§7.1

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5. a. $E(X)=\sum_{i=1}^{2} x_i p(x_i)=2-\theta$, $\bar{X}=\frac{1}{3}\sum_{i=1}^{3}X_i=\frac{5}{3}$

令 $E(X)=\bar{X}$，则 $2-\hat{\theta}=\frac{5}{3} \Rightarrow \hat{\theta}=\frac{1}{3}$

b. $L(\theta;x_1,x_2,x_3)=P(X_1=x_1,X_2=x_2,X_3=x_3)=\prod_{i=1}^{3}\theta^{2-x_i}(1-\theta)^{x_i-1}=\theta^{6-\sum_{i=1}^{3}x_i}(1-\theta)^{\sum_{i=1}^{3}x_i-3}=\theta(1-\theta)^2$
$(\theta\in[0,1])$.

c. $\frac{d\ln L(\theta;x_1,x_2,x_3)}{d\theta}=\frac{1}{\theta}-\frac{2}{1-\theta}=0 \Rightarrow \hat{\theta}=\frac{1}{3}$

补充题:

1. $E(X)=\int_0^{\theta}x\frac{2}{\theta^2}(\theta-x)dx=\frac{2}{\theta^2}[\frac{\theta}{2}x^2-\frac{1}{3}x^3]_0^{\theta}=\frac{\theta}{3}$, $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

令 $E(X)=\bar{X}$，则 $\hat{\theta}=3\bar{X}=\frac{3}{n}\sum_{i=1}^{n}X_i$

2. (1) $L(\theta;x_1\dots x_n)=\prod_{i=1}^{n}\left(\frac{\theta^{x_i}}{x_i!}e^{-\theta}\right)=e^{-n\theta}\cdot\frac{\theta^{\sum_{i=1}^{n}x_i}}{\prod_{i=1}^{n}(x_i!)}$, $\ln L=-n\theta+n\bar{x}\ln\theta-\ln(\prod_{i=1}^{n}(x_i!))$

$\frac{d\ln L}{d\theta}=-n+\frac{n\bar{x}}{\theta}=0 \Rightarrow \hat{\theta}=\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

(2) $L(\theta;x_1\dots x_n)=\prod_{i=1}^{n}(\theta\alpha x_i^{\alpha-1}e^{-\theta x_i^{\alpha}})=\theta^n\alpha^n(\prod_{i=1}^{n}x_i)^{\alpha-1}e^{-\theta\sum_{i=1}^{n}x_i^{\alpha}}$.

$\ln L=n\ln\theta+n\ln\alpha+(\alpha-1)\sum_{i=1}^{n}\ln x_i-\theta\sum_{i=1}^{n}x_i^{\alpha}$

$\frac{d\ln L}{d\theta}=\frac{n}{\theta}-\sum_{i=1}^{n}x_i^{\alpha}=0 \Rightarrow \hat{\theta}=\frac{n}{\sum_{i=1}^{n}X_i^{\alpha}}$

3. 矩估计:

$E(X)=\int_0^1 x\theta(1-x)^{\theta-1}dx=[-x(1-x)^{\theta}]_0^1+\int_0^1(1-x)^{\theta}d\theta=\left[\frac{-(1-x)^{\theta+1}}{\theta+1}\right]_0^1=\frac{1}{\theta+1}$

$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

令 $E(X)=\bar{X}$，则 $\hat{\theta}=\frac{n}{\sum_{i=1}^{n}X_i}-1$

最大似然估计:

$L(\theta;x_1\dots x_n)=\prod_{i=1}^{n}(\theta(1-x_i)^{\theta-1})=\theta^n\left(\prod_{i=1}^{n}(1-x_i)\right)^{\theta-1}$,

$\ln L=n\ln\theta+(\theta-1)\sum_{i=1}^{n}\ln(1-x_i)$

$\frac{d\ln L}{d\theta}=\frac{n}{\theta}+\sum_{i=1}^{n}\ln(1-x_i)=0 \Rightarrow \hat{\theta}=-\frac{n}{\sum_{i=1}^{n}\ln(1-X_i)}$