Qualitative Spatio-temporal Reasoning about Moving Objects in Three-dimensional Space

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Abstract. To represent and reason about moving objects in three-dimensional space qualitatively, we need a right fundamental logic system to provide us with a criterion of logical validity for reasoning as well as a formal representation language. In order to reason about new spatio-temporal knowledge with incomplete or sometime even inconsistent knowledge, the fundamental logic must be able to underlie relevant and truth-preserving reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and spatio-temporal reasoning. This paper proposes a new family of three-dimensional spatio-temporal relevant logics as a hopeful candidate for the fundamental logic, and shows that the logics can satisfy all the requirements for the fundamental logic we need.

Keywords: Relevant and truth-preserving reasoning, Ampliative reasoning, Paracomplete and paraconsistent reasoning, Spatio-temporal reasoning, Three-dimensional spatio-temporal relevant logic.

1 Introduction

Time and space are two essential and primitive parts of the fundamental structure of the universe as well as the fundamental intellectual structure of our human beings. Temporal, spatial, and spatio-temporal notions, i.e., time, schedule, time zone, shape, size, distance, orientation, relative position, connectivity, motion, speed, etc, play many important roles in our cognition and understanding of the real world and our communication and cooperation in the human society. Our temporal, spatial, and spatio-temporal knowledge are fundamentals and sources from which we can represent, reason about, and derive new knowledge. There are many applications that need means for representing and reasoning about temporal, spatial, and spatio-temporal knowledge, such as robotics, motion planning, motion capture, machine vision, solid modeling, natural language understanding, temporal database systems, spatial database systems, geographic information systems, distributed systems, air traffic control systems, train control systems, expressway control systems, etc. Therefore, it is certainly a very important task to provide the means dealing with temporal, spatial, and spatio-temporal knowledge required by various applications.

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To develop and maintain any intelligent system dealing with moving objects in three-dimensional space, we have to consider not only plane (two-dimensional) space but also solid (three-dimensional) space. To represent and reason about moving objects in three-dimensional space, we need a right fundamental logic system to provide us with a criterion of logical validity for reasoning as well as a formal representation language. In order to reason about new spatio-temporal knowledge with incomplete or sometime even inconsistent knowledge, the fundamental logic must be able to underlie relevant and truth-preserving reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, three-dimensional spatial reasoning, temporal reasoning, and three-dimensional spatio-temporal reasoning.

There are many classical or modal logic systems proposed for representing and reasoning about temporal, spatial, and spatio-temporal knowledge [3, 11-14, 16, 17, 19-21]. All of these logics are some how based on classical mathematical logic, and therefore, they cannot underlie relevant and truth-preserving reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning. Relevant logics cannot underlie spatial reasoning, temporal reasoning, and spatio-temporal reasoning, even though them can underlie relevant and truth-preserving reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning well [1, 2, 15, 18]. Two-dimensional spatio-temporal relevant logics cannot underlie three-dimensional spatial reasoning and three-dimensional spatio-temporal reasoning [8]. Therefore, no existing logic system can satisfy all of the above requirements. This paper proposes a new family of three-dimensional spatio-temporal relevant logics as a hopeful candidate for the fundamental logic, and shows that the logics can satisfy all the requirements for the fundamental logic we need.

2 The Logical Basis for Qualitative Spatio-temporal Reasoning about Moving Objects in Three-dimensional Space

The question, “Which is the right logic?” invites the immediate counter-question “Right for what?” Only if we certainly know what we need, we can make a good choice. The present author considers that the fundamental logic system to underlie representing and reasoning about moving objects in three-dimensional space must satisfy all the following essential requirements.

First, as a general logical criterion for the validity of reasoning, the logic must be able to underlie relevant and truth-preserving reasoning in the sense of conditional, i.e., for any reasoning based on the logic to be valid, if its premises are true in the sense of conditional, then its conclusion must be relevant to the premises and true in the sense of conditional.

Second, the logic must be able to underlie ampliative reasoning in the sense that the truth of conclusion of the reasoning should be recognized after the completion of the reasoning process but not be invoked in deciding the truth of premises of the reasoning. From the viewpoint to regard reasoning as the process of drawing new conclusions from given premises, any meaningful reasoning must be ampliative but not circular and/or tautological.

Third, the logic must be able to underlie paracomplete and paraconsistent reasoning. In particular, the so-called principle of Explosion that everything follows from a contradiction should not be accepted by the logic as a valid principle. In general, our knowledge about a domain as well as a scientific discipline may be incomplete and/or inconsistent in many ways, i.e., it gives us no evidence for deciding the truth of either a proposition or its negation, and/or it directly or indirectly includes some contradictions. Therefore, reasoning with incomplete and/or inconsistent knowledge is the rule rather than the exception in our everyday lives and almost all scientific disciplines.

Finally, the logic must be able to underlie temporal, three-dimensional spatial, and three-dimensional spatio-temporal reasoning. In an application system dealing with mobile objects in three-dimensional space, any reasoning may somehow depend on notions of time and/or three-dimensional space. Not only propositions (statements) about mobile objects but also relevant relationships among mobile objects may be dependent on three-dimensional spatial regions and/or points, and may change over time. Some properties of mobile objects, i.e., motion and speed, are intrinsically dependent on both time and three-dimensional space. This naturally requires that the logic must be able to underlie temporal, three-dimensional spatial, and three-dimensional spatio-temporal reasoning.

