Relevant Reasoning Based on Strong Relevant Logic

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"Logic is the science of sciences, and the art of arts."

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-- John Duns Scotus, 13th century.

"Nothing can be more important than the art of formal reasoning according to true logic."

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-- Gottfried Wilhelm Leibniz

"Logic is the basis for all other sciences"

- "There is a special discipline, called logic, which is considered to be the basis for all other sciences."
  "Logic evolved into an independent science long ago, earlier even than arithmetic and geometry."
  -- A. Tarski, 1941.
- "Mathematical Logic, it is a science prior to all others, which contains the ideas and principles underlying all sciences."
  -- K. Gödel, 1944.

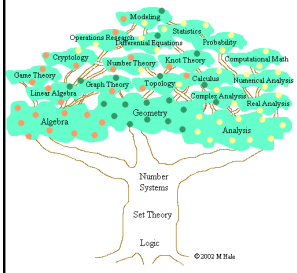
"Logic is the basis for all other sciences"

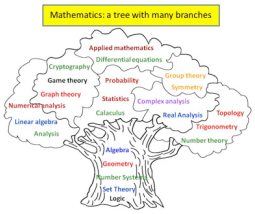
- "The development of Western Science has been based on two great achievements, the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility of finding out causal relationships by systematic experiment (at the Renaissance)."
  -- A. Einstein, 1953.

"Fields of Science and Technology" by UNESCO

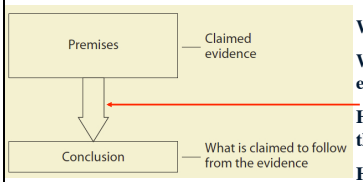
- "Proposed International Standard Nomenclature for Fields of Science and Technology," UNESCO/ NS/ROU/257 rev.1, 1988.
- 11. Logic, 12. Mathematics
- 21. Astronomy and Astrophysics, 22. Physics, 23. Chemistry, 24. Life Sciences, 25. Earth and Space Science
- 31. Agricultural Sciences, 32. Medical Sciences, 33. Technological Sciences
- 1203. Computer Science
  3304. Computer Technology

Logic as the Fundamental Basis for all Mathematics





Logic: What Is It?



[From P. J. Hurley, "A Concise Introduction to Logic"]

What entails what?

What follows from what?

Why? What are the evaluation criteria?

How to establish/define the evaluation criteria?

How to evaluate arguments/reasoning?

It is LOGIC to answer these fundamental questions.

Relevant Reasoning Based on Strong Relevant Logic

- Introduction
- Fallacies of Relevance
- Relevant (Relevance) Logic: What Is It and Why Study It?
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- Various Applications of Relevant Reasoning Based on SRL
- Research Directions and Challenging Problems
- Bibliography

An Example [E. D. Mares, 2004]

♣ **A short proof of Fermat's Last Theorem**

- Since 1993, when Andrew Wiles completed his difficult proof of Fermat's Last Theorem, mathematicians have wanted a shorter, easier proof.
- Suppose when someone addressing a conference of number theorists, suggests the following proof of the theorem:

  The sky is blue

  ∴ There is no integer n greater than or equal to 3 such that for any non-zero integers x, y, z, $x^n = y^n + z^n$.
- This proof would not be well received. It is a non-sequitur - its conclusion does not, in any intuitive sense, follow from its premise. It is a bad proof.

♣ **The classical validity and soundness of the proof**

- Validity: It is impossible for the premises all to be true and the conclusion false.
- Soundness: The premise is true.

A Modification of Mares's Example [J. Cheng, 2005]

♣ **Classical valid proofs of Fermat's Last Theorem**

- Taniyama-Shimura conjecture & The sky is blue & …

  ∴ There is no integer n greater than or equal to 3 such that for any non-zero integers x, y, z, $x^n = y^n + z^n$.
- Taniyama-Shimura conjecture

  ∴ There is no integer n greater than or equal to 3 such that for any non-zero integers x, y, z, $x^n = y^n + z^n$ or The sky is blue or … .

♣ **The classical and weak-relevant validity of the proofs**

- If a proof is valid, then a new proof that is obtained by adding any conjunct into the premises of the proof is still valid.
- If a proof is valid, then a new proof that is obtained by adding any disjunct into the conclusion of the proof is still valid.

The Propositions This Tutorial Will Show You

♣ **Logic in general**

- Reasoning is intrinsically different from proving.
- The major role of logic is to underlie valid reasoning rather than to provide a formal language for representation.
- The notion of conditional is the heart of logic (and hence, mathematics).
- Any valid reasoning in scientific discoveries as well as our everyday lives should be both truth-preserving and relevant in the sense of conditional.

The Propositions This Tutorial Will Show You

♣ **Relevant reasoning and relevant (relevance) logic**

- Relevant (relevance) logic is the logic for reasoning (and hence, for discovery and prediction) rather than proving; while classical mathematical logic (and hence, its various classical conservative extensions) is the logic for (describing) proving rather than reasoning.
- Strong relevance in the sense of conditional is indispensable to any valid reasoning for discovery and prediction.
- You should invoke relevant reasoning, if you really want to discover some new things or predict some future events by reasoning.
- You indeed did relevant reasoning when you discovered some new things or predicted some future events.

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Fallacies of Relevance

♣ **Fallacies of relevance and non sequiturs**

- Fallacies of relevance occur when the premises of an argument are simply not relevant to the conclusion.
- Such arguments often called *non sequiturs* (from the Latin phrase 'non sequitur', meaning "it does not follow")

♣ **High inductive probability with low relevance**

- The simplest cases of high inductive probability with low relevance occur among arguments whose conclusions are *logically necessary*.
- A *logically necessary* statement is a statement whose very conception or meaning requires its truth; its falsehood, in other words, is logically impossible.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ **Ad horninem arguments**

- *Ad horninem* arguments try to discredit a claim or proposal by attacking its proponents instead of providing a reasoned examination of the proposal itself.
- 'Ad horninem' means "*against the person*."

