7.1(b)

without deduction

$$proof: \underset{L}{\vdash} A \to (\neg(\neg A))$$

1. $(\neg(\neg(\neg A))) \to (\neg A)\{conclusion\ from\ 7.1(a)\}$
2. $((\neg(\neg(\neg A))) \to (\neg A)) \to (A \to (\neg(\neg A)))\{as3\}$
3. $A \to (\neg(\neg A))\{MP\ from\ 1,2\}$

with deduction

$$proof: A \underset{L}{\vdash} (\neg(\neg A))$$

1. $(\neg(\neg(\neg A))) \to (\neg A)\{conclusion\ from\ 7.1(a)\}$
2. $((\neg(\neg(\neg A))) \to (\neg A)) \to (A \to (\neg(\neg A)))\{as3\}$
3. $A \to (\neg(\neg A))\{MP\ from\ 1,2\}$
4. $A\{premise\}$
5. $(\neg(\neg A))\{MP\ from\ 3,4\}$

by the deduction theorem ,$\vdash_L A \to (\neg(\neg A))$.

7.2(b)

without deduction

proof:

$$proof: \underset{HB}{\vdash} (A \wedge B) \leftrightarrow (B \wedge A)$$

1. $((A \wedge B) \to A) \to ((A \wedge B) \to B) \to ((A \wedge B) \to (B \wedge A)))$
$\{((A \to B) \to ((A \to C) \to (A \to (B \wedge C))\}$
2. $(A \wedge B) \to A\{(A \wedge B) \to A\}$
3. $(A \wedge B) \to B\{(A \wedge B) \to B\}$
4. $(A \wedge B) \to B) \to ((A \wedge B) \to (B \wedge A))\{MP\ from\ 1,2\}$
5. $(A \wedge B) \to (B \wedge A)\{MP\ from\ 3,4\}$

finally we can get $\vdash_{HB} (A \wedge B) \to (B \wedge A)$,then

1. $(A \wedge B) \to (B \wedge A)\{the\ conclusion\ above\}$
2. $(B \wedge A) \to (A \wedge B)\{reverse\ the\ positions\ of\ A\ and\ B\}$
3. $((A \wedge B) \to (B \wedge A)) \to ((B \wedge A) \to (A \wedge B)) \to ((A \wedge B) \leftrightarrow (B \wedge A))$
$\{(X \to Y) \to (Y \to X) \to (X \leftrightarrow Y)\}$
4. $((B \wedge A) \to (A \wedge B)) \to ((A \wedge B) \leftrightarrow (B \wedge A))\{MP\ from\ 1,3\}$
5. $((A \wedge B) \leftrightarrow (B \wedge A))\{MP\ from\ 2,4\}$

with deduction

$proof$: $A \wedge B \underset{HB}{\vdash} (B \wedge A)$

1. $A \wedge B\{premise\}$
2. $(A \wedge B) \rightarrow A\{(X \wedge Y) \rightarrow X\}$
3. $(A \wedge B) \rightarrow B\{(X \wedge Y) \rightarrow Y\}$
4. $A\{MP\ from 1,2\}$
5. $B\{MP\ from 1,3\}$
6. $B \rightarrow (A \rightarrow (B \wedge A))\{the\ conclusion\ from 7.2(a)\}$
7. $A \rightarrow (B \wedge A)\{MP\ from 5,6\}$
8. $B \wedge A\{MP\ from 4,7\}$

finally we can get $\vdash_{HB} (A \wedge B) \rightarrow (B \wedge A)$

1. $(A \wedge B) \rightarrow (B \wedge A)\{the\ conclusion\ above\}$
2. $(B \wedge A) \rightarrow (A \wedge B)\{reverse\ the\ positions\ of\ A\ and\ B\}$
3. $((A \wedge B) \rightarrow (B \wedge A)) \rightarrow ((B \wedge A) \rightarrow (A \wedge B)) \rightarrow ((A \wedge B) \leftrightarrow (B \wedge A))$
$\{(X \rightarrow Y) \rightarrow (Y \rightarrow X) \rightarrow (X \leftrightarrow Y)\}$
4. $((B \wedge A) \rightarrow (A \wedge B)) \rightarrow ((A \wedge B) \leftrightarrow (B \wedge A))\{MP\ from\ 1,3\}$
5. $((A \wedge B) \leftrightarrow (B \wedge A))\{MP\ from\ 2,4\}$