§2.2

33、证：$F(x)=\begin{cases}0, & x<0\\ 1-e^{-\alpha x^{\beta}}, & x\geq 0.\end{cases}$

$x\geq 0$，$F'(x)=\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}>0$ 恒成立

故$F(x)$在R上非减.

$\because x\geq 0$，$e^{-\alpha x^{\beta}}>0$ 恒成立　$\therefore x\in R$，$0\leq F(x)\leq 1$ 恒成立

$\lim\limits_{x\to-\infty}F(x)=0$，$\lim\limits_{x\to+\infty}F(x)=1$

$x<0$与$x>0$时$F(x)$分别左右连续.

在$x=0$处，$F(0+0)=\lim\limits_{t\to 0^+}F(t)=\lim\limits_{t\to 0^+}(1-e^{-\alpha x^{\beta}})=0=F(0)$

故$F(x)$满足cdf的三性质，所以$F(x)$是cdf.

相应的密度函数为：

$f(x)=F'(x)=\begin{cases}0, & x<0\\ \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}, & x\geq 0.\end{cases}$

40. a. $\because \Sigma f(x)=1$　$\therefore \int_0^1 cx^2dx+0=1\Rightarrow \frac{c}{3}=1\Rightarrow c=3.$

b. $P(X<0)=\int_{-\infty}^{0}f(x)dx=0$

$P(X>1)=\int_1^{\infty}f(x)dx=0$

$P(0\leq X\leq 1)=\int_0^x f(t)dt=\int_0^x 3t^2dt=x^3$

$\therefore$ cdf $F(x)=\begin{cases}0, & x<0\\ x^3, & 0\leq x\leq 1\\ 0, & x>1\end{cases}$

c. $P(0.1\leq X\leq 0.5)=\int_{0.1}^{0.5}f(x)dx=\int_{0.1}^{0.5}3x^2dx=0.125-0.001=0.124.$

45、设电子元件的寿命为X，则$X\sim EXP(0.1)$. 密度函数$f(x)=\begin{cases}\lambda e^{-\lambda x}, & x>0\\ 0, & x\leq 0\end{cases}$ $(\lambda=0.1)$.

a. $P(X<10)=\int_{-\infty}^{10}f(x)dx=0+\int_0^{10}\lambda e^{-\lambda x}dx=[-e^{-\lambda x}]_0^{10}=1-e^{-10\lambda}=1-\frac{1}{e}$

b. $P(5\leq X\leq 15)=\int_5^{15}f(x)dx=\int_5^{15}\lambda e^{-\lambda x}dx=[-e^{-\lambda x}]_5^{15}=e^{-5\lambda}-e^{-15\lambda}=\frac{e-1}{e^{1.5}}$

c. $\because P(X>t)=\int_t^{+\infty}f(x)dx=[-e^{-\lambda x}]_t^{+\infty}=\lim\limits_{b\to+\infty}(e^{-\lambda t}-e^{-\lambda b})=e^{-\lambda t}=0.01$　$\therefore t=20\ln 10$

52. 设个体身高为X，则$X\sim N(70,9)$. $\mu=70, \sigma=3$.　1 ft =12 in.

则$\frac{X-70}{3}\sim N(0,1)$.

a. $P(X>6ft)=P(X>72in)=P\left(\frac{X-70}{3}>\frac{72-70}{3}\right)=1-\Phi(\frac{2}{3})=1-0.7454=0.2546$

b. 均是正态分布

厘米表示：$X\sim N(177.8, 7.62^2)$

米表示：$X\sim N(1.778, 0.0762^2)$

53. $X\sim N(5,100)$. $\mu=5, \sigma=10$. 故$\frac{X-5}{10}\sim N(0,1)$

(a) $P(X>10)=1-P(X\leq 10)=1-P\left(\frac{X-5}{10}\leq\frac{10-5}{10}\right)=1-\Phi(0.5)=1-0.6915=0.3085$

(b) $P(-20<X<15)=P(X<15)-P(X<-20)=P\left(\frac{X-5}{10}\leq 1\right)-P\left(\frac{X-5}{10}\leq -2.5\right)=\Phi(1)-\Phi(-2.5)=0.8413-1+0.9938$

$=0.8351$

(c) $P(X>x)=0.05\Rightarrow P(X\leq x)=0.95\Rightarrow P\left(\frac{X-5}{10}\leq\frac{x-5}{10}\right)=0.95\Rightarrow\Phi\left(\frac{x-5}{10}\right)=0.95$

$\because \Phi(1.65)=0.9505>0.95$　$\therefore \frac{x-5}{10}=1.65\Rightarrow x=21.5$

补充题：

1、$f(x)=Ae^{-|x|}$，$x\in R$.

(1) $\int_{-\infty}^{\infty}f(x)dx \Rightarrow 2\int_{0}^{\infty}Ae^{-x}dx=2A=1 \quad \therefore A=\frac{1}{2}$

(2) $P\{0<X<1\}=\int_0^1 f(x)dx=\int_0^1 Ae^{-|x|}dx=[-Ae^{-x}]_0^1=A-Ae^{-1}=\frac{e-1}{2e}$

(3) $F(x)=\int_{-\infty}^{x}f(t)dt$

$x<0$，$\int_{-\infty}^{x}f(t)dt=\int_{-\infty}^{x}Ae^{t}dt=[Ae^{t}]_{-\infty}^{x}=Ae^{x}=\frac{e^{x}}{2}$

$x\geq 0$，$\int_{-\infty}^{x}f(t)dt=\int_{-\infty}^{0}Ae^{t}dt+\int_{0}^{x}Ae^{-t}dt=2A-Ae^{-x}=\frac{1}{2}(2-e^{-x})$

故 $F(x)=\begin{cases}\frac{1}{2}e^{x}, & x<0\\ \frac{1}{2}(2-e^{-x}), & x\geq 0.\end{cases}$

2. $X\sim E(\frac{1}{5})$. 密度函数 $f(x)=\begin{cases}\lambda e^{-\lambda x}, & x>0\\ 0, & x\leq 0\end{cases}$ $(\lambda=\frac{1}{5})$

$p=P(X>10)=\int_{10}^{+\infty}f(x)dx=e^{-2}$.

$Y\sim b(5,e^{-2})$

$P(Y=k)=C_5^k(e^{-2})^k(1-e^{-2})^{5-k} \quad (k=0,1,2,3,4,5)$.

$P(Y\geq 1)=1-P(Y=0)=1-C_5^0(e^{-2})^0(1-e^{-2})^5=1-(1-e^{-2})^5$.