♣ **Five varieties of ad horninem arguments**

- Ad hominem abusive arguments.
- Guilt by association arguments.
- Tu quoque ("you too") arguments
- Vested interest arguments.
- Circumstantial ad horninern arguments.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ **Ad horninem abusive arguments**

- *Ad horninem abusive* arguments attack a person's age, character, family, gender, ethnicity, social or economic status, personality, appearance, dress, behavior, or professional, political, or religious affiliations.
- The implication is that there is no reason to take the person's views seriously.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ **Example**

- Jones advocates fluoridation of the city water supply.
- Jones is a convicted thief.
- Therefore, we should not fluoridate the city water supply.

♣ **What is wrong?**

- Even if Jones is a convicted thief, this has no bearing on whether the water supply should be fluoridated.
- To dismiss Jones' view simply because Jones is reprehensible is to commit the ad honzinern abusive fallacy.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ Guilt by association arguments

- The fallacy of *guilt by association* is the attempt to repudiate a claim by attacking not the claim's proponent, but the company he or she keeps, or by questioning the reputations of those with whom he or she agrees.
- This is also known as poisoning the well, which suggests a fitting scenario.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ Example

- Jones advocates fluoridation of the city water supply.
- Jones spends much of his free time hanging around with known criminals, drug addicts, and deviants.
- Therefore, we should not fluoridate the city water supply.

♣ What is wrong?

- The premises are irrelevant to the conclusion; even if Jones has detestable friends, what he advocates may well be true.
- Notice that it would not be a fitting reply merely to contest the second premise ("Jones actually spends most of his free time helping the elderly and performing volunteer work in hospitals").
- The central logical issue here is not whether Jones is the victim of a smear, but the failure to produce any premises germane to the conclusion.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ Tu quoque ("you too") arguments

- *Tu quoque* ("you too") arguments attempt to refute a claim by attacking its proponent on the grounds that he or she is a hypocrite, upholds a double standard of conduct, or is selective and therefore inconsistent in enforcing a principle.
- The implication is that the arguer is unqualified to make the claim, and hence that there is no reason to take the claim seriously.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ Example

- Jones believes we should abstain from liquor.
- Jones is a habitual drunkard.
- Therefore, we should not abstain from liquor.

♣ What is wrong?

- Jones' actions have no bearing on the truth or falsity of his belief, even if he holds the belief hypocritically; the argument commits a tu quoyue fallacy.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ Vested interest arguments

- *Vested interest* arguments attempt to refute a claim by arguing that its proponents are motivated by the desire to gain something (or avoid losing something).
- The implication is that were it not for this vested interest, the claim's proponents would hold a different view, and hence that we should discount their argument.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

♣ Example

- Jones supports the fluoridation bill pending in Congress.
- He does so because he owns a major fluoridation firm, which will reap huge dividends if the bill passes.
- Therefore, we should not support this bill.

♣ What is wrong?

- The premises are irrelevant to the conclusion.
- Fluoridation may well be justified independently of Jones' allegedly selfish motives.
- The argument commits the vested interest fallacy.
- Whether Jones stands to gain or lose from fluoridation is immaterial.
- What counts is whether fluoridation is hygienically desirable, whether it is cost-effective, and so forth.

Fallacies of Relevance: Ad Horninem Arguments [N&R&V]

- ♣ Circumstantial ad horninern arguments
  - ◆ *Circumstantial ad horninern* arguments are the attempt to refute a claim by arguing that its proponents endorse two or more conflicting propositions.
  - ◆ The implication is that we may therefore safely disregard one or all of those propositions.
- ♣ Example
  - ◆ Jones says he abhors all forms of superstition.
  - ◆ Jones also says that breaking a mirror brings bad luck.
  - ◆ Therefore, there probably is something to superstition after all.
- ♣ What is wrong?
  - ◆ Jones' claims, whether consistent or not, have by themselves no bearing on the truth of the conclusion.

Fallacies of Relevance: Ad Baculum Arguments [N&R&V]

- ♣ Ad baculum arguments
  - ◆ *Ad baculum* arguments (*appeals to force*, *appeals to the stick*) are attempts to establish a conclusion by threat or intimidation.
- ♣ Example
  - ◆ If you do not vote for me, Ms. Jones, I'll tell everybody you are a liar.
  - ◆ Therefore, you ought to vote for me.
- ♣ What is wrong?
  - ◆ The premise is irrelevant to the justification of the conclusion.
  - ◆ Note that it makes no difference how Jones responds to this threat; even if she capitulates, that does not change the fact that this sort of "reasoning" is logically unacceptable.

Fallacies of Relevance: Ad Verecundiam Arguments [N&R&V]

- ♣ Ad verecundiam arguments
  - ◆ *Ad verecundiam* arguments (*appeals to authority*) occur when we accept (or reject) a claim merely because of the prestige, status, or respect we accord its proponents (or opponents).
- ♣ Example
  - ◆ My teacher says that I should be proud to be an American.
  - ◆ Therefore, I should be proud to be an American.
- ♣ What is wrong?
  - ◆ Without some further evidence that the teacher's statement is correct or justified, the premise is simply irrelevant to the conclusion.
  - ◆ Saying something does not make it so, no matter how eminent the speaker.

Fallacies of Relevance: Ad Populum Arguments [N&R&V]

- ♣ Ad populum arguments
  - ◆ *Ad populum* arguments (*appeals to the people*) occur when we infer a conclusion merely on the grounds that most people accept it.
  - ◆ This fallacy has the form: X says that P; therefore, P.
- ♣ Example
  - ◆ Everybody believes that premarital sex is wrong.
  - ◆ Therefore, premarital sex is wrong.
- ♣ What is wrong?
  - ◆ The issue is not whether "everybody" in fact believes this (though that premise is in fact false), but what we may infer from such a premise.
  - ◆ The argument fails to establish any relevant connection between premises and conclusion and hence it is fallacious.

Fallacies of Relevance: Ad Misericordiam Arguments [N&R&V]

- ♣ Ad misericordiam arguments
  - ◆ *Ad misericordiam* arguments (*appeals to pity*) ask us to excuse or forgive an action on the grounds of extenuating circumstances.
  - ◆ They seek clemency for breaches of duty, or sympathy for someone whose poor conduct or noncompliance with a rule is already established.
- ♣ Example
  - ◆ Oh, officer, you see my baby here was crying for some candy and I took her to the candy store before I came back to my car.
  - ◆ Therefore, you should not give me a parking ticket.
- ♣ What is wrong?
  - ◆ The arguer appeals to the officer's pity, but the appeal is clearly irrelevant.

