# 8.1(g) and 8.2(g)

引理 1

$$\mathrm{A} \to \mathrm{B}, \mathrm{B} \to \mathrm{C} \vdash_L A \to C$$

| | | |
|---|---|---|
| 1. | $(\mathrm{B} \to \mathrm{C}) \to (\mathrm{A} \to (\mathrm{B} \to \mathrm{C}))$ | AS1 |
| 2. | $\mathrm{B} \to \mathrm{C}$ | Premise |
| 3. | $\mathrm{A} \to (\mathrm{B} \to \mathrm{C})$ | MP 1, 2 |
| 4. | $(\mathrm{A} \to (\mathrm{B} \to \mathrm{C})) \to ((A \to B) \to (A \to C))$ | AS2 |
| 5. | $(A \to B) \to (A \to C)$ | MP 3, 4 |
| 6. | $A \to B$ | Premise |
| 7. | $A \to C$ | MP 5, 6 |

引理 2

$$\vdash_L (\mathrm{A} \to (\mathrm{B} \to \mathrm{C})) \to (\mathrm{B} \to (\mathrm{A} \to \mathrm{C}))$$

| | | |
|---|---|---|
| 1. | $(\mathrm{A} \to (\mathrm{B} \to \mathrm{C})) \to ((A \to B) \to (A \to C))$ | AS2 |
| 2. | $(\mathrm{A} \to (\mathrm{B} \to \mathrm{C})) \to (B \to ((A \to B) \to (A \to C)))$ | AS1+引理 1 |
| 3. | $(B \to ((A \to B) \to (A \to C))) \to ((B \to (A \to B)) \to (B \to (A \to C)))$ | AS2 |

易证$\vdash_L (T \to B) \to B$ hint: $\vdash_L (T \to B) \to (T \to B)$

| | | |
|---|---|---|
| 4. | $((B \to (A \to B)) \to (B \to (A \to C))) \to (B \to (A \to C))$ | |
| 5. | $(B \to ((A \to B) \to (A \to C))) \to (B \to (A \to C))$ | 引理 1 |
| 6. | $(\mathrm{A} \to (\mathrm{B} \to \mathrm{C})) \to (B \to (A \to C))$ | 引理 1 |

引理 3

$$\mathrm{A} \to \mathrm{B} \vdash_L (B \to C) \to (A \to C)$$

| | | |
|---|---|---|
| 1. | $(\mathrm{B} \to \mathrm{C}) \to (\mathrm{A} \to (\mathrm{B} \to \mathrm{C}))$ | AS1 |
| 2. | $(\mathrm{A} \to (\mathrm{B} \to \mathrm{C})) \to ((A \to B) \to (A \to C))$ | AS2 |
| 3. | $(\mathrm{B} \to \mathrm{C}) \to ((\mathrm{A} \to \mathrm{B}) \to (\mathrm{A} \to \mathrm{C}))$ | 引理 1 |
| 4. | $(A \to B) \to ((B \to C) \to (A \to C))$ | 引理 2+MP |
| 5. | $(\mathrm{A} \to \mathrm{B})$ | Premise |
| 6. | $(B \to C) \to (A \to C)$ | MP 4, 5 |

8.1(g) 不使用 deduction theorem

$$\vdash_L (B \to C) \to (((\neg B) \to C) \to C)$$

| | | |
|---|---|---|
| 1. | $(B \to C) \to (\neg C \to \neg B)$ | 8.1(e) |
| 2. | $(\neg C \to \neg B) \to (((\neg C) \to B) \to C)$ | AS3 |
| 3. | $(B \to C) \to (((\neg C) \to B) \to C)$ | 引理 1 1, 2 |

下证:$\vdash_L ((\neg B) \to C) \to ((\neg C) \to B)$

| | | |
|---|---|---|
| 4. | $((\neg B) \to C) \to ((\neg C) \to (\neg\neg B))$ | AS3 |
| 5. | $((\neg\neg B) \to B) \to ((\neg C) \to ((\neg\neg B) \to B))$ | AS1 |
| 6. | $(\neg\neg B) \to B$ | 8.1(a) |
| 7. | $((\neg C) \to ((\neg\neg B) \to B))$ | MP 5, 6 |
| 8. | $((\neg C) \to (\neg\neg B)) \to ((\neg C) \to B)$ | AS2+MP |
| 9. | $((\neg B) \to C) \to ((\neg C) \to B)$ | 引理 1 4, 8 |
| 10. | 接下来只要证$(((\neg C) \to B) \to C) \to (((\neg B) \to C) \to C)$就可以得到结论 | |

| | | |
|---|---|---|
| 11. | $(((\neg C) \to B) \to C) \to (((\neg B) \to C) \to C)$ | 引理 3 9 |
| 12. | $(B \to C) \to (((\neg B) \to C) \to C)$ | 引理 1 3, 11 |

