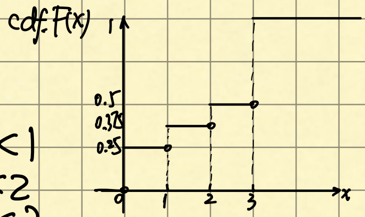
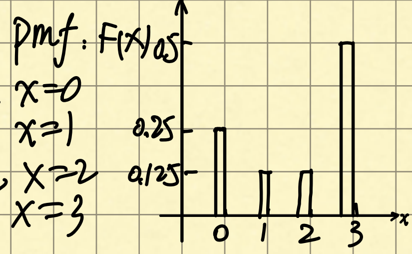
§2.1

1. cdf:
$$F(x)=\begin{cases}0, & x<0\\ 0.25, & 0\le x<1\\ 0.375, & 1\le x<2\\ 0.5, & 2\le x<3\\ 1, & x\ge 3\end{cases}$$



pmf: $f(x)=\begin{cases}0.25, & x=0\\ 0.125, & x=1\\ 0.125, & x=2\\ 0.5, & x=3\end{cases}$



3. 由累积分布函数得频率函数

$F(k=0)=F(k\le 0)=0$, $F(k=1)=F(k\le 1)-F(k\le 0)=0.1$,
$F(k=2)=F(k\le 2)-F(k\le 1)=0.2$, $F(k=3)=F(k\le 3)-F(k\le 2)=0.4$
$F(k=4)=F(k\le 4)-F(k\le 3)=0.1$, $F(k=5)=F(k\le 5)-F(k\le 4)=0.2$

即 $F(k)=\begin{cases}0, & k=0\\ 0.1, & k=1\\ 0.2, & k=2\\ 0.4, & k=3\\ 0.1, & k=4\\ 0.2, & k=5.\end{cases}$

7. 伯努利随机变量的频率函数:
$F(x)=\begin{cases}1-p, & x=0\\ p, & x=1\\ 0, & x\ne 0 \text{ 且 } x\ne 1.\end{cases}$

则pmf → cdf. $x<0$, $F(x)=0$; $0\le x<1$, $F(x)=1-p$; $x\ge 1$, $F(x)=1$.
故伯努利随机变量的cdf为 $F(x)=\begin{cases}0, & x<0\\ 1-p, & 0\le x<1\\ 1, & x\ge 1.\end{cases}$

15. 5局3胜:
$P_1=C_5^5(0.4)^5+C_5^4(0.4)^4 0.6+C_5^3(0.4)^3(0.6)^2=0.31744$
7局4胜:
$P_2=C_7^7(0.4)^7+C_7^6(0.4)^6(0.6)+C_7^5(0.4)^5(0.6)^2+C_7^4(0.4)^4(0.6)^3=0.2898$
$\because P_1>P_2$ $\therefore$ 有利的是5局3胜制.

31. a. ∵每小时内被叫电话次数服从 $\lambda=2$ 的泊松过程
∴10分钟内被叫次数服从 $\omega=\frac{\lambda}{6}=\frac{1}{3}$ 的泊松过程.
设事件A={期间电话铃响起}.
$P(A)=1-P(\bar{A})=1-e^{-\frac{1}{3}}=0.283$
b. 设可以洗浴m分钟, 则服从 $\omega=\frac{m}{60}\lambda=\frac{m}{30}$ 的泊松分布.
$P(\bar{A})=e^{-\frac{m}{30}}\le 0.5 \Rightarrow m=30\ln 2\approx 20.794$
故可洗20.794min.

补充题:

1. ∵ $\sum_{x=1}^{3} P(X=x) = \frac{2}{3}c + \frac{4}{9}c + \frac{8}{27}c = 1$

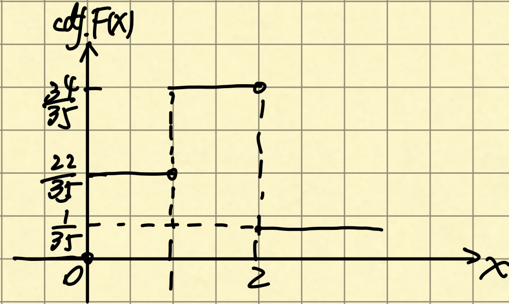
∴ $c = \frac{27}{38}$

2. (1) $P(X=0)=\frac{C_{13}^3}{C_{15}^3} = \frac{13\times12\times11}{15\times14\times13} = \frac{22}{35}$, $P(X=1)=\frac{C_{13}^2 C_2^1}{C_{15}^3} = \frac{13\times12\times2\times3}{15\times14\times13} = \frac{12}{35}$

$P(X=2) = \frac{C_{13}^1}{C_{15}^3} = \frac{13\times6}{15\times14\times13} = \frac{1}{35}$

故频率函数为 $F(x)=\begin{cases} \frac{22}{35}, & x=0 \\ \frac{12}{35}, & x=1 \\ \frac{1}{35}, & x=2 \end{cases}$

(2) cdf: $F(x)=\begin{cases} 0, & x<0 \\ \frac{22}{35}, & 0\le x<1 \\ \frac{34}{35}, & 1\le x<2 \\ \frac{1}{35}, & x\ge 2 \end{cases}$



3. X服从泊松分布,则

$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda} \Rightarrow P(X=k+1)=\frac{\lambda^{k+1}}{(k+1)!}e^{-\lambda}$

∴ $\frac{P(X=k+1)}{P(X=k)} = \frac{\lambda}{k+1}$

当 $k<\lambda-1$ 时. $P(X=k+1)>P(X=k)$, 递增

当 $k>\lambda-1$ 时. $P(x=k+1)<P(X=k)$, 递减

因此, 当 $k=[\lambda-1]$ 时, $P(X=k)$ 最大.

4、$n=2500$, $p=0.002$. $\lambda=np=5$, 令 $X\sim Possion(5)$. 设死人 m.

(1) 亏本概率:

$P(2000m>12\times2500)=P(m>15)\approx P(X>15)\approx 0$.

(2) $P(12\times2500-2000m\ge 10000)=P(m\le 10)\approx P(X\le 10)\approx 0.986$.

$P(12\times2500-2000m\ge 20000)=P(m\le 5)\approx P(X\le 5)\approx 0.616$.