§1.5

46. 设事件A={硬币正面朝上}, B={硬币正面朝下}, C={抽到红球}, D={抽到白球}.

a. $P(C)=P(C|A)P(A)+P(C|B)P(B)$

$=\frac{3}{5}\times\frac{1}{2}+\frac{2}{7}\times\frac{1}{2}=\frac{31}{70}$

b. $P(A|C)=\frac{P(C|A)P(A)}{P(C)}=\frac{(\frac{3}{5}\times\frac{1}{2})}{(31/70)}=\frac{21}{31}$

53. 设事件A={客户索赔}, B_i={i类客户}, i=1为高, 2为中, 3为低.

$P(A)=\sum_{i=1}^{3}P(A|B_i)P(B_i)=0.1\times0.02+0.2\times0.01+0.7\times0.0025=0.00575$

$P(B_1|A)=\frac{P(A|B_1)P(B_1)}{P(A)}=\frac{0.02\times0.1}{0.00575}=\frac{2}{5.75}=\frac{8}{23}$

54. a. $P(R_2)=P(R_2|R_1)P(R_1)+P(R_2|R_1^c)P(R_1^c)=\alpha p+(1-\beta)(1-p)=(\alpha+\beta-1)p+1-\beta$

b. $P(R_3)=P(R_3|R_2)P(R_2)+P(R_3|R_2^c)P(R_2^c)=\alpha[\alpha p+(1-\beta)(1-p)]+(1-\beta)[\beta+p-\alpha p-\beta p]$

$=[(\alpha-1)^2+(\beta-1)^2+2\alpha\beta-1]p+(\alpha+\beta-\alpha\beta-\beta^2)=(\alpha+\beta-1)^2p+(\alpha+\beta)(1-\beta)$

c. 令 $p_n=P(R_n)$, 则 $p_{n+1}=\alpha p_n+(1-\beta)(1-p_n)=(\alpha+\beta-1)p_n+(1-\beta)$

$=(\alpha+\beta-1)^n p+(1-\beta)\cdot\sum_{i=1}^{n}(\alpha+\beta-1)^{i-1}$

$\lim_{n\to\infty}p_n=\frac{1-\beta}{2-\alpha-\beta}$

63. 设事件A={活到70岁}, B={活到80岁},

$P(B|A)=\frac{P(AB)}{P(A)}=\frac{P(B)}{P(A)}=\frac{1}{3}$

补充题:

1. 设事件M_i={车在i门}, N_i={开i门}, L_i={选i门} (i=A, B, C).

$P(N_BL_A)=P(N_BL_AM_A)+P(N_BL_AM_B)+P(N_BL_AM_C)=\frac{1}{18}+0+\frac{1}{9}=\frac{1}{6}$

$P(M_A|N_BL_A)=\frac{P(M_AN_BL_A)}{P(N_BL_A)}=\frac{1}{3}$, $P(M_C|N_BL_A)=\frac{P(M_CN_BL_A)}{P(N_BL_A)}=\frac{2}{3}$

故应改变选择

2. 设事件A={孩子得病}, B={母亲得病}, C={父亲得病}

则 $P(A)=0.6$, $P(B|A)=0.5$, $P(C|AB)=0.4$.

$P(AB)=P(B|A)P(A)=0.3$, $P(ABC)=P(C|AB)P(AB)=0.12$

$P(AB\bar{C})=P(AB-ABC)=P(AB)-P(ABC)=0.3-0.12=0.18$

3. 设事件A={机器良好}, B={产品合格}. 则 $P(B|A)=0.98$, $P(B|\bar{A})=0.55$, $P(A)=0.95$.

$P(A|B)=\frac{P(B|A)\cdot P(A)}{P(B|A)P(A)+P(B|\bar{A})P(\bar{A})}=\frac{0.98\times0.95}{0.98\times0.95+0.55\times0.05}\approx\frac{1862}{1917}\approx0.971$

§1.6

68. 该陈述为假.

