10.1 Pick out the free and bound occurrences of variables in the following formulas.  
(a) (∀x) P(x) ∧ ¬P(y)

Free: the occurrence of y in the latter ¬P(y)

Bound: the occurrence of x in the former P(x)

(b) (∀y) P(y) ↔ P(y)

Free: the occurrence of y in the latter P(y)

Bound: the occurrence of y in the former P(y)

(c) (∀y) (∃x) Q(x, y)

Free: null

Bound: the occurrence of x and y in Q(x, y)

(d) (∀x)(∀u)(∀v)P(x) → (P(y) ∧ ¬P(x))

Free: the occurrence of x in the latter ¬P(x); the occurrence of y in the latter P(y)

Bound: the occurrence of x in the former P(x)

(e) (∀x)(∀y)(∀z)P(x) ∨ (∃y)(¬(∀z)Q(z, y))

Free: null

Bound: the occurrence of x in the former P(x); the occurrence of y and z in the latter Q(z, y)

(f) (∀x)(¬P(x)) → T(x, x, y) ∨ (∀x)P(x)

Free: the occurrence of x and y in the latter T(x, x, y)

Bound: the occurrence of x in the former ¬P(x); the occurrence of x in the latter P(x)

(g) ¬(∀x)P(x) → (∃y)P(y) → Q(x, y) ∧ P(y)

Free: the occurrence of x in the latter Q(x, y); the occurrence of y in the latter Q(x, y) ∧ P(y)

Bound: the occurrence of y in the middle P(y); the occurrence of x in the former P(x)

(h) (((∀x)(P(x) → P(x))) ∨ ((∃x)P(x)))

Free: null

Bound: the occurrence of x in P(x) → P(x) and P(x)

(i) ((¬((∃y)(P(y) ∨ P(a)))) ↔ P(y))

Free: the occurrence of a in P(a); the occurrence of y in the latter P(y)

Bound: the occurrence of y in the former P(y)

(j) (((∀x)(¬(¬P(a)))) → (P(x) → P(y)))

Free: the occurrence of a in the former ¬(¬P(a)); the occurrence of x in the latter P(x); the occurrence of y in the latter P(y)

Bound: the occurrence of x in the former ¬(¬P(a))

(k) (∀z)(((∀x) Q(x, y)) → Q(z, a))

Free: the occurrence of y in Q(x, y); the occurrence of a in Q(z, a)

Bound: the occurrence of x in Q(x, y); the occurrence of z in Q(z, a)

(l) (∀y)Q(z, y) → (∀z)Q(z, y)

Free: the occurrence of z in the former Q(z, y); the occurrence of y in the latter Q(z, y)

Bound: the occurrence of y in the former Q(z, y); the occurrence of z in the latter Q(z, y)

(m) ((∀y)(∃x)T(x, y, g(x, y))) ∨ ¬(∀x)Q(y, f(x))

Free: the occurrence of y in Q(y, f(x))

Bound: the occurrence of x and y in T(x, y, g(x, y)); the occurrence of x in Q(y, f(x))

10.2 Indicate the free and bound variables in the formulas of Exercise 10.1

(a) (∀x) P(x) ∧ ¬P(y)

Free: y in the latter ¬P(y)

Bound: x in the former P(x)

(b) (∀y) P(y) ↔ P(y)

Free: y in the latter P(y)

Bound: y in the former P(y)

(c) (∀y) (∃x) Q(x, y)

Free: null

Bound: x and y in Q(x, y)

(d) (∀x)(∀u)(∀v)P(x) → (P(y) ∧ ¬P(x))

Free: x in the latter ¬P(x); y in the latter P(y)

Bound: x in the former P(x)

(e) (∀x)(∀y)(∀z)P(x) ∨ (∃y)(¬(∀z)Q(z, y))

Free: null

Bound: x in the former P(x); y and z in the latter Q(z, y)

(f) (∀x)(¬P(x)) → T(x, x, y) ∨ (∀x)P(x)

Free: x and y in the latter T(x, x, y)

Bound: x in the former ¬P(x); x in the latter P(x)

(g) ¬(∀x)P(x) → (∃y)P(y) → Q(x, y) ∧ P(y)

Free: x in the latter Q(x, y); y in the latter Q(x, y) ∧ P(y)

Bound: y in the middle P(y); x in the former P(x)

(h) (((∀x)(P(x) → P(x))) ∨ ((∃x)P(x)))

Free: null

Bound: x in P(x) → P(x) and P(x)

(i) ((¬((∃y)(P(y) ∨ P(a)))) ↔ P(y))

Free: a in P(a); y in the latter P(y)

Bound: y in the former P(y)

(j) (((∀x)(¬(¬P(a)))) → (P(x) → P(y)))

Free: a in the former ¬(¬P(a)); x in the latter P(x); y in the latter P(y)

Bound: x in the former ¬(¬P(a))

(k) (∀z)(((∀x) Q(x, y)) → Q(z, a))

Free: y in Q(x, y); a in Q(z, a)

Bound: x in Q(x, y); z in Q(z, a)

(l) (∀y)Q(z, y) → (∀z)Q(z, y)

Free: z in the former Q(z, y); y in the latter Q(z, y)

Bound: y in the former Q(z, y); z in the latter Q(z, y)