§1.3

4、$n=1$ 时 $P(\bigcup_{i=1}^{n}A_i)=P(A_1)=\sum_{i=1}^{n}P(A_i)$ 成立

设$n=k$时,$P(\bigcup_{i=1}^{k}A_i)\le\sum_{i=1}^{k}P(A_i)$

当$n=k+1$时,$P(\bigcup_{i=1}^{k+1}A_i)=P(\bigcup_{i=1}^{k}A_i\cup A_{k+1})=P(\bigcup_{i=1}^{k}A_i)+P(A_{k+1})-P(\bigcup_{i=1}^{k}A_i\cap A_{k+1})$

$\le\sum_{i=1}^{k}P(A_i)+P(A_{k+1})-P(\bigcup_{i=1}^{k}A_i\cap A_{k+1})=\sum_{i=1}^{k+1}P(A_i)-P(\bigcup_{i=1}^{k}A_i\cap A_{k+1})$

$\because P\in[0,1]$ $\therefore P(\bigcup_{i=1}^{k+1}A_i)\le\sum_{i=1}^{k+1}P(A_i)$

综上:$P(\bigcup_{i=1}^{n}A_i)\le\sum_{i=1}^{k+1}P(A_i)$

7. 由加法定律得:$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

即 $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

又 $P(A\cup B)\le1$,即$-P(A\cup B)\ge-1$

故 $P(A\cap B)\ge P(A)+P(B)-1$

补充题:

1. $\because ABC\subset AB$ $\therefore P(ABC)\le P(AB)=0$ 又 $P\in[0,1]$ $\therefore P(ABC)=0$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)$

$=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}-0-0-\frac{1}{8}+0=\frac{5}{8}$

2. 由加法定律得:$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

故 $P(\bar{A}\cap\bar{B})=P(\bar{A})+P(\bar{B})-P(\bar{A}\cup\bar{B})$

又 $P(A\cap B)=P(\bar{A}\cap\bar{B})=P(\bar{A})+P(\bar{B})-P(\bar{A}\cup\bar{B})=2-P(A)-P(B)-[1-P(A\cap B)]$

$=1-P(A)-P(B)+P(A\cap B)$

故 $P(A)+P(B)=1$

又 $P(A)=p$ $\therefore P(B)=1-p$.

§1.4

28. $C_{52}^{5}C_{47}^{5}C_{42}^{5}C_{37}^{5}C_{32}^{5}$

29. $P=\frac{C_{10}^{2}}{C_{47}^{2}}$

补充题:

1、(1) $P(A)=\frac{C_n^{2r}2^{2r}}{C_{2n}^{2r}}$

(2) $P(B)=\frac{C_n^1C_{n-1}^{2r-2}2^{2r-2}}{C_{2n}^{2r}}$

(3) $P(C)=\frac{C_n^r}{C_{2n}^{2r}}$

2、设事件A={至少有一人能打开门}

$P(A)=1-P(\bar{A})=1-\frac{9}{24}=\frac{5}{8}$