Classical mathematical logic (**CML** for short) was established based on a number of fundamental assumptions and/or principles. Among them, the most characteristic one is the classical account of validity (i.e., an argument/reasoning is valid if and only if it is impossible for all its premises to be true while its conclusion is false) that is the logical validity criterion of **CML** by which one can decide whether the conclusion of an argument/reasoning really does follow from its premises or not in the framework of **CML**. Because relevance between premises and conclusion of an argument/reasoning is not accounted for by the classical validity criterion, a reasoning based on **CML** is not necessarily relevant. On the other hand, in **CML** the notion of conditional, which is intrinsically intensional but not truth-functional, is represented by the notion of material implication, which is intrinsically an extensional truth-function. This leads to the problem of 'implicational paradoxes' [1, 2, 15, 18] (therefore any reasoning based on **CML** is not truth-preserving in the sense of conditional) as well as the problem that a reasoning based on **CML** must be circular and/or tautological but not ampliative. Moreover, because **CML** accepts the principle of Explosion (i.e., everything follows from a contradiction), reasoning under inconsistency is impossible within the framework of **CML**. The above three facts are also true to those classical conservative extensions or non-classical alternatives of **CML** where the classical account of validity is adopted as the logical validity criterion and the notion of conditional is directly or indirectly represented by the material implication. Finally **CML** does not provide explicit means to deal with the notion of time and/or the notion of space. Therefore, **CML** cannot satisfy any of the essential requirements for the fundamental logic system.

Temporal (classical) logics was established in order to represent and reason about notions, relations, and properties of time-related entities within a logical framework, and therefore to underlie temporal reasoning, i.e., reasoning about those propositions and/or formulas whose truth-values may depend on time [4, 22]. However, because any temporal (classical) logic is a classical conservative extension of **CML** in the sense that it is based on the classical account of validity and it represents the notion of conditional directly or indirectly by the material implication, all problems in **CML** caused by the classical account of validity and the material implication also remain in temporal (classical) logic. As a result, no temporal (classical) logic can satisfy the first three of the essential requirements for the fundamental logic system.

Spatial (classical) logics was proposed in order to deal with geometric and/or topological entities, notions, relations, and properties, and therefore to underlie spatial reasoning, i.e., reasoning about those propositions and formulas whose truth-values may depend on a location [11-14, 20]. However, these existing spatial logics are classical conservative extensions of **CML** in the sense that they are based on the classical account of validity and they represent the notion of conditional directly or indirectly by the material implication. Therefore, similar to the case of temporal (classical) logic, these spatial logics cannot satisfy the first three of the essential requirements for the fundamental logic system.

Traditional relevant (relevance) logics were constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called ‘paradoxes’ of material and strict implication [1, 2, 15, 18]. Some major traditional relevant logic systems are ‘system **E** of entailment’, ‘system **R** of relevant implication’, and ‘system **T** of ticket entailment’. A major characteristic of the relevant logics is that they have a primitive intensional connective to represent the notion of conditional and their logical theorems include no implicational paradoxes. The underlying principle of the relevant logics is the relevance principle, i.e., for any entailment provable in **E**, **R**, or **T**, its antecedent and consequent must share a propositional variable. Variable-sharing is a formal notion designed to reflect the idea that there be a meaning-connection between the antecedent and consequent of an entailment. It is this relevance principle that excludes those implicational paradoxes from logical axioms or theorems of relevant logics. In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present author has proposed some strong relevant (relevance) logics, named **Rc**, **Ec**, and **Tc** [5, 6, 9]. The logics require that the premises of an argument represented by a conditional include no unnecessary and needless conjuncts and the conclusion of that argument includes no unnecessary and needless disjuncts. As a modification of traditional relevant logics **R**, **E**, and **T**, strong relevant logics **Rc**, **Ec**, and **Tc** rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in **R**, **E**, and **T**, respectively. What underlies the strong relevant logics is the strong relevance principle: If *A* is a theorem of **Rc**, **Ec**, or **Tc**, then every propositional variable in *A* occurs at least once as an antecedent part and at least once as a consequent part. Although strong relevant logics can satisfy the first three of the essential requirements for the fundamental logic system, they do not provide explicit means to deal with the notion of time and/or the notion of space and therefore they cannot satisfy the fourth essential requirement.

Two-dimensional spatio-temporal relevant logics [8], which are obtained by introducing region connection predicates and axiom schemata of spatial logic RCC [3, 11-14], point position predicates and axiom schemata, and point adjacency predicates and axiom schemata into temporal relevant logics [7], can satisfy the first three of the essential requirements for the fundamental logic system but only can partly satisfy the fourth essential requirement because they cannot deal with spatio-temporal notions (such as motion and speed) and three-dimensional space.

Therefore, no existing logic system can satisfy all of the essential requirements.

3 Three-Dimensional Spatio-Temporal Relevant Logic

We now propose a new family of relevant logic systems, named ***three-dimensional spatio-temporal relevant logic***, which can satisfy all the essential requirements for the fundamental logic system to underlie representing and reasoning about mobile three-dimensional geometric objects. The logics are obtained by introducing predicates and axiom schemata about solid-region connection, predicates and axiom schemata about point position, and predicates and axiom schemata about motion of mobile objects into temporal relevant logics [7]. Therefore, they are conservative extensions of temporal relevant logics as well as strong relevant logics. On the other hand, we do not introduce primitive predicates about the notion of distance but define the notion of distance by predicates about point position and adjacency, predicates about movement of mobile objects, and temporal operators.

