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Strong Relevant Logic as the Universal Basis of Various Applied Logics for Knowledge Representation and Reasoning

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Abstract. In many applications in computer science and artificial intelligence, in order to represent, specify, verify, and reason about various objects and relationships among them, we often need a right fundamental logic system to provide us with a criterion of logical validity for reasoning as well as a formal representation and specification language. Although different applications may require different logic systems, the fundamental logics must be able to underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete reasoning, and paraconsistent reasoning. Based on our experiences, this paper shows that strong relevant logic can be used as the universal basis to construct various applied logics to satisfy the requirements. The paper discusses why any of the classical mathematical logic, its various classical conservative extensions, and its non-classical alternatives is not a suitable candidate for the universal basis to construct various applied logics, shows that strong relevant logic is a more hopeful candidate for the purpose, and presents our experiences on constructions of temporal relevant logics, deontic relevant logics, spatial relevant logics, and spatial-temporal relevant logics.

1. Introduction

In many applications in computer science and artificial intelligence, in order to represent, specify, verify, and reason about various objects and relationships among them, we often need a right fundamental logic system to provide us with a criterion of logical validity for reasoning as well as a formal representation and specification language. The question, “Which is the right logic?” invites the immediate counter-question “Right for what?” Only if we certainly know what we need, we can make a good choice. It is obvious that different applications may require different characteristics of logic. However, can we have a set of essential requirements as the universal core requirements for various applications and a logic system satisfying the essential requirements as the universal core logic such that we can construct various applied logics by extending the logic?

The present author considers that we should consider the following three essential requirements for the universal core logic. First, as a general logical criterion for the validity of reasoning as well as proving, the logic must be able to underlie relevant reasoning as well as truth-preserving reasoning in the sense of conditional, i.e., for any reasoning based on the logic to be valid, if its premises are true in the sense of conditional, then its conclusion must be relevant to the premises and must be true in the sense of conditional. Second, the logic must be able to underlie ampliative reasoning in the sense that the truth of conclusion of the reasoning should be recognized after the completion of the reasoning process but not be invoked in deciding the truth of premises of the reasoning. From the viewpoint to regard reasoning as the process of drawing new conclusions from given premises, any meaningful reasoning must be ampliative but not circular and/or tautological. Third, the logic must be able to underlie paracomplete reasoning and paraconsistent reasoning. In particular, the so-called principle of Explosion that everything follows from a contradiction cannot be accepted by the logic as a valid principle. In general, our knowledge about a domain as well as a scientific discipline may be incomplete and sometime even inconsistent in many ways, i.e., it gives us no evidence for deciding the truth of either a proposition or its negation, and it directly or indirectly includes some contradictions. Therefore, reasoning with incomplete and inconsistent knowledge is the rule rather than the exception in our everyday lives and almost all scientific disciplines.

Based on our experiences, this paper shows that strong relevant logic can be used as the universal basis to construct various applied logics to satisfy the three essential requirements. The paper discusses why any of the classical mathematical logic, its various classical conservative extensions, and its non-classical alternatives is not a suitable candidate for the universal basis to construct various applied logics, shows that strong relevant logic is a more hopeful candidate for the purpose, and presents our experiences on constructions of temporal relevant logics, deontic relevant logics, spatial relevant logics, and spatial-temporal relevant logics.

2. Basic Notions

Reasoning is the ***process*** of drawing ***new conclusions*** from given premises, which are already known facts or previously assumed hypotheses to provide some ***evidence*** for the conclusions (Note that how to define the notion of 'new' formally and satisfactorily is still a difficult open problem until now). Therefore, reasoning is intrinsically ampliative, i.e., it has the function of enlarging or extending some things, or adding to what is already known or assumed. In general, a reasoning consists of a number of arguments ***in some order***. An ***argument*** is a set of ***statements*** (or ***declarative sentences***) of which one statement is intended as the ***conclusion***, and one or more statements, called "***premises***," are intended to provide some evidence for the conclusion. An argument is a conclusion standing in relation to its supporting evidence. In an argument, a claim is being made that there is some sort of ***evidential relation*** between its premises and its conclusion: the conclusion is supposed to ***follow from*** the premises, or equivalently, the premises are supposed to ***entail*** the conclusion. Therefore, the correctness of an argument is a matter of the ***connection*** between its premises and its conclusion, and concerns the ***strength*** of the relation between them (Note that the correctness of an argument depends neither on whether the premises are really true or not, nor on whether the conclusion is really true or not). Thus, there are some fundamental questions: What is the criterion by which one can decide whether the conclusion of an argument or a reasoning really does follow from its premises or not? Is there the only one criterion, or are there many criteria? If there are many criteria, what are the intrinsic differences between them? It is logic that deals with the validity of argument and reasoning in a general theory.

