)P(B)$
   D. $P(\bar{A})+P(\bar{B})$

   Assume the events ***A*** and ***B*** are independent to each other, then $P(A\cup B)=$

   A. $P(A)+P(B)$
   B. $P(A)+P(B)-P(A)P(B)$
   C. $P(A)P(B)$
   D. $P(\bar{A})+P(\bar{B})$

B

2. 设$F(x,y)$是二维随机变量(X,Y)的分布函数，下面四个结论中错误的是：

   A. $F(+\infty,+\infty)=1$
   B. $F(-\infty,-\infty)=0$
   C. $F(+\infty,y)=1$
   D. $F(x,-\infty)=0$

   Assume $F(x,y)$ is the distribution function of a two-dimensional random variable $(\boldsymbol{X},\boldsymbol{Y})$**,** which conclusion is wrong:

   A. $F(+\infty,+\infty)=1$
   B. $F(-\infty,-\infty)=0$
   C. $F(+\infty,y)=1$
   D. $F(x,-\infty)=0$

$X<1=\frac{4}{9}=C_2^0p^0(1-p)^2\Rightarrow p=\frac{1}{3}$

B

3. 设随机变量$X\sim b\,(2,p),Y\sim b\,(3,p)$，若$P(X\geq 1)=\frac{5}{9}$，则$P(Y\geq 1)=$____ .

   A. $\frac{8}{27}$ B. $\frac{19}{27}$ C. $\frac{5}{9}$ D. $\frac{4}{9}$.

$1-C_3^0\left(\frac{2}{3}\right)^3=\frac{19}{27}$

Assume the random variables $X \sim b\,(2,p), Y \sim b\,(3,p)$. If $P(X \geq 1) = \frac{5}{9}$, $P(Y \geq 1) =$____ .

A. $\frac{8}{27}$ B. $\frac{19}{27}$ C. $\frac{5}{9}$ D. $\frac{4}{9}$.

A

4. 设$X_i(i=1,2,3)$为三个正态随机变量，且$X_1 \sim N\,(0,1), X_2 \sim N\,(0,2^2), X_3 \sim N\,(5,3^2)$，记$p_i = P(-2 < X_i < 2),\ i=1,2,3$，则_____

$\frac{X_2}{2} \sim N(0,1)$

$-2.5 < u < -1$ $\frac{X_3-5}{3} \sim N(0,1)$

A. $p_1 > p_2 > p_3$
B. $p_3 > p_1 > p_2$
C. $p_2 > p_1 > p_3$
D. $p_1 > p_3 > p_2$.

Let $X_i(i=1,2,3)$ be three normal distributed random variables $X_1 \sim N\,(0,1),\ X_2 \sim N\,(0,2^2),\ X_3 \sim N\,(5,3^2)$. Let $p_i = P(-2 < X_i < 2),\ i=1,2,3$, then______

A. $p_1 > p_2 > p_3$
B. $p_3 > p_1 > p_2$
C. $p_2 > p_1 > p_3$
D. $p_1 > p_3 > p_2$.

C

5. 设 *X* 与 *Y* 为二随机变量，下面叙述正确的是 ____

A. 若 *X* 与 *Y* 均为一维正态随机变量，则(X,Y)是二维正态随机向量；
B. 若 *X* 与 *Y* 均为一维均匀随机变量，则(X,Y)是二维均匀随机向量；
C. 若(X,Y)是二维正态随机向量，则 *X* 与 *Y* 均为一维正态随机变量；
D. 若(X,Y)是二维均匀随机向量，则 *X* 与 *Y* 均为一维均匀随机变量.

Let *X and Y* be two random variables, which statement of the following is true ______

A. If *X and Y* are both one-dimensional normal distributions, (X,Y) is a bivariate normal distribution;
B. If *X and Y* are both one-dimensional uniform distributions，(X,Y) is a two-dimensional uniform distribution；
C. If (X,Y) is a bivariate normal distribution, *X* and *Y* are both one-dimensional normal distributions；
D. If (X,Y) is a two-dimensional uniform distribution, *X* and *Y* are both one-dimensional uniform distributions

第二部分 填空题（每题 2 分，总共 20 分）

Part Two Fill in the boxes for each Question (2 marks each box, in total 20 marks)

1. 随机地把C, S, S, T, U五个字母排成一排，计算得到SUSTC的概率$\frac{1}{30}$ $\frac{2}{A_5^5/2}$

   Line up five letters C, S, S, T, U randomly, the probability you get SUSTC is________.

