

# An Introduction to Mathematical Logic

(Guidance)

## Jingde Cheng

Southern University of Science and Technology

### CS104/CS108: An Introduction to Mathematical Logic

♣ **Teaching professor**

- ◆ Jingde Cheng (程京德), [chengjd@sustech.edu.cn](mailto:chengjd@sustech.edu.cn)
- ◆ HP: <http://www.aise.ics.saitama-u.ac.jp/~cheng/index.html>
- ◆ Blog: <http://blog.sciencenet.cn/u/JingdeCheng>  
(科学网博客: 计算机科学, 数理逻辑, 人工智能, 等等)
- ◆ Use Google or 百度 to find more information about me.

♣ **Teaching assistant teacher**

- ◆ Yun Shen (沈昀), [sheny@mail.sustech.edu.cn](mailto:sheny@mail.sustech.edu.cn)
- ◆ Answering class (optional): Will be announced.

♣ **QQ group**

- ◆ CS104-CS108-12ML: 659130328

3/23/21 2 \*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



### Who Should be Here and Who Should NOT be Here?


♣ **Who should be here? 谁应该在这里?**

- ◆ Want to be a world-class (CS, IS, AI) scientist or mathematician.  
想成为世界级的 (计算机、智能、人工智能) 科学家或数学家。
- ◆ Want to know what is (mathematical) logic and why study it.  
想知道什么是 (数理) 逻辑和为什么学习研究它。
- ◆ Want to know what is the fundamental basis for all sciences.  
想知道什么是所有科学的基础。

♣ **Who should NOT be here? 谁不应该在这里?**

- ◆ Fear of difficulties, especially in mathematics; fear of hard work, want to get a good score easily.  
畏惧困难, 尤其是数学的困难; 畏惧努力, 希冀轻松获取好成绩。
- ◆ Want to get some logic knowledge to become a good engineer.  
想获取一些逻辑学知识以成为一名好工程师。
- ◆ Want to get some logic knowledge to help getting a high salary job.  
想获取一些逻辑学知识以帮助找到一个高薪工作。

3/23/21 3 \*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*




### Formal Method: Why Study It ? [B&K-08]

*“Formal methods should be part of the education of every computer scientist and software engineer, just as the appropriate branch of applied mathematics is a necessary part of the education of all other engineers.”*

– FAA (Federal Aviation Authority) and NASA (National Aeronautics and Space Administration), USA

3/23/21 4 \*\*\*\*\* Jingde Cheng / Saitama University \*\*\*\*\*



### CS104/CS108: An Introduction to Mathematical Logic


♣ **Pre-requisites**

- ◆ CS101A/CS101B: Introduction to Computer Science
- ◆ Those who haven’t learned naive set theory and the theory of computation should learn them by themselves in advance.

♣ **Note**

- ◆ Because of time limit, I do not answer questions about the set theory and the theory of computation in classes.

3/23/21 5 \*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*




### CS104/CS108: An Introduction to Mathematical Logic

◆ **Guidance**

- ◆ Logic: What Is It and Why Study It?
- ◆ Formal Logic Systems and Formal Theories
- ◆ Classical Propositional Calculus (CPC)
- ◆ Classical First Order Predicate Calculus (CFOPC)
- ◆ Gödel’s Incompleteness Theorems
- ◆ Various Modal Logics (CS108 class only)
- ◆ Relevant Logics (CS108 class only)

3/23/21 6 \*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*




The BEST Course on “Introduction to ML” in China

I will provide you with the BEST course on “Introduction to Mathematical Logic” in China !

3/23/21

7

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



My Hope/Statement and Your Attitude

♣ My hope/statement

◆ I really like to teach and guide you to become world-class scientists and/or engineers in the future, by laying a good foundation for you.

♣ Your attitude

◆ (I want you) to listen carefully, think deeply, ask questions actively, and read text books and reference materials carefully.