A ***logically valid reasoning*** is a reasoning such that its arguments are justified based on some ***logical validity criterion*** provided by a logic system in order to obtain correct conclusions (Note that here the term 'correct' does not necessarily mean 'true'). Today, there are so many different logic systems motivated by various philosophical considerations. As a result, a reasoning may be valid on one logical validity criterion but invalid on another. For example, the ***classical account of validity***, which is one of fundamental principles and assumptions underlying classical mathematical logic and its various conservative extensions, is defined in terms of ***truth-preservation*** (in some certain sense of truth) as: an argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false. Therefore, a classically valid reasoning must be ***truth-preserving***. On the other hand, for any correct argument in scientific reasoning as well as our everyday reasoning, its premises must somehow be ***relevant*** to its conclusion, and vice versa. The ***relevant account of validity*** is defined in terms of ***relevance*** as: for an argument to be valid there must be some connection of meaning, i.e., some relevance, between its premises and its conclusion. Obviously, the relevance between the premises and conclusion of an argument is not accounted for by the classical logical validity criterion, and therefore, a classically valid reasoning is not necessarily relevant.

Proving is the process of finding a justification for an explicitly ***specified statement*** from given ***premises***, which are already known facts or previously assumed hypotheses to provide some ***evidence*** for the specified statement. A ***proof*** is a description of a found justification. A ***logically valid proving*** is a proving such that it is justified based on some logical validity criterion provided by a logic system in order to obtain a correct proof. The most intrinsic difference between reasoning and proving is that the former is intrinsically prescriptive and predictive while the latter is intrinsically descriptive and non-predictive. The purpose of reasoning is to find some new conclusion previously unknown or unrecognized, while the purpose of proving is to find a justification for some specified statement previously given. Proving has an explicitly given target as its goal while reasoning does not. Unfortunately, until now, many studies in Computer Science and Artificial Intelligence disciplines still confuse proving with reasoning.

Logic deals with ***what entails what*** or ***what follows from what***, and aims at determining which are the correct conclusions of a given set of premises, i.e., to determine which arguments are valid. Therefore, the most essential and central concept in logic is the ***logical consequence relation*** that relates a given set of premises to those conclusions, which validly follow from the premises. To define a logical consequence relation is nothing else but to provide a logical validity criterion by which one can decide whether the conclusion of an argument or a reasoning really does follow from its premises or not. Moreover, to answer the question what is the correct conclusion of given premises, we have to answer the question: correct for what? Based on different philosophical motivations, one can define various logical consequence relations and therefore establish various logic systems.

In logic, a sentence in the form of ‘if ... then ...’ is usually called a ***conditional proposition*** or simply ***conditional*** which states that there exists a relation of sufficient condition between the ‘if’ part and the ‘then’ part of the sentence. In general, a conditional must concern two parts which are connected by the connective ‘if ... then ...’ and called the ***antecedent*** and the ***consequent*** of that conditional, respectively. The truth of a conditional depends not only on the truth of its antecedent and consequent but also, and more essentially, on a necessarily relevant and conditional relation between them. The notion of conditional plays the most essential role in reasoning because any reasoning form must invoke it, and therefore, it is historically always the most important subject studied in logic and is regarded as the heart of logic [1].

When we study and use logic, the notion of conditional may appear in both the ***object logic*** (i.e., the logic we are studying) and the ***meta-logic*** (i.e., the logic we are using to study the object logic). In the object logic, there usually is a connective in its formal language to represent the notion of conditional, and the notion of conditional, usually represented by a meta-linguistic symbol, is also used for representing a logical consequence relation in its proof theory or model theory. On the other hand, in the meta-logic, the notion of conditional, usually in the form of natural language, is used for defining various meta-notions and describing various meta-theorems about the object logic.

From the viewpoint of object logic, there are two classes of conditionals. One class is empirical conditionals and the other class is logical conditionals. For a logic, a conditional is called an ***empirical conditional*** of the logic if its truth-value, in the sense of that logic, depends on the contents of its antecedent and consequent and therefore cannot be determined only by its abstract form (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be empirical); a conditional is called a ***logical conditional*** of the logic if its truth-value, in the sense of that logic, depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true or false (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be logical). A logical conditional that is considered to be universally true, in the sense of that logic, is also called an ***entailment*** of that logic. Indeed, the most intrinsic difference between various different logic systems is to regard what class of conditionals as entailments, as Diaz pointed out: "The problem in modern logic can best be put as follows: can we give an explanation of those conditionals that represent an entailment relation?" [14]

3. Strong Relevant Logics

As we have mentioned, the universal core logic must be able to underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning.

Classical mathematical logic (**CML** for short) was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them; its principal aim is a precise and adequate understanding of the notion of mathematical proof. **CML** was established based on a number of fundamental assumptions. Among them, the most characteristic one is the classical account of validity that is the logical validity criterion of **CML** by which one can decide whether the conclusion of an argument or a reasoning really does follow from its premises or not in the framework of **CML**. However, since the relevance between the premises and conclusion of an argument is not accounted for by the classical validity criterion, a reasoning based on **CML** is not necessarily relevant. On the other hand, in **CML** the notion of conditional, which is intrinsically intensional but not truth-functional, is represented by the notion of material implication, which is intrinsically an extensional truth-function. This leads to the problem of 'implicational paradoxes' [1, 2, 15, 19-21, 23].

CML cannot satisfy any of the above three essential requirements because of the following facts: a reasoning based on **CML** is not necessarily relevant; the classical truth-preserving property of a reasoning based on **CML** is meaningless in the sense of conditional; a reasoning based on **CML** must be circular and/or tautological but not ampliative; reasoning under inconsistency is impossible within the framework of **CML** [6]. These facts are also true to those classical conservative extensions or non-classical alternatives of **CML** where the classical account of validity is adopted as the logical validity criterion and the notion of conditional is directly or indirectly represented by the material implication [6].