Let $\{r_1, r_2, r_3, ...\}$ be a countably infinite set of individual variables, called ***solid-region variables***. Atomic formulas of the form $\boldsymbol{C}(r_1, r_2)$ are read as 'region r_1 connects with region r_2.' Let $\{p_1, p_2, p_3, ...\}$ be a countably infinite set of individual variables, called ***point variables***. Let ***TCP*** be an individual constant of point, called ***the central point***. Atomic formulas of the form $\boldsymbol{I}(p_1, r_1)$ are read as 'point p_1 is included in region r_1.' Atomic formulas of the form $\boldsymbol{Id}(p_1, p_2)$ are read as 'point p_1 is identical with p_2.' Atomic formulas of the form $\boldsymbol{Arc}(p_1, p_2)$ are read as 'points p_1, p_2 are adjacent such that there is an arc from point p_1 to point p_2, or more simply, point p_1 is adjacent to point p_2.' Note that an arc has a direction. Atomic formulas of the form $\boldsymbol{Reachable}(p_1, p_2)$ are read as 'there is at least one directed path (i.e., a sequence of arcs such that one connects to the next one) from point p_1 to point p_2.' Atomic formulas of the form $\boldsymbol{NH}(p_1, p_2)$ are read as 'taking ***TCP*** as the reference point, the position of point p_1 is not higher than the position of point p_2, i.e., the length of the vertical line from p_1 to ***TCP*** is not longer than the length of the vertical line from p_2 to ***TCP***. Here, $\boldsymbol{C}(r_1, r_2)$, $\boldsymbol{I}(p_1, r_1)$, $\boldsymbol{Id}(p_1, p_2)$, $\boldsymbol{Arc}(p_1, p_2)$, $\boldsymbol{Reachable}(p_1, p_2)$ and $\boldsymbol{NH}(p_1, p_2)$ are primitive binary predicates to represent three-dimensional geometric relationships between three-dimensional geometric regions and points. Note that here we use a many-sorted language.

Let $\{o_1, o_2, o_3, ...\}$ be a countably infinite set of individual variables, called ***object variables***. Atomic formulas of the form $\boldsymbol{A}(o_1, p_1)$ are read as 'object o_1 arrives at point p_1.' Atomic formulas of the form $\boldsymbol{NS}(o_1, o_2)$ are read as 'the speed of object o_1 is not faster than the speed of object o_2.' Here, $\boldsymbol{A}(o_1, p_1)$ and $\boldsymbol{NS}(o_1, o_2)$ are primitive binary predicates to represent motion relationships between mobile objects in a three-dimensional geometric space.

The symbols (logical connectives, quantifiers, individual variables, individual constants, solid-region variables, point variables, object variables, predicates, temporal operators), region connection predicates, point position predicates, object movement predicates, axiom schemata, and inference rules are as follows:

Symbols:

$\{\neg, \Rightarrow, \wedge, \forall, \exists, x_1, x_2, ..., x_n, ..., \boldsymbol{TCP}, c_1, c_2, ..., c_n, ..., r_1, r_2, r_3, ..., r_n, ..., p_1, p_2, p_3, ..., p_n, ..., o_1, o_2, o_3, ..., o_n, ..., p_0^1, ..., p_0^n, ..., p_1^1, ..., p_1^n, ..., p_2^1, ..., p_2^n, ..., p_k^1, ..., p_k^n, ..., (,), \boldsymbol{G}, \boldsymbol{H}, \boldsymbol{F}, \boldsymbol{P}\}$

Primitive and defined logical connectives:

$\Rightarrow$ (entailment),
$\neg$ (negation),
$\otimes$ (intensional conjunction, $A\otimes B =_{\mathrm{df}} \neg(A\Rightarrow\neg B)$),
$\oplus$ (intensional disjunction, $A\oplus B =_{\mathrm{df}} \neg A\Rightarrow B$),
$\Leftrightarrow$ (intensional equivalence, $A\Leftrightarrow B =_{\mathrm{df}} (A\Rightarrow B)\otimes(B\Rightarrow A)$),
$\wedge$ (extensional conjunction),
$\vee$ (extensional disjunction, $A\vee B =_{\mathrm{df}} \neg(\neg A\wedge\neg B)$),
$\rightarrow$ (material implication, $A\rightarrow B =_{\mathrm{df}} \neg(A\wedge\neg B)$ or $\neg A\vee B$),
$\leftrightarrow$ (extensional equivalence, $A\leftrightarrow B =_{\mathrm{df}} (A\rightarrow B)\wedge(B\rightarrow A)$).

Temporal operators:

G (future-tense always or henceforth operator, ***G****A* means 'it will always be the case in the future from now that *A*'),
H (past-tense always operator, ***H****A* means 'it has always been the case in the past up to now that *A*'),
F (future-tense sometime or eventually operator, ***F****A* means 'it will be the case at least once in the future from now that *A*'),
P (past-tense sometime operator, ***P****A* means 'it has been the case at least once in the past up to now that *A*').

Note that these temporal operators are not independent and can be defined as follows: $\boldsymbol{G}A =_{\mathrm{df}} \neg\boldsymbol{F}\neg A$, $\boldsymbol{H}A =_{\mathrm{df}} \neg\boldsymbol{P}\neg A$, $\boldsymbol{F}A =_{\mathrm{df}} \neg\boldsymbol{G}\neg A$, $\boldsymbol{P}A =_{\mathrm{df}} \neg\boldsymbol{H}\neg A$.