♣ “青出于蓝而胜于蓝”


◆ “青，取之于蓝，而胜于蓝；冰，水为之，而寒于水。”  
— 荀子, “劝学”, 约公元前230年

◆ 我将把你们视为清华的学生来教授你们知识和能力；  
期待着你们 “青出于蓝而胜于蓝”。

3/23/21

8

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



My Teaching Style

♣ For important notions/concepts

◆ What is it?

◆ Why study it?

♣ For important assumptions and/or principles

◆ Why LOGIC needs it?

◆ Why it is fundamental? / What is underlain by it?

♣ For important problems

◆ What is its background?

◆ Why it is interesting?


♣ Using QQ groups

◆ Send various advices/materials/notices to you, and answer your questions.

3/23/21

9

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Some Rules You Should Follow

♣ Preview

◆ I will distribute courseware, teaching main points (in Chinese), and reading materials one week before lectures every week.

◆ You should preview the materials before the lectures every week.

◆ Note: It is very difficult to understand all the teaching contents at once just by listening in the class !!!

♣ Ask questions at classes but not at break time


◆ I will give you time at classes to let you ask questions.

◆ You should ask questions at classes, but not at break time, because my answers at classes are beneficial and useful to all students.

3/23/21

10

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Assessment Grading

♣ Homework (50% of final score, 100/130 points)

◆ Answer questions, read materials, and submit reports

◆ Penalty for answer and report copies: Give minus points to all similar answers and reports.

◆ You can (should!) get full points if you do homework hard.

♣ Final examination (50% of final score, 100/120 points)


◆ Open-book examination (two hours)

◆ Do not consider that the open-book examination is easy !!!  
(Only those students who have studied and reviewed the course contents very seriously can get good score)

3/23/21

11

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Major Text and Reference Books for Undergraduates

♣ Logic

◆ I. M. Copi and C. Cohen, “Introduction to Logic,” Macmillan, 1953, 1961 (I. M. Copi), 1968, 1972, 1978, 1982, 1986, 1990, Prentice-Hall, 1994, 1998, 2002, 2005, Pearson Education, 2008 (I. M. Copi and C. Cohen), Routledge, 2010 (with K. D. McMahon) (14th Edition), 2019 (with V. Rodych) (15th Edition). [C&C]


◆ P. J. Hurley, “A Concise Introduction to Logic,” Wadsworth, 1982, 1985, 1988, 1991, 1993, 1997, 1999, 2003, 2005, 2008, 2012, 2014, 2016 (with L. Watson) (13th Edition). [Hurley]

◆ D. Kelley, “The Art of Reasoning: An Introduction to Logic and Critical Thinking,” W. W. Norton & Company, 1988, 1990, 1994, 1998, 2014 (4th Edition). [Kelley]

3/23/21

12

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Major Text and Reference Books for Undergraduates

♣ Mathematical Logic

- ♦ E. Mendelson, "Introduction to Mathematical Logic," Chapman & Hall, 1964, 1979, 1987, 1997, 2010, **2015** (6th Edition). [Mendelson]
- ♦ R. M. Smullyan, "A Beginner's Guide to Mathematical Logic," Dover Publications, **2014**. [Smullyan]
- ♦ M. Ben-Ari, "Mathematical Logic for Computer Science," Springer, 1993, 2001, **2012** (3rd Edition). [Ben-Ari]
- ♦ W. Routenberg, "A Concise Introduction to Mathematical Logic," Springer, 1979, 2006, **2010** (3rd Edition). [Routenberg]
- ♦ S. Reeves and M. Clarke, "Logic for Computer Science," Addison-Wesley, 1990-**2003**. [R&C]