Traditional ***relevant*** (or ***relevance***) ***logics*** were constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called ‘paradoxes’ of material and strict implication [1, 2, 15, 19-21, 23]. Some major traditional relevant logic systems are ‘system **E** of entailment’, ‘system **R** of relevant implication’, and ‘system **T** of ticket entailment’. A major characteristic of the relevant logics is that they have a primitive intensional connective to represent the notion of conditional (entailment) and their logical theorems include no implicational paradoxes. The underlying principle of the relevant logics is the relevance principle, i.e., for any entailment provable in **E**, **R**, or **T**, its antecedent and consequent must share a propositional variable. Variable-sharing is a formal notion designed to reflect the idea that there be a meaning-connection between the antecedent and consequent of an entailment. It is this relevance principle that excludes those implicational paradoxes from logical axioms or theorems of relevant logics. Also, since the notion of entailment is represented in the relevant logics by a primitive intensional connective but not an extensional truth-function, a reasoning based on the relevant logics is ampliative but not circular and/or tautological. Moreover, because the relevant logics reject the principle of Explosion, they can certainly underlie paraconsistent reasoning.

In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present author has proposed some ***strong relevant*** (or ***relevance***) ***logics***, named **Rc**, **Ec**, and **Tc** [6]. The logics require that the premises of an argument represented by a conditional include no unnecessary and needless conjuncts and the conclusion of that argument includes no unnecessary and needless disjuncts. As a modification of traditional relevant logics **R**, **E**, and **T**, strong relevant logics **Rc**, **Ec**, and **Tc** rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in **R**, **E**, and **T**, respectively. What underlies the strong relevant logics is the strong relevance principle: If A is a theorem of **Rc**, **Ec**, or **Tc**, then every propositional variable in A occurs at least once as an antecedent part and at least once as a consequent part. Since the strong relevant logics are free of not only implicational paradoxes but also conjunction-implicational and disjunction-implicational paradoxes, in the framework of strong relevant logics, if a reasoning is valid, then both the relevance between its premises and its conclusion and the validity of its conclusion in the sense of conditional can be guaranteed in a certain sense of strong relevance.

The logical connectives, axiom schemata, and inference rules of relevant logics are as follows:

Primitive logical connectives: $\Rightarrow$ (entailment), $\neg$ (negation), $\wedge$ (extensional conjunction).

Defined logical connectives: $\otimes$ (intensional conjunction, $A\otimes B =_{\text{df}} \neg(A\Rightarrow\neg B)$), $\oplus$ (intensional disjunction, $A\oplus B =_{\text{df}} \neg A\Rightarrow B$), $\Leftrightarrow$ (intensional equivalence, $A\Leftrightarrow B =_{\text{df}} (A\Rightarrow B)\otimes(B\Rightarrow A)$), $\vee$ (extensional disjunction, $A\vee B =_{\text{df}} \neg(\neg A\wedge\neg B)$), $\rightarrow$ (material implication, $A\rightarrow B =_{\text{df}} \neg(A\wedge\neg B)$ or $A\rightarrow B =_{\text{df}} \neg A\vee B$), $\leftrightarrow$ (extensional equivalence, $A\leftrightarrow B =_{\text{df}} (A\rightarrow B)\wedge(B\rightarrow A)$).

Axiom schemata: E1: $A\Rightarrow A$, E2: $(A\Rightarrow B)\Rightarrow((C\Rightarrow A)\Rightarrow(C\Rightarrow B))$, E2′: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow C))$, E3: $(A\Rightarrow(A\Rightarrow B))\Rightarrow(A\Rightarrow B)$, E3′: $(A\Rightarrow(B\Rightarrow C))\Rightarrow((A\Rightarrow B)\Rightarrow(A\Rightarrow C))$, E3″: $(A\Rightarrow B)\Rightarrow((A\Rightarrow(B\Rightarrow C))\Rightarrow(A\Rightarrow C))$, E4: $(A\Rightarrow((B\Rightarrow C)\Rightarrow D))\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow D))$, E4′: $(A\Rightarrow B)\Rightarrow(((A\Rightarrow B)\Rightarrow C)\Rightarrow C)$, E4″: $((A\Rightarrow A)\Rightarrow B)\Rightarrow B$, E4‴: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(((A\Rightarrow C)\Rightarrow D)\Rightarrow D))$, E5: $(A\Rightarrow(B\Rightarrow C))\Rightarrow(B\Rightarrow(A\Rightarrow C))$, E5′: $A\Rightarrow((A\Rightarrow B)\Rightarrow B)$, N1: $(A\Rightarrow(\neg A))\Rightarrow(\neg A)$, N2: $(A\Rightarrow(\neg B))\Rightarrow(B\Rightarrow(\neg A))$, N3: $(\neg(\neg A))\Rightarrow A$, C1: $(A\wedge B)\Rightarrow A$, C2: $(A\wedge B)\Rightarrow B$, C3: $((A\Rightarrow B)\wedge(A\Rightarrow C))\Rightarrow(A\Rightarrow(B\wedge C))$, C4: $(\boldsymbol{L}A\wedge\boldsymbol{L}B)\Rightarrow\boldsymbol{L}(A\wedge B)$, where $\boldsymbol{L}A =_{\text{df}} (A\Rightarrow A)\Rightarrow A$, D1: $A\Rightarrow(A\vee B)$, D2: $B\Rightarrow(A\vee B)$, D3: $((A\Rightarrow C)\wedge(B\Rightarrow C))\Rightarrow((A\vee B)\Rightarrow C)$, DCD: $(A\wedge(B\vee C))\Rightarrow((A\wedge B)\vee C)$, C5: $(A\wedge A)\Rightarrow A$, C6: $(A\wedge B)\Rightarrow(B\wedge A)$, C7: $((A\Rightarrow B)\wedge(B\Rightarrow C))\Rightarrow(A\Rightarrow C)$, C8: $(A\wedge(A\Rightarrow B))\Rightarrow B$, C9: $\neg(A\wedge\neg A)$, C10: $A\Rightarrow(B\Rightarrow(A\wedge B))$, IQ1: $\forall x(A\Rightarrow B)\Rightarrow$