Primitive binary predicates:

$\boldsymbol{C}(r_1, r_2)$ (connection), $\boldsymbol{I}(p_1, r_1)$ (inclusion), $\boldsymbol{Id}(p_1, p_2)$ (the same point), $\boldsymbol{Arc}(p_1, p_2)$ (arc), $\boldsymbol{Reachable}(p_1, p_2)$ (reachable), $\boldsymbol{NH}(p_1, p_2)$ (not higher than), $\boldsymbol{A}(o_1, p_1)$ (arrives at), $\boldsymbol{NS}(o_1, o_2)$ (not speedier than).

Defined binary predicates:

$\boldsymbol{DC}(r_1, r_2) =_{\mathrm{df}} \neg\boldsymbol{C}(r_1, r_2)$, $\boldsymbol{DC}(r_1, r_2) =_{\mathrm{df}} \neg(\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{I}(p_1, r_2)))$ ($\boldsymbol{DC}(r_1, r_2)$ means 'r_1 is disconnected from r_2')

$\boldsymbol{Pa}(r_1, r_2) =_{\mathrm{df}} \forall r_3(\boldsymbol{C}(r_3, r_1)\Rightarrow\boldsymbol{C}(r_3, r_2))$, $\boldsymbol{Pa}(r_1, r_2) =_{\mathrm{df}} \forall p_1(\boldsymbol{I}(p_1, r_1)\Rightarrow\boldsymbol{I}(p_1, r_2))$ ($\boldsymbol{Pa}(r_1, r_2)$ means 'r_1 is a part of r_2')

$\boldsymbol{PrPa}(r_1, r_2) =_{\mathrm{df}} \boldsymbol{Pa}(r_1, r_2)\wedge(\neg\boldsymbol{Pa}(r_2, r_1))$ ($\boldsymbol{PrPa}(r_1, r_2)$ means 'r_1 is a proper part of r_2')

$\boldsymbol{EQ}(r_1, r_2) =_{\mathrm{df}} \boldsymbol{Pa}(r_1, r_2)\wedge\boldsymbol{Pa}(r_2, r_1)$ ($\boldsymbol{EQ}(r_1, r_2)$ means 'r_1 is identical with r_2')

$\boldsymbol{O}(r_1, r_2) =_{\mathrm{df}} \exists r_3(\boldsymbol{Pa}(r_3, r_1)\wedge\boldsymbol{Pa}(r_3, r_2))$ ($\boldsymbol{O}(r_1, r_2)$ means 'r_1 overlaps r_2')

$\boldsymbol{DR}(r_1, r_2) =_{\mathrm{df}} \neg\boldsymbol{O}(r_1, r_2)$ ($\boldsymbol{DR}(r_1, r_2)$ means 'r_1 is discrete from r_2')

$\boldsymbol{PaO}(r_1, r_2) =_{\mathrm{df}} \boldsymbol{O}(r_1, r_2)\wedge(\neg\boldsymbol{Pa}(r_1, r_2))\wedge(\neg\boldsymbol{Pa}(r_2, r_1))$ ($\boldsymbol{PaO}(r_1, r_2)$ means 'r_1 partially overlaps r_2')

$\boldsymbol{EC}(r_1, r_2) =_{\mathrm{df}} \boldsymbol{C}(r_1, r_2)\wedge(\neg\boldsymbol{O}(r_1, r_2))$ ($\boldsymbol{EC}(r_1, r_2)$ means 'r_1 is externally connected to r_2')

$\boldsymbol{TPrPa}(r_1, r_2) =_{\mathrm{df}} \boldsymbol{PrPa}(r_1, r_2)\wedge\exists r_3(\boldsymbol{EC}(r_3, r_1)\wedge\boldsymbol{EC}(r_3, r_2))$ ($\boldsymbol{TPrPa}(r_1, r_2)$ means 'r_1 is a tangential proper part of r_2')