3/23/21

13

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Reference Books on Logic

- ♦ V. Klenk, "Understanding Symbolic Logic," Prentice-Hall, 1983, 1989, 1994, **2002**, 2010 (5th Edition). [Klenk]
- ♦ R. M. Johnson, "A Logic Book: Fundamentals of Reasoning," Wadsworth, 1986, 1991, 1999, 2002, **2007** (5th Edition). [Johnson]
- ♦ R. Jeffrey, "Formal Logic: Its Scope and Limits," McGraw-Hill, 1967, 1981, 1991, Hackett Publishing, **2006** (with J. P. Burgess) (4th Edition). [Jeffrey]
- ♦ G. Restall, "Logic: An Introduction," Routledge, 2006. [Restall]
- ♦ C. S. Layman, "The Power of Logic," Mayfield, 1999, 2002, McGraw-Hill, **2005** (3rd Edition). [Layman]
- ♦ C. Allen & M. Hand, "Logic Primer," The MIT Press, 1992, **2001** (2nd Edition). [A&H]
- ♦ G. Priest, "Logic: A Very Short Introduction," Oxford University Press, 2000. [Priest]
- ♦ J. N. Nolt, D. Rohatyn, and A. Varzi, "Schaum's Outline of Theory and Problems of Logic," McGraw-Hill, 1988, **1998** (2nd Edition). [N&R&V]

3/23/21

14

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Reference Books on Mathematical Logic

- ♦ B. Meltzer (translation) and R. B. Braithwaite (Introduction), K. Gödel, "On formally undecidable propositions of Principia Mathematica and related systems I," Basic Books, 1962, Dover Publications, **1992**. [Gödel]
- ♦ R. M. Smullyan, "A Beginner's Guide to Mathematical Logic," Dover Publications, **2014**; "A Beginner's Further Guide to Mathematical Logic," World Scientific, **2017**. [Smullyan]
- ♦ D. van Dalen, "Logic and Structure," Springer, 1980, 1983, 1994, **2004**, 2013 (5th Edition). [Dalen]
- ♦ Y. I. Manin, "A Course in Mathematical Logic for Mathematicians," Springer, 1977, **2010** (2nd Edition). [Manin]
- ♦ G. S. Boolos, J. P. Burgess, and R. C. Jeffrey, "Computability and Logic," Cambridge University Press, 1974, 1980, 1990, 2002, **2007** (5th Edition). [B&B&J]
- ♦ H. B. Enderton, "A Mathematical Introduction to Logic," Academic Press, 1972, **2001** (2nd Edition). [Enderton]
- ♦ H.-D. Ebbinghaus, J. Flum, and W. Thomas, "Mathematical Logic," Springer, 1978, **1994** (2nd Edition). [E&F&T]
- ♦ A. G. Hamilton, "Logic for Mathematicians," Cambridge University Press, **1978**, 1989 (2nd Revised Edition). [Hamilton]
- ♦ J. R. Shoenfield, "Mathematical Logic," Association for Symbolic Logic / Addison-Wesley, 1967. [Shoenfield]

3/23/21

15

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Reference Books on Mathematical Logic for CS

- ♦ J. H. Gallier, "Logic for Computer Science – Foundations of Automatic Theorem Proving," Harper & Row, 1986, **2003** (Revised Online Edition), Dover Publications, 2015 (2nd Edition). [Gallier]
- ♦ M. Ben-Ari, "Mathematical Logic for Computer Science," Springer, 1993, 2001, **2012** (3rd Edition). [Ben-Ari]
- ♦ M. Huth and M. Ryan, "Logic in Computer Science: Modelling and Reasoning about Systems," **2004** (2nd Edition). [H&R]
- ♦ S. Reeves and M. Clarke, "Logic for Computer Science," Addison-Wesley, 1990-**2003**. [R&C]
- ♦ A. Nerode and R. A. Shore, "Logic for Applications," Springer, **1993**, 1997 (2nd Edition). [N&S]
- ♦ M. Fitting, "First-Order Logic and Automated Theorem Proving," Springer, 1990, **1996** (2nd Edition). [Fitting]

3/23/21

16

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Reference Books on Philosophical Logic