8.1(g) 使用 deduction theorem

$$\vdash_L (B \to C) \to (((\neg B) \to C) \to C)$$

| | | |
|---|---|---|
| 1. | $B \to C$ | Premise |
| 2. | $(\neg B) \to C$ | Premise |
| 3. | $(B \to C) \to ((\neg C) \to (\neg B))$ | 8.1(e) |
| 4. | $(\neg C) \to (\neg B)$ | MP 1, 3 |
| 5. | $(\neg C \to \neg B) \to (((\neg C) \to B) \to C)$ | AS3 |
| 6. | $((\neg C) \to B) \to C$ | MP 4, 5 |
| 7. | $((\neg B) \to C) \to ((\neg C) \to B)$ | 上证 |
| 8. | $(\neg C) \to B$ | MP 2, 7 |
| 9. | $C$ | MP 6, 8 |
| 10. | $B \to C, (\neg B) \to C \vdash_L C$ | |
| 11. | $\vdash_L (B \to C) \to (((\neg B) \to C) \to C)$ | Deduction Theorem |

8.2(g) 不使用 deduction theorem

首先证明←：

$$\vdash_{HB} ((A \wedge B) \vee (A \wedge C)) \to (A \wedge (B \vee C))$$

观察到结论和 AS3.3 很像，因此尝试证明

$(A \wedge B) \to ((A \wedge (B \vee C)))$和$(A \wedge C) \to (A \wedge (B \vee C))$

| | | |
|---|---|---|
| 1. | $((A \wedge B) \to A) \to (((A \wedge B) \to (B \vee C)) \to ((A \wedge B) \to ((A \wedge (B \vee C))))))$ | AS2.3 |
| 2. | $((A \wedge B) \to A)$ | AS2.1 |
| 3. | $((A \wedge B) \to (B \vee C)) \to ((A \wedge B) \to (A \wedge (B \vee C)))$ | MP 1, 2 |
| 4. | $((A \wedge B) \to B)$ | AS2.2 |
| 5. | $B \to (B \vee C)$ | AS3.1 |
| 6. | $(A \wedge B) \to (B \vee C)$ | AS1.3<br>MP 4, 5 |
| 7. | $(A \wedge B) \to (A \wedge (B \vee C))$ | MP 3, 6 |
| 8. | 同理可证，$(A \wedge C) \to (A \wedge (B \vee C))$ | |
| 9. | $((A \wedge B) \vee (A \wedge C)) \to (A \wedge (B \vee C))$ | AS3.3<br>MP 7, 8 |

接下来证明→（不使用 deduction theorem）：

$$\vdash_{HB} (A \wedge (B \vee C)) \to ((A \wedge B) \vee (A \wedge C))$$

首先对箭头右边式子使用分配律得到: 8.2 (f)

$$((A \wedge B) \vee (A \wedge C)) \leftrightarrow (((A \wedge B) \vee A) \wedge ((A \wedge B) \vee C)))$$

结论被改写成了X ∧ Y的形式，因此联想到 AS2.3，只需证明：

$(A \wedge (B \vee C)) \to ((A \wedge B) \vee A)$和$(A \wedge (B \vee C)) \to ((A \wedge B) \vee C)$

| | | |
|---|---|---|
| 1. | $(A \wedge (B \vee C)) \to A$ | AS2.1 |
| 2. | $A \to ((A \wedge B) \vee A)$ | AS3.2 |
| 3. | $(A \wedge (B \vee C)) \to ((A \wedge B) \vee A)$ | AS1.3 |

| | | |
|---|---|---|
| | | MP 1, 2 |
| | 接下来证明$(A \wedge (B \vee C)) \to ((A \wedge B) \vee C)$，注意到：<br>$((A \wedge B) \vee C) \leftrightarrow ((A \vee C) \wedge (B \vee C))$ | 8.2(f) |
| | 因此只要证明$(A \wedge (B \vee C)) \to ((A \vee C) \wedge (B \vee C))$ | |
| 4. | $(A \wedge (B \vee C)) \to (B \vee C)$ | AS2.2 |
| 5. | $A \to (A \vee C)$ | AS3.1 |
| 6. | $(A \wedge (B \vee C)) \to (A \vee C)$ | AS1.3 |
| | | MP 1, 5 |
| 7. | $(A \wedge (B \vee C)) \to ((A \vee C) \wedge (B \vee C))$ | AS2.3 |
| | | MP 4, 6 |
| 8. | $(A \wedge (B \vee C)) \to ((A \wedge B) \vee C)$ | AS2.3 |
| | | MP 7, (f) |
| 9. | $(A \wedge (B \vee C)) \to ((A \wedge B) \vee (A \wedge C))$ | AS2.3 |
| | | MP 3, 8 |

deduction theorem 对证明帮助不大，略。