8.1 Let A, B, and C be formulas., and to show a formal proof for each of the following logical theorems of CPC:

(d) $\vdash_L (\neg C \rightarrow \neg B) \rightarrow (B \rightarrow C)$

Proof without deduction theorem

1. $(\neg C \Rightarrow \neg B) \Rightarrow (B \Rightarrow C)$ Axiom (A3)

Using the deduction theorem

1. $((\neg C) \rightarrow (\neg B)) \rightarrow (B \rightarrow C)$	AS3$\{(\neg A\rightarrow\neg B)\rightarrow(B\rightarrow A)$, C=A, B=B $\}$
2. $((\neg C) \rightarrow (\neg B))$	premise
3. $B \rightarrow C$	follow from 1 and 2 by MP
4. B	premise
5. C	follow from 3 and 4 by MP
6. $\{ ((\neg C) \rightarrow (\neg B)), B \} \vdash_L C$	deduction theorem
7. $\{(\neg C) \rightarrow (\neg B)\} \vdash_L (B \rightarrow C)$	deduction theorem
8. $\vdash_L ((\neg C) \rightarrow (\neg B)) \rightarrow (B \rightarrow C)$	deduction theorem

8.2 Let A, B, and C be formulas. Using MP and derived rules, the deduction theorem, and axiom/theorem schemata of CPC, to show a formal proof (different from your answer in problem 8.1) for each of the following logical theorems of CPC.

(d) $\vdash_{HB} (A \rightarrow B) \rightarrow ((C \vee A) \rightarrow (C \vee B))$

Lemma: $[A\rightarrow(B\rightarrow C), B] \vdash A\rightarrow C$

1. $A\rightarrow(B\rightarrow C)$ premise
2. A premise
3. $B\rightarrow C$ 1, 2, MP
4. B premise
5. C 3, 4, MP

6. [A→(B→C), B, A] ⊢ C deduction theorem

7. [A→(B→C), B] ⊢ A→C deduction theorem

Proof without deduction theorem

1. (C→(C∨B))→((A→(C∨B))→((C∨A)→(C∨B))

[AS((A →C)→((B→C)→(A∨B)→C)), C=A,(C∨B)=C,A=B]

2. C→(C∨B) [AS(A→(A∨B))]

3. ((A→(C∨B))→((C∨A)→(C∨B)) [1, 2, MP]

4. (A→B)→(((B→(C∨B))→((A→(C∨B)))

[AS((A→B) →((B→C) →(A→C))), A=A, B=B, (C∨B)=C]

5. B→(C∨B) [AS(A→(A∨B))]

6. (A→B)→(A→(C∨B)) [4, 5, Lemma]

7. (A→B)→((C∨A)→(C∨B)) [3, 6, MP]

Using the deduction theorem

⊢HB (A → B) → ((C ∨ A) → (C ∨ B))

1. (A→B)→((B→(C∨B))→(A→(C∨B)))

[AS (A→B)→((B→C)→(A→C)) , A=A,B=B, (C∨B)=C]

2. A→B [premise]

3. (B→(C∨B))→(A→(C∨B)) [1, 2, MP]

4. B→(C∨B) [AS(B→(A∨B)) , C=A, B=B]

5. A→(C∨B) [3, 4, MP]

6. (C→(C∨B))→((A→(C∨B))→((C∨A)→(C∨B)))

[AS (A→C)→((B→C)→((A∨B)→C)) ,C=A,A=B, (C∨B)=C]

7. C→(C∨B) [AS A→(A ∨ B), C=A,B=B]

8. (A→(C∨B))→((C∨A)→(C∨B)) [7, 6, MP]

9. (C∨A)→(C∨B) [5, 8, MP]

10. $\{(A \rightarrow B), (C \vee A)\} \vdash_{HB} (C \vee B)$ [deduction theorem]

11. $\{(A \rightarrow B)\} \vdash_{HB} (C \vee A) \rightarrow (C \vee B)$ [deduction theorem]

12. $\vdash_{HB} (A \rightarrow B) \rightarrow ((C \vee A) \rightarrow (C \vee B))$ [deduction theorem]