Logic: What Is It and Why Study It?

- Logic: What Is It and Why Study It?
- Reasoning, Proving, Discovery, and Prediction
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The Notion of a Conditional: What Is It and Why Study It ?

The Notion of a Conditional: What Is It and Why Study It ?

Object Logic/Language and Meta-logic/language

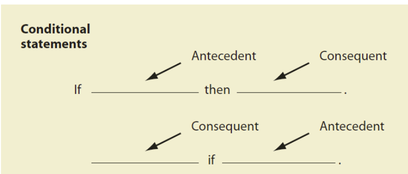
- **Object logic**
  - The logic we are studying.
- **Object language**
  - The formal language of the object logic.
- **Meta-logic (Observer's logic)**
  - The logic we are using to study the object logic.
- **Meta-language (Observer's language)**
  - The language we are using to study the object logic.
- **Note**
  - Maybe you have used Chinese (meta-language) to learn English (object language), do not you?

The Notion of a Conditional

- **Definitions of 'conditional' in dictionary** [The OED, 2nd Edition]
  - "A word or clause expressing a condition."
  - "A conditional conjunction; the conditional mood of the verb."
  - "A conditional proposition or syllogism."

The Notion of a Conditional

- **The Notion of a Conditional**
  - A statement of the form '*IF ... THEN ...*' is usually called a *conditional proposition* or simply *conditional* which states that there exists a relationship of *sufficient condition* between the 'IF' part (called the *antecedent*) and the 'THEN' part (called the *consequent*) of the statement.
  - '*... IF ... THEN ...*' , '*... IF ...*'



The Roles of Conditionals

- **The roles of conditional in various scientific disciplines**
  - Mathematical, natural, and social scientists always use conditionals in their descriptions of various definitions, propositions, theorems, and laws to connect a concept, fact, situation or conclusion and its sufficient conditions.
  - Conditionals are objects of scientific discovery.
- **The roles of conditional in our daily life**
  - The notion of a conditional plays the most essential role in human logical thinking because any reasoning must invoke it.
- **The conditional in deduction, induction, and abduction**
  - *If* A *then* B, A, therefore B.
  - *If* A_1 is a B, A_2 is a B, …, A_n is a B, *then* maybe all A are B.
  - *If* A *then* C, C, therefore maybe A.

The Notion of a Conditional as the Heart of Logic

♣ The conditional as the most important subject studied in logic
- ◆ There is no reasoning that does not invoke the notion of conditional.
- ◆ The notion of a conditional was discussed by the Greek ancients, and it is historically always the most important subject studied in logic.
- ◆ Any logical consequence relation is nothing other than a conditional.

♣ The conditional as the heart of logic
- ◆ Relevant logicians' claim: "We take the heart of logic to lie in the notion 'if … then …'." [Anderson and Belnap, 1975]

The Conditional as the Heart of Mathematics

♣ The conditional as the heart of mathematics
- ◆ The notion of a conditional is also the heart of mathematics, because there is no branch of mathematics which does not use logic as the most general correct criterion for reasoning and proving.

♣ The conditional and pure mathematics
[B. Russell, "The Principles of Mathematics," 1903]
- ◆ "Pure Mathematics is the class of all propositions of the form 'p implies q,' where p and q are propositions *containing one or more variables, the same in the two propositions*, and neither p nor q contains any constants except logical constants. And logical constants are all notions definable in terms of the following: Implication, the relation of a term to a class of which it is a member, the notion of such that, the notion of relation, and such further notions as may be involved in the general notion of propositions of the above form. In addition to these, mathematics uses a notion which is not a constituent of the propositions which it considers, namely the notion of truth."

The Conditional in Object Logic and Meta-logic

♣ The conditional in the object logic
- ◆ In the object language, there usually is a *logic connective* to represent the notion of conditional.
- ◆ Both the *logical consequence relation* in the proof theory (usually denoted by '|-') and the *logical consequence relation* in the model theory (usually denoted by '|=') can be considered as a representation of the notion of conditional, respectively.

♣ The conditional in the meta-logic
- ◆ The notion of a conditional, usually in the form of natural language, is used for defining various meta-notions and describe various meta-theorems about the object logic.

Truth of a Conditional

♣ Truth of a conditional
- ◆ In general, the truth of a conditional depends not only on the truths of its antecedent and consequent but also more essentially on a *necessarily relevant and conditional relation* between its antecedent and consequent.

♣ Notes
- ◆ The truth of the consequent (or the falsity of the antecedent) of a conditional is by itself insufficient for the truth of that conditional.
- ◆ Discussion on the notion of conditional is still continuous in modern philosophy and logic.
- ◆ How to define the truth of a conditional (and hence *entailment*) is always the most fundamental issue in logic.