0.06 0.14 0.09

$0.4\times0.5\times0.5+0.4\times0.5\times0.7+0.3\times0.5\times0.6$

2. A、B、C 3 位同学同时独立参加数学补考考试，不及格的概率分别为 0.4, 0.3, 0.5. 恰有 2 位同学不及格的概率是0.29. 如果已经知道这 3 位同学中有 2 位不及格，那么其中 1 位是 B 同学不及格的概率是0.52 $\frac{0.06+0.08}{0.29}$

   Students A, B, C independently attend the mathematics resit examination at the same time. The probability of their failure is 0.4, 0.3, 0.5. The probability that exactly two students fail is ______. If it is known that two out of three students fail, the probability of student B fails is ______.

$1-(0.6)^3-(0.4)^3=1-0.216-0.064=0.72$

3. 在一场五局三胜制的游戏中，双方每局的胜率分别是60%和40%，且每局之间相互独立。则游戏结束时每边至少赢了一局的概率是0.72

   In the best three-out-of-five games, the probability that each side wins a game is 60%and40%respectively, and all games are independent. The probability that each side wins at least one game before the whole games end is _____.

$1=\frac{4}{3}-P(AB)$ $P(\overline{AB})=1-P(AB)$

4. 设事件A和B满足$P(A)=P(B)=\frac{2}{3}$, $P(A\cup B)=1$，则$P(\bar{A}\cup\bar{B})=$$\frac{2}{3}$

   Suppose two events A and B satisfies $P(A)=P(B)=\frac{2}{3}$, $P(A\cup B)=1$, then $P(\bar{A}\cup\bar{B})=$_______.

$P(3<X<4)=\int_3^4\lambda e^{-\lambda x}dx=[-e^{-\lambda x}]_3^4=e^{-3\lambda}-e^{-4\lambda}$

5. 设随机变量$X\sim EXP(\lambda)$服从指数分布。则$P(4>X>3|X>2)=$________. 当参数

$e^{-\lambda}-e^{-2\lambda}$ $P(X>2)=1-\int_0^2\lambda e^{-\lambda x}dx=1-[-e^{-\lambda x}]_0^2=e^{-2\lambda}$

$u = e^{-\lambda}$, $u - u^2 = u(1-u)$ $u = e^{-\lambda} = \frac{1}{2}$

$-\lambda = -\ln 2$

$\lambda =$ $\ln 2$ 时这个概率取到最大值。

Suppose random variable X satisfies exponential distribution $X \sim EXP(\lambda)$. $P(4 > X > 3|X > 2) =$_______. When the parameter $\lambda =$____, this probability reaches its maximum.

$1 - P(X \leq 2)\, P(Y \leq 2) = 1 - \frac{1}{2} \cdot \frac{1}{2}$

6. 设随机变量X和Y独立，且均匀分布在$[1,3]$，则$P(\max(X,Y) > 2) =$ $\frac{3}{4}$.

$\frac{1}{2}$

Suppose two random variables X and Y independently uniformly distributed on $[1,3]$. $P(\max(X,Y) > 2) =$______.

$Z = Y - X \sim N(-\mu, 2\sigma^2)$

7. 设$X \sim N(\mu, \sigma^2)$, $Y \sim N(2\mu, \sigma^2)$是相互独立的正态分布的随机变量且$P(X - Y \geq 2) = \frac{1}{2}$。则$\mu =$ -2.

$P(Z \leq 2) = \frac{1}{2}$

Let $X \sim N(\mu, \sigma^2)$, $Y \sim N(2\mu, \sigma^2)$ be two independent normal distributed random variables and $P(X - Y \geq 2) = \frac{1}{2}$. Then $\mu =$______.