- ♦ G. Priest, "An Introduction to Non-Classical Logic: From If to Is," Cambridge University Press, 2001, 2008 (2nd Edition).
- ♦ G. E. Hughes and M. J. Cresswell, "A New Introduction to Modal Logic," Routledge, 1968.
- ♦ P. Blackburn, M. de Rijke, and Y. Venema, "Modal Logic," Cambridge University Press, 2001.
- ♦ P. Ohlstrom and P. F. V. Hasle, "Temporal Logic: From Ancient Ideas to Artificial Intelligence," Kluwer Academic, 1995.
- ♦ J.-J. Ch. Meyer and W. van der Hoek, "Epistemic Logic for AI and Computer Science," Cambridge University Press, 1995.
- ♦ N. Rescher, "Epistemic Logic: A Survey of the Logic of Knowledge," University of Pittsburgh Press, 2005.
- ♦ H. van Ditmarsch, W. van der Hoek, and B. Kooi, "Dynamic Epistemic Logic," Springer, 2007.
- ♦ A. R. Anderson and N. D. Belnap Jr., "Entailment: The Logic of Relevance and Necessity," Vol. I, Princeton University Press, Princeton, 1975.
- ♦ A. R. Anderson, N. D. Belnap Jr., and J. M. Dunn, "Entailment: The Logic of Relevance and Necessity," Vol. II, Princeton University Press, Princeton, 1992.
- ♦ E. D. Mares, "Relevant Logic: A Philosophical Interpretation," Cambridge University Press, Cambridge, 2004.

3/23/21

17

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



Classical/Important Reference Books on Logic

- ♦ A. Tarski, "Introduction to Logic and to the Methodology of the Deductive Sciences," Oxford University Press, 1941, 1946, 1965, 1994 (4th Edition, Revised).
- ♦ W. Kneale and M. Kneale, "The Development of Logic," Clarendon Press, 1962, 1984 (Paperback Edition with Corrections).
- ♦ P. H. Nidditch, "The Development of Mathematical Logic," Thoemmes, 1962.
- ♦ H. DeLong, "A Profile of Mathematical Logic," Addison-Wesley, 1970, Dover Publications, 2004.
- ♦ J. N. Crossley, et al., "What is Mathematical Logic?" Oxford University Press, 1972, Dover Publications 1990.
- ♦ H. Wang, "Popular Lectures on Mathematical Logic," Van Nostrand Reinhold, 1981, Dover Publications, 1993.
- ♦ D. M. Gabbay (ed.), "What is a Logical System?" Oxford University Press, 1994.

3/23/21

18

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*



程研究室のスローガン (Saitama University, Japan)



3/23/21

19

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

Improve Your Position/Value Forever

- ♣ **Acknowledging your past honestly**
  - ◆ It is all over. No effort of yours can change what has passed.
- ♣ **The past does not tell you the future at all**
  - ◆ Self-abandonment will drag you to quagmire.
  - ◆ The polarization of students after entering the school is the characteristic of a school like SUSTech !!!
- ♣ **Your future is in your hand**
  - ◆ Life lies in evolution continually; Success lies in diligent correctly.
  - ◆ “Raise your quality standards as high as you can live with, ..... Do this, because it is the only way of discovering how that boundary should be moved forward.” -- E. W. Dijkstra

3/23/21

20

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

Dijkstra's Three Golden Rules for Successful Scientific Research

Raise your standards as high as you can live with, avoid wasting your time on routine problems, and always try to work as closely as possible at the boundary of your abilities. Do this because it is the only way of discovering how that boundary could be moved forward.

We all like our work to be socially relevant and intellectually sound. If we can find a topic satisfying both desires, we are lucky; if the two targets are in conflict with each other, let the requirement of scientific soundness prevail.

Never tackle a problem of which you can be pretty sure that (now or in the near future) it will be tackled by others who are, in relation to that problem, at least as competent and well-equipped as you are.