Truth of a Conditional: Examples

♣ Examples (relevant conditionals)
- ◆ If ShenYang is in LiaoNing, then ShenYang is in DongBei.
- ◆ If ShenYang is in JiLin, then ShenYang is in DongBei.
- ◆ If ShenYang is in LiaoNing, then ShenYang is in Asia.
- ◆ If ShenYang is in JiLin (China), then ShenYang is in Asia.

♣ Examples (irrelevant conditionals)
- ◆ If today is Friday, then ShenYang is in DongBei.
- ◆ If today is Sunday, then ShenYang is in DongBei.
- ◆ If today is Friday, then ShenYang is in Asia.
- ◆ If today is Sunday, then ShenYang is in Asia.

Truth of a Conditional: Examples

♣ Examples (relevant conditionals)
- ◆ If ShenZhen is in GuangDong, then ShenZhen is in HuaNan.
- ◆ If ShenZhen is in FuJian, then ShenZhen is in HuaNan.
- ◆ If ShenZhen is in GuangDong, then ShenZhen is in Asia.
- ◆ If ShenZhen is in FuJian (China), then ShenZhen is in Asia.

♣ Examples (irrelevant conditionals)
- ◆ If today is Friday, then ShenZhen is in HuaNan.
- ◆ If today is Sunday, then ShenZhen is in HuaNan.
- ◆ If today is Friday, then ShenZhen is in Asia.
- ◆ If today is Sunday, then ShenZhen is in Asia.

Conditionals as Arguments

- **A conditional as an argument**
  - For a conditional, taking all conjuncts of its antecedent as the premises, and its consequent as the conclusion, the conditional can be regarded as an argument.
- **Examples**
  - (A∧(A ⇒ B)) ⇒ B: A and A ⇒ B, therefore, B.
  - (A∧¬A) ⇒ B: A and ¬A, therefore, B.

Empirical and Logical Conditionals

- **Empirical conditionals**
  - For a logic, a conditional is called an ***empirical conditional*** of the logic if its truth-value, in the sense of that logic, depends on the contents of its antecedent and consequent, and therefore, it cannot be determined only by its abstract form.
  - From the viewpoint of that logic, the necessarily relevant and conditional relation between the antecedent and the consequent of an empirical conditional is considered to be ***empirical*** but not logical.
- **Examples**
  - A ⇒ B
  - (A∧B) ⇒ C

Empirical and Logical Conditionals

- **Logical conditionals**
  - For a logic, a conditional is called a ***logical conditional*** of the logic if its truth-value, in the sense of that logic, depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true or false.
  - From the viewpoint of that logic, the necessarily relevant and conditional relation between the antecedent and the consequent of that conditional is considered to be ***logical*** but not empirical.
- **Examples**
  - (A∧(A ⇒ B)) ⇒ B
  - (A∧¬A) ⇒ B
  - A ⇒ (A∧B)

Entailments: Logically Valid Conditionals

- **Valid conditionals**
  - For a logic, a logical conditional is called a ***valid conditional*** (or ***entailment***) of the logic if its truth, in the sense of that logic, is considered to be universally true.
- **Invalid conditionals**
  - For a logic, a logical conditional is called an ***invalid conditional*** of the logic if its truth, in the sense of that logic, is considered to be universally false.
- **Intrinsic difference between different logic systems**
  - The most intrinsic difference between different logics is to consider what class of conditionals as entailments.
- **The most important problem in modern logic**
  - "The problem in modern logic can best be put as follows: can we give an explanation of those conditionals that represent an entailment relation?" [Diaz, 1981]

The Notion of Conditional (Entailment) in CML

- **The notion of material implication [Philo of Megara, 400 B.C.]**
  - The notion of conditional is represented in CML by the extensional notion of ***material implication*** (denoted by '→' or '⊃') which is defined as an extensional truth-functional connective as follows:

$$A \rightarrow B =_{df} \neg(A \wedge \neg B)\ ,\quad A \rightarrow B =_{df} \neg A \vee B$$

- **Notes**
  - The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication (i.e., NOT in the sense of conditional).
  - '$\neg(A \wedge \neg B)$' and/or '$\neg A \vee B$' is necessary but not sufficient to a conditional A⇒B, because the relevance between A and B is not accounted.