$\left[\frac{\lambda^k}{k!} e^{-\lambda}\right]^3$ $P(y=k) = \left(\frac{\lambda^k}{k!} e^{-\lambda}\right)^3$

8. 设$X \sim P(\lambda)$服从参数为λ的泊松分布，则$Y = X^3$的频率函数为______. $(k = 0, 1, 2 \cdots)$

Suppose $X \sim P(\lambda)$ has a Poisson distribution with parameter λ. Then the frequency function for $Y = X^3$ is ______.

第三部分 问答题（每题 10 分，总共 60 分）

Part Three Questions and Answers (10 marks each question, in total 60 marks)

1. 设随机变量X表示某个人打靶的准心情况，其概率分布密度函数

$P\{X\le\frac{2}{3}\}=\int_0^{\frac{2}{3}}3x^2dx=\frac{8}{27}$

$$f(x)=\begin{cases}3x^2 & x\in(0,1)\\ 0 & x\notin(0,1)\end{cases}$$

$P\{Y=1\}=C_3^1(\frac{8}{27})(\frac{19}{27})^2$

设定事件$\{X\le\frac{2}{3}\}$发生为打靶成功。另 Y 代表三次打靶成功的次数，求刚好成功一次的概率。

$=3\times\frac{8}{27}\times\frac{19}{27}\times\frac{19}{27}=$

Let X stand for the result of shooting target practice from someone. The density function of X is

$$f(x)=\begin{cases}3x^2 & x\in(0,1)\\ 0 & x\notin(0,1)\end{cases}$$

Assume $\{X\le\frac{2}{3}\}$ as a successful practice. Let Y stand for the number of successful practice in total 3 shots. Find the probability when Y=1.

2. (X, Y) 是两个离散随机变量，有如下联合概率分布

(1) $\frac{1}{6}+a+\frac{1}{18}+\frac{1}{3}+\frac{2}{9}+b=a+b+\frac{7}{9}=1\Rightarrow a+b=\frac{2}{9}$

X / Y	1	2	3
0	1/6	a	1/18
1	1/3	2/9	b

(2) $\frac{1}{3}=(\frac{1}{3}+\frac{2}{9}+b)(\frac{1}{6}+\frac{1}{3})=\frac{1}{2}(\frac{5}{9}+b)\Rightarrow b=\frac{1}{9}$

求：（1）**a** 与 **b** 存在的关系；（2）若 **X** 与 **Y** 独立，求 **a** 与 **b** 的值。

$\therefore a=\frac{1}{9}$.

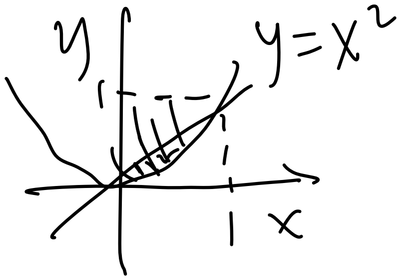
Suppose (X,Y) be a two-dimensional discrete random variable with the following distribution

X / Y	1	2	3
0	1/6	a	1/18
1	1/3	2/9	b

(1) find the relationship between **a** and **b**; (2) given **X** and **Y** independent, find **a** and **b**.

(1) $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,dx\,dy = \int_0^1\int_{x^2}^{1} cx^2y\,dy\,dx$

$= \frac{c}{2}\int_0^1 (x^2 - x^6)\,dx = \frac{c}{2}\left(\frac{1}{3} - \frac{1}{7}\right) = \frac{2c}{21} = 1$



3. 设二维随机变量(X, Y)的联合概率密度为 $\Rightarrow c = \frac{21}{2}$

$$f(x,y) = \begin{cases} cx^2y & x^2 \le y \le 1, \\ 0 & \text{其他} \end{cases}$$

(2) $P\{X>Y\} = \iint_{x>y} f(x,y)\,dx\,dy = \int_0^1\int_{x^2}^{x} \frac{21}{2}x^2y\,dy\,dx$

求：（1）常数 c ； （2）求P{X > Y}.

