Assignment 1

Read the following reading materials and write a report (in English, at least 1,500 words, or in Chinese, at least 3000 words) to answer:
(1) Which definition(s) of Logic (show original statements) is (are) the most convincing to you? Why?
(2) Show your consideration about why UNESCO recommended that Logic should be classified as the first basic discipline in the classification of science and technology. (You can refer some references)
(3) Show your considerations about why you should study Logic.
(4) To illustrate, with examples, the concrete applications of logic in your daily study.
(5) To illustrate, with examples, the possible applications of logic in your future professional field.

Reading materials:
JD.Cheng_L_Logic-What.is.it.pdf
D.Kelley_The.Art.of.Reasoning.4Ed_2014-Introduction4S.pdf
G.Restall_Logic_An.Introduction_2006-Introduction4S.pdf
I.M.Copi_C.Cohen_K.McMahon_Introduction2Logic.14Ed_2016-Forward4S.pdf
P.J.Hurley_L.Watson_A.Concise.Introduction.to.Logic.13Ed_2016-Why4S.pdf
S.Gimbel_An.Introduction.to.Formal.Logic-Course.Guidebook_2016-Introduction4S.pdf
Solution: 略

Assignment 2

(1) About concepts of reasoning, proving, discovery, prediction, arguments, deduction, induction, and abduction, explain the changes in your understanding before and after learning them in this course.
Solution: 略

(2) Answer “true” or “false” to the following statements and explain your reasons:
1. The purpose of the premise or premises is to set forth the reasons or evidence given in support of the
conclusion.
2. Some arguments have more than one conclusion.
3. All arguments must have more than one premise.
4. The words “therefore,” “hence,” “so,” “since,” and “thus” are all conclusion indicators.
5. The words “for,” “because,” “as,” and “for the reason that” are all premise indicators.
6. In the strict sense of the terms, inference and argument have exactly the same meaning.
7. In most (but not all) arguments that lack indicator words, the conclusion is the first statement.

8. Any sentence that is either true or false is a statement.
9. Every statement has a truth value.
10. Aristotle is the person usually credited with being the father of logic.
11. Any passage that contains an argument must contain a claim that something is supported by evidence or
reasons.
12. In an argument, the claim that something is supported by evidence or reasons is always explicit.
13. Passages that contain indicator words such as "thus," "since," and "because" are always arguments.
14. In deciding whether a passage contains an argument, we should always keep an eye out for indicator words
and the presence of an inferential relationship between the statements.
15. Some expository passages can be correctly interpreted as arguments.
16. Some passages containing "for example" can be correctly interpreted as arguments.
17. In deciding whether an expository passage or an illustration should be interpreted as an argument, it helps to
note whether the claim being developed or illustrated is one that is accepted by everyone.
18. Some conditional statements can be reexpressed to form arguments.
19. In an inductive argument, it is intended that the conclusion contain more information than the premises.
20. In a deductive argument, the conclusion is not supposed to contain more information than the premises.
21. The form of argumentation the arguer uses may allow one to determine whether an argument is inductive or
deductive.
22. The actual strength of the link between premises and conclusion may allow one to determine whether an
argument is inductive or deductive.
23. A geometrical proof is an example of an inductive argument.
24. Most arguments based on statistical reasoning are deductive.
25. If the conclusion of an argument follows merely from the definition of a word used in a premise, the
argument is deductive.
26. An argument that draws a conclusion about a thing based on that thing's similarity to something else is a
deductive argument.
27. An argument that draws a conclusion that something is true because someone has said that it is, is a
deductive argument.
28. An argument that presents two alternatives and eliminates one, leaving the other as the conclusion, is an
inductive argument.

29. An argument that proceeds from knowledge of a cause to knowledge of an effect is an inductive argument.
30. If an argument contains the phrase "it definitely follows that," then we know for certain that the argument is
deductive.
31. An argument that predicts what will happen in the future, based on what has happened in the past, is an
inductive argument.
32. Inductive arguments always proceed from the particular to the general.
33. Deductive arguments always proceed from the general to the particular.