The Notion of Conditional (Entailment) in CML

- **Proof-theoretical deduction theorems in CML**
  - $\Gamma \cup \{A\} \vdash_{CML} B$ IFF $\Gamma \vdash_{CML} A \rightarrow B$
  - $\{A\} \vdash_{CML} B$ IFF $\vdash_{CML} A \rightarrow B$
  - $\Gamma \cup \{A_1, ..., A_n\} \vdash_{CML} B$ IFF $\Gamma \vdash_{CML} A_1 \rightarrow (\ldots(A_n \rightarrow B)...)$
  - $\Gamma \cup \{A_1, ..., A_n\} \vdash_{CML} B$ IFF $\Gamma \vdash_{CML} (A_1 \wedge \ldots \wedge A_n) \rightarrow B$
- **Model-theoretical deduction theorems in CML**
  - $\Gamma \cup \{A\} \models_{CML} B$ IFF $\Gamma \models_{CML} A \rightarrow B$
  - $\{A\} \models_{CML} B$ IFF $\models_{CML} A \rightarrow B$
  - $\Gamma \cup \{A_1, ..., A_n\} \models_{CML} B$ IFF $\Gamma \models_{CML} A_1 \rightarrow (\ldots(A_n \rightarrow B)...)$
  - $\Gamma \cup \{A_1, ..., A_n\} \models_{CML} B$ IFF $\Gamma \models_{CML} (A_1 \wedge \ldots \wedge A_n) \rightarrow B$
- **Note**
  - The notion of material implication is "equivalent" to the logical consequence relation '$\vdash_{CML}$' and/or '$\models_{CML}$'.

Comparison of Conditional and Material Implication

♣ The notion of conditional

- The notion of conditional is intrinsically intensional but not truth-functional.
- The notion of conditional requires that there is a necessarily relevant and conditional relation between its antecedent and consequent.
- The truth of a conditional depends not only on the truth of its antecedent and consequent but also, and more essentially, on a necessarily relevant and conditional relation between them.
- The truth of the consequent (or the falsity of the antecedent) of a conditional is by itself insufficient for the truth of that conditional.

Comparison of Conditional and Material Implication

♣ The notion of material implication

- The notion of material implication is no more than an extensional truth-function of its antecedent and consequent, and therefore, the truth of a material implication depends totally on the truth of its antecedent and consequent.
- The notion of material implication does not require that there is a necessarily relevant and conditional relation between its antecedent and consequent.
- The truth-value of a material implication depends only on the truth-values of its antecedent and consequent, without regard to any relevance between them.
- The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication.

Comparison of Conditional and Material Implication

antecedent A	consequent B	necessarily relevant relation between A and B	conditional "if A then B" $A \Rightarrow B$	material implication "A implies B" $A \rightarrow B$
T	*T*	Existence	*T*	*T*
T	*T*	Not existence	*F*	*T*
T	*F*	Existence	*F*	*F*
T	*F*	Not existence	*F*	*F*
F	*T*	Existence	*T*	*T*
F	*T*	Not existence	*F*	*T*
F	*F*	Existence	*T*	*T*
F	*F*	Not existence	*F*	*T*

Paradoxes of Material Implication in CML

♣ The problem of implicational paradox

- If one considers the material implication as the notion of conditional and considers every logical theorem of CML as a valid reasoning form or entailment, then a great number of logical theorems of CML present some paradoxical properties and therefore they have been referred to in the literature as "*implicational paradoxes*."

♣ Note

- "If one considers …… and considers ……, then …… ."
- It is to think of the notion of material implication as the notion of conditional, or in other words, it is to use material implication in the sense of conditional, that leads to the problem of implicational paradoxes.
- If we use a material implication as an extensional truth-value function (and hence not conditional) in the sense of its original definition in CML, then no problem occurs.

Paradoxes of Material Implication in CML

♣ Paradoxes of material implication as empirical conditionals

snow is white → 1 + 1 = 2
snow is black → 1 + 1 = 2
snow is black → 1 + 1 = 3

♣ Paradoxes of material implication as entailments

- $A\rightarrow(B\rightarrow A)$, $B\rightarrow(\neg A\vee A)$, $\neg A\rightarrow(A\rightarrow B)$, $(\neg A\wedge A)\rightarrow B$ (ECQ!), $(A\rightarrow B)\vee(\neg A\rightarrow B)$, $(A\rightarrow B)\vee(A\rightarrow\neg B)$, $(A\rightarrow B)\vee(B\rightarrow A)$, $((A\wedge B)\rightarrow C)\rightarrow((A\rightarrow C)\vee(B\rightarrow C))$
- $B\rightarrow(\neg A\vee A)$, $(\neg A\wedge A)\rightarrow B$, Nothing is shared by antecedent and consequent, i.e., they are each other NOT relevant at all!
- Do you think that from 'if *A* and *B* then *C*' you can say 'if *A* then *C* or if *B* then *C*'?

Implicational Paradoxes: Problems and Results

♣ Necessary but not sufficient to the notion of conditional

- '$\neg(A\wedge\neg B)$' or '$\neg A\vee B$' (definitions of material implication '$A\rightarrow B$') is necessary but not sufficient to the notion of conditional '$A\Rightarrow B$' because the relevance between A and B, another necessary condition required by conditional, is not accounted.

♣ Result

- The notion of material implication cannot be used for distinguishing conditionals from implicational statements.
- Those paradoxes of material implication cannot be regarded conditionals.