$\boldsymbol{NTPrPa}(r_1, r_2) =_{df} \boldsymbol{PrPa}(r_1, r_2)\wedge(\neg\exists r_3(\boldsymbol{EC}(r_3, r_1)\wedge\boldsymbol{EC}(r_3, r_2)))$ ($\boldsymbol{NTPrPa}(r_1, r_2)$ means 'r_1 is a nontangential proper part of r_2')
$\boldsymbol{SA}(p_1, p_2) =_{df} \boldsymbol{NH}(p_1, p_2)\wedge\boldsymbol{NH}(p_2, p_1)$ ($\boldsymbol{SA}(p_1, p_2)$ means 'the position of point p_1 and the position of point p_2 are in the same altitude')
$\boldsymbol{Hi}(p_1, p_2) =_{df} \neg\boldsymbol{NH}(p_1, p_2)$ ($\boldsymbol{Hi}(p_1, p_2)$ means 'the position of point p_1 is higher than the position of point p_2')
$\boldsymbol{SS}(o_1, o_2) =_{df} \boldsymbol{NS}(o_1, o_2)\wedge\boldsymbol{NS}(o_2, o_1)$ ($\boldsymbol{SS}(o_1, o_2)$ means 'the motion of object o_1 and the motion of object o_2 are in the same speed')
$\boldsymbol{Sp}(o_1, o_2) =_{df} \neg\boldsymbol{NS}(o_1, o_2)$ ($\boldsymbol{Sp}(o_1, o_2)$ means 'the motion of object o_1 is faster than the motion of object o_2')
$\boldsymbol{ND}(p_1, p_2, p_3) =_{df} \exists o_1\exists o_2((\boldsymbol{A}(o_1, p_1)\wedge\boldsymbol{A}(o_2, p_2)\wedge\boldsymbol{Reachable}(p_1, p_3)\wedge\boldsymbol{Reachable}(p_2, p_3)\wedge\boldsymbol{SS}(o_1, o_2))\Rightarrow\boldsymbol{G}(\boldsymbol{A}(o_1, p_3)\Rightarrow\boldsymbol{A}(o_2, p_3)))$ ($\boldsymbol{ND}(p_1, p_2, p_3)$ means 'the distance of between point p_2 and point p_3 is not more distant than the distance of between point p_1 and point p_3')
$\boldsymbol{SD}(p_1, p_2, p_3) =_{df} \boldsymbol{ND}(p_1, p_2, p_3)\wedge\boldsymbol{ND}(p_2, p_1, p_3)$ ($\boldsymbol{SD}(p_1, p_2, p_3)$ means 'the distance of between point p_1 and point p_3 is equal to the distance of between point p_2 and point p_3')
$\boldsymbol{Ne}(p_1, p_2, p_3) =_{df} \neg\boldsymbol{ND}(p_1, p_2, p_3)$ ($\boldsymbol{Ne}(p_1, p_2, p_3)$ means 'the distance of between point p_1 and point p_3 is nearer than the distance of between point p_2 and point p_3')

Axiom schemata:

E1: $A\Rightarrow A$,
E2: $(A\Rightarrow B)\Rightarrow((C\Rightarrow A)\Rightarrow(C\Rightarrow B))$,
E2′: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow C))$,
E3: $(A\Rightarrow(A\Rightarrow B))\Rightarrow(A\Rightarrow B)$,
E3′: $(A\Rightarrow(B\Rightarrow C))\Rightarrow((A\Rightarrow B)\Rightarrow(A\Rightarrow C))$,
E3″: $(A\Rightarrow B)\Rightarrow((A\Rightarrow(B\Rightarrow C))\Rightarrow(A\Rightarrow C))$,
E4: $(A\Rightarrow((B\Rightarrow C)\Rightarrow D))\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow D))$,
E4′: $(A\Rightarrow B)\Rightarrow(((A\Rightarrow B)\Rightarrow C)\Rightarrow C)$,
E4″: $((A\Rightarrow A)\Rightarrow B)\Rightarrow B$,
E4‴: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(((A\Rightarrow C)\Rightarrow D)\Rightarrow D))$,
E5: $(A\Rightarrow(B\Rightarrow C))\Rightarrow(B\Rightarrow(A\Rightarrow C))$,
E5′: $A\Rightarrow((A\Rightarrow B)\Rightarrow B)$,
N1: $(A\Rightarrow(\neg A))\Rightarrow(\neg A)$, N2: $(A\Rightarrow(\neg B))\Rightarrow(B\Rightarrow(\neg A))$, N3: $(\neg(\neg A))\Rightarrow A$,
C1: $(A\wedge B)\Rightarrow A$, C2: $(A\wedge B)\Rightarrow B$, C3: $((A\Rightarrow B)\wedge(A\Rightarrow C))\Rightarrow(A\Rightarrow(B\wedge C))$,
C4: $(\boldsymbol{L}A\wedge\boldsymbol{L}B)\Rightarrow\boldsymbol{L}(A\wedge B)$, where $\boldsymbol{L}A =_{df} (A\Rightarrow A)\Rightarrow A$,
D1: $A\Rightarrow(A\vee B)$, D2: $B\Rightarrow(A\vee B)$, D3: $((A\Rightarrow C)\wedge(B\Rightarrow C))\Rightarrow((A\vee B)\Rightarrow C)$,
DCD: $(A\wedge(B\vee C))\Rightarrow((A\wedge B)\vee C)$,
C5: $(A\wedge A)\Rightarrow A$, C6: $(A\wedge B)\Rightarrow(B\wedge A)$, C7: $((A\Rightarrow B)\wedge(B\Rightarrow C))\Rightarrow(A\Rightarrow C)$,
C8: $(A\wedge(A\Rightarrow B))\Rightarrow B$, C9: $\neg(A\wedge\neg A)$, C10: $A\Rightarrow(B\Rightarrow(A\wedge B))$,
T1: $\boldsymbol{G}(A\Rightarrow B)\Rightarrow(\boldsymbol{G}A\Rightarrow\boldsymbol{G}B)$, T2: $\boldsymbol{H}(A\Rightarrow B)\Rightarrow(\boldsymbol{H}A\Rightarrow\boldsymbol{H}B)$,
T3: $A\Rightarrow\boldsymbol{G}(\boldsymbol{P}A)$, T4: $A\Rightarrow\boldsymbol{H}(\boldsymbol{F}A)$, T5: $\boldsymbol{G}A\Rightarrow\boldsymbol{G}(\boldsymbol{G}A)$,
T6: $(\boldsymbol{F}A\wedge\boldsymbol{F}B)\Rightarrow(\boldsymbol{F}(A\wedge\boldsymbol{F}B)\vee\boldsymbol{F}(A\wedge B)\vee\boldsymbol{F}(\boldsymbol{F}A\wedge B))$,
T7: $(\boldsymbol{P}A\wedge\boldsymbol{P}B)\Rightarrow(\boldsymbol{P}(A\wedge\boldsymbol{P}B)\vee\boldsymbol{P}(A\wedge B)\vee\boldsymbol{P}(\boldsymbol{P}A\wedge B))$,
T8: $\boldsymbol{G}A\Rightarrow\boldsymbol{F}A$, T9: $\boldsymbol{H}A\Rightarrow\boldsymbol{P}A$, T10: $\boldsymbol{F}A\Rightarrow\boldsymbol{F}(\boldsymbol{F}A)$,
T11: $(A\wedge\boldsymbol{H}A)\Rightarrow\boldsymbol{F}(\boldsymbol{H}A)$, T12: $(A\wedge\boldsymbol{G}A)\Rightarrow\boldsymbol{P}(\boldsymbol{G}A)$,