Let (X, Y) be a two-dimensional distribution with the joint density function

$= \frac{21}{4}\int_0^1 (x^4 - x^6)\,dx = \frac{21}{4}\left(\frac{1}{5} - \frac{1}{7}\right) = \frac{21}{70} = \frac{3}{10}$

$$f(x,y) = \begin{cases} cx^2y & x^2 \le y \le 1, \\ 0 & \text{otherwise} \end{cases}$$

(1) find the constant c; (2) compute P(X>Y).

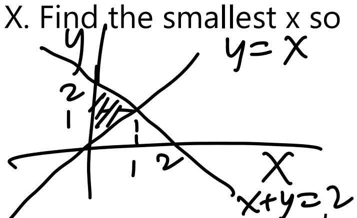
$X \sim N(110, 12^2) \Rightarrow \frac{X-110}{12} \sim N(0,1)$

4. 某地区 18 岁女青年的血压（收缩压，以 mmHg 计）服从$N(110,\ 12^2)$，在该地区任选一 18 岁女青年，测量她的血压X. 确定最小的x, 使得$P\{X > x\} \le 0.05$.（可能用到的参数：$\Phi(1.645) = 0.95$）

$P\{X>x\} = P\left\{\frac{X-110}{12} > \frac{x-110}{12}\right\}$

$= 1 - \Phi\left(\frac{x-110}{12}\right) \le 0.05 \Rightarrow \Phi\left(\frac{x-110}{12}\right) \ge 0.95 = \Phi(1.645) \Rightarrow \frac{x-110}{12} \ge 1.645 \Rightarrow x = 129.74$

Suppose the blood pressure (systolic pressure, measured in mmHg) of 18 years old women somewhere has a normal distribution $N(110,\ 12^2)$. Randomly select a 18 years old woman and measure her systolic pressure X. Find the smallest x so that $P\{X > x\} \le 0.05$. (it might be used $\Phi(1.645) = 0.95$)



$f(x,y) = \begin{cases} 1, & (x,y) \in G \\ 0, & (x,y) \notin G \end{cases}$

(1) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} \int_x^{2-x} dy = 2-2x, & x \in (0,1) \\ 0, & \text{其他} \end{cases}$

5. 设随机变量(X, Y)在区域 G 上服从均匀分布，其中 G 由$x - y = 0,\ x + y = 2$ 与 $y = 0$ 围成.

(2) $f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} \int_0^y dx + \int_0^{2-y} dx = 2, & y \in (0,2) \\ 0, & \text{其他} \end{cases}$

(1) 求边际密度$f_X(x)$.

(2) 求条件密度$f_{X|Y}(x|y)$.

$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} 1-x, & x \in (0,1), y \in (0,2) \\ 0, & \text{其他} \end{cases}$

Suppose a two-dimensional random variable (X, Y) is uniformly distributed in the region G where G is formed by the three lines $x - y = 0,\ x + y = 2\ and\ y = 0.$

(1) Find the marginal density function $f_X(x)$

(2) Find the conditional density function $f_{X|Y}(x|y)$

(1) $F_Y(y) = P\{Y \le y\}$
$x \le 1,\ P\{Y \le y\} = P\{2 \le y\}$

6. 设随机变量X的概率密度为

$$f(x) = \begin{cases} \frac{1}{9}x^2, & 0 < x < 3, \\ 0, & 其它. \end{cases}$$

令随机变量
$$Y = \begin{cases} 2 & X \le 1, \\ X & 1 < X < 2, \\ 1 & X \ge 2. \end{cases}$$

(1) 求Y的累积分布函数；

(2) 求概率$P\{X \le Y\}$.

Let X be a random variable with the density function

$$f(x) = \begin{cases} \frac{1}{9}x^2, & 0 < x < 3, \\ 0, & \text{others} \end{cases}$$

Define
$$Y = \begin{cases} 2 & X \le 1, \\ X & 1 < X < 2, \\ 1 & X \ge 2. \end{cases}$$

(1) Find the cumulative distribution function of Y.
(2) Compute $P\{X \le Y\}$.