Implicational Paradoxes: Problems and Results

- **CML cannot underlie relevant and truth-preserving in the sense of conditional**
  - In the framework of CML (or any of its classical conservative extensions), even if a reasoning is classically valid/sound, both the relevance relationship between its premises and conclusion and the truth of its conclusion in the sense of conditional cannot be guaranteed necessarily.
- **Reason: material implication and implicational paradoxes**
  - We cannot directly accept a conclusion of a reasoning with implicational paradoxes as a relevant and true conclusion in the sense of conditional, even if all premises of the sound reasoning are true and the conclusion is true in the sense of material implication.

Implicational Paradoxes: Problems and Results

- **Examples**
  - From A, we can infer $B\rightarrow A$, $C\rightarrow A$, ... where B, C, ... are arbitrary formulas, by using logical axiom $A\rightarrow(B\rightarrow A)$ of CML and Modus Ponens for material implication (from A and $A\rightarrow B$ to infer B).
  - However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be considered to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B and A, C and A, …, and therefore we cannot say 'if B then A', 'if C then A', and so on.

Implicational Paradoxes: Problems and Results

- **CML cannot underlie ampliative reasoning**
  - Any reasoning based on CML (or any of its classical conservative extensions) is circular and/or tautological but not ampliative.
- **Reason: material implication is an extensional truth-function**
  - Since any material implication is an extensional truth-function of its antecedent and consequent, the truth of a material implication depends totally on the truth of its antecedent and consequent, i.e., one cannot determine the truth of a material implication without knowing truths of its antecedent and consequent.
  - On the other hand, the truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication. However, when we reason, we do not know the truth of the consequent and do not use false antecedent.

Implicational Paradoxes: Problems and Results

- **Examples (reasoning by Modus Ponens for material implication)**
  - Modus Ponens: If A holds then B holds, now A holds, therefore B holds.
  - Before the reasoning is performed, we do not know whether B holds or not. (If we do, we do not need reasoning at all.)
  - Modus Ponens in CML: From A and $A\rightarrow B$ to infer B.
  - According to the extensional truth-functional semantics of the material implication, if we know 'A is true' but do not know the truth-value of B, then we cannot decide the truth-value of '$A\rightarrow B$'.
  - In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is performed!

Implicational Paradoxes: Problems and Results

- **Paraconsistent logic (NOT allow ECQ)**
  - For a paraconsistent logic with Modus Ponens as an inference rule, the paraconsistence requires that the logic does not have $(\neg A\wedge A)\Rightarrow B$ as a logical theorem where A and B are any two different formulas and '$\Rightarrow$' is the notion of conditional used in Modus Ponens.
  - If a logic is not paraconsistent, then infinite propositions (even negations of those logical theorems of the logic) may be reasoned out based on the logic from a set of premises that directly or indirectly include a contradiction.
- **Paraconsistent reasoning based on CML is impossible**
  - CML (or any of its classical conservative extensions) is explosive but not paraconsistent.
  - CML uses Modus Ponens for material implication as its inference rule, and has "$(\neg A\wedge A)\rightarrow B$" as a logical theorem.

The Notion of Conditional in Lewis's Modal Logics

- **Lewis's work on modal logic**
  - The main aim of Lewis's work beginning in 1912 on the establishment of modern modal logic was to find a satisfactory theory of implication which is better than CML in that it can avoid those implicational paradoxes.

The Notion of Conditional in Lewis's Modal Logics

♣ Lewis's strict implication

- Lewis's idea is to define the notion of conditional by using necessity operator.
- $A \rightarrow 3\ B =_{df} L(A \rightarrow B)$ where L is necessity operator

♣ Paradoxes of strict implication

- Lewis's plan was not successful in the sense that some implicational paradoxes in terms of strict implication remained in his modal logics (S1 ~ S5).
- Ex.

$LA \rightarrow (B \rightarrow 3A)$	$[\ A \rightarrow (B \rightarrow A)\ ]$
$L\neg A \rightarrow (A \rightarrow 3B)$	$[\ \neg A \rightarrow (A \rightarrow B)\ ]$
$(\neg A \wedge A) \rightarrow 3B$	$[\ (\neg A \wedge A) \rightarrow B\ ]$
$B \rightarrow 3(\neg A \vee A)$	$[\ B \rightarrow (\neg A \vee A)\ ]$

Sugihara's Characterization of Implication Paradoxes

♣ Strongest and weakest formulas [Sugihara, 1955]

- Relative to a given connective, →, intended as implication, a formula A is said to be *strongest* if one can prove $A \rightarrow B$ for every formula B, and a formula A is said to be *weakest* if $B \rightarrow A$ is provable for all B.

♣ Paradoxical logic systems

- A logic system is *paradoxical in the sense of Sugihara* just in case it has either a weakest or a strongest formula.
- The existence of a proposition that is implied by all or implies all others does not fit the concept of implication as a logical connection between two propositions.

♣ CML is paradoxical in the sense of Sugihara

- CML (and Lewis's modal logics S1 ~ S5) has $\neg A \wedge A$ as strongest formula and $\neg A \vee A$ as weakest formula, and therefore, it is paradoxical.