$(\forall xA \Rightarrow \forall xB)$, IQ2: $(\forall xA \land \forall xB) \Rightarrow \forall x(A \land B)$, IQ3: $\forall xA \Rightarrow A[t/x]$ (if x may appear free in A and t is free for x in A, i.e., free variables of t do not occur bound in A), IQ4: $\forall x(A \Rightarrow B) \Rightarrow (A \Rightarrow \forall xB)$ (if x does not occur free in A), IQ5: $\forall x_1 \ldots \forall x_n(((A \Rightarrow A) \Rightarrow B) \Rightarrow B)$ ($n \geq 0$).

Inference rules: $\Rightarrow$E: from A and $A \Rightarrow B$ to infer B (Modus Ponens), $\land$I: from A and B to infer $A \land B$ (Adjunction), $\forall$I: if A is an axiom, so is $\forall xA$ (Generalization of axioms).

Thus, various relevant logic systems may now defined as follows, where we use 'A | B' to denote any choice of one from two axiom schemata A and B: $\mathbf{T}_{\Rightarrow} =_{df}$ {E1, E2, E2′, E3 | E3″} + $\Rightarrow$E, $\mathbf{E}_{\Rightarrow} =_{df}$ {E1, E2 | E2′, E3 | E3′, E4 | E4′} + $\Rightarrow$E, $\mathbf{E}_{\Rightarrow} =_{df}$ {E2′, E3, E4″} + $\Rightarrow$E, $\mathbf{E}_{\Rightarrow} =_{df}$ {E1, E3, E4‴} + $\Rightarrow$E, $\mathbf{R}_{\Rightarrow} =_{df}$ {E1, E2 | E2′, E3 | E3′, E5 | E5′} + $\Rightarrow$E, $\mathbf{T}_{\Rightarrow,\neg} =_{df} \mathbf{T}_{\Rightarrow}$ + {N1, N2, N3}, $\mathbf{E}_{\Rightarrow,\neg} =_{df} \mathbf{E}_{\Rightarrow}$ + {N1, N2, N3}, $\mathbf{R}_{\Rightarrow,\neg} =_{df} \mathbf{R}_{\Rightarrow}$ + {N2, N3}, $\mathbf{T} =_{df} \mathbf{T}_{\Rightarrow,\neg}$ + {C1~C3, D1~D3, DCD} + $\land$I, $\mathbf{E} =_{df} \mathbf{E}_{\Rightarrow,\neg}$ + {C1~C4, D1~D3, DCD} + $\land$I, $\mathbf{R} =_{df} \mathbf{R}_{\Rightarrow,\neg}$ + {C1~C3, D1~D3, DCD} + $\land$I, $\mathbf{Tc} =_{df} \mathbf{T}_{\Rightarrow,\neg}$ + {C3, C5~C10}, $\mathbf{Ec} =_{df} \mathbf{E}_{\Rightarrow,\neg}$ + {C3~C10}, $\mathbf{Rc} =_{df} \mathbf{R}_{\Rightarrow,\neg}$ + {C3, C5~C10}, $\mathbf{TQ} =_{df} \mathbf{T}$ + {IQ1~IQ5} + $\forall$I, $\mathbf{EQ} =_{df} \mathbf{E}$ + {IQ1~IQ5} + $\forall$I, $\mathbf{RQ} =_{df} \mathbf{R}$ + {IQ1~IQ5} + $\forall$I, $\mathbf{TcQ} =_{df} \mathbf{Tc}$ + {IQ1~IQ5} + $\forall$I, $\mathbf{EcQ} =_{df} \mathbf{Ec}$ + {IQ1~IQ5} + $\forall$I, $\mathbf{RcQ} =_{df} \mathbf{Rc}$ + {IQ1~IQ5} + $\forall$I. Here, $\mathbf{T}_{\Rightarrow}$, $\mathbf{E}_{\Rightarrow}$, and $\mathbf{R}_{\Rightarrow}$ are the purely implicational fragments of **T**, **E**, and **R**, respectively, and the relationship between $\mathbf{E}_{\Rightarrow}$ and $\mathbf{R}_{\Rightarrow}$ is known as $\mathbf{R}_{\Rightarrow} = \mathbf{E}_{\Rightarrow} + \mathrm{A} \Rightarrow \boldsymbol{L}\mathrm{A}$; $\mathbf{T}_{\Rightarrow,\neg}$, $\mathbf{E}_{\Rightarrow,\neg}$, and $\mathbf{R}_{\Rightarrow,\neg}$ are the implication-negation fragments of **T**, **E**, and **R**, respectively; **Tc**, **Ec**, **Rc, TcQ**, **EcQ**, and **RcQ** are strong relevant (relevance) logics.