Solution:
1. T.
2. T/F.
3. F.
4. F.
5. T.
6. T.
7. T/F.
8. T.
9. T.
10. T.
11. T.
12. F.
13. T.
14. T.
15. T (if it includes indicator words)/F (if it includes no indicator words).
16. T (if it includes indicator words)/F (if it includes no indicator words).
17. F.
18. T.
19. T.
20. T.
21. T.
22. T.
23. F.
24. F.
25. T.
26. F.
27. F.
28. F.
29. F.
30. T.
31. T.
32. T.

33. T.

Assignment 3

About concepts of truth and validity, explain the changes in your understanding before and after learning them in this course (in English, at least 1,000 words, or in Chinese, at least 2,000 words).
Solution: 略

(2) Answer “true” or “false” to the following statements and explain your reasons:
1. Some arguments, while not completely valid, are almost valid.
2. Inductive arguments allow for varying degrees of strength and weakness.
3. Invalid deductive arguments are basically the same as inductive arguments.
4. If a deductive argument has true premises and a false conclusion, it is necessarily invalid.
5. A valid argument may have a false premise and a false conclusion.
6. A valid argument may have a false premise and a true conclusion.
7. A sound argument may be invalid.
8. A sound argument may have a false conclusion.
9. A strong argument may have false premises and a probably false conclusion.
10. A strong argument may have true premises and a probably false conclusion.
11. A cogent argument may have a probably false conclusion.
12. A cogent argument must be inductively strong.
13. If an argument has true premises and a true conclusion, we know that it is a perfectly good argument.
Solution:
1. F.
2. T.
3. F.
4. T.
5. T.
6. T.
7. F.
8. F.
9. T.
10. T.
11. F.
12. T.
13. F.

(3) Use the counter-example method to prove each of the following arguments invalid.
1. All galaxies are structures that contain black holes in the center, so all galaxies are quasars, since all quasars
are structures that contain black holes in the center.
2. Some farmworkers are not people who are paid decent wages, because no

undocumented individuals are
people who are paid decent wages, and some undocumented individuals are not farmworkers.
3. No patrons of fast-food restaurants are health-food addicts. Consequently, no patrons of fast-food restaurants
are connoisseurs of fine desserts, since no connoisseurs of fine desserts are health-food addicts.
4. Some school boards are not groups that oppose values clarification, because some school boards are not
organizations with vision, and some groups that oppose values clarification are not organizations with vision.
5. Some improvers of humankind are not exploiters of personal information. As a result, some corporate social
networks are not improvers of humankind, seeing that all corporate social networks are exploiters of personal
information.
6. If animal species are fixed and immutable, then evolution is a myth. Therefore, evolution is not a myth, since
animal species are not fixed and immutable.
7. If energy taxes are increased, then either the deficit will be reduced or conservation will be taken seriously. If
the deficit is reduced, then inflation will be checked. Therefore, if energy taxes are increased, then inflation will
be checked.
8. All community colleges with low tuition are either schools with large enrollments or institutions supported
by taxes. Therefore, all community colleges are institutions supported by taxes.
9. All FHA loans are living-standard enhancers for the following reasons. All reverse mortgages that are FHA
loans are either living-standard enhancers or home-equity depleters, and all reverse mortgages are home-equity
depleters.
Solution:

1.	All G are S	All cats are animals. (T)
	All Q are S	All dogs are animals. (T)
	Therefore, All G are Q.	Therefore, All cats are dogs. (F)
2.	No U are P.	No fish are mammals. (T)
	Some U are not F.	Some fish are not cats. (T)
	Therefore, Some F are not P.	Therefore, Some cats are not mammals. (F)
3.	No P are H.	No dogs are fish. (T)
	No C are H.	No mammals are fish. (T)
	Therefore, No P are C.	Therefore, No dogs are mammals. (F)
4.	Some S are not O.	Some dogs are not fish. (T)
	Some G are not O.	Some animals are not fish. (T)