Ackermann's Rigorous Implication

♣ Rigorous implication [Ackermann, 1956]

- "*Rigorous implication*, which we write as $A \rightarrow B$, should express the fact that a logical connection holds between A and B, that the content of B is part of that of A. … That has nothing to do with the truth of falsity of A or B. Thus one would *reject* the validity of the formula $A \rightarrow (B \rightarrow A)$, since it permits the inference from A of $B \rightarrow A$, and since the truth of A has nothing to do with whether a logical connection holds between B and A."

♣ Logical connection and relevance

- For an entailment (argument) to be valid there should be some connection of meaning, i.e. some relevance, between its premises and its conclusion, among other things.

The von Wright-Geach-Smiley Criterion for Entailment

- "A entails B, if and only if, by means of logic, it is possible to come to know the truth of $A \rightarrow B$ without coming to know the falsehood of A or the truth of B" [von Wright, 1957]
- "A entails B if and only if there is an a priori way of getting to know that $A \rightarrow B$ which is not a way of getting to know whether A or whether B" [Geach, 1958]
- "$A_1 \&\ \dots\ \& A_n \rightarrow B$ should not only be itself a tautology, but should also be a substitution instance of some more general implication $A_1' \&\ \dots\ \& A_n' \rightarrow B'$, where neither B' nor $\neg(A_1' \&\ \dots\ \& A_n')$ are themselves tautologies" [Smiley, 1959]
- However, it is hard until now to know exactly how to formally interpret such epistemological phrases as 'coming to know' and 'getting to know' in the context of logic.

Traditional Relevant (Relevance) Logics

♣ Motivation of the traditional relevant (or relevance) logics

- *Relevant (relevance) logics* ware constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called 'paradoxes' of material and strict implication.

♣ Paradoxes of material and strict implication

- $A \rightarrow (B \rightarrow A)$, $B \rightarrow (\neg A \vee A)$, $\neg A \rightarrow (A \rightarrow B)$, $(\neg A \wedge A) \rightarrow B$, $(A \rightarrow B) \vee (\neg A \rightarrow B)$, $(A \rightarrow B) \vee (A \rightarrow \neg B)$, $(A \rightarrow B) \vee (B \rightarrow A)$, $((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
- $(\neg A \wedge A) \rightarrow 3B$, $B \rightarrow 3(\neg A \vee A)$

Traditional Relevant (Relevance) Logics

♣ Well-known main relevant logics

- system Π' of *rigorous implication* [Ackermann, 1956]
- system E of *entailment* [Anderson and Belnap, 1958]
- system R of *relevant implication* [Belnap, 1967]
- system T of *ticket entailment* (*entailment shorn of modality*) [Anderson, 1960]

♣ Characteristic features of the relevant logics

- A primitive intensional connective to represent the notion of conditional (entailment)
- Variable-sharing and the relevance principle
- Free from the paradoxes of material and strict implication
- Relevant (in the sense of weak relevance!) reasoning
- Ampliative reasoning

The Notion of Conditional (Entailment) in RL

♣ **Conditional as a primitive intensional connective**

- RL has a primitive intensional connective (relevant implication, entailment, ……) to represent the notion of conditional (entailment).
- In RL, the principle connective to represent the notion of conditional is the primitive intensional connective but not material implication.

♣ **RL includes CML certainly**

- The notion of material implication can be defined in RL as an extensional truth-function of its antecedent and consequent in the same way as that in CML.
- Both the language and the logical theorems of RL are a conservative extension of that of CML.

Variable-Sharing and the Relevance Principle

♣ **Variable-sharing as a necessary condition for relevance**

- The antecedent and consequent of an entailment should share some propositional variable(s).
- Variable-sharing is a necessary, but by no means sufficient, formal notion designed to reflect the idea that there be a meaning connection, i.e., relevance, between the antecedent and consequent of an entailment.

♣ **The relevance principle** [A&B-E1-1975]

- The relevance principle: If $A \Rightarrow B$, where $\Rightarrow$ denotes the notion of entailment, is a logical theorem of a relevant logic, for any two propositional formulas A and B, then A and B share at least one propositional variable.

♣ **The relevance principle and implicational paradoxes**

- As a result of the relevance principle, all relevant logics are free from the paradoxes of material and strict implication.

Relationship between RL and CML

♣ **ZDF theorem [Anderson and Belnap, 1959]**

- The zero degree (i.e., includes no entailment or relevant implication) formulas provable in E (R, T) are precisely the theorems of CML.

♣ **CML is exactly the extensional fragment of E (R, T)**

- CML is exactly the extensional fragment of E, R, or T in the sense that if one define the material implication as $A \rightarrow B =_{df} \neg(A \wedge \neg B)$ or $A \rightarrow B =_{df} \neg A \vee B$ then all tautologies of CML are theorems of E, R, or T.
- RL is a conservative extension of CML.