IQ1: $\forall x(A\Rightarrow B)\Rightarrow(\forall xA\Rightarrow\forall xB)$, IQ2: $(\forall xA\wedge\forall xB)\Rightarrow\forall x(A\wedge B)$,
IQ3: $\forall xA\Rightarrow A[t/x]$ (if x may appear free in A and t is free for x in A, i.e., free variables of t do not occur bound in A),
IQ4: $\forall x(A\Rightarrow B)\Rightarrow(A\Rightarrow\forall xB)$ (if x does not occur free in A),
IQ5: $\forall x_1 \ldots \forall x_n(((A\Rightarrow A)\Rightarrow B)\Rightarrow B)$ ($n\geq 0$),
RCC1: $\forall r_1(\boldsymbol{C}(r_1, r_1))$, RCC2: $\forall r_1\forall r_2(\boldsymbol{C}(r_1, r_2)\Rightarrow\boldsymbol{C}(r_2, r_1))$,
PRCC1: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{DC}(r_1, r_2))\Rightarrow\neg\boldsymbol{I}(p_1, r_2))$,
PRCC2: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{Pa}(r_1, r_2))\Rightarrow\boldsymbol{I}(p_1, r_2))$,
PRCC3: $\forall r_1\forall r_2(\boldsymbol{O}(r_1, r_2)\Rightarrow\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{I}(p_1, r_2)))$,
PRCC4: $\forall r_1\forall r_2(\boldsymbol{PaO}(r_1, r_2)\Rightarrow(\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{I}(p_1, r_2))\wedge$
$\exists p_2(\boldsymbol{I}(p_2, r_1)\wedge\neg\boldsymbol{I}(p_2, r_2))\wedge\exists p_3(\neg\boldsymbol{I}(p_3, r_1)\wedge\boldsymbol{I}(p_3, r_2))))$,
PRCC5: $\forall r_1\forall r_2(\boldsymbol{EC}(r_1, r_2)\Rightarrow\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{I}(p_1, r_2)\wedge$
$\forall p_2(\neg\boldsymbol{Id}(p_2, p_1)\Rightarrow(\neg\boldsymbol{I}(p_2, r_1)\wedge\neg\boldsymbol{I}(p_2, r_2)))))$,
PRCC6: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{TPrPa}(r_1, r_2))\Rightarrow\boldsymbol{I}(p_1, r_2))$,
PRCC7: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{NTPrPa}(r_1, r_2))\Rightarrow\boldsymbol{I}(p_1, r_2))$,
RC1: $\forall p_1\forall p_2(\boldsymbol{Arc}(p_1, p_2)\Rightarrow\boldsymbol{Reachable}(p_1, p_2))$,
RC2: $\forall p_1\forall p_2\forall p_3((\boldsymbol{Reachable}(p_1, p_2)\wedge\boldsymbol{Reachable}(p_2, p_3))\Rightarrow\boldsymbol{Reachable}(p_1, p_3))$,
HC1: $\forall p_1(\boldsymbol{NH}(p_1, p_1))$, HC2: $\forall p_1\forall p_2\forall p_3((\boldsymbol{NH}(p_1, p_2)\wedge\boldsymbol{NH}(p_2, p_3))\Rightarrow\boldsymbol{NH}(p_1, p_3))$,
MC1: $\forall o_1(\boldsymbol{NS}(p_1, p_1))$, MC2: $\forall o_1\forall o_2\forall o_3((\boldsymbol{NS}(o_1, o_2)\wedge\boldsymbol{NS}(o_2, o_3))\Rightarrow\boldsymbol{NS}(o_1, o_3))$,
DC1: $\forall p_1\forall p_3(\boldsymbol{ND}(p_1, p_1, p_3))$,
DC2: $\forall p_1\forall p_2\forall p_3\forall p_4((\boldsymbol{ND}(p_1, p_2, p_4)\wedge\boldsymbol{ND}(p_2, p_3, p_4))\Rightarrow\boldsymbol{ND}(p_1, p_3, p_4))$.

Inference rules:
$\Rightarrow$E: from A and $A\Rightarrow B$ to infer B (Modus Ponens),
$\wedge$I: from A and B to infer $A\wedge B$ (Adjunction),
TG: from A to infer $\boldsymbol{G}A$ and $\boldsymbol{H}A$ (Temporal Generalization),
$\forall$I: if A is an axiom, so is $\forall xA$ (Generalization of axioms).

