6.1

(a) p1

(b) ¬p1

(c) (p1 ∧ (¬p2)) ∨ (p2 ∧ (¬p1))

(d) p1 ∨ p2

(e) p1 ∨ (¬p2)

(f) ¬(p1 ∧ p2)

(g) (¬p1) → p2

(h) ¬(p1 → p2)

(i) ¬(p2 → p1)

(j) p1 ↔ (¬p2)

(k) (¬p1) ∧ (¬p2)

(l) (p1 ∧ p2) → p2

(m) (¬p1) → (p1 ∧ p2)

(n) p1 ∨ (p1 ∧ p2)

(o) (p1 ∧ p2) ∨ ((¬p1) ∧ p2)

6.2

(a) ¬A = A↑A, or

¬A = A↓A

(b) A ∨ B = (A ↑ A) ↑ (B ↑ B), or

A ∨ B = (A ↓ B) ↓ (A ↓ B)

(c) A ∧ B = (A ↓ A) ↓ (B ↓ B), or

$A \wedge B = (A \uparrow B) \uparrow (A \uparrow B)$

(d) $A \rightarrow B = ((A \uparrow A) \uparrow B) \uparrow ((A \uparrow A) \uparrow B)$, or

$A \rightarrow B = ((A \downarrow A) \downarrow B) \downarrow ((A \downarrow A) \downarrow B)$

6.3

(a)

A	∨	(¬A)
T	T	FT
F	T	TF

tautology

(b)

A	∧	(¬A)
T	F	FT
F	F	TF

contradiction

(c)

(¬A)	∨	B
FT	T	T
TF	T	T
TF	T	F
FT	F	F

contingency

(d)

(	A	∨	B	)	∧	(¬ (	A	∧	B	))
	T	T	T		F	F	T	T	T	
	T	T	F		T	T	T	F	F	
	F	T	T		T	T	F	F	T	
	F	F	F		F	T	F	F	F	

contingency

(e) ((¬ A) ∨ (¬ B)) ↔ (A ∧ B)

FT	T	FT	T	T	T	T
FT	T	TF	T	T	T	F
TF	F	FT	F	F	T	T
TF	T	TF	F	F	F	F

contingency

(f) A → (B ∨ (¬ C))

T	T	T	T	FT
T	T	T	T	TF
T	F	F	F	FT
T	T	F	T	TF
F	T	T	T	FT
F	T	T	T	TF
F	T	F	F	FT
F	T	F	T	TF

contingency

(g) ((A ∧ B) ∧ (C ∧ D)) → A

T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T
T	F	F	F	F	F	T	T	T
T	F	F	F	F	F	F	T	T
T	T	T	F	F	F	F	T	T
T	T	T	F	T	F	F	T	T
T	F	F	F	T	F	F	T	T
T	T	T	F	F	F	T	T	T
F	F	T	F	T	T	T	T	F
F	F	F	F	T	T	T	T	F
F	F	F	F	F	F	T	T	F
F	F	F	F	F	F	F	T	F
F	F	T	F	F	F	F	T	F
F	F	T	F	T	F	F	T	F
F	F	F	F	T	F	F	T	F
F	F	T	F	F	F	T	T	F

tautology

(h)	(A	↔	((¬ B)	∨	C))	→	((¬ A)	→	B)
	T	T	FT	T	T	T	FT	T	T
	T	F	FT	F	F	T	FT	T	T
	T	T	TF	T	T	T	FT	T	F
	T	T	TF	T	F	T	FT	T	F
	F	F	FT	T	T	T	TF	T	T
	F	T	FT	F	F	T	TF	T	T
	F	F	TF	T	T	T	TF	F	F
	F	F	TF	T	F	T	TF	F	F

tautology

6.4

(a) Tautology

$$((A \rightarrow C) \vee B \vee C) \rightarrow ((A \vee B) \rightarrow (D \vee E \vee \neg E))$$