Relationship between RL and CML

♣ **Entailments of E (R, T) is a "proper subsystem" of CML**

- The theoremhood of E (R, T) can be regarded as a "proper subset" of the theoremhood of CML, in the sense that if all entailment connectives in a theorem of E (R, T) are replaced by material implication connectives then the resultant formula must be a theorem of CML, and E (R, T) rejects some theorems (like implicational paradoxes) of CML.

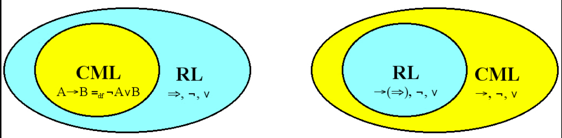
Relationship between RL and CML

The fact:

RL is a conservative extension of CML

If replacing "⇒" by "→":

Entailments of E (R, T) is a "proper subsystem" of CML



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Logic: What Is It All About ?

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Logical Validity of Reasoning and Logic

- Logical validity as the only general criterion any reasoning must act according to
  - Since a reasoning has no previously explicitly defined target, the only general criterion it must act according to is to reason correct new conclusions from its premises, i.e., the logical validity criterion.
  - It is logic that can underlie valid reasoning generally.
- What is the right logic(s) ?
  - Right for what?
  - Is there 'the only one logic (The One True Logic)' right for any reasoning, or 'various logics' right for various types of reasoning?
  - From the viewpoint of application, practices are the only criterion for testing any theory and truth !!!

How to Define Logical Validity?

- Argument form
  - An *argument form* is an abstract pattern which common to many different concrete arguments.
- Examples
  - (1) All H are m.
    (2) S is H.
    Therefore, (3) S is m.
  - (1) If P, then Q.
    (2) P.
    Therefore, (3) Q.
- Form vs. content ("Formal" logic)
  - Logic deals with forms rather than contents of arguments.
  - Logics define logical validity criteria by investigating abstract argument forms rather than concrete arguments.

Logic: What Is It all About?

- The notion of logical consequence relation
  - The most essential and central notion in logic is the *logical consequence relation* that relates a given set of premises to those conclusions, which *follow validly from* the premises.
  - To define a logical consequence relation is nothing else but to provide a logical validity criterion by which one can decide whether the conclusion of an argument or a reasoning really does follow from its premises or not.
- Various logic systems
  - Any science is established based on some fundamental principles and assumptions.
  - Different philosophical motivations on fundamental principles and assumptions lead to different logical validity criteria and different logic systems.

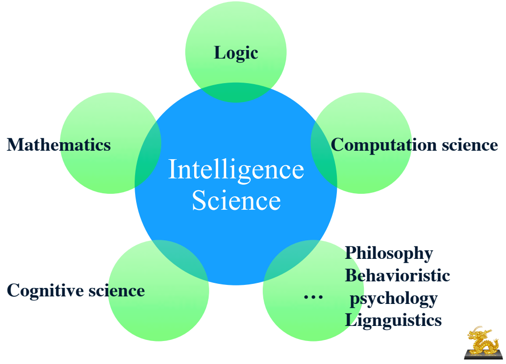
Logic and Sciences

- Logic is the fundamentals of all sciences
  - Logic is a science prior to all others, which contains the ideas and principles underlying all sciences.
  - No science is possible if it does not invoke logic as the fundamental validity criterion for scientific reasoning, scientific discovery, and scientific prediction.
- Logic is the most indispensable methodology/tool
  - Logic is the most indispensable methodology/tool for scientific reasoning as well as our everyday reasoning.
  - No scientist can work well if he/she does not reason new knowledge from known knowledge.
  - No body can life happy if he/she does not reason new fact from known facts.

Logic and Computation Science

- Logic was the mathematical root of Computation Science
  - It was the studies in logic that arose ideas concerning the notion of "effectively computable" and theoretical models of computation.
  - The theory of computation was born in logic.
- Logic is one of the fundamentals of Computer Science
  - All computers and programming languages depend, to a greater or lesser extent, on logic.
- Logic is one of the partners of Computer Science
  - Logic and Computer Science have each contributed to the growth of the other and still are contributing.

Intelligence Science as an Interdiscipline [Cheng, 2018]



Logic and Intelligence Science

- **Reasoning ability is the most fundamental attribute of human intelligence**
  - The great ability to reason, in particular, to reason conceptually, is the most fundamental attribute of human intelligence, and therefore, the most intrinsic difference between human beings and animals.
- **Basic requirements for the fundamental logic for IS**
  - Providing absolute correct evaluation criteria for deductive reasoning.
  - Underlying relevant reasoning and truth-preserving reasoning in the sense of conditional.
  - Providing high degree correct evaluation criteria for inductive and abductive reasoning.
  - Underliying ampliative reasoning.
  - Underliying paracomplete and paraconsistent reasoning.