Various relevant logic systems are defined as follows, where we use 'X | Y' to denote any choice of one from two axiom schemata X and Y:
$\mathbf{T}_{\Rightarrow} =_{\mathrm{df}} \{$E1, E2, E2′, E3 | E3″$\} + \Rightarrow$E,
$\mathbf{E}_{\Rightarrow} =_{\mathrm{df}} \{$E1, E2 | E2′, E3 | E3′, E4 | E4′$\} + \Rightarrow$E,
$\mathbf{E}_{\Rightarrow} =_{\mathrm{df}} \{$E2′, E3, E4″$\} + \Rightarrow$E,
$\mathbf{E}_{\Rightarrow} =_{\mathrm{df}} \{$E1, E3, E4‴$\} + \Rightarrow$E,
$\mathbf{R}_{\Rightarrow} =_{\mathrm{df}} \{$E1, E2 | E2′, E3 | E3′, E5 | E5′$\} + \Rightarrow$E,
$\mathbf{T}_{\Rightarrow,\neg} =_{\mathrm{df}} \mathbf{T}_{\Rightarrow} + \{$N1, N2, N3$\}$,
$\mathbf{E}_{\Rightarrow,\neg} =_{\mathrm{df}} \mathbf{E}_{\Rightarrow} + \{$N1, N2, N3$\}$,
$\mathbf{R}_{\Rightarrow,\neg} =_{\mathrm{df}} \mathbf{R}_{\Rightarrow} + \{$N2, N3$\}$,
$\mathbf{T} =_{\mathrm{df}} \mathbf{T}_{\Rightarrow,\neg} + \{$C1~C3, D1~D3, DCD$\} + \wedge$I,
$\mathbf{E} =_{\mathrm{df}} \mathbf{E}_{\Rightarrow,\neg} + \{$C1~C4, D1~D3, DCD$\} + \wedge$I,
$\mathbf{R} =_{\mathrm{df}} \mathbf{R}_{\Rightarrow,\neg} + \{$C1~C3, D1~D3, DCD$\} + \wedge$I,
$\mathbf{Tc} =_{\mathrm{df}} \mathbf{T}_{\Rightarrow,\neg} + \{$C3, C5~C10$\}$,
$\mathbf{Ec} =_{\mathrm{df}} \mathbf{E}_{\Rightarrow,\neg} + \{$C3~C10$\}$,
$\mathbf{Rc} =_{\mathrm{df}} \mathbf{R}_{\Rightarrow,\neg} + \{$C3, C5~C10$\}$,
$\mathbf{TQ} =_{\mathrm{df}} \mathbf{T} + \{$IQ1~IQ5$\} + \forall$I,
$\mathbf{EQ} =_{\mathrm{df}} \mathbf{E} + \{$IQ1~IQ5$\} + \forall$I,
$\mathbf{RQ} =_{\mathrm{df}} \mathbf{R} + \{$IQ1~IQ5$\} + \forall$I,

TcQ $=_{df}$ **Tc** + {IQ1~IQ5} + ∀I,
EcQ $=_{df}$ **Ec** + {IQ1~IQ5} + ∀I,
RcQ $=_{df}$ **Rc** + {IQ1~IQ5} + ∀I.

The minimal or weakest propositional temporal relevant logics are as:
$\mathbf{T_0Tc}$ = **Tc** + {T1~T4} + TG,
$\mathbf{T_0Ec}$ = **Ec** + {T1~T4} + TG,
$\mathbf{T_0Rc}$ = **Rc** + {T1~T4} + TG.

Note that the minimal or weakest temporal classical logic $\mathbf{K_t}$ = all axiom schemata for **CML** + →E + {T1~T4} + TG. Other characteristic axioms such as T5~T12 that correspond to various assumptions about time can be added to $\mathbf{T_0Tc}$, $\mathbf{T_0Ec}$, and $\mathbf{T_0Rc}$ respectively to obtain various propositional temporal relevant logics. Various predicate temporal relevant logics then can be obtained by adding axiom schemata IQ1~IQ5 and inference rule ∀I into the propositional temporal relevant logics. For examples, minimal or weakest predicate temporal relevant logics are as follows:
$\mathbf{T_0TcQ}$ = $\mathbf{T_0Tc}$ + {IQ1~IQ5} + ∀I,
$\mathbf{T_0EcQ}$ = $\mathbf{T_0Ec}$ + {IQ1~IQ5} + ∀I,
$\mathbf{T_0RcQ}$ = $\mathbf{T_0Rc}$ + {IQ1~IQ5} + ∀I.

Now, we can obtain various three-dimensional spatio-temporal relevant logics by adding axiom schemata about region connection, point position, and motion of mobile objects into the various predicate temporal relevant logics. For examples:
$\mathbf{ST_0TcQ}$ = $\mathbf{T_0TcQ}$ + {RCC1, RCC2, PRCC1~PRCC7, RC1, RC2, HC1, HC2, MC1, MC2, DC1, DC2},
$\mathbf{ST_0EcQ}$ = $\mathbf{T_0EcQ}$ + {RCC1, RCC2, PRCC1~PRCC7, RC1, RC2, HC1, HC2, MC1, MC2, DC1, DC2},
$\mathbf{ST_0RcQ}$ = $\mathbf{T_0RcQ}$ + {RCC1, RCC2, PRCC1~PRCC7, RC1, RC2, HC1, HC2, MC1, MC2, DC1, DC2}.