Fallacies of Relevance: Ad Ignorantiam Arguments [N&R&V]

- ♣ Ad ignorantiam arguments
  - ◆ *Ad ignorantiam* arguments (*appeals to ignorance*) have one of the following two forms:
  - ◆ It has not been proved that P; therefore, NOT P.
  - ◆ It has not been proved that NOT P; therefore, P.
- ♣ Examples
  - ◆ No one has ever proved the God exists; therefore, God does not exist.
  - ◆ No one has ever proved the God does not exist; therefore, God exists.
- ♣ What is wrong?
  - ◆ Nothing about the existence of God follows from our inability to prove God's existence or nonexistence.

Fallacies of Relevance: Ignoratio Elenchi Arguments [N&R&V]

- **Ignoratio elenchi arguments**
  - *Ignoratio elenchi* (*missing the point*) occurs when the premises of an argument warrant a different conclusion from the one the arguer draws.
  - This can be very embarrassing, especially if the conclusion which does follow contradicts or undermines the one actually drawn.
  - The expression "missing the point" is also used as a general catchphrase to describe the fallacies of relevance.
  - However, we shall reserve it for the precise mistake just mentioned.

Fallacies of Relevance: Ignoratio Elenchi Arguments [N&R&V]

- **Example**
  - Any amount of inflation is bad for the economy.
  - Last month inflation was running at an annual rate of 10 percent.
  - This month the inflation rate is only 7 percent.
  - Therefore, the economy is on the upswing.
- **What is wrong?**
  - Given the premises, what follows is only that the rate of inflation is slowing down.
  - Inflation is still occurring (i.e., things are still getting worse), but more slowly than before.
  - This is very different from maintaining that the economy is improving; indeed, it suggests the opposite conclusion.

Fallacies of Relevance: Red Herring Arguments [N&R&V]

- **Red herring arguments**
  - A *red herring* is an extraneous or tangential matter used purely to divert attention away from the issue posed by an argument. (The phrase stems from a method used to train hunting dogs to track down a scent.)
  - Because it is irrelevant, a red herring contributes nothing to an argument, though it may mislead its audience into thinking otherwise.
  - Red herrings are rhetorical devices. They enable those who use them to mask other defects hampering an argument, and thus to evade the real issue.

Fallacies of Relevance: Red Herring Arguments [N&R&V]

- **Example**
  - Some members of the police force may be corrupt, but there are corrupt politicians, corrupt plumbers, corrupt salespeople, and even corrupt preachers.
  - There are also lots of honest cops on the job.
  - Therefore, let's put police corruption in perspective (the implication being, of course, that police corruption is not as bad as it may seem).
- **What is wrong?**
  - Rhetorical grandiloquence is here employed to throw us "off the scent" of the real issue, which is what to do about police who accept bribes.
  - The argument attempts to lull its audience into complacency about this issue.

Fallacies of Relevance: A Summary [Layman]

- **Ad horninem (against the person)**
  - Premises: Instead of providing a rational critique of a statement (or argument), attack the person who advances it.
  - Conclusion: The statement is false or dubious. (Or the argument is unsound or uncogent.)
- **Straw man fallacy**
  - Premises: A misrepresentation of the view is false.
  - Conclusion: The view itself is false.
- **Ad baculum (appeals to force)**
  - Premises: You can avoid harm by accepting this statement.
  - Conclusion: This statement is true.

Fallacies of Relevance: A Summary [Layman]

- **Ad populum (appeals to the people)**
  - Premises: You will be accepted or valued if you believe this statement.
  - Conclusion: This statement is true.
- **Ad misericordiam (appeals to pity)**
  - Premises: You have reason to pity this person (or group).
  - Conclusion: You should do X for the benefit of this person (or group), although doing X is not called for logically by the reason given.

Fallacies of Relevance: A Summary [Layman]

- ♣ Ad ignorantiam (appeals to ignorance)
  - Premises: This statement has not been proven true.
  - Conclusion: This statement is false (or may be reasonably believed false).
  - Premises: This statement has not been proven false.
  - Conclusion: This statement is true (or may be reasonably believed true).

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Fundamental Principles/Assumptions Underlying CML

- ♣ The classical abstraction
  - The only properties of a proposition that matter to logic are its form and its truth-value.
- ♣ The Fregean assumption
  - The truth-value of a proposition is determined by its form and the truth-values of its constituents.
- ♣ The principle of bivalence
  - There are exactly two truth-values, TRUE and FALSE. Every declarative sentence has one or other, but not both, of these truth-values.

Fundamental Principles/Assumptions Underlying CML

- ♣ The classical account of validity (CAV)
  - An argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false.
- ♣ Notes
  - Usually, classical mathematical logic (CML) is often called to be 'classical' in the sense of the principle of bivalence.
  - From the viewpoint of conditional (entailment), it is the CAV that makes CML 'classical'.

Fundamental Principles/Assumptions Underlying CML

- ♣ Fallacies of relevance and CML
  - CML regards all fallacies of relevance to be informal fallacies but NOT formal fallacies, and therefore, CML does NOT regard fallacies of relevance as the object of study.
- ♣ Facts
  - CML includes a lot of fallacies of relevance (e.g., implicational paradoxes and so on) as logical theorems, and therefore, it cannot distinguish those fallacies of relevance from its logical theorems.
  - "many of the most acute logicians in the past thirty years have marched under a philosophical banner reading 'Down with relevance, meanings, and intensions generally!' " [A&B-E1-1975]

The Notion of Conditional (Entailment) in CML

- ♣ The notion of material implication [Philo of Megara, 400 B.C.]
  - The notion of conditional is represented in CML by the extensional notion of *material implication* (denoted by '→' or '⊃') which is defined as an extensional truth-functional connective as follows:

$$A \rightarrow B =_{df} \neg(A \wedge \neg B), \quad A \rightarrow B =_{df} \neg A \vee B$$

- ♣ Notes
  - The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication.
  - '$\neg(A \wedge \neg B)$' and/or '$\neg A \vee B$' is necessary but not sufficient to a conditional A⇒B, because the relevance between A and B is not accounted.