A	C	B	D	E	$(A \Rightarrow C) \vee B \vee C \Rightarrow (A \vee B \Rightarrow D \vee E \vee \neg E)$
T	T	T	T	T	T
T	T	T	T	F	T
T	T	T	F	T	T
T	T	T	F	F	T
T	T	F	T	T	T
T	T	F	T	F	T
T	T	F	F	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	T	T	F	T
T	F	T	F	T	T
T	F	T	F	F	T
T	F	F	T	T	T
T	F	F	T	F	T
T	F	F	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	T	T	F	T
F	T	T	F	T	T
F	T	T	F	F	T
F	T	F	T	T	T
F	T	F	T	F	T
F	T	F	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	T	T	F	T
F	F	T	F	T	T
F	F	T	F	F	T
F	F	F	T	T	T
F	F	F	T	F	T
F	F	F	F	T	T
F	F	F	F	F	T

(b) Contingency

$$(B \vee (\neg B) \vee (A \rightarrow D) \vee (C \rightarrow E)) \rightarrow ((A \vee C) \rightarrow (D \vee E))$$

B	A	D	C	E	$B \vee \neg B \vee (A \Rightarrow D) \vee (C \Rightarrow E) \Rightarrow (A \vee C \Rightarrow D \vee E)$
T	T	T	T	T	T
T	T	T	T	F	T
T	T	T	F	T	T
T	T	T	F	F	T
T	T	F	T	T	T
T	T	F	T	F	F
T	T	F	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	T	T	F	T
T	F	T	F	T	T
T	F	T	F	F	T
T	F	F	T	T	T
T	F	F	T	F	F
T	F	F	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	T	T	F	T
F	T	T	F	T	T
F	T	T	F	F	T
F	T	F	T	T	T
F	T	F	T	F	F
F	T	F	F	T	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	T	T	F	T
F	F	T	F	T	T
F	F	T	F	F	T
F	F	F	T	T	T
F	F	F	T	F	F
F	F	F	F	T	T
F	F	F	F	F	T

(c) Contradiction

$$(D \vee A \vee (\neg D) \vee (\neg B) \vee (\neg A)) \rightarrow (E \wedge (\neg E) \wedge C \wedge (\neg C))$$

D	A	B	E	C	$D \vee A \vee \neg D \vee \neg B \vee \neg A \Rightarrow E \wedge \neg E \wedge C \wedge \neg C$
T	T	T	T	T	F
T	T	T	T	F	F
T	T	T	F	T	F
T	T	T	F	F	F
T	T	F	T	T	F
T	T	F	T	F	F
T	T	F	F	T	F
T	T	F	F	F	F
T	F	T	T	T	F
T	F	T	T	F	F
T	F	T	F	T	F
T	F	T	F	F	F
T	F	F	T	T	F
T	F	F	T	F	F
T	F	F	F	T	F
T	F	F	F	F	F
F	T	T	T	T	F
F	T	T	T	F	F
F	T	T	F	T	F
F	T	T	F	F	F
F	T	F	T	T	F
F	T	F	T	F	F
F	T	F	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	T	T	F	F
F	F	T	F	T	F
F	F	T	F	F	F
F	F	F	T	T	F
F	F	F	T	F	F
F	F	F	F	T	F
F	F	F	F	F	F

6.5

(a) { $A \rightarrow B, B \rightarrow C, \quad (C \vee D) \quad \leftrightarrow \quad (\neg B) \quad \}$

A	B	C	D	A→B	B→C	C∨D	¬B	(C∨D) ↔ (¬B)
T	T	T	T	T	T	T	F	F
T	T	T	F	T	T	T	F	F
T	T	F	F	T	F	F	F	T
T	F	F	F	F	T	F	T	F
T	T	F	T	T	F	T	F	F
T	F	F	T	F	T	T	T	T
T	F	T	T	F	T	T	T	T
T	F	T	F	F	T	T	T	T
F	T	T	T	T	T	T	F	F
F	T	T	F	T	T	T	F	F
F	T	F	F	T	F	T	F	F
F	F	F	F	T	T	F	T	F
F	T	F	T	T	F	T	F	F
F	F	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T

Satisfiable for three situations, no unsatisfiable. There exist three truth valuations making the formulas always true. (yellow background)

(b) { $\neg(\quad (\neg B) \quad \vee \quad A), A \vee \quad (\neg C) \quad , B \rightarrow \quad (\neg C) \quad \}$

A	B	C	¬B	¬C	(¬B)∨A	¬((¬B)∨A)	A∨(¬C)	B→(¬C)
T	T	T	F	F	T	F	T	F
T	T	F	F	T	T	F	T	T
T	F	F	T	T	T	F	T	T
T	F	T	T	F	T	F	T	T
F	T	T	F	F	F	T	F	F
F	T	F	F	T	F	T	T	T
F	F	F	T	T	T	F	T	T
F	F	T	T	F	T	F	F	T

Satisfiable.

