6. $\overline{X}=\frac{1}{6}(1.5+2+2.5+3+3.5+1.5)=\frac{7}{3}$

$S^2=\frac{1}{6}($

7. $X\sim P(\lambda)$, $p(X=k)=\begin{cases}\frac{\lambda^k}{k!}e^{-\lambda}, & k>0\\ 0, & k\le 0\end{cases}$

(1) $f(x_1,x_2,\cdots,x_n)=\prod_{i=1}^{n}f(x_i)=\prod_{i=1}^{n}p(x_i=k)=\prod_{i=1}^{n}\frac{\lambda^k}{k!}e^{-\lambda}=\left(\frac{\lambda^k}{k!}\right)^n e^{-n\lambda}$

(2) $E(\overline{X})=\frac{1}{n}\sum_{i=1}^{n}E(X_i)=\lambda$, $D(\overline{X})=\frac{1}{n^2}\sum_{i=1}^{n}D(X_i)=\frac{\lambda}{n}$

$E(S_n^2)=E\left(\frac{1}{n}\sum_{i=1}^{n}X_i^2-\overline{X}^2\right)=E(X^2)-E(\overline{X}^2)=D(X)+[E(X)]^2-D(\overline{X})-[E(\overline{X})]^2=\lambda+\lambda^2-\frac{\lambda}{n}-\lambda^2=\frac{n-1}{n}\lambda$

$E(S_n^{*2})=E\left(\frac{n}{n-1}S_n^2\right)=\lambda$

8. $\because X_i\sim N(0,\sigma^2)$ $\therefore \frac{X_i}{\sigma}\sim N(0,1)$

$U=\sum_{i=1}^{n}\left(\frac{X_i}{\sigma}\right)^2\sim\chi^2(n)$, $V=\sum_{i=n+1}^{n+m}\left(\frac{X_i}{\sigma}\right)^2\sim\chi^2(m)$

$F=\frac{m\sum_{i=1}^{n}X_i^2}{n\sum_{i=n+1}^{m+n}X_i^2}=\frac{U/n}{V/m}\sim F(n,m)$

9. $\overline{X}_1\sim N(\mu,\frac{\sigma^2}{n})$, $\overline{X}_2\sim N(\mu,\frac{\sigma^2}{n})$, $\overline{X}_1-\overline{X}_2\sim N(0,\frac{2\sigma^2}{n})$

$P\{|\overline{X}_1-\overline{X}_2|>\sigma\}=0.01 \Rightarrow P\{-\sigma\le\overline{X}_1-\overline{X}_2\le\sigma\}=0.99$

$\Rightarrow P\left\{\frac{-\sigma}{\sqrt{\frac{2\sigma^2}{n}}}\le\frac{\overline{X}_1-\overline{X}_2}{\sqrt{\frac{2\sigma^2}{n}}}\le\frac{\sigma}{\sqrt{\frac{2\sigma^2}{n}}}\right\}=P\left\{-\sqrt{\frac{n}{2}}\le\frac{\overline{X}_1-\overline{X}_2}{\sqrt{\frac{2\sigma^2}{n}}}\le\sqrt{\frac{n}{2}}\right\}=0.99$

$\Rightarrow \Phi(\sqrt{\frac{n}{2}})-\Phi(-\sqrt{\frac{n}{2}})=0.99 \Rightarrow \Phi(\sqrt{\frac{n}{2}})=0.995$

10. $\overline{X}\sim N(\mu,\frac{\sigma^2}{n})$, $\frac{nS_n^2}{\sigma}\sim\chi^2(n-1)$

$X_{n+1}-\overline{X}\sim N(0,(1+\frac{1}{n})\sigma^2)$, $\frac{X_{n+1}-\overline{X}}{\sqrt{\frac{n+1}{n}}\sigma}\sim N(0,1)$

$T=\frac{X_{n+1}-\overline{X}}{S_n}\sqrt{\frac{n-1}{n+1}}=\frac{(X_{n+1}-\overline{X})/(\sqrt{\frac{n+1}{n}}\sigma)}{\sqrt{\frac{nS_n^2}{\sigma^2}/(n-1)}}\sim t(n-1)$

11. $\sum_{i=1}^{4}X_i\sim N(0,4\sigma^2)\Rightarrow\frac{\sum_{i=1}^{4}X_i}{2\sigma}\sim N(0,1)$

$\sum_{i=5}^{10}X_i^2\sim\chi^2(6)$

$Y=a\frac{\sum_{i=1}^{4}X_i}{\sqrt{\sum_{i=5}^{10}X_i^2}}=\sqrt{12}\sigma a\frac{\sum_{i=1}^{4}X_i/2\sigma}{\sqrt{\sum_{i=5}^{10}X_i^2/6}}$

若$Y\sim t$，则$\sqrt{12}\sigma a=1\Rightarrow a=\frac{1}{\sqrt{12}\sigma}$