The Notion of Conditional (Entailment) in CML

♣ **Model-theoretical deduction theorems in CML**

- $\Gamma \cup \{A\} \models_{CML} B$ IFF $\Gamma \models_{CML} A \rightarrow B$
- $\{A\} \models_{CML} B$ IFF $\models_{CML} A \rightarrow B$
- $\Gamma \cup \{A_1, ..., A_n\} \models_{CML} B$ IFF $\Gamma \models_{CML} A_1 \rightarrow (\ldots (A_n \rightarrow B) \ldots)$
- $\Gamma \cup \{A_1, ..., A_n\} \models_{CML} B$ IFF $\Gamma \models_{CML} (A_1 \wedge \ldots \wedge A_n) \rightarrow B$

♣ **Proof-theoretical deduction theorems in CML**

- $\Gamma \cup \{A\} \vdash_{CML} B$ IFF $\Gamma \vdash_{CML} A \rightarrow B$
- $\{A\} \vdash_{CML} B$ IFF $\vdash_{CML} A \rightarrow B$
- $\Gamma \cup \{A_1, ..., A_n\} \vdash_{CML} B$ IFF $\Gamma \vdash_{CML} A_1 \rightarrow (\ldots (A_n \rightarrow B) \ldots)$
- $\Gamma \cup \{A_1, ..., A_n\} \vdash_{CML} B$ IFF $\Gamma \vdash_{CML} (A_1 \wedge \ldots \wedge A_n) \rightarrow B$

♣ **Note**

- The notion of material implication is "equivalent" to the logical consequence relation '$\models_{CML}$' and/or '$\vdash_{CML}$'.

Comparison of Conditional and Material Implication

♣ **The notion of conditional**

- The notion of conditional is intrinsically intensional but not truth-functional.
- The notion of conditional requires that there is a necessarily relevant and conditional relation between its antecedent and consequent.
- The truth of a conditional depends not only on the truth of its antecedent and consequent but also, and more essentially, on a necessarily relevant and conditional relation between them.
- The truth of the consequent (or the falsity of the antecedent) of a conditional is by itself insufficient for the truth of that conditional.

Comparison of Conditional and Material Implication

♣ **The notion of material implication**

- The notion of material implication is no more than an extensional truth-function of its antecedent and consequent, and therefore, the truth of a material implication depends totally on the truth of its antecedent and consequent.
- The notion of material implication does not require that there is a necessarily relevant and conditional relation between its antecedent and consequent.
- The truth-value of a material implication depends only on the truth-values of its antecedent and consequent, without regard to any relevance between them.
- The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication.

Comparison of Conditional and Material Implication

antecedent A	consequent B	necessarily relevant relation between A and B	conditional 'if A then B' $A \Rightarrow B$	material implication 'A implies B' $A \rightarrow B$
T	T	Existence	T	T
T	T	Not existence	F	T
T	F	Existence	F	F
T	F	Not existence	F	F
F	T	Existence	T	T
F	T	Not existence	F	T
F	F	Existence	T	T
F	F	Not existence	F	T

Paradoxes of Material Implication in CML

♣ **The problem of implicational paradox**

- If one considers the material implication as the notion of conditional and considers every logical theorem of CML as a valid reasoning form or entailment, then a great number of logical theorems of CML present some paradoxical properties and therefore they have been referred to in the literature as "*implicational paradoxes*."

♣ **Note**

- "If one considers …… and considers ……, then …… ."
- It is to think of the notion of material implication as the notion of conditional, or in other words, it is to use material implication in the sense of conditional, that leads to the problem of implicational paradoxes.
- If we use a material implication as an extensional truth-value function (and hence not conditional) in the sense of its original definition in CML, then no problem occurs.

Paradoxes of Material Implication in CML

♣ **Paradoxes of material implication as empirical conditionals**

snow is white → 1 + 1 = 2
snow is black → 1 + 1 = 2
snow is black → 1 + 1 = 3

♣ **Paradoxes of material implication as entailments**

- $A \rightarrow (B \rightarrow A)$, $B \rightarrow (\neg A \vee A)$, $\neg A \rightarrow (A \rightarrow B)$, $(\neg A \wedge A) \rightarrow B$ (ECQ!), $(A \rightarrow B) \vee (\neg A \rightarrow B)$, $(A \rightarrow B) \vee (A \rightarrow \neg B)$, $(A \rightarrow B) \vee (B \rightarrow A)$, $((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
- $B \rightarrow (\neg A \vee A)$, $(\neg A \wedge A) \rightarrow B$, Nothing is shared by antecedent and consequent, i.e., they are each other NOT relevant at all!
- Do you think that from 'if A and B then C' you can say 'if A then C or if B then C'?

Implicational Paradoxes: Problems and Results

♣ **Necessary but not sufficient to the notion of conditional**

- ‘$\neg(A\wedge\neg B)$’ or ‘$\neg A\vee B$’ (definitions of material implication ‘$A\rightarrow B$’, CAV) is necessary but not sufficient to the notion of conditional ‘$A\Rightarrow B$’ because the relevance between A and B, another necessary condition required by conditional, is not accounted.

♣ **Necessary but not sufficient to the notion of conditional**

- The notion of material implication cannot be used for distinguishing conditionals from implicational statements.

Implicational Paradoxes: Problems and Results

♣ **CML cannot underlie relevant and truth-preserving in the sense of conditional**

- In the framework of CML (or any of its classical conservative extensions), even if a reasoning is classically valid, both the relevance relationship between its premises and conclusion and the truth of its conclusion in the sense of conditional cannot be guaranteed necessarily.

♣ **Reason: material implication and implicational paradoxes**

- We cannot directly accept a conclusion of a reasoning with implicational paradoxes of entailment as a relevant and true conclusion in the sense of conditional, even if all premises of the reasoning are true and the conclusion is true in the sense of material implication.