My Claims and/or Predictions [Cheng, 2018]

- **The fundamental logic to underlie Intelligence Science must be the strong relevant logic.**
- **The unified fundamental theory to underlie Intelligence Science (and Artificial Intelligence) must be an extension of the strong relevant logic.**

Logic and Artificial Intelligence

- **"Artificial" Intelligence is based on Computer Science**
  - "Artificial" means "computing" on computers.
- **Logic is the most indispensable tool for KR&R**
  - Logic is the most indispensable tool for knowledge representation and reasoning in Artificial Intelligence.
- **Reasoning ability is the most fundamental attribute of human intelligence**
  - The great ability to reason, in particular, to reason conceptually, is the most fundamental attribute of human intelligence, and therefore, the most intrinsic difference between human beings and animals.
  - No intelligent system can be said to be completely intelligent if it has no reasoning ability.

Logic and Artificial Intelligence

- **The fundamental theory underlying Artificial Intelligence**
  - Until now, Artificial Intelligence has no unified fundamental theory.
- **Logic must be a (part of) fundamental theory of Artificial Intelligence**
  - Any unified fundamental theory must be general, no theory may be more general than logic.
  - Any unified fundamental theory must be abstract and formal, no theory may be more abstract and formal than logic.
  - Any theory without regard to validity of reasoning cannot be a unified fundamental theory to underlie Artificial Intelligence.

Logic: What Is It and Why Study It?

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Mathematical Logic: What Is It and Why Study It ?

Mathematical Logic: What Is It and Why Study It ?

(Classical) Mathematical Logic: What Is It?

- "Mathematical logic, also call symbolic logic or logistic, is an extension of the formal method of mathematics to the field of logic. It employs for logic a symbolic language like that which has long been in use to express mathematical relations." "The purpose of the symbolic language in mathematical logic is to achieve in logic what it has achieved in mathematics, namely, an exact scientific treatment of its subject-matter."
  -- D. Hilbert and W. Ackermann, "Principles of Mathematical Logic," (Translation into English of the second Edition of the "Grundzuge der Theoretischen Logik") Julius Springer, 1928, 1938, Chelsea Publishing Company, 1950.

(Classical) Mathematical Logic: What Is It?

- "Mathematical Logic, which is nothing else but a precise and complete formulation of formal logic, has two quite different aspects. On the one hand, it is a section of Mathematics, treating of classes, relations, combinations of symbols, etc. instead of numbers, functions, geometric figures, etc. On the other hand, it is a science prior to all others, which contains the ideas and principles underlying all sciences. It was in the second sense that Mathematical Logic was first conceived by Leibniz in his Characteristica universalis, of which it would have formed a central part."
  -- K. Gödel, "Russell's Mathematical Logic," in Schilpp (Ed.) "The Philosophy of Bertrand Russell," Open Court Publishing Company, 1994.

(Classical) Mathematical Logic: What Is It? [E&F&T]

♣ **Mathematical logic and Mathematics**

- While traditional logic can be considered as part of philosophy, mathematical logic is more closely related to mathematics.

♣ **Motivation and Goals**

- Investigations in mathematical logic arose mainly from questions concerning the foundations of mathematics.
- Frege intended to base mathematics on logical and set-theoretical principles.
- Russell tried to eliminate contradictions that arose in Frege's system.
- Hilbert's goal was to show that "the generally accepted methods of mathematics taken as a whole do not lead to a contradiction" (known as Hilbert's program).

(Classical) Mathematical Logic: What Is It? [E&F&T]

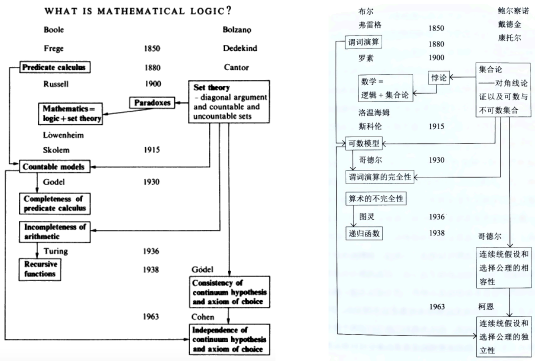
♣ **Methods**

- In mathematical logic the methods used are primarily mathematical.
- This is exemplified by the way in which new concepts are formed, definitions are given, and arguments are conducted.

♣ **Applications in Mathematics/TCS**

- The methods and results obtained in mathematical logic are not only useful for treating foundational problems; they also increase the stock of tools available in mathematics itself.
- There are applications in many areas of mathematics, but also in various parts of theoretical computer science.

What is Mathematical Logic? [J. N. Crossley, et al.]



(Classical) Mathematical Logic: What Is It? [E&F&T]

♣ Mathematical logic vs. Traditional logic

- In mathematical logic, as in traditional logic, deduction and proofs are central objects of investigation.
- However, it is the methods of deduction and the types of argument as used in mathematical proofs which are considered in mathematical logic.