The strong relevant logics can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning. Therefore, they can satisfy all three essential requirements for the universal core logic.

4. Temporal Relevant Logics

Time is one of the most fundamental notions in our cognition about the real world. The ability of representing and reasoning about temporal knowledge conceptually is one of the most intrinsic characteristics of human intelligence. Therefore, no account of reasoning can properly be considered to be complete if it does not say something about how we reason about change.

Classical temporal logics was established in order to represent and reason about notions, relations, and properties of time-related entities within a logical framework, and therefore to underlie temporal reasoning, i.e., reasoning about those propositions and formulas whose truth-values may depend on time. As a conservative extension of **CML**, they have remarkably expanded the uses of logic to reasoning about human (and hence computer) time-related activities [5, 16, 25, 27, 28]. However, because any of classical temporal logics is a classical conservative extension of **CML** in the sense that it is based on the classical account of validity and it represents the notion of conditional directly or indirectly by the material implication, no classical temporal logic can satisfy the three essential requirements.

The present author has proposed ***temporal relevant logics*** which are obtained by introducing the following temporal operators and related axiom schemata and inference rules into strong relevant logics [7].

Temporal operators: ***G*** (future-tense always or henceforth operator, ***G****A* means 'it will always be the case in the future from now that *A*'), ***H*** (past-tense always operator, ***H****A* means 'it has always been the case in the past up to now that *A*'), ***F*** (future-tense sometime or eventually operator, ***F****A* means 'it will be the case at least once in the future from now that *A*'), ***P*** (past-tense sometime operator, ***P****A* means 'it has been the case at least once in the past up to now that *A*'). Note that these temporal operators are not independent and can be defined as follows: $\boldsymbol{G}A =_{df} \neg\boldsymbol{F}\neg A$, $\boldsymbol{H}A =_{df} \neg\boldsymbol{P}\neg A$, $\boldsymbol{F}A =_{df} \neg\boldsymbol{G}\neg A$, $\boldsymbol{P}\mathrm{A} =_{df} \neg\boldsymbol{H}\neg A$.

Axiom schemata: T1: $\boldsymbol{G}(A\Rightarrow B)\Rightarrow(\boldsymbol{G}A\Rightarrow\boldsymbol{G}B)$, T2: $\boldsymbol{H}(A\Rightarrow B)\Rightarrow(\boldsymbol{H}A\Rightarrow\boldsymbol{H}B)$, T3: $A\Rightarrow\boldsymbol{G}(\boldsymbol{P}A)$, T4: $A\Rightarrow\boldsymbol{H}(\boldsymbol{F}A)$, T5: $\boldsymbol{G}A\Rightarrow\boldsymbol{G}(\boldsymbol{G}A)$, T6: $(\boldsymbol{F}A\wedge\boldsymbol{F}B)\Rightarrow\boldsymbol{F}(A\wedge\boldsymbol{F}B)\vee\boldsymbol{F}(A\wedge B)\vee\boldsymbol{F}(\boldsymbol{F}A\wedge B)$, T7: $(\boldsymbol{P}A\wedge\boldsymbol{P}B)\Rightarrow\boldsymbol{P}(A\wedge\boldsymbol{P}B)\vee\boldsymbol{P}(A\wedge B)\vee\boldsymbol{P}(\boldsymbol{P}A\wedge B)$, T8: $\boldsymbol{G}A\Rightarrow\boldsymbol{F}A$, T9: $\boldsymbol{H}A\Rightarrow\boldsymbol{P}A$, T10: $\boldsymbol{F}A\Rightarrow\boldsymbol{F}(\boldsymbol{F}A)$, T11: $(A\wedge\boldsymbol{H}A)\Rightarrow\boldsymbol{F}(\boldsymbol{H}A)$, T12: $(A\wedge\boldsymbol{G}A)\Rightarrow\boldsymbol{P}(\boldsymbol{G}A)$.

Inference rules: TG : from A to infer $\boldsymbol{G}A$ and $\boldsymbol{H}A$ (Temporal Generalization)

We can obtain minimal or weakest propositional temporal relevant logics as follows: $\mathbf{T_0Tc}$ = $\mathbf{Tc}$ + {T1~T4} + TG, $\mathbf{T_0Ec}$ = $\mathbf{Ec}$ + {T1~T4} + TG, $\mathbf{T_0Rc}$ = $\mathbf{Rc}$ + {T1~T4} + TG. Note that the minimal or weakest temporal classical logic $\mathbf{K_t}$ = all axiom schemata for **CML** + →E + {T1~T4} + TG. Other characteristic axioms such as T5~T12 that correspond to various assumptions about time can be added to $\mathbf{T_0Tc}$, $\mathbf{T_0Ec}$, and $\mathbf{T_0Rc}$ respectively to obtain various propositional temporal relevant logics. Various predicate temporal relevant logics then can be obtained by adding axiom schemata IQ1~IQ5 and inference rule ∀I into the propositional temporal relevant logics. For examples, minimal or weakest predicate temporal relevant logics are as follows: $\mathbf{T_0TcQ}$ = $\mathbf{T_0Tc}$ + {IQ1~IQ5} + ∀I, $\mathbf{T_0EcQ}$ = $\mathbf{T_0Ec}$ + {IQ1~IQ5} + ∀I, and $\mathbf{T_0RcQ}$ = $\mathbf{T_0Rc}$ + {IQ1~IQ5} + ∀I.

