§5.1

刘乐奇

12011327

1、$E(X)=E(Y)=2$，$D(X)=1$，$D(Y)=4$，$\rho_{XY}=0.5$

$E(X-Y)=E(X)-E(Y)=0$.

$\rho_{XY}=\dfrac{cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} \Rightarrow cov(X,Y)=1$

$D(X-Y)=D(X)+D(Y)-2cov(X,Y)=3$

由切比雪夫不等式：

$P\{|X-Y|>6\}=P\{|(X-Y)-E(X-Y)|>6\}\le\dfrac{D(X-Y)}{36}=\dfrac{1}{12}$，即式子上限为$\frac{1}{12}$

2、设$X_i=\{$第i次掷出的点数$\}$. $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$E(X_i)=\sum_{k=1}^{6}kP(k)=\frac{1}{6}(1+2+3+4+5+6)=\frac{7}{2}$，$E(X_i^2)=\sum_{k=1}^{6}k^2P(k)=\frac{1}{6}(1+4+9+16+25+36)$
$=\frac{91}{6}$

$D(X_i)=E(X_i^2)-[E(X_i)]^2=\frac{91}{6}-\frac{49}{4}=\frac{35}{12}$

由辛钦大数定律：

$\forall\varepsilon>0$，$P\{|\bar{X}-\mu|>\varepsilon\}=P\{|\frac{1}{n}\sum_{i=1}^{n}X_i-\mu|\ge\varepsilon\}\to 0$ 当$n\to\infty$，

其中，$\mu=E(X_1)=\frac{7}{2}$

故$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i\xrightarrow{P}\frac{7}{2}$

§5.2

1、设第i只元件寿命为X_i，则$X_i\sim EXP(0.01)$，$i=1,2,\cdots,16$.

且X_i互相独立. $E(X_i)=100$，$D(X_i)=10000$

$P\{\sum_{i=1}^{16}X_i>1920\}=1-P\{\sum_{i=1}^{16}X_i\le1920\}$

$=1-P\left\{\dfrac{\sum_{i=1}^{16}X_i-16\cdot100}{\sqrt{16\cdot10000}}\le\dfrac{1920-16\cdot100}{\sqrt{16\cdot10000}}\right\}$

$=1-\Phi(\frac{4}{5})$

2、设第i个人死亡为X_i，则$X_i\sim B(p)$，$p=0.017$.

且X_i互相独立

保险公司亏本为$\sum_{i=1}^{10000}X_i>200$

$P\{\sum_{i=1}^{10000}X_i>200\}=P\left\{\dfrac{\sum_{i=1}^{10000}X_i-np}{\sqrt{np(1-p)}}>\dfrac{200-np}{\sqrt{np(1-p)}}\right\}$

$=1-\Phi\left(\dfrac{200-np}{\sqrt{np(1-p)}}\right)=1-\Phi\left(\dfrac{30}{\sqrt{17\times983}}\right)\approx1-\Phi(2.32)$