(Classical) Mathematical Logic

♣ Set Theory

- Set Theory is the mathematical science of the infinite.
- Set Theory studies properties of sets, abstract objects that pervade the whole of modern mathematics.
- The language of set theory, in its simplicity, is sufficiently universal to formalize all mathematical concepts and thus set theory, along with Predicate Calculus, constitutes the true Foundations of Mathematics.

♣ Predicate Calculus

- The Predicate Logic, the Mathematical Logic.
- Predicate Logic is the study of the type of reasoning done by mathematicians; it provides formal languages for describing the structures with which mathematicians work, and the methods of proof available to them.

(Classical) Mathematical Logic

♣ Model Theory

- Model theory is the study of formal languages and their interpretations.

♣ Proof Theory

- Proof theory is the study of the general structure of mathematical proofs, and of arguments with demonstrative force as encountered in logic.

♣ Recursive Functions

- Recursive Functions is (one of) the study of “what is computable?”

Fundamental Assumptions/Principles Underlying CML

♣ The classical abstraction

- The only properties of a proposition that matter to logic are its form and its truth-value.

♣ The Fregean assumption / the principle of extensionality

- The truth-value of a (composite) proposition depends only on its (composition) form and the truth-values of its constituents, not on their meaning.

♣ The principle of bivalence

- There are exactly two truth-values, TRUE and FALSE. Every proposition has one or other, but not both, of these truth-values.

Fundamental Principles/Assumptions Underlying CML

♣ The classical account of validity (CAV)

- An argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false.

♣ Notes

- Usually, classical mathematical logic (CML) is often called to be ‘classical’ in the sense of the principle of bivalence.
- From the viewpoint of conditional (entailment), it is the CAV that makes CML ‘classical’.

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Philosophical Logics: What Are They and Why Study Them ?

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Various Philosophical (Non-Classical) Logics

- Philosophical (Non-Classical) Logics: What are they?
  - Philosophical Logics are the study of the more specifically philosophical aspects of logic.
- Various modal logics
  - A modal is an expression (like “necessarily” or “possibly”) that is used to qualify the truth of a judgement.
  - Modal logic is the study of the deductive behavior of the expressions “it is necessary that” and “it is possible that”.
  - Logics for belief, knowledge, tense and other temporal expressions, deontic (moral) expressions, and many others.
- Other philosophical logics
  - Intuitionistic Logic, Many-Valued Logic, Relevant (Relevance) Logic, Paraconsistent Logics, …

Various Modalities

- Alethic Modalities (Modalities of necessity and possibility)
  - “it is necessary”, “it is possible”
- Tense
  - “at all future times”, “eventually”
- Temporal
  - “henceforth”, “sometimes”
- Deontic
  - “it is obligatory”, “it is permitted”
- Epistemic
  - “it is known”, “it does not contradict to what is known”

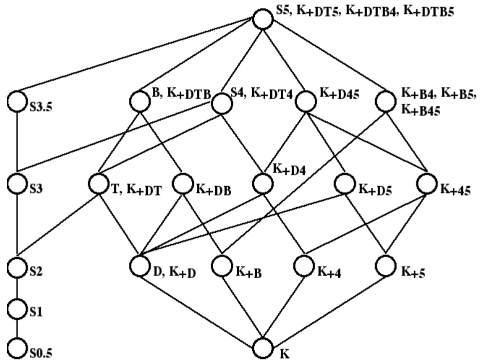
Various Modalities

- Provable
  - “it is provable in a given formal theory”
  - “it is consistent with the theory”
- Dynamic
  - “after every execution of the program”
  - “after some execution of the program”
- Cognitive
  - “it is believed”
  - “it is consistent with the current knowledge base”

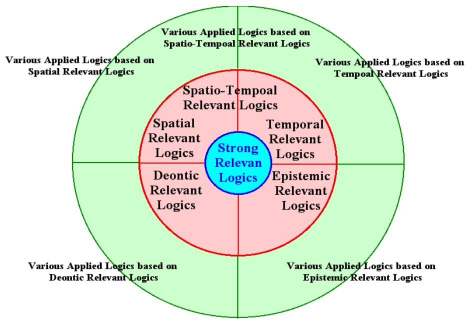
Various Modal Logics

- Ancient modal logic
  - A modal syllogistic by Aristotle (384-322 B.C.) in Book I of his Prior Analytics.
  - Modal logic is the study of the deductive behavior of the expressions “it is necessary that” and “it is possible that”.
- Modern modal logics
  - Modal logics of strict implication S1, ..., S5 [C. I. Lewis (1883-1964), 1912-1932].
  - Kripke model, possible world model [S. A. Kripke, 1959-1965].
  - Modal axioms: K, T, D, B, 4, 5(E), …

Various Modal Logics



Strong Relevant Logics as the Core Logic



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