3/23/21

21

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

Dijkstra's Three Golden Rules for Successful Scientific Research

- ♣ **E. W. Dijkstra (The winner of 1972 ACM Turing Award)**
  - ◆ Edsger Wybe Dijkstra (Rotterdam, Netherlands, May 11, 1930 – Nuenen, Netherlands, August 6, 2002)
- ♣ **First, “internal” one**
  - ◆ “Raise your quality standards as high as you can live with, avoid wasting your time on routine problems, and always try to work as closely as possible at the boundary of your abilities. Do this, because it is the only way of discovering how that boundary should be moved forward.”
  - ◆ “It has nothing to do with your relation with others, it concerns you yourself in isolation.”

3/23/21

22

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

Dijkstra's Three Golden Rules for Successful Scientific Research

- ♣ **Second, “external” one**
  - ◆ “We all like our work to be socially relevant and scientifically sound. If we can find a topic satisfying both desires, we are lucky; if the two targets are in conflict with each other, let the requirement of scientific soundness prevail.”
  - ◆ “It deals with the relation between “the scientific world” and “the real world.”
- ♣ **Third, “internal/external” one**
  - ◆ “Never tackle a problem of which you can be pretty sure that (now or in the near future) it will be tackled by others who are, in relation to that problem, at least as competent and well-equipped as you.”
  - ◆ “It deals with the relation between you and your scientific colleagues.”

3/23/21

23

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

为学日益，为道日损

- ◆ 为学日益，为道日损。
  - 老子，道德经，第四十八章，约公元前600年。  
な ひび ま な ひび せん
- ◆ 学を為すは日に益す。道を為すは日に損ず。
  - 老子，道德经，第四十八章，纪元前600年頃。
- ◆ **To attain knowledge, add things every day.  
To attain wisdom, remove things every day.**
  - Lao Tzu, Tao Te Ching, ch. 48, about 600 B.C.

3/23/21

24

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

猎枪与干粮；授人以鱼不如授人以渔

♣ 何谓大学

◆ “所谓大学者，非谓有大楼之谓也，有大师之谓也。”  
— 梅贻琦，清华大学校长就任演说，1931

♣ 在大学里应该学习什么？

◆ “猎枪与干粮”  
— 蒋南翔，清华大学校长，1950s

◆ “授人以鱼不如授人以渔” — 中国古代谚语

◆ “Give a man a fish and you feed him for a day; teach a man to fish and you feed him for a lifetime.”


♣ 切不可“知其然不知其所以然”

◆ “知其然”不难，难在“知其所以然”。

◆ 如若仅知其然却不知其所以然，则不能成大事。

3/23/21

25

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

CS104/CS108: An Introduction to Mathematical Logic

♣ Teaching professor

◆ Jingde Cheng (程京德), chengjd@sustech.edu.cn

◆ HP: <http://www.aise.ics.saitama-u.ac.jp/~cheng/index.html>

◆ Blog: <http://blog.sciencenet.cn/u/JingdeCheng>  
(科学网博客： 计算机科学，数理逻辑，人工智能，等等)

◆ Use Google or 百度 to find more information about me.

♣ Teaching assistant teacher

◆ Yun Shen (沈昀), sheny@mail.sustech.edu.cn


◆ Answering class (optional): Will be announced.

♣ QQ group

◆ CS104-CS108-12ML: 659130328

3/23/21

26

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

An Introduction to Mathematical Logic

◆ Logic: What Is It and Why Study It?

◆ Formal Logic Systems and Formal Theories

◆ Classical Propositional Calculus (CPC)

◆ Classical First Order Predicate Calculus (CFOPC)


◆ Gödel’s Incompleteness Theorems

◆ Various Modal Logics (CS108 class only)

◆ Relevant Logics (CS108 class only)

3/23/21

27

\*\*\*\*\* Jingde Cheng / SUSTech \*\*\*\*\*

5