	Therefore, Some S are not G.	Therefore, Some dogs are not animals. (F)
5.	Some I are not E.	Some animals are not mammals. (T)
	All C are E.	All cats are mammals. (T)
	Therefore, Some C are not I.	Therefore, Some cats are not animals. (F)
6.	If A then E.	If George Washington was assassinated, then George Washington is dead. (T)
	Not A.	George Washington was not assassinated. (T)
	Therefore, Not E.	Therefore, George Washington is not dead. (F)
7.	If E, then either D or C.	If Tom Cruise is a man, then he is either a mouse or a human. (T)
	If D, then I.	If Tom Cruise is a mouse, then he has a tail. (T)
	Therefore, If E, then I.	Therefore, If Tom Cruise is a man, then he has a tail. (F)
8.	All C with L are either S or I.	All cats with fur are either mammals or dogs. (T)
	Therefore, All C are I.	Therefore, All cats are dogs. (F)
9.	All R that are F are either L or H.	All cats that are mammals are either dogs or animals. (T)
	All R are H.	All cats are animals. (T)
	Therefore, All F are L.	Therefore, All mammals are dogs. (F)

Assignment 4

(1) About concepts of object logic, meta-logic, object language, meta-language, conditional, antecedent, consequent, sufficient condition, necessary condition, sufficient and necessary condition, and material implication, explain the changes in your understanding before and after learning them in this course (in English, at least 2,000 words, or in Chinese, at least 4,000 words).
(2) About logic (including mathematical logic and philosophical logics), explain the changes in your understanding before and after learning it in this course (in English, at least 1,000 words, or in Chinese, at least 2,000 words).
Solution: 略

Assignment 5

Read the following reading materials, find reference materials on the Internet, and write a report (in English, at least 1,000 words, or in Chinese, at least 2,000 字) to answer:
(1) Why Mathematical Logic is indispensable to Mathematics, Computer Science, Intelligence Science, and Artificial Intelligence?
(2) What theories and applications you known (or found in reference materials) are based on Mathematical Logic? Why?

Solution: 略

Assignment 6

6.1 Interpreting the propositional letter 'p1' as 'It is raining' and the letter 'p2' as 'It is snowing', express the form of each of the following English sentences in the language of CPC (the outermost brackets can be omitted):

(a) It is raining.
(b) It is not raining.
(c) It is either raining or snowing.
(d) It is both raining and snowing.
(e) It is raining, but it is not snowing.
(f) It is not both raining and snowing.
(g) If it is not raining, then it is snowing.
(h) It is not the case that if it is raining then it is snowing.
(i) It is not the case that if it is snowing then it is raining.
(j) It is raining if and only if it is not snowing.
(k) It is neither raining nor snowing.
(l) If it is both snowing and raining, then it is snowing.
(m) If it is not raining, then it is not both snowing and raining.
(n) Either it is raining, or it is both snowing and raining.
(o) Either it is both raining and snowing or it is snowing but not raining.

Solution

(a) p1
(b) ¬p1
(c) p1 ∨ p2 or p1⊕p2

(e) p1 ∧ p2

(f) p1 ∧ (¬p2)

(g) ¬(p1 ∧ p2)

(h) ¬(p1 → p2)
(i) ¬(p2 → p1)
(j) p1 ↔ (¬p2)

(k) ¬(p1 ∨ p2) or (¬p1) ∧ (¬p2)

(l) (p2 ∧ p1) → p2

(m) (¬p1) → (¬(p2 ∧ p1))

(n) p1 ∨ (p2 ∧ p1)

(o) (p1 ∧ p2) ∨ (p2 ∧ (¬p1))

6.2 Prove that all the connectives '$\neg$','$\wedge$','$\vee$', and '$\rightarrow$' can be defined using either logical connective '$\uparrow$' ($\neg\wedge$, nand) or logical connective '$\downarrow$' ($\neg\vee$, nor), respectively.

Solution:

$\neg A = (A \uparrow A)$,

$A \vee B = ((A \uparrow A) \uparrow (B \uparrow B))$,

$A \wedge B = \neg(\neg A \vee \neg B) = [((A \uparrow A) \uparrow (A \uparrow A)) \uparrow ((B \uparrow B) \uparrow (B \uparrow B))] \uparrow [((A \uparrow A) \uparrow (A \uparrow A)) \uparrow ((B \uparrow B) \uparrow (B \uparrow B))]$,

$A \rightarrow B = \neg A \vee B = ((A\uparrow A)\uparrow(A\uparrow A)) \uparrow (B \uparrow B)$