当A与C事件相同时,

满足A与B独立, B与C独立, 但A与C不独立.

71. 证: ∵A, B, C相互独立 ∴ $P(AB)=P(A)P(B)$, $P(AC)=P(C)P(A)$, $P(BC)=P(B)P(C)$

$P(ABC)=P(A)P(B)P(C)$

∴ $P(ABC)=P(A)P(B)P(C)=P(AB)P(C)$

$P[(A\cup B)\cap C] = P[(AC)\cup(BC)] = P(AC)+P(BC)-P(ABC) = P(A)P(C)+P(B)P(C)-P(A)P(B)P(C)$

$=[P(A)+P(B)-P(AB)]P(C) = P(A\cup B)P(C)$

故 $A\cap B$与C独立, $A\cup B$与C独立.

74. $P = 1-\bar{P} = 1-\{2[p^2+2(1-p)p]+p\} = 1+2p^2-5p$

77. ∵ $P = 1-(1-0.05)^n = 1-0.95^n = 0.5$

∴ $0.95^n = 0.5 \Rightarrow n = \frac{\ln 0.5}{\ln 0.95} = 14$

79. a. $P\{aa\}=\frac{1}{4}$, $P\{AA\}=\frac{1}{4}$, $P\{Aa\}=\frac{1}{2}$

b. $P\{Aa\}=\frac{2}{3}$

c. $P\{aa\}=\frac{2}{3}\times p\times\frac{1}{4}=\frac{1}{6}p$

$P\{Aa\}=\frac{1}{3}\times p\times\frac{1}{2}+\frac{2}{3}\times(1-p)\times\frac{1}{2}+\frac{2}{3}\times p\times\frac{1}{2}=\frac{1}{3}+\frac{1}{6}p$

$P\{AA\}=\frac{1}{3}\times(1-p)+\frac{1}{3}\times p\times\frac{1}{2}+\frac{2}{3}\times(1-p)\times\frac{1}{2}+\frac{2}{3}\times p\times\frac{1}{4}=\frac{2}{3}-\frac{1}{3}p$

d. $P(\text{父辈为携带者})=\frac{\frac{2}{3}-\frac{1}{6}p}{1-\frac{1}{6}p}=\frac{4-p}{6-p}$

补充题:

1. $P(\bar{A}\bar{B})=\frac{1}{9}$, $P(A\bar{B})=P(\bar{A}B)$

∵A, B互相独立 ∴ $P(A\bar{B})=P(A)P(\bar{B})=P(\bar{B})-P(\bar{A})P(\bar{B})=P(\bar{B})-P(\bar{A}\bar{B})$

$P(\bar{A}B)=P(\bar{A})P(B)=P(\bar{A})-P(\bar{A})P(\bar{B})=P(\bar{A})-P(\bar{A}\bar{B})$

∵ $P(A\bar{B})=P(\bar{A}B)$ ∴ $P(\bar{A})=P(\bar{B})$ ∵ $P(\bar{A}\bar{B})=P(\bar{A})P(\bar{B})=\frac{1}{9}$ 又 $P\in[0,1]$ ∴ $P(\bar{A})=\frac{1}{3}$

∴ $P(A)=1-P(\bar{A})=\frac{2}{3}$.

2. $P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(AB)-P(BC)-P(AC)+P(ABC)$

∵A, B, C两两相互独立 ∴ $P(AB)=P(A)P(B)$, $P(BC)=P(B)P(C)$, $P(AC)=P(A)P(C)$, $P(ABC)=P(A)P(B)P(C)$

令 $P(A)=P(B)=P(C)=p$, 则 $P(A\cup B\cup C)=3p-3p^2=\frac{9}{16}$ ∴ $p=\frac{3}{4}$ 或 $p=\frac{1}{4}$.

∵ $A\subset(A\cup B\cup C)$ ∴ $P(A)=p\le P(A\cup B\cup C)$ ∴ $P(A)=p=\frac{1}{4}$.