(c) { D → B, A ∨ (¬B) , ¬(D ∧ A), D }

A	B	¬B	D	D → B	A∨(¬B)	¬(D∧A)
T	T	F	T	T	T	F
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	T	T	F	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	T	T	F	T	T

Unsatisfiable.

6.6

(a) { A→B , A } |=CPC B

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

Holds.

(b) { A→B , B } |=CPC A

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

Holds.

(c) { A→B , B→C } |=CPC A→C

A	B	C	A→B	B→C	A→C
T	T	T	T	T	T
T	T	F	T	F	F
T	F	F	F	T	F
T	F	T	F	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T
F	F	T	T	T	T

Holds.

(d) { A→B , A→(¬B) } |=CPC ¬A

A	B	¬B	¬A	A→B	A→(¬B)
T	T	F	F	T	F
T	F	T	F	F	T
F	T	F	T	T	T
F	F	T	T	T	T

Holds.

(e) A→B |=CPC ¬(B→A)

A	B	B→A	A→B	¬(B→A)
T	T	T	T	F
T	F	T	F	F
F	T	F	T	T
F	F	T	T	F

Does not hold.

(f) { A∨B , B∨C } |=CPC A∨C

A	B	C	A∨B	B∨C	A∨C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	F	T	F	T

T	F	T	T	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	F	F	F	F
F	F	T	F	T	T

Holds.

(g) $\{ A , \neg A \} \models_{CPC} B$

A	¬A	B
T	F	T
T	F	F
F	T	T
F	T	F

Holds.

(h) $C \models_{CPC} A \leftrightarrow (A \vee (A \wedge B))$

A	B	C	A∧B	A ∨ (A∧B)	A ↔ (A ∨ (A∧B))
T	T	T	T	T	T
T	T	F	T	T	T
T	F	F	F	T	T
T	F	T	F	T	T

Holds.

6.7

(1)

$$((A \vee B) \wedge C) \vee ((\neg C) \wedge B \wedge D \wedge (\neg D)) \models_{CPC} A$$

A	B	C	D	$((A \vee B) \wedge C) \vee (\neg\, C \wedge B \wedge D \wedge \neg\, D)$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	F
T	T	F	F	F
T	F	T	T	T
T	F	T	F	T
T	F	F	T	F
T	F	F	F	F
F	T	T	T	T
F	T	T	F	T
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

Doesn’t hold.

(2)

$$(D \vee A \vee (\neg D) \vee (\neg B) \vee (\neg A)) \rightarrow (E \wedge (\neg E) \wedge C \wedge (\neg C))$$

$$|\!=\!\text{CPC } D$$

D	A	B	E	C	$D \vee A \vee \neg D \vee \neg B \vee \neg A \Rightarrow E \wedge \neg E \wedge C \wedge \neg C$
T	T	T	T	T	F
T	T	T	T	F	F
T	T	T	F	T	F
T	T	T	F	F	F
T	T	F	T	T	F
T	T	F	T	F	F
T	T	F	F	T	F
T	T	F	F	F	F
T	F	T	T	T	F
T	F	T	T	F	F
T	F	T	F	T	F
T	F	T	F	F	F
T	F	F	T	T	F
T	F	F	T	F	F
T	F	F	F	T	F
T	F	F	F	F	F
F	T	T	T	T	F
F	T	T	T	F	F
F	T	T	F	T	F
F	T	T	F	F	F
F	T	F	T	T	F
F	T	F	T	F	F
F	T	F	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	T	T	F	F
F	F	T	F	T	F
F	F	T	F	F	F
F	F	F	T	T	F
F	F	F	T	F	F
F	F	F	F	T	F
F	F	F	F	F	F

holds