$\neg A = (A \downarrow A)$,

$A \wedge B = ((A \downarrow A) \downarrow (B \downarrow B))$,

$A \vee B = \neg(\neg A \wedge \neg B) = [((A \downarrow A) \downarrow (A \downarrow A)) \downarrow ((B \downarrow B) \downarrow (B \downarrow B))] \downarrow [((A \downarrow A) \downarrow (A \downarrow A)) \downarrow ((B \downarrow B) \downarrow (B \downarrow B))]$

$A \rightarrow B = \neg(A \wedge \neg B) = [(A\downarrow A)\downarrow((B\downarrow B)\downarrow(B\downarrow B))] \downarrow [(A\downarrow A)\downarrow((B\downarrow B)\downarrow(B\downarrow B))]$

6.3 Let A, B, C, and D be formulas. Construct (and show) a truth-table for each of following formulas to
determine whether the formula is a tautology, contradiction, or contingency.

(a) $A \vee (\neg A)$

(b) $A \wedge (\neg A)$

(c) $(\neg A) \vee B$

(d) $(A \vee B) \wedge (\neg(A \wedge B))$

(e) $((\neg A) \vee (\neg B)) \leftrightarrow (A \wedge B)$

(f) $A \rightarrow (B \vee (\neg C))$

(g) $((A \wedge B) \wedge (C \wedge D)) \rightarrow A$

(h) $(A\leftrightarrow((\neg B) \vee C)) \rightarrow ((\neg A) \rightarrow B)$

Solution (must check the truth-table!)
(a) tautology
(b) contradiction
(c) contingency
(d) contingency
(e) contradiction
(f) contingency

(g) tautology
(h) tautology

6.4 Let A, B, C, D, and E be formulas.
(a) Use each of all letters at least twice to construct (and show) a formula and its truth-table such that it is a
tautology.
(b) Use each of all letters at least twice to construct (and show) a formula and its truth-table such that it is a
contradiction.
(c) Use each of all letters at least twice to construct (and show) a formula and its truth-table such that it is a
contingency.
6.5 Let A, B, C, and D be formulas. For the following sets of formulas, construct truth-tables of formulas, and
then check their consistency (satisfiability) (show your reasons).

(a) $\{ A \rightarrow B, B \rightarrow C, (C \lor D) \leftrightarrow (\neg B) \}$

(b) $\{ \neg((\neg B) \lor A), A \lor (\neg C), B \rightarrow (\neg C) \}$

(c) $\{ D \rightarrow B, A \lor (\neg B), \neg(D \land A), D \}$

Solution (must check the truth-table!)
(a) consistent, let A, B, and C be F, and D be T.
(b) consistent, let A and C be F, and B be T.
(c) inconsistent, (must show impossibility!, if D and $D \rightarrow B$ are T, then B must be T, …)
6.6 Let A, B, and C be formulas. Construct (and show) a truth-table for each of the following semantic (model-
theoretical, logical) consequence relations and use it to verify that whether the relation holds or not.
(a) $\{ A\rightarrow B , A \} \models_{CPC} B$
(b) $\{ A\rightarrow B , B \} \models_{CPC} A$
(c) $\{ A\rightarrow B , B\rightarrow C \} \models_{CPC} A\rightarrow C$
(d) $\{ A\rightarrow B , A\rightarrow(\neg B) \} \models_{CPC} \neg A$
(e) $A\rightarrow B \models_{CPC} \neg(B\rightarrow A)$

(f) $\{ A \lor B , B \lor C \} \models_{CPC} A \lor C$

(g) $\{ A , \neg A \} \models_{CPC} B$

(h) $C \models_{CPC} A \leftrightarrow (A \lor (A \land B))$

Solution (must check the truth-table!)
(a) holds
(b) not holds
(c) holds

(d) holds
(e) not holds
(f) not holds
(g) holds
(h) holds
6.7 Let A, B, C, D, and E be formulas.
(a) Use each of all letters at least twice to construct (and show) some formulas and their truth-tables for a semantic (model-theoretical, logical) consequence relation that holds.
(b) Use each of all letters at least twice to construct (and show) some formulas and their truth-tables for a semantic (model-theoretical, logical) consequence relation that does not hold.
Solution: 略

