10.1 Pick out the free and bound occurrences of variables in the following formulas.

(a) $(\forall x)\ P(x) \wedge \neg P(y)$

Free: the occurrence of y in the latter $\neg P(y)$

Bound: the occurrence of x in the former $P(x)$

(b) $(\forall y)\ P(y) \leftrightarrow P(y)$

Free: the occurrence of y in the latter $P(y)$

Bound: the occurrence of y in the former $P(y)$

(c) $(\forall y)\ (\exists x)\ Q(x, y)$

Free: null

Bound: the occurrence of x and y in $Q(x, y)$

(d) $(\forall x)(\forall u)(\forall v)P(x) \rightarrow (P(y) \wedge \neg P(x))$

Free: the occurrence of x in the latter $\neg P(x)$; the occurrence of y in the latter $P(y)$

Bound: the occurrence of x in the former $P(x)$

(e) $(\forall x)(\forall y)(\forall z)P(x) \vee (\exists y)(\neg(\forall z)Q(z, y))$

Free: null

Bound: the occurrence of x in the former P(x); the occurrence of y and z in the latter Q(z, y)

(f) $(\forall x)(\neg P(x)) \rightarrow T(x, x, y) \vee (\forall x)P(x)$

Free: the occurrence of x and y in the latter T(x, x, y)

Bound: the occurrence of x in the former ¬P(x); the occurrence of x in the latter P(x)

(g) $\neg(\forall x)P(x) \rightarrow (\exists y)P(y) \rightarrow Q(x, y) \wedge P(y)$

Free: the occurrence of x in the latter Q(x, y); the occurrence of y in the latter $Q(x, y) \wedge P(y)$

Bound: the occurrence of y in the middle P(y); the occurrence of x in the former P(x)

(h) $(((\forall x)(P(x) \rightarrow P(x))) \vee ((\exists x)P(x)))$

Free: null

Bound: the occurrence of x in $P(x) \rightarrow P(x)$ and P(x)

(i) ((¬((∃y)(P(y) ∨ P(a))) ↔ P(y))

Free: the occurrence of a in P(a); the occurrence of y in the latter P(y)

Bound: the occurrence of y in the former P(y)

(j) (((∀x)(¬(¬P(a))) → (P(x) → P(y)))

Free: the occurrence of a in the former ¬(¬P(a)); the occurrence of x in the latter P(x); the occurrence of y in the latter P(y)

Bound: the occurrence of x in the former ¬(¬P(a))

(k) (∀z)(((∀x) Q(x, y)) → Q(z, a))

Free: the occurrence of y in Q(x, y); the occurrence of a in Q(z, a)

Bound: the occurrence of x in Q(x, y); the occurrence of z in Q(z, a)

(l) (∀y)Q(z, y) → (∀z)Q(z, y)

Free: the occurrence of z in the former Q(z, y); the occurrence of y in the latter Q(z, y)

Bound: the occurrence of y in the former Q(z, y); the occurrence of z in the latter Q(z, y)

(m) ((∀y)(∃x)T(x, y, g(x, y))) ∨ ¬(∀x)Q(y, f(x))

Free: the occurrence of y in Q(y, f(x))

Bound: the occurrence of x and y in T(x, y, g(x, y)); the occurrence of x in Q(y, f(x))

10.2 Indicate the free and bound variables in the formulas of Exercise 10.1

(a) (∀x) P(x) ∧ ¬P(y)

Free: y in the latter ¬P(y)

Bound: x in the former P(x)

(b) (∀y) P(y) ↔ P(y)

Free: y in the latter P(y)

Bound: y in the former P(y)

(c) (∀y) (∃x) Q(x, y)

Free: null

Bound: x and y in Q(x, y)

(d) $(\forall x)(\forall u)(\forall v)P(x) \rightarrow (P(y) \wedge \neg P(x))$

Free: x in the latter $\neg P(x)$; y in the latter $P(y)$

Bound: x in the former $P(x)$

(e) $(\forall x)(\forall y)(\forall z)P(x) \vee (\exists y)(\neg(\forall z)Q(z, y))$

Free: null

Bound: x in the former $P(x)$; y and z in the latter $Q(z, y)$

(f) $(\forall x)(\neg P(x)) \rightarrow T(x, x, y) \vee (\forall x)P(x)$

Free: x and y in the latter $T(x, x, y)$

Bound: x in the former $\neg P(x)$; x in the latter $P(x)$

(g) $\neg(\forall x)P(x) \rightarrow (\exists y)P(y) \rightarrow Q(x, y) \wedge P(y)$

Free: x in the latter $Q(x, y)$; y in the latter $Q(x, y) \wedge P(y)$

Bound: y in the middle $P(y)$; x in the former $P(x)$

(h) $(((\forall x)(P(x) \rightarrow P(x))) \vee ((\exists x)P(x)))$

Free: null

Bound: x in P(x) → P(x) and P(x)

(i) ((¬((∃y)(P(y) ∨ P(a)))) ↔ P(y))

Free: a in P(a); y in the latter P(y)

Bound: y in the former P(y)

(j) (((∀x)(¬(¬P(a)))) → (P(x) → P(y)))

Free: a in the former ¬(¬P(a)); x in the latter P(x); y in the latter P(y)

Bound: x in the former ¬(¬P(a))

(k) (∀z)(((∀x) Q(x, y)) → Q(z, a))

Free: y in Q(x, y); a in Q(z, a)

Bound: x in Q(x, y); z in Q(z, a)

(l) (∀y)Q(z, y) → (∀z)Q(z, y)

Free: z in the former Q(z, y); y in the latter Q(z, y)

Bound: y in the former Q(z, y); z in the latter Q(z, y)

(m) $((\forall y)(\exists x)T(x, y, g(x, y))) \vee \neg(\forall x)Q(y, f(x))$

Free: y in $Q(y, f(x))$

Bound: x and y in $T(x, y, g(x, y))$; x in $Q(y, f(x))$