Implicational Paradoxes: Problems and Results

♣ **Examples**

- From A, we can infer $B\rightarrow A$, $C\rightarrow A$, ... where B, C, ... are arbitrary formulas, by using logical axiom $A\rightarrow(B\rightarrow A)$ of CML and Modus Ponens for material implication (from A and $A\rightarrow B$ to infer B).
- However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be considered to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B and A, C and A, …, and therefore we cannot say ‘if B then A’, ‘if C then A’, and so on.

Implicational Paradoxes: Problems and Results

♣ **CML cannot underlie ampliative reasoning**

- Any reasoning based on CML (or any of its classical conservative extensions) is circular and/or tautological but not ampliative.

♣ **Reason: material implication is an extensional truth-function**

- Since any material implication is an extensional truth-function of its antecedent and consequent, the truth of a material implication depends totally on the truth of its antecedent and consequent, i.e., one cannot determine the truth of a material implication without knowing truths of its antecedent and consequent.
- On the other hand, the truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication. However, when we reason, we do not know the truth of the consequent and do not use false antecedent.

Implicational Paradoxes: Problems and Results

♣ **Examples (reasoning by Modus Ponens for material implication)**

- Modus Ponens: If A holds then B holds, now A holds, therefore B holds.
- Before the reasoning is performed, we do not know whether B holds or not. (If we do, we do not need reasoning at all.)
- Modus Ponens in CML: From A and $A\rightarrow B$ to infer B.
- According to the extensional truth-functional semantics of the material implication, if we know ‘A is true’ but do not know the truth-value of B, then we cannot decide the truth-value of ‘$A\rightarrow B$’.
- In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is performed!

Implicational Paradoxes: Problems and Results

♣ **Paraconsistent logic (NOT allow ECQ)**

- For a paraconsistent logic with Modus Ponens as an inference rule, the paraconsistence requires that the logic does not have $(\neg A\wedge A)\Rightarrow B$ as a logical theorem where A and B are any two different formulas and ‘$\Rightarrow$’ is the notion of conditional used in Modus Ponens.
- If a logic is not paraconsistent, then infinite propositions (even negations of those logical theorems of the logic) may be reasoned out based on the logic from a set of premises that directly or indirectly include a contradiction.

♣ **Paraconsistent reasoning based on CML is impossible**

- CML (or any of its classical conservative extensions) is explosive but not paraconsistent.
- CML uses Modus Ponens for material implication as its inference rule, and has “$(\neg A\wedge A)\rightarrow B$” as a logical theorem.

The Notion of Conditional in Lewis's Modal Logics

♣ Lewis's work on modal logic

- The main aim of Lewis's work beginning in 1912 on the establishment of modern modal logic was to find a satisfactory theory of implication which is better than CML in that it can avoid those implicational paradoxes.

The Notion of Conditional in Lewis's Modal Logics

♣ Lewis's strict implication

- Lewis's idea is to define the notion of conditional by using necessity operator.
- $A > B =_{df} L(A \rightarrow B)$ where L is necessity operator

♣ Paradoxes of strict implication

- Lewis's plan was not successful in the sense that some implicational paradoxes in terms of strict implication remained in his modal logics (S1 ~ S5).
- Ex.

$LA \rightarrow (B > A)$	$[\, A \rightarrow (B \rightarrow A)\,]$
$L\neg A \rightarrow (A > B)$	$[\, \neg A \rightarrow (A \rightarrow B)\,]$
$(\neg A \wedge A) > B$	$[\, (\neg A \wedge A) \rightarrow B\,]$
$B > (\neg A \vee A)$	$[\, B \rightarrow (\neg A \vee A)\,]$

Sugihara's Characterization of Implication Paradoxes

♣ Strongest and weakest formulas [Sugihara, 1955]

- Relative to a given connective, $\rightarrow$, intended as implication, a formula A is said to be *strongest* if one can prove $A \rightarrow B$ for every formula B, and a formula A is said to be *weakest* if $B \rightarrow A$ is provable for all B.

♣ Paradoxical logic systems

- A logic system is *paradoxical in the sense of Sugihara* just in case it has either a weakest or a strongest formula.
- The existence of a proposition that is implied by all or implies all others does not fit the concept of implication as a logical connection between two propositions.

♣ CML is paradoxical in the sense of Sugihara

- CML (and Lewis's modal logics S1 ~ S5) has $\neg A \wedge A$ as strongest formula and $\neg A \vee A$ as weakest formula, and therefore, it is paradoxical.

Ackermann's Rigorous Implication

♣ Rigorous implication [Ackermann, 1956]

- "*Rigorous implication*, which we write as $A \rightarrow B$, should express the fact that a logical connection holds between A and B, that the content of B is part of that of A. … That has nothing to do with the truth of falsity of A or B. Thus one would *reject* the validity of the formula $A \rightarrow (B \rightarrow A)$, since it permits the inference from A of $B \rightarrow A$, and since the truth of A has nothing to do with whether a logical connection holds between B and A."

♣ Logical connection and relevance

- For an entailment (argument) to be valid there should be some connection of meaning, i.e. some relevance, between its premises and its conclusion, among other things.

The von Wright-Geach-Smiley Criterion for Entailment

- "A entails B, if and only if, by means of logic, it is possible to come to know the truth of $A \rightarrow B$ without coming to know the falsehood of A or the truth of B" [von Wright, 1957]
- "A entails B if and only if there is an a priori way of getting to know that $A \rightarrow B$ which is not a way of getting to know whether A or whether B" [Geach, 1958]
- "$A_1 \& \ldots \& A_n \rightarrow B$ should not only be itself a tautology, but should also be a substitution instance of some more general implication $A_1' \& \ldots \& A_n' \rightarrow B'$, where neither B' nor $\neg(A_1' \& \ldots \& A_n')$ are themselves tautologies" [Smiley, 1959]
- However, it is hard until now to know exactly how to formally interpret such epistemological phrases as 'coming to know' and 'getting to know' in the context of logic.

Traditional Relevant (Relevance) Logics

♣ Motivation of the traditional relevant (or relevance) logics

- *Relevant (relevance) logics* ware constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called 'paradoxes' of material and strict implication.