Assignment 7

Solution 略

Assignment 8

8.1 Interpreting the symbols “F(x)” and “G(x)” as the predicates “x is a frog” and “x is green”, respectively,
formalize each of the following sentences:
(a) Frogs are green.
(b) There is at least one green frog.
(c) Some frogs are not green.
(d) There are not any green frogs.
(e) No frogs are green.
(f) Frogs are not green.
(g) Not everything that is a frog is green.
Solution
(a) $(\forall x)((F(x) \rightarrow G(x))$

(b) $(\exists x)(F(x) \wedge G(x))$

(c) $(\exists x)(F(x) \wedge \neg G(x))$

(d) $\neg(\exists x)(F(x) \wedge G(x))$

(e) $(\forall x)(F(x) \rightarrow \neg G(x))$ or (d) $\neg(\exists x)(F(x) \wedge G(x))$

(f) (e) $(\forall x)(F(x) \rightarrow \neg G(x))$ or (d) $\neg(\exists x)(F(x) \wedge G(x))$

(g) $\neg(\forall x)(F(x) \rightarrow G(x))$
8.2 Interpreting the symbols “R” as the sentence “It is raining” and the symbols “F(x)”, “G(x)”, “H(x)”, and “I(x)” as the predicates “x is a frog”, “x is green”, “x is hopping”,

and “x is iridescent”, respectively, formalize each of the following sentences:
(a) Everything is a frog.
(b) Something is a frog.
(c) Not everything is a frog.
(d) Nothing is a frog.
(e) Green frogs exist.
(f) Everything is either green or iridescent.
(g) Everything is a green frog.
(h) It is raining and some frogs are hopping.
(i) If it is raining, then all frogs are hopping.
(j) Some things are green and some are not.
(k) Some things are both green and iridescent.
(l) Either everything is a frog or nothing is a frog.
(m) Everything is either a frog or not a frog.
(n) All frogs are frogs.
(o) Only frogs are green.
(p) Iridescent frogs do not exist.
(q) All green frogs are hopping.
(r) Some green frogs are not hopping.
(s) It is not true that some green frogs are hopping.
(t) If nothing is green, then green frogs do not exist.
(u) Green frogs hop if and only if it isn't raining.
Solution:
(a) $(\forall x)F(x)$
(b) $(\exists x)F(x)$
(c) $\neg(\forall x)F(x)$
(d) $(\forall x)\neg F(x)$ or $\neg(\exists x)F(x)$

(e) $(\exists x)(G(x) \wedge F(x))$

(f) $(\forall x)(G(x) \vee I(x))$

(g) $(\forall x)(G(x) \wedge F(x))$

(h) $R \wedge (\exists x)(F(x) \wedge H(x))$

(i) $R \rightarrow (\forall x)(F(x) \rightarrow H(x))$

(j) $(\exists x)G(x) \wedge (\exists x)\neg G(x)$

(k) $(\exists x)(G(x) \wedge I(x))$

(1) $(\forall x)F(x) \vee (\forall x)\neg F(x)$

(m) $(\forall x)(F(x) \vee \neg F(x))$

(n) $(\forall x)(F(x) \to F(x))$
(0) $(\forall x)(G(x) \to F(x))$

(p) $\neg(\exists x)(I(x) \wedge F(x))$

(q) $(\forall x)((G(x) \wedge F(x)) \to H(x))$

(r) $(\exists x)((G(x) \wedge F(x)) \wedge \neg H(x))$

(s) $\neg(\exists x)((G(x) \wedge F(x)) \wedge H(x))$

(t) $(\forall x)(\neg G(x)) \to \neg(\exists x)(G(x) \wedge F(x))$

(u) $(\forall x)((Gx \& Fx) \to (Hx \leftrightarrow \neg R))$

8.3 Interpreting the symbols 'F(x)' and 'G(x)' as the predicates 'x is a frog' and 'x is green', respectively,
formalize each of the following sentences:
(a) If something is a frog, then it is green.
(b) If anything at all is a frog, then something is green.
(c) Anything that is a frog is green.
(d) If anything is green, then frogs are green.
(e) If everything is green, then frogs are green.
(f) Invariably, frogs are green.
(g) Occasionally, frogs are green.
(h) A frog is green.
(i) A frog is always green.
(j) Only frogs are green.
Solution
(a) $(\forall x)(F(x) \to G(x))$
(b) $(\exists x)F(x) \to (\exists x)G(x)$
(c) $(\forall x)(F(x) \to G(x))$
(d) $(\exists x)G(x) \to (\forall x)(F(x) \to G(x))$
(e) $(\forall x)G(x) \to (\forall x)(F(x) \to G(x))$
(f) $(\forall x)(F(x) \to G(x))$
(g) $(\exists x)(F(x) \wedge G(x))$
(h) This sentence is ambiguous. It may be interpreted as a general statement about frogs (as in (a) and (c)), or as
an existential statement of (g).
(i) $(\forall x)(F(x) \to G(x))$
(j) $(\forall x)(G(x) \to F(x))$