The temporal relevant logics can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and temporal reasoning (in particular, anticipatory reasoning). Therefore, they can be used as the fundamental logic to underlie reasoning about dynamics of a knowledge-based system where not only truth-vules of propositions and/or formulas but also relevant relationships between them may depend on time. Probably, the most important application of temporal relevant logics is to underlie anticipatory reasoning in prediction [7, 17, 24].

5. Deontic Relevant Logics

In general, the actual behavior (as it is) of a computing system in its running is somewhat different from the ideal (or normative) behavior (as it should be) of the system which is specified by requirements of the system. Therefore, to distinguish between ideal behavior and actual behavior of a computing system is important to defining what behavior is illegal and specifying what should be done if such illegal but possible behavior occurs. On the other hand, information security and assurance problems are intrinsically concerning the following questions: Who is authorized (permitted, or has right) to access and modify what information by what way from what machine or site in what time or state, and who does not be authorized to do so. Therefore, information security and assurance engineering is intrinsically an engineering discipline to deal with normative requirements and their implementation and verification (validation) techniques for secure information systems. Any approach to specifying and reasoning about information security and assurance cannot be considered to be adequate if it does not deal with normative notions explicitly, soundly and completely.

Deontic logic is a branch of philosophical logic to deal with normative notions such as obligation (ought), permission (permitted), and prohibition (may not) for underlying normative reasoning [3, 4, 18, 22]. Informally, it can also be considered as a logic to reason about ideal versus actual states or behaviour. It seems to be an adequate tool to specify and verify information security and assurance requirements. However, classical deontic logic has the problem of deontic paradoxes as well as the problem of implicational paradoxes. No classical deontic logic can satisfy the three essential requirements.

Tagawa and the present author have proposed ***deontic relevant logics*** based on strong relevant logics to remove those deontic paradoxes from classical deontic logic [26]. The deontic relevant logics are obtained by introducing the following deontic operators and related axiom schemata and inference rules into strong relevant logics.

Deontic operators: $\boldsymbol{O}$ (obligation operator, $\boldsymbol{O}A$ means "It is obligatory that A"), $\boldsymbol{P}$ (permission operator, $\boldsymbol{P}A =_{df} \neg\boldsymbol{O}(\neg A)$, $\boldsymbol{P}A$ means "It is permitted that A").

Axiom schemata: DR1: $\boldsymbol{O}(A\Rightarrow B)\Rightarrow(\boldsymbol{O}A\Rightarrow\boldsymbol{O}B)$, DR2: $\boldsymbol{O}A\Rightarrow\boldsymbol{P}A$, DR3: $\neg(\boldsymbol{O}A\wedge\boldsymbol{O}\neg A)$, DR4: $\boldsymbol{O}(A\wedge B)\Rightarrow(\boldsymbol{O}A\wedge\boldsymbol{O}B)$, DR5: $\boldsymbol{P}(A\wedge B)\Rightarrow(\boldsymbol{P}A\wedge\boldsymbol{P}B)$.

Inference rules: $\boldsymbol{O}$-necessitation: "if A is a logical theorem, then so is $\boldsymbol{O}A$" (Deontic Generalization)

We can obtain propositional deontic relevant logics as follows: **DTc** = **Tc** + {DR1~DR5} + $\boldsymbol{O}$-necessitation, **DEc** = **Ec** + {DR1~DR5} + $\boldsymbol{O}$-necessitation, **DRc** = **Rc** + {DR1~DR5} + $\boldsymbol{O}$-necessitation. Various predicate deontic relevant logics then can be obtained by adding axiom schemata IQ1~IQ5 and inference rule ∀I into the propositional deontic relevant logics.

The deontic relevant logics provide a formal language with normative notions which can be used as a formal specification language for specifying information security and assurance requirements. The deontic relevant logics also provide a sound logical basis for proving systems, and therefore it can be used as a analysis and verification tool for verifying information security and assurance requirements. The deontic relevant logics also provide a sound logical basis for reasoning systems, and therefore it can underlie reasoning about ideal versus actual, present versus potential states or behaviour of a computing system. Finally, the deontic relevant logics provide a unified, explicit, logical criterion for all the three parties of users, developers, and managers of information systems when they are involved in some legal actions or cases.

6. Spatial Relevant Logics

Space is another one of the most fundamental notions in our cognition about the real world. The ability of representing and reasoning about spatial knowledge conceptually is another one of the most intrinsic characteristics of human intelligence.

Classical spatial logics was proposed in order to deal with geometric and/or topological entities, notions, relations, and properties, and therefore to underlie spatial reasoning, i.e., reasoning about those propositions and formulas whose truth-values may depend on a location [10-13, 16, 25]. However, these existing spatial logics are classical conservative extensions of **CML** in the sense that they are based on the classical account of validity and they represent the notion of conditional directly or indirectly by the material implication. Therefore, these spatial logics cannot satisfy the three essential requirements for the fundamental logics.

