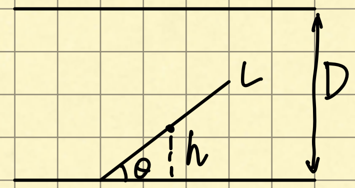
§3.3



5. 要想正好与一条直线相交，则 $h \le \frac{D}{2}$

又 $h = \frac{L}{2}\sin\theta$

则 $p = \dfrac{\int_0^{\pi} \frac{L}{2}\sin\theta\, d\theta}{\frac{D}{2}\cdot \pi} = \dfrac{2L}{\pi D}$

当投针次数非常大时，p 将会趋近于某个值 ε，而 L、D 又已知，故可通过 $p = \frac{2L}{\pi D} = \varepsilon$ 求出

$\pi = \frac{2L}{\varepsilon D}$.

6. 从椭圆内部随机选一点，可看作均匀分布。

密度为 $f_{XY}(x,y) = \begin{cases} \frac{1}{\pi ab}, & |x| \le a, |y| \le b \\ 0, & 其它 \end{cases}$

边际密度为 $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \begin{cases} \int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}} \frac{1}{\pi ab}\,dy = \frac{2}{\pi a}\sqrt{1-\frac{x^2}{a^2}} & (x \in [-a,a]) \\ 0, & 其他 \end{cases}$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \begin{cases} \int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} \frac{1}{\pi ab}\,dx = \frac{2}{\pi b}\sqrt{1-\frac{y^2}{b^2}} & (y \in [-b,b]) \\ 0, & 其他 \end{cases}$

7. cdf. $F(x,y) = (1-e^{-\alpha x})(1-e^{-\beta y})$ $(x, y \ge 0,\ \alpha, \beta > 0)$

$f_{XY}(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y) = \alpha\beta e^{-(\alpha x + \beta y)}$

$f_X(x) = \frac{\partial}{\partial x} F(x, +\infty) = \alpha e^{-\alpha x}$ $(x \ge 0)$

$f_Y(y) = \frac{\partial}{\partial y} F(+\infty, y) = \beta e^{-\beta y}$ $(y \ge 0)$

8. $f(x,y) = \frac{6}{7}(x+y)^2$ $(0 \le x \le 1,\ 0 \le y \le 1)$

a. (i) $P(X > Y) = \int_0^1 \int_y^1 f(x,y)\,dx\,dy = \frac{1}{2}$

(ii) $P(X+Y \le 1) = \int_0^1 \int_0^{1-y} f(x,y)\,dx\,dy = \frac{3}{14}$

(iii) $P(X \le \frac{1}{2}) = \int_0^1 \int_0^{1/2} f(x,y)\,dx\,dy = \frac{2}{7}$

b. $f_X(x) = \begin{cases} \int_0^1 \frac{6}{7}(x+y)^2\,dy = \frac{6}{7}x^2 + \frac{6}{7}x + \frac{2}{7}, & 0 \le x \le 1 \\ 0, & 其他 \end{cases}$

$f_Y(y) = \begin{cases} \int_0^1 \frac{6}{7}(x+y)^2\,dx = \frac{6}{7}y^2 + \frac{6}{7}y + \frac{2}{7}, & 0 \le y \le 1 \\ 0, & 其他 \end{cases}$

c. $0 \le x \le 1,\ 0 \le y \le 1$

$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{3(x+y)^2}{3y^2+3y+1}$

$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{3(x+y)^2}{3x^2+3x+1}$

§3.4

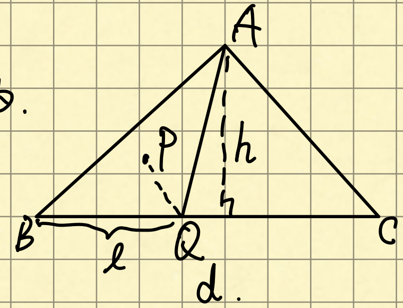
19. $T_1 \sim EXP(\alpha)$, $T_2 \sim EXP(\beta)$

$f_{T_1}(t)=\alpha e^{-\alpha t}\ (t>0)$, $f_{T_2}(s)=\beta e^{-\beta s}\ (s>0)$. $f_{T_1T_2}(t,s)=\alpha\beta e^{-(\alpha t+\beta s)}\ (t>0, s>0)$

(a) $P(T_1>T_2)=\int_0^\infty\int_s^\infty \alpha\beta e^{-(\alpha t+\beta s)}\,dt\,ds=\frac{\beta}{\alpha+\beta}$

(b) $P(T_1>2T_2)=\int_0^\infty\int_{2s}^\infty \alpha\beta e^{-(\alpha t+\beta s)}\,dt\,ds=\frac{\beta}{2\alpha+\beta}$

补充题:



设△ABC以BC为底的高为h. |BQ|=l. |BC|=d.

在△ABC内部取一点P的均匀分布, 密度为 $f_P(p)=\begin{cases}\frac{2}{hd}, & p\text{在}\triangle ABC\text{内}.\\ 0, & \text{其他}\end{cases}$

在底边BC取一点Q的均匀分布, 密度为 $f_Q(q)=\begin{cases}\frac{1}{d}, & 0\le l\le d.\\ 0, & \text{其他}\end{cases}$

∵取P与取Q相互独立

∴ $f_{PQ}(p,q)=f_P(p)f_Q(q)$

若让 $PQ\cap AB\neq\phi$, 则P要在△ABQ中.

$P(PQ\cap AB\neq\phi)=\iint_{\{p\in\triangle ABQ,\ 0\le l\le d\}} f_{PQ}(p,q)\,dp\,dq=\int_0^d f_Q(q)\,dq\int_{p\in\triangle ABQ} f_P(p)\,dp=\frac{2}{hd}\cdot\frac{1}{d}\cdot\int_0^d\frac{1}{2}h\cdot l\,dl$

$=\frac{2}{hd^2}\cdot\frac{1}{4}hd^2=\frac{1}{2}$