(1)

Mathematical Logic as a Base of Disciplines

As a base of the science study and discovery, logic can be seen in many disciplines, which cannot be independent with the logic. Mathematical logic is used as a tool or a language that is convenient for the people to study the problems in proofs and computation.

Mathematics is a highly abstract and rigorous science, the correctness of whose theorems, laws, principles and so on cannot be proved by specific experiments and empirical practice as chemistry and biology. However, it can only be confirmed by strict mathematical logic. Without mathematical logic, the subgrade of mathematics would be not stable, and the edifice of mathematics cannot be constructed. Namely, mathematics in the modern sense is supposed to be impossible to exist.

The development of mathematics has its own regularity, but its stage of development is also accompanied by the development of mathematical logic. The formation of theory in mathematics requires a process of reasoning and refinement, and finally forms a system of mathematics. During this process, mathematical logic provides a rigorous standard for the correct mathematical reasoning. Hence, there is no doubt that it needs mathematical logic throughout mathematics.

As for computer science, mathematical logic has a very close relationship with it. From the Turing Machine of digital electronic computer to the Boolean algebra of digital circuit, mathematical logic plays an important role in them, and they are inseparable from mathematical logic. In the logical digital, it requires us to properly arrange the AND gate and the NOT gate and so on; in the assembly language editing,

it requires us to carry out the inverse and compliment operation in many situations. All that needs mathematical logic to guide it correctly. Mathematical logic is so basic for computer science that it is no exaggeration to say that without mathematical logic and its development, there would not appear the Turing Machine and so that the modern computer would no longer exist.

In intelligence science, lots of information about the study objects is incomplete. That is, for some specific problem, we cannot obtain more complete data from the real world. At this time, based on the given incomplete data, we can use mathematical intelligence to perform modeling analysis so that deal with the problem effectively.

At last, for artificial intelligence, we need huge amount of data to build up the knowledge of the artificial intelligence so that it can deal with the practical problems. However, in essence, the artificial intelligence is an information receiving processor. After the certain calculating rules is defined, it can do the calculations and acquire effective knowledge constantly in the observation of the real world. At this time, if we still insisted to input the statistic we collected, it would cost an enormous payment of time and energy. Nevertheless, by utilizing mathematical logic, the artificial intelligence can do some deduction by itself, which will greatly reduce the cost and even sometimes we can acquire some data that is hard to get. And then, the artificial intelligence can learn by itself.

(2)

Theories and Applications Based on Mathematical Logic

The decidability of elementary algebra and geometry ---- Alfred Tarski

Tarski used quantifier elimination method to prove that the first-order theory of real numbers with only addition and multiplication is decidable.

Model Theory, Proof Theory

Two important branches of mathematical logic.

Model theory is essentially semantic logic, as opposed to proof theory, which is essentially syntactic.

Model theory is generally concerned with first-order logic. Model theory treats formulae, sentences, theories (sets of sentences) and models in formal languages as mathematical objects.

By expressing mathematical proofs as formal mathematical objects, the analysis of mathematical proofs can be simplified through mathematical logic. Proof is usually expressed in terms of data structures that are defined inductively.

Axiomatic set theory

It is one of the main branches of mathematical logic, and it is the study of reconstructing naive set theory with axiomatic methods, and the study of the meta-mathematics of set theory and the new axioms of set theory.

Recursion theory

An important branch of mathematical logic that deals with feasible computation methods and computation complexity for solving problems, especially recursive

functions and their generalizations.