We have propose a new family of relevant logic systems, named ***spatial relevant logic*** [9]. The logics are obtained by introducing region connection predicates and axiom schemata of RCC [10-13], point position predicates and axiom schemata, and point adjacency predicates and axiom schemata into predicate strong relevant logics.

Let $\{r_1, r_2, r_3, \ldots\}$ be a countably infinite set of individual variables, called ***region variables***. Atomic formulas of the form $\boldsymbol{C}(r_1, r_2)$ are read as 'region r_1 connects with region r_2.' Let $\{p_1, p_2, p_3, \ldots\}$ be a countably infinite set of individual variables, called ***point variables***. Atomic formulas of the form $\boldsymbol{I}(p_1, r_1)$ are read as 'point p_1 is included in region r_1.' Atomic formulas of the form $\boldsymbol{Id}(p_1, p_2)$ are read as 'point p_1 is identical with p_2.' Atomic formulas of the form $\boldsymbol{Arc}(p_1, p_2)$ are read as 'points p_1, p_2 are adjacent such that there is an arc from point p_1 to point p_2, or more simply, points p_1 is adjacent to point p_2.' Note that an arc has a direction. Atomic formulas of the form $\boldsymbol{Path}(p_1, p_2)$ are read as 'there is a directed path from point p_1 to point p_2.' Here, $\boldsymbol{C}(r_1, r_2)$, $\boldsymbol{I}(p_1, r_1)$, $\boldsymbol{Id}(p_1, p_2)$,

Arc(p_1, p_2), and ***Path***(p_1, p_2) are primitive binary predicates to represent relationships between geometric objects. Note that here we use a many-sorted language.

The logical connectives, region connection predicates, point position predicates, point adjacency predicates, axiom schemata, and inference rules are as follows:

Primitive binary predicate: ***C*** (connection, ***C***(r_1, r_2) means 'r_1 connects with r_2'), ***I*** (inclusion, ***I***(p_1, r_1) means 'p_1 is included in r_1'), ***Id*** (the same point, ***Id***(p_1, p_2) means 'p_1 is identical with p_2'), ***Arc*** (arc, ***Arc***(p_1, p_2) means 'p_1 is adjacent to p_2'), ***Path*** (path, ***Path***(p_1, p_2) means 'there is a directed path from p_1 to p_2').

Defined binary predicates: ***DC***$(r_1, r_2) =_{df} \neg$***C***(r_1, r_2) (***DC***(r_1, r_2) means 'r_1 is disconnected from r_2'), ***Pa***$(r_1, r_2) =_{df} \forall r_3($***C***$(r_3, r_1)\Rightarrow$***C***$(r_3, r_2))$ (***Pa***(r_1, r_2) means 'r_1 is a part of r_2'), ***PrPa***$(r_1, r_2) =_{df}$ ***Pa***$(r_1, r_2)\wedge(\neg$***Pa***$(r_2, r_1))$ (***PrPa***(r_1, r_2) means 'r_1 is a proper part of r_2'), ***EQ***$(r_1, r_2) =_{df}$ ***Pa***$(r_1, r_2)\wedge$***Pa***(r_2, r_1) (***EQ***(r_1, r_2) means 'r_1 is identical with r_2'), ***O***$(r_1, r_2) =_{df} \exists r_3($***Pa***$(r_3, r_1)\wedge$***Pa***$(r_3, r_2))$ (***O***(r_1, r_2) means 'r_1 overlaps r_2'), ***DR***$(r_1, r_2) =_{df} \neg$***O***(r_1, r_2) (***DR***(r_1, r_2) means 'r_1 is discrete from r_2'), ***PaO***$(r_1, r_2) =_{df}$ ***O***$(r_1, r_2)\wedge(\neg$***Pa***$(r_1, r_2))\wedge(\neg$***Pa***$(r_2, r_1))$ (***PaO***(r_1, r_2) means 'r_1 partially overlaps r_2'), ***EC***$(r_1, r_2) =_{df}$ ***C***$(r_1, r_2)\wedge(\neg$***O***$(r_1, r_2))$ (***EC***(r_1, r_2) means 'r_1 is externally connected to r_2'), ***TPrPa***$(r_1, r_2) =_{df}$ ***PrPa***$(r_1, r_2)\wedge\exists r_3($***EC***$(r_3, r_1)\wedge$***EC***$(r_3, r_2))$ (***TPrPa***(r_1, r_2) means 'r_1 is a tangential proper part of r_2'), ***NTPrPa***$(r_1, r_2) =_{df}$ ***PrPa***$(r_1, r_2)\wedge(\neg\exists r_3($***EC***$(r_3, r_1)\wedge$***EC***$(r_3, r_2)))$ (***NTPrPa***(r_1, r_2) means 'r_1 is a nontangential proper part of r_2').