♣ Paradoxes of material and strict implication

- $A \rightarrow (B \rightarrow A)$, $B \rightarrow (\neg A \vee A)$, $\neg A \rightarrow (A \rightarrow B)$, $(\neg A \wedge A) \rightarrow B$, $(A \rightarrow B) \vee (\neg A \rightarrow B)$, $(A \rightarrow B) \vee (A \rightarrow \neg B)$, $(A \rightarrow B) \vee (B \rightarrow A)$, $((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
- $(\neg A \wedge A) > B$, $B > (\neg A \vee A)$

Traditional Relevant (Relevance) Logics

- ♣ Well-known main relevant logics
  - ◆ system Π' of *rigorous implication* [Ackermann, 1956]
  - ◆ system E of *entailment* [Anderson and Belnap, 1958]
  - ◆ system R of *relevant implication* [Belnap, 1967]
  - ◆ system T of *ticket entailment* (*entailment shorn of modality*) [Anderson, 1960]
- ♣ Characteristic features of the relevant logics
  - ◆ A primitive intensional connective to represent the notion of conditional (entailment)
  - ◆ Variable-sharing and the relevance principle
  - ◆ Free from the paradoxes of material and strict implication
  - ◆ Relevant (in the sense of weak relevance!) reasoning
  - ◆ Ampliative reasoning

The Notion of Conditional (Entailment) in RL

- ♣ Conditional as a primitive intensional connective
  - ◆ RL has a primitive intensional connective (relevant implication, entailment, ……) to represent the notion of conditional (entailment).
  - ◆ In RL, the principle connective to represent the notion of conditional is the primitive intensional connective but not material implication.
- ♣ RL includes CML certainly
  - ◆ The notion of material implication can be defined in RL as an extensional truth-function of its antecedent and consequent in the same way as that in CML.
  - ◆ Both the language and the logical theorems of RL are a conservative extension of that of CML.

Variable-Sharing and the Relevance Principle

- ♣ Variable-sharing as a necessary condition for relevance
  - ◆ The antecedent and consequent of an entailment should share some propositional variable(s).
  - ◆ Variable-sharing is a necessary, but by no means sufficient, formal notion designed to reflect the idea that there be a meaning connection, i.e., relevance, between the antecedent and consequent of an entailment.
- ♣ The relevance principle [A&B-E1-1975]
  - ◆ The relevance principle: If $A \Rightarrow B$, where $\Rightarrow$ denotes the notion of entailment, is a logical theorem of a relevant logic, for any two propositional formulas A and B, then A and B share at least one propositional variable.
- ♣ The relevance principle and implicational paradoxes
  - ◆ As a result of the relevance principle, all relevant logics are free from the paradoxes of material and strict implication.

Relationship between RL and CML

- ♣ ZDF theorem [Anderson and Belnap, 1959]
  - ◆ The zero degree (i.e., includes no entailment or relevant implication) formulas provable in E (R, T) are precisely the theorems of CML.
- ♣ CML is exactly the extensional fragment of E (R, T)
  - ◆ CML is exactly the extensional fragment of E, R, or T in the sense that if one define the material implication as $A \rightarrow B =_{df} \neg(A \wedge \neg B)$ or $A \rightarrow B =_{df} \neg A \vee B$ then all tautologies of CML are theorems of E, R, or T.
  - ◆ RL is a conservative extension of CML.

Relationship between RL and CML

- ♣ Entailments of E (R, T) is a "proper subsystem" of CML
  - ◆ The theoremhood of E (R, T) can be regarded as a "proper subset" of the theoremhood of CML, in the sense that if all entailment connectives in a theorem of E (R, T) are replaced by material implication connectives then the resultant formula must be a theorem of CML, and E (R, T) rejects some theorems (like implicational paradoxes) of CML.

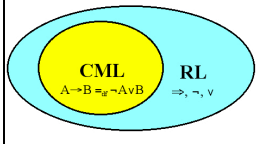
Relationship between RL and CML

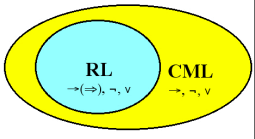
The fact:

RL is a conservative extension of CML

If replacing "⇒" by "→":

Entailments of E (R, T) is a "proper subsystem" of CML





Relevant Reasoning

- Introduction
- Fallacies of Relevance
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Paradoxes of Relevant Implication [Cheng, 1991]

♣ Conjunction-implicational paradoxes [Cheng, 1991]

- The antecedent includes some conjuncts that are not relevant to the consequent.
- Ex. $(A \wedge B) \Rightarrow A$, $(A \wedge B) \Rightarrow B$, $(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow B)$
- Ex. If snow is white and 1+1=2(even 3!), then snow is white.

♣ Disjunction-implicational paradoxes [Cheng, 1991]

- The consequent includes some disjuncts that are not relevant to the antecedent.
- Ex. $A \Rightarrow (A \vee B)$, $B \Rightarrow (A \vee B)$, $(A \Rightarrow B) \Rightarrow (A \Rightarrow (B \vee C))$
- Ex. If snow is white, then snow is white or 1+1=2(even 3!).

Paradoxes of Relevant Implication [Cheng, 1991]

♣ Paradoxes of relevant implication (with conjunction and disjunction)

- In traditional (weak) relevant logics, there are still some paradoxes in the sense of conditional.
- 'The relevance principle' is necessary but still not sufficient to the notion of conditional.
- Traditional (weak) relevant logics are certainly 'relevant' but not so strongly relevant.

♣ The problem of equivalence between the notion of entailment and the logical consequence relations

- In traditional (weak) relevant logics, $\{A, B\} \vdash_{RL} A$ does not hold, but $(A \wedge B) \Rightarrow A$ is an entailment.
- $(A \wedge B) \Rightarrow A$ not should be regarded as an entailment.

Paradoxes of Relevant Implication

♣ The relevance principle is necessary but not sufficient to the notion of conditional

- The relevance principle is necessary but not sufficient to the notion of conditional $A \Rightarrow B$ because the relevance between A and B is not fully accounted.
- The notion of relevant implication cannot be used for distinguishing conditionals from implicational statements.

♣ The root cause of the implicational paradox problem

- In general, a necessary condition for something is not necessarily a sufficient condition, and vice versa.
- It is to consider one of necessary conditions for the notion of conditional as the sufficient condition that leads to the problem of implicational paradoxes.

Reasoning Based on RL: Validity?

