

§8.2

$$1. X \sim N(62, \sigma^2), \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i = 67.1, S^2 = \frac{1}{10} \sum_{i=1}^{10} (X_i - \bar{X})^2 = 6.34^2, n = 10$$

在显著水平 $\alpha = 0.05$ 下, 检验假设:

$$H_0: \mu = \mu_0 = 62, H_1: \mu \neq \mu_0.$$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1), H_0 \text{ 的拒绝域为 } \{ |T| \geq t_{\frac{\alpha}{2}}(9) \}$$

代入 $\bar{X} = 67.1, S^2 = 6.34^2$ 有

$$|T| = 2.39 > t_{\frac{\alpha}{2}}(9) = 2.262$$

因此拒绝 H_0 , 认为患者脉搏与正常人有显著差异.

2. 设方法 A 的吸热量为 $X \sim N(\mu_1, \sigma^2)$, 方法 B 吸热量为 $Y \sim N(\mu_2, \sigma^2)$

在显著水平 $\alpha = 0.05$ 下, 检验假设

$$H_0: \mu_1 = \mu_2, H_1: \mu_1 \neq \mu_2$$

依题意得: $\bar{X} = 80.02, n_1 = 13, S_1^2 = 0.000574$

$$\bar{Y} = 79.98, n_2 = 8, S_2^2 = 0.000961$$

$$S_w^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = 0.0007166$$

$$\text{取 } t = \frac{\bar{X} - \bar{Y}}{S_w \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1+n_2-2), H_0 \text{ 的拒绝域为 } \{ |t| > t_{1-\frac{\alpha}{2}}(n_1+n_2-2) \}$$

代入数据得:

$$|t| = 3.325 > t_{1-\frac{\alpha}{2}}(19) = 1.729$$

故接受 H_0 , 两种方法总体均值相等.

3. 设零件尺寸为 $X \sim N(\mu, \sigma^2), n = 19, S^2 = 1.2^2$.

在显著水平 $\alpha = 0.05$ 下, 检验假设

$$H_0: \sigma \leq \sigma_0 = 0.9, H_1: \sigma > \sigma_0$$

$$\text{取 } \chi = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1), H_0 \text{ 的拒绝域为 } \{ |\chi| > \chi_{\alpha}^2(n-1) \}$$

$$\text{代入数据得: } |\chi| = 32 > \chi_{1-\alpha}^2(n-1) = 28.87$$

故拒绝 H_0 , 认为标准差变大.

4. 设学生智商为 $X_1 \sim N(\mu_1, \sigma_1^2), n_1 = 16, \bar{X}_1 = 107, S_1 = 10$

$$X_2 \sim N(\mu_2, \sigma_2^2), n_2 = 16, \bar{X}_2 = 112, S_2 = 8$$

在显著水平 $\alpha = 0.05$ 下, 检验假设

$$H_0: \sigma_1 = \sigma_2, H_1: \sigma_1 \neq \sigma_2$$

$$\text{取 } F = \frac{S_1^2}{S_2^2} \sim F(n_1-1, n_2-1), \text{ 此时 } H_0 \text{ 的拒绝域为 } \{ |F| > F_{1-\alpha}(n_1-1, n_2-1) \}$$

$$\text{代入数据得: } |F| = 1.562 < F_{1-\alpha}(n_1-1, n_2-1) = 2.86$$

故接受 H_0 , 认为智商无差异

5. 设由3个字母组成的词的比例为 $X_1 \sim N(\mu_1, \sigma^2)$, $X_2 \sim N(\mu_2, \sigma^2)$
在显著水平 $\alpha = 0.05$ 下, 检验假设

$$H_0: \mu_1 = \mu_2, H_1: \mu_1 \neq \mu_2.$$

已知 $n_1 = 8, \bar{X}_1 = \dots, S_1 = \dots$

$n_2 = 10, \bar{X}_2 = \dots, S_2 = \dots$

$$\text{则 } S_W = \frac{(n_1-1)S_1 + (n_2-1)S_2}{n_1+n_2-2} = \dots$$

$$\text{取 } t = \frac{\bar{X}_1 - \bar{X}_2}{S_W \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1+n_2-2)$$

H_0 的拒绝域为 $\{|t| \geq t_{1-\alpha/2}(n_1+n_2-2)\}$

代入数据得: $|t| = 3.9 > t_{1-\alpha/2}(n_1+n_2-2) = 2.12$

拒绝 H_0 , 故两者有明显差异.