8.4 Formalize the following statements, interpreting the symbols 'a', 'b', and 'c' as the proper names 'Alex', 'Bob',
and 'Cathy'; 'M(x)' and 'N(x)' as the one-place predicates 'x is a mechanic' and 'x is a

nurse'; 'L(x,y)' and 'T(x,y)'
as the two-place predicates 'x likes y' and 'x is taller than y'; and 'I(x,y,z)' as the three-place predicate 'x
introduced y to z'.

(a) Cathy is a mechanic.
(b) Bob is a mechanic.
(c) Cathy and Bob are mechanics.
(d) Either Cathy or Bob is a mechanic.
(e) Cathy is either a mechanic or a nurse (or both).
(f) If Cathy is a mechanic, then she isn't a nurse.
(g) Cathy is taller than Bob.
(h) Bob likes Cathy.
(i) Bob likes himself.
(j) Cathy likes either Bob or Alex.
(k) Alex introduced Cathy to Bob.
(l) Cathy introduced herself to Bob but not to Alex.

Solution:

(a) M(c)
(b) M(b)
(c) $\mathrm{M}(\mathrm{c}) \wedge \mathrm{M}(\mathrm{b})$
(d) $M(c) \vee M(b)$
(e) $M(c) \vee N(c)$
(f) $\mathrm{M(c)} \rightarrow \neg\mathrm{N(c)}$
(g) T(c,b)
(h) L(b,c)
(i) L(b,b)
(j) $L(c,b) \vee L(c,a)$
(k) I(a,b,c)
(l) $I(c,c,b) \wedge \neg I(c,c,a)$

8.5 Formalize the following statements using the same interpretation as in Problem 9.4.

(a) Bob likes nothing.
(b) Nothing likes Bob.
(c) Something likes itself.
(d) There is something which Cathy does not like.
(e) Cathy likes something which Bob likes.
(f) There is something which both Bob and Cathy like.
(g) There is something which Bob likes and something which Cathy likes.
(h) If Bob likes himself, then he likes something.
(i) If Bob does not like himself, then he likes nothing.
(j) If Bob likes something, then he likes everything.
(k) Everything likes everything.
(l) There is some one which is liked by everything.
(m) Everything likes at least one thing.

Solution:
(a) $(\forall x)\ \neg L(b, x)$
(6) $(\forall x)\ \neg L(x, b)$
(c) $(\exists x)\ L(x, x)$
(d) $(\exists x)\ \neg L(c, x)$
(e) $(\exists x)(L(c,x) \ \wedge\ L(b,x))$
(f) $(\exists x)(L(b,x) \ \wedge\ L(c,x))$
(g) $(\exists x)L(b,x) \ \wedge\ (\exists x)L(c,x)$
(h) $L(b,b) \rightarrow (\exists x)L(b,x)$
(i) $\neg L(b,b) \rightarrow (\forall x)\neg L(b,x)$
(1) $(\exists x)L(b,x) \rightarrow (\forall x)L(b,x)$
(k) $(\forall x)(\forall y)L(x,y)$
(1) $(\exists x)(\forall y)L(y,x)$
(m) $(\forall x)(\exists y)L(x,y)$

8.6 Formalize the following statements using the same interpretation as in Problem 9.4.
(a) A mechanic likes Bob.
(b) A mechanic likes herself.
(c) Every mechanic likes Bob.
(d) Bob likes a nurse.
(e) Some mechanic likes every nurse.
(f) There is a mechanic who is liked by every nurse.
(g) Bob introduced a mechanic to Cathy.
(h) A mechanic introduced Bob to Alex.
(i) A mechanic introduced herself to Bob and Alex.
(j) Cathy introduced a mechanic and a nurse to Bob.
(k) Cathy introduced a mechanic to a nurse.
(l) A mechanic introduced a nurse to Cathy.
(m) Some mechanic introduced a nurse to a mechanic.
Solution:
(a) $(\exists x)(M(x) \ \wedge\ L(x,b))$
(b) $(\exists x)(M(x) \ \wedge\ L(x,x))$
(c) $(\forall x)(M(x) \ \rightarrow\ L(x,b))$
(d) $(\exists x)(N(x) \ \wedge\ L(b,x))$
(e) $(\exists x)(M(x) \ \wedge\ \forall y(N(y) \ \rightarrow\ L(x,y)))$
(f) $(\exists x)(M(x) \ \wedge\ \forall y(N(y) \ \rightarrow\ L(y,x)))$
(g) $(\exists x)(M(x) \ \wedge\ I(b,x,c))$
(h) $(\exists x)(M(x) \ \wedge\ I(x,b,a))$
(i) $(\exists x)(M(x) \ \wedge\ (I(x,x,b) \ \wedge\ I(x,x,a)))$
(1) $(\exists x)(M(x) \ \wedge\ I(c,x,b)) \ \wedge\ (\exists x)(N(x) \ \wedge\ I(c,x,b))$
(k) $(\exists x)(\exists y)((M(x) \ \wedge\ N(y)) \ \wedge\ I(c,x,y))$
(1) $(\exists x)(\exists y)((M(x) \ \wedge\ N(y)) \ \wedge\ I(x,y,c))$
(m) $(\exists x)(\exists y)(\exists z)((M(x) \ \wedge\ N(y)) \ \wedge\ (M(z) \ \wedge\ I(x,y,z)))$

8.7 Let SUST = F ∪ A ∪ S , where F is the set of faculties, A is the set of administrative staffs, and S is the set of students. Define your predicates, functions, and constants on(in) SUST at first, and then formalize ten sentences about various propositions in SUST (write formula and give its English or Chinese interpretations).
Solution: 略

Assignment 9

9.1 Recall five you known properties about natural numbers (number theory) at first, then based on PA, use L_A to formalize the properties (write formula and give its English or Chinese interpretations). You can define new predicates and functions if they are necessary.
Solution: 略

9.2 Based on NBG, formalize the following concepts/notions of set theory (write formula and give its English
or Chinese interpretations):
(a) Reflexive relation
(b) Irreflexive relation
(c) Symmetric relation
(d) Antisymmetric relation
(e) Connected relation
(f) Transitive relation
(g) Equivalence relation
(h) Partial order relation
(i) Partial function/mapping
(j) Total function/mapping
(k) Injective function/mapping
(l) Surjective function/mapping
(m) Bijective function/mapping
Solution: 略

Assignment 10

10.1 Pick out the free and bound occurrences of variables in the following formulas.
(a) $(\forall x)P(x) \land \neg P(y)$
(b) $(\forall y)P(y) \leftrightarrow P(y)$
(c) $(\forall y)(\exists x)Q(x, y)$
(d) $(\forall x)(\forall u)(\forall v)P(x) \to (P(y) \land \neg P(x))$
(e) $(\forall x)(\forall y)(\forall z)P(x) \lor (\exists y)(\neg(\forall z)Q(z, y))$
(f) $(\forall x)(\neg P(x)) \to T(x, x, y) \lor (\forall x)P(x)$
(g) $\neg(\forall x)P(x) \to (\exists y)P(y) \to Q(x, y) \land P(y)$
(h) $(((\forall x)(P(x) \to P(x))) \lor ((\exists x)P(x)))$
(i) $((\neg((\exists y)(P(y) \lor P(a)))) \leftrightarrow P(y))$

(j) $(((\forall x)(\neg(\neg P(a)))) \rightarrow (P(x) \rightarrow P(y)))$
(k) $(\forall z)(((\forall x)Q(x, y)) \rightarrow Q(z, a))$
(l) $(\forall y)Q(z, y) \rightarrow (\forall z)Q(z, y)$
(m) $((\forall y)(\exists x)T(x, y, g(x, y))) \vee \neg(\forall x)Q(y, f(x))$
Solution: 略

10.2 Indicate the free and bound variables in the formulas of Exercise 11.1.
Solution: 略