♣ Reasonable validity of reasoning in the sense of conditional [Cheng, 1998]

- For any meaningful/useful reasoning, one must can directly accept the conclusion of the reasoning as correct/true conclusion in the sense of conditional, if all premises of the reasoning are true in the sense of conditional.

♣ Reasoning based on RL are not relevant and/or truth-preserving in the sense of conditional

- One cannot directly accept a conclusion of a reasoning with (conjunction or disjunction) implicational paradoxes of entailment as a correct/true conclusion in the sense of conditional, even if all premises of the reasoning are true in the sense of relevant implication (even conditional!).

Reasoning Based on RL: Motivating Examples for SRL

♣ Examples

- From any given premise $A \Rightarrow B$, we can infer $(A \wedge C) \Rightarrow B$, $(A \wedge C \wedge D) \Rightarrow B$, …, and so on by using logical theorem $(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow B)$ of traditional (weak) relevant logics and Modus Ponens for entailment (from A and $A \Rightarrow B$ to infer B).
- However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded as valid in the sense of conditional because there may be no necessarily relevant and conditional relation between C and B, D and B, …, and therefore we cannot say 'if A and C then B', 'if A and C and D then B', …, and so on.

Relevant Logic and Relevant Reasoning

- **Relevant logic**
  - Relevant logic is intended to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, but did not pay attentions so much to relevant reasoning.
- **Relevant reasoning (in the sense of strong relevance)** [Cheng, 1998]
  - For a reasoning to be valid (in the sense of strong relevance), in its arguments, the premises must not contain conjuncts irrelevant to the conclusion, and the conclusion must not contain disjuncts irrelevant to the premises.
  - Relevant (and ampliative) reasoning is the heart of discovery and prediction.
  - Traditional (weak) relevant logics cannot underlie relevant reasoning in the sense of strong relevance.

Strong Relevant (Relevance) Logic

- **Strong relevant (relevance) logics** [Cheng, 1992]
  - As a modification of R, E, and T, strong relevant logics Rc, Ec, and Tc rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in R, E, and T, respectively.
  - What underlies the strong relevant logics Rc, Ec, and Tc is the strong relevance principle.
- **The strong relevance principle**
  - If A is a theorem of Rc, Ec, and Tc, then every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part in A.

Relevant Reasoning Based on Strong Relevance Logic

- **Reasoning based on SRL are relevant in the sense of strong relevance**
  - A reasoning based on a strong relevant logic is relevant in the sense of strong relevance.
- **Reasoning based on SRL are truth-preserving in the sense of conditional**
  - A reasoning based on a strong relevant logic is truth-preserving in the sense of conditional.
  - The truth in the sense of conditional is guaranteed necessarily.
- **A fundamental question: Are there other paradoxes?**
  - In strong relevant logic, are there some other unknown type of paradoxes that is not natural in the sense of conditional?

Relevant Reasoning

Applications of Relevant Reasoning Based on SRL

- **Conditional as the heart of knowledge/intelligence science**
  - The notion of conditional (entailment) plays the most essential role in knowledge representation and reasoning.
  - The notion of conditional (entailment) is the heart of logic and mathematics, and therefore, is the heart of knowledge/ intelligence science.
- **The logical basis for discovery and prediction**
  - Any discovery and prediction must ask relevant and ampliative reasoning.
  - It is the primitive intensional notion of conditional (entailment) that plays the most fundamental role in any relevant and ampliative reasoning for discovery and prediction.

Ampliative Reasoning

- **Ampliative reasoning**
  - From the viewpoint to consider reasoning as the process of drawing new conclusions from given premises, any meaningful/useful reasoning should be ampliative, i.e., the conclusion of a meaningful/useful reasoning must be NEW, in some certain meaning, to the premises of that reasoning.
  - An ampliative reasoning must be non-circular or non-tautological, i.e., the "truth" of conclusion of any ampliative reasoning should be recognized by the reasoning itself after the end of the reasoning process but not be invoked in deciding the "truth" of premises of the reasoning.
- **A fundamental problem (open problem)**
  - How can we define what is an ampliative reasoning formally?

Conditional in Ampliative Modus Ponens

♣ **Modus Ponens**

- If A holds then B holds, now A holds, therefore, B holds.

♣ **Modus Ponens should be ampliative and non-circular**

- When we reason using Modus Ponens, what we know are 'if A holds then B holds' and 'A holds'.
- Before the reasoning, we do not know whether or not 'B holds'.
- If we know, then we would not need to reason at all.

Conditional in Ampliative Modus Ponens

♣ **How does Modus Ponens work?**

- Indeed, by using Modus Ponens, we can know 'B holds', which is unknown until the reasoning is done, based on the following reasons:
  (i) 'A holds'
  (ii) 'There is no case such that A holds but B does not hold'
  (iii) we know (ii) without investigating either 'whether A holds or not' or 'whether B holds or not'.

♣ **Note**

- The von Wright-Geach-Smiley criterion for entailment is corresponding to the above (ii) and (iii).

Conditional in Tautological Modus Ponens

- Modus Ponens for material implication: from A and $A \rightarrow B$ to infer B
- Note: $A \rightarrow B =_{df} \neg(A \wedge \neg B)$ or $A \rightarrow B =_{df} \neg A \vee B$.
- According to the extensional truth-functional definition of the material implication, if we know 'A is true' but do not know the truth-value of B, then we cannot decide the truth-value of $A \rightarrow B$.
- In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is done!
- Modus Ponens for material implication is non-ampliative, circular, or tautological.

Ampliative Reasoning Based on RL

♣ **Modus Ponens for entailment**

- From A and $A \Rightarrow B$ to infer B.
- The notion of entailment ($\Rightarrow$) is primitive intensional.

♣ **Ampliative reasoning based on RL**

- Because relevant logics have a primitive intensional connective to represent the notion of conditional (entailment) which satisfies the von Wright-Geach-Smiley criterion, Modus Ponens for entailment is ampliative, non-circular or non-tautological in the sense of relevance.

Paraconsistent Reasoning

♣ **Fundamental problem**

- Our knowledge about a domain or a problem may be incomplete and even inconsistent in many ways.
- Reasoning with inconsistent knowledge is the rule rather than the exception in our everyday lives and all scientific disciplines.