4 Reasoning about Moving Objects Based on Three-dimensional Spatio-temporal Relevant Logic

We now show that the three-dimensional spatio-temporal relevant logics proposed in Section 3 can satisfy all the requirements present in Section 2 for the fundamental logic system to underlie representing and reasoning about moving objects in three-dimensional space.

First of all, we have the following facts about relevant logics including strong relevant logics:

The strong relevant logics provide a logical validity criterion for relevant reasoning in the sense of strong relevance, i.e. for any valid reasoning based on a strong relevant logic, its premises include no irrelevant and unnecessary conjuncts and its conclusion includes no irrelevant or unnecessary disjuncts (Note that the logical validity criterion provided by traditional relevant logics is not necessarily relevant in this sense). Therefore, in the framework of strong relevant logic, if a reasoning is valid, then the strong relevance between its premises and its conclusion can be guaranteed necessarily, i.e. the logics can certainly underlie relevant reasoning in the sense of strong relevance [5, 6, 9]. On the other hand, because the strong relevant logics are free of not only implicational paradoxes but also conjunction-implicational and disjunction-implicational paradoxes, the logical validity criterion provided by strong relevant logics is truth-preserving in the sense of conditional. Note that the logical validity criterion provided by **CML** is truth-preserving only in the sense of material implication; it is not truth-preserving in the sense of conditional. Also note that the logical validity criterion provided by traditional relevant logics is truth-preserving only in the sense of relevant implication; it is not truth-preserving in the sense of conditional. Therefore, in the framework of strong relevant logic, if a reasoning is valid, then the truth of its conclusion in the sense of conditional can be guaranteed necessarily, i.e. the logics can certainly underlie truth-preserving in the sense of conditional [5, 6, 9].

A reasoning based on any of relevant logics including strong relevant logics is ampliative but not circular and/or tautological. This is because the notion of entailment (conditional) that plays the most intrinsic role in any reasoning is represented in relevant logics by a primitive intensional connective satisfying the Wright-Geach-Smiley criterion, i.e. to come to know the truth of an entailment without coming to know the falsehood of its antecedent or the truth of consequent [1, 2, 5, 6, 9, 15, 18].

All relevant logics including strong relevant logics reject the principle of Explosion, and therefore, they are paraconsistent but not explosive [1, 2, 5, 6, 9, 15, 18]. All relevant logics can certainly underlie paracomplete and paraconsistent reasoning.

Now, although the three-dimensional spatio-temporal relevant logics are conservative extensions of strong relevant logics, they extended temporal relevant logics and/or strong relevant logics by introducing only predicates and axiom schemata about solid-region connection, point position, and motion of mobile objects but nothing about the classical account of validity and the material implication. Therefore, any reasoning based on the three-dimensional spatio-temporal relevant logics must be relevant and truth-preserving in the sense of conditional as well as ampliative. The logics can also underlie paracomplete and paraconsistent reasoning in the way as the same as relevant logics.

On the other hand, the predicates and axiom schemata about solid-region connection and point position provide the means to represent and reason about three-dimensional geometric relationships among three-dimensional geometric regions and points. The temporal operators and related axiom schemata provide the means to represent and reason about those propositions and/or formulas whose truth-values may depend on time. The predicates and axiom schemata about motion of mobile objects provide the means to represent and reason about motion relationships among mobile objects in a three-dimensional geometric space. Therefore, the three-dimensional spatio-temporal relevant logics can also underlie three-dimensional spatial reasoning, temporal reasoning, and three-dimensional spatio-temporal reasoning. In particular, the logics can also underlie anticipatory reasoning about mobile three-dimensional geometric objects. An anticipatory reasoning is a reasoning to draw new, previously unknown and/or unrecognized conclusions about some future event or events whose occurrence and truth are uncertain at the point of time when the reasoning is being performed. By using temporal operators and predicates about motion of mobile objects, one can easily represent and reason about future situations of mobile objects moving in a three-dimensional geometric space.

Consequently, the three-dimensional spatio-temporal relevant logics provide an explicit way to underlie representing and reasoning about qualitative knowledge concerning time, space, motion, speed, and distance.

5 Concluding Remarks

We have proposed the three-dimensional spatio-temporal relevant logics as the fundamental logic system to underlie representing and reasoning about moving objects in three-dimensional space. The logics are obtained by introducing predicates and axiom schemata about solid-region connection, predicates and axiom schemata about point position, and predicates and axiom schemata about motion of mobile objects into temporal relevant logics. The notion of distance are defined by predicates about point position and adjacency, predicates about movement of mobile objects, and temporal operators. The logics can underlie relevant and truth-preserving reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, three-dimensional spatial reasoning, temporal reasoning, and three-dimensional spatio-temporal reasoning. To our knowledge, no other logic proposed for spatio-temporal reasoning has these advantages.

The three-dimensional spatio-temporal relevant logics also provide a foundation for constructing more powerful logic systems to underlie other reasoning issues in various intelligent systems dealing with mobile three-dimensional geometric objects. For examples, we can add epistemic operators and related axiom schemata into the logics in order to represent and reason about cognitive and/or epistemic processes of human being about mobile three-dimensional geometric objects and their motions.

We are working on applying the three-dimensional spatio-temporal relevant logics to developing anticipatory reasoning-reacting systems [7] dealing with mobile objects moving in three-dimensional space, e.g., anticipatory reasoning-reacting systems for air traffic control, train control, expressway control, and so on [10].

A future work that is important in theory is to establish a semantic (model) theory of the three-dimensional spatio-temporal relevant logics.

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