Axiom schemata: RCC1: $\forall r_1\forall r_2($***C***$(r_1, r_2)\Rightarrow$***C***$(r_2, r_1))$, RCC2: $\forall r_1($***C***$(r_1, r_1))$, PRCC1: $\forall p_1\forall r_1\forall r_2(($***I***$(p_1, r_1)\wedge$***DC***$(r_1, r_2))\Rightarrow\neg$***I***$(p_1, r_2))$, PRCC2: $\forall p_1\forall r_1\forall r_2(($***I***$(p_1, r_1)\wedge$***Pa***$(r_1, r_2))\Rightarrow$ ***I***$(p_1, r_2))$, PRCC3: $\forall r_1\forall r_2($***O***$(r_1, r_2)\Rightarrow\exists p_1($***I***$(p_1, r_1)\wedge$***I***$(p_1, r_2)))$, PRCC4: $\forall r_1\forall r_2($***PaO***(r_1, r_2) $\Rightarrow\exists p_1($***I***$(p_1, r_1)\wedge$***I***$(p_1, r_2))\wedge\exists p_2($***I***$(p_2, r_1)\wedge\neg$***I***$(p_2, r_2))\wedge\exists p_3(\neg$***I***$(p_3, r_1)\wedge$***I***$(p_3, r_2)))$, PRCC5: $\forall r_1\forall r_2($***EC***$(r_1, r_2)\Rightarrow\exists p_1($***I***$(p_1, r_1)\wedge$***I***$(p_1, r_2)\wedge\forall p_2(\neg$***Id***$(p_2, p_1)\Rightarrow\neg$***I***$(p_2, r_1)\wedge\neg$***I***$(p_2, r_2)))$, PRCC6: $\forall p_1\forall r_1\forall r_2(($***I***$(p_1, r_1)\wedge$***TPrPa***$(r_1, r_2))\Rightarrow$***I***$(p_1, r_2))$, PRCC7: $\forall p_1\forall r_1\forall r_2(($***I***$(p_1, r_1)\wedge$ ***NTPrPa***$(r_1, r_2))\Rightarrow$***I***$(p_1, r_2))$, APC1: $\forall p_1\forall p_2($***Arc***$(p_1, p_2)\Rightarrow$***Path***$(p_1, p_2))$, APC2: $\forall p_1\forall p_2\forall p_3(($***Path***$(p_1, p_2)\wedge$***Path***$(p_2, p_3))\Rightarrow$***Path***$(p_1, p_3))$.

We can now obtain some spatial relevant logics as follows: **RTcQ** $=_{df}$ **TcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, **REcQ** $=_{df}$ **EcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, **RRcQ** $=_{df}$ **RcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}

The spatial relevant logics can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and spatial reasoning. Therefore, they can be used to underlie reasoning about geometric and/or topological entities, notions, relations, and their properties.

7. Spatio-temporal Relevant Logics

To represent, specify, verify, and reason about spatial objects and relationships among them that may change over time, e.g., mobile agents, a right fundamental logic to underlie both spatial reasoning and temporal reasoning is indispensable [16, 25].

The present author has proposed a new family of relevant logic systems, named ***spatio-temporal relevant logic*** [8]. The logics are obtained by introducing region connection predicates and axiom schemata of RCC, point position predicates and axiom schemata, and point adjacency predicates and axiom schemata into various predicate temporal relevant logics. Therefore, they are conservative extensions of strong relevant logics, temporal relevant logics, and spatial relevant logics. For examples: **ST$_0$TcQ** = **T$_0$TcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, **ST$_0$EcQ** = **T$_0$EcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, **ST$_0$RcQ** = **T$_0$RcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}.

The spatio-temporal relevant logics have the following characteristics. First, as conservative extensions of strong relevant logics satisfying the strong relevance principle, the logics underlie relevant reasoning as well as truth-preserving reasoning in the sense of conditional, ampliative reasoning, paracomplete reasoning, and paraconsistent reasoning. Second, the logics underlie spatial reasoning and temporal reasoning. We can select any one of them according our purpose in an application from various aspects of relevance, temporality, and spatiality.

The spatio-temporal relevant logics themselves have various possible applications. For example, because the logics can underlie relevant, truth-preserving, ampliative, paracomplete, paraconsistent, spatial, and temporal reasoning, they provide us with criteria of logical validity for reasoning about behavior of mobile agents with incomplete or even inconsistent knowledge acting concurrently in spatial regions changing over time. On the other hand, the spatio-temporal relevant logics provide us with a foundation for constructing more powerful applied logics in order to deal with various notions and issues in application systems. For examples, we can add epistemic operators and related axiom schemata into the logics in order to reason about interaction among agents as well as epistemic states of agents; we can also add deontic operators and related axiom schemata into the logics in order to reason about information security and assurance in mobile multi-agent systems.

8. Concluding Remarks

We have shown that strong relevant logic can be used as the universal basis to construct various applied logics to underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and other special reasoning. We presented our experiences on constructions of temporal relevant logics, deontic relevant logics, spatial relevant logics, and spatial-temporal relevant logics.

The work presented in this paper is our first step for establishing a fundamental logic system as a universal basis to construct various applied logics. There are many challenging theoretical and technical problems that have to be solved in order to apply the strong relevant logics and their various extensions to practices in the real world. A challenging open problem, named '***NRT problem***' by the present author, is as follows: Although it is necessary to deal with all notions of normativeness, relevance, and temporality explicitly and soundly in specifying, verifying, and reasoning about information systems, until now there is no formal logic system can be used as a unified logical basis for the purpose. Can we establish a unified formal logic system, which takes all notions of normativeness, relevance, and temporality into account, to underlie specifying, verifying, and reasoning about information systems satisfactorily?

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