♣ **Logical basis for paraconsistent reasoning**

- Any logic to underlie paraconsistent reasoning (reasoning under inconsistency) must reject the principle of explosion (ECQ).
- The principle of explosion (ECQ): Everything follows from a contradiction.

Paraconsistent Reasoning Based on RL

♣ **CML is explosive**

- '$(A \wedge \neg A) \rightarrow B$,' a typical paradox of material implication, is a logical theorem of CML, and therefore, CML is explosive.
- Any logic based on the principle of explosion, e.g., CML and its various classical conservative extensions, cannot underlie paraconsistent reasoning.

♣ **Paraconsistent reasoning based on relevant logic**

- Relevant logics are paraconsistent because they use Modus Ponens for entailment ($\Rightarrow$) as their inference rule but do not accept '$(\neg A \wedge A) \Rightarrow B$' as a logical theorem.

Abductive Reasoning Based on RL

- **Abductive reasoning [Peirce]**
  - The surprising fact, *C*, is observed.
    But if *A* were true, *C* would be a matter of course.
    Hence, there is reason to suspect that *A* is true.
  - From '*C*' and 'if *A* then *C*' to infer '*A*'
  - Note: From the viewpoint of deduction, abduction is a formal fallacy.
- **The notion of conditional in abduction**
  - Since '*C*' is an observed fact and '*A*' is the result of the inference, the logical "validity" of abduction is totally determined by the validity of conditional 'if *A* then *C*.'
  - The key point in abduction is how to get and use genuine conditionals that are certainly relevant to the observed fact.
  - Those implicational paradoxes used as conditionals in abduction are not only useless but also harmful.

Conditional and Nonmonotonic Reasoning

- **Two fundamental facts in CML**
  - (1) in CML, if *B* is right then so is $A \rightarrow B$
    if $P \vdash_{CML} B$ then $P \vdash_{CML} A \rightarrow B$
    if $P \models_{CML} B$ then $P \models_{CML} A \rightarrow B$
  - (2) deduction theorem of CML
    $P \cup \{A\} \vdash_{CML} B$ IFF $P \vdash_{CML} A \rightarrow B$
    $P \cup \{A\} \models_{CML} B$ IFF $P \models_{CML} A \rightarrow B$
- **Why CML is monotonic?**
  - As a direct result of the two facts, CML has the following proof-theoretical and model-theoretical monotonicity:

    if $P \vdash_{CML} B$ then $P \cup \{A\} \vdash_{CML} B$
    if $P \models_{CML} B$ then $P \cup \{A\} \models_{CML} B$

Nonmonotonic Reasoning Based on RL

- **Natural nonmonotonicity of relevant logics**
  - The relevant logic R is naturally nonmonotonic.
  - Its deducibility relation is already nonmonotonic and no new logical primitives need be introduced to get the desired effect.
- **Nonmonotonic reasoning based on RL**
  - To get a nonmonotonic deducibility relation by formalizing the notion of entailment satisfactorily is an important research direction for establishing a satisfactory logic system underlying nonmonotonic reasoning.

Anticipatory Reasoning based on Temporal Relevant Logic

- **Anticipatory reasoning for prediction**
  - An anticipatory reasoning is a reasoning to draw new, previously unknown and/or unrecognized conclusions about some future event or events whose occurrence and truth are uncertain at the point of time when the reasoning is being performed.
- **Anticipatory reasoning based on temporal relevant logic**
  - Temporal relevant logic can underlie relevant, truth-preserving, ampliative, paraconsistent, and temporal reasoning.

Other Applications of Relevant Reasoning Based on RLs

- **Automated theorem finding (rather then proving)** [Wos, 1988]
  - "What properties can be identified to permit an automated reasoning program to find new and interesting theorems, as opposed to proving conjectured theorems?"
- **Inference rule generation**
  - Inadequacy of the current knowledge-based systems is that they cannot autonomously generate new and valid inference rules from those existing rules and facts that are programmed or inputted in the systems by their developers or users.
  - Autonomous evolution of a knowledge-based system is impossible if it has no ability of autonomous generation of new and valid inference rules.

Other Applications of Relevant Reasoning Based on RLs

- **Epistemic process modeling and automation**
  - New conditionals are epistemic goals of any scientific discovery: Any scientific discovery process must include an epistemic process to gain knowledge of or to ascertain the existence of some empirical or logical conditionals previously unknown or unrecognized.
- **Epistemic programming**
  - Provide scientists with a computational methodology and its computational tools to program their epistemic processes in scientific discovery and prediction, and therefore, make scientific discovery and prediction become a 'science' and/or an 'engineering.'
  - Regards conditionals as the subject of computing, takes primary epistemic operations as basic operations of computing, and regards epistemic processes as the subject of programming.

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Research Directions and Challenging Problems

♣On relevant logic and relevant reasoning

- The most satisfactory way of grasping the notion of entailment.
- The most satisfactory logic to underlie truth-preserving and relevant reasoning in the sense of conditional.
- Model theory and decision problems for strong relevant logics.
- Normal form(s) to represent equivalence classes of formulas in a logic where the notion of conditional is represented by a primitive connective.
- Automated relevant reasoning

Research Directions and Challenging Problems

♣On extensions of relevant logics

- Temporal relevant logics
- Spatial relevant logics
- Spatial-temporal relevant logics
- Deontic relevant logics
- Epistemic relevant logics
- Many-valued and/or fuzzy relevant logics
- …… relevant logics

Research Directions and Challenging Problems

♣On applications of relevant (relevance) logic

- Mathematical knowledge representation and reasoning based on strong relevant logic, Automated theorem finding.
- Autonomous generation of inference rules.
- Epistemic programming: Programming epistemic processes for computing conditionals.

♣On applications of extensions of relevant (relevance) logic

- Anticipatory reasoning based on temporal relevant logic.
- Specifying and reasoning about information assurance and security based on deontic relevant logic.
- Representing and reasoning about spatial knowledge based on spatial relevant logic.
- Specifying, verifying, and reasoning about mobile malti-agent systems based on spatio-temporal relevant logic.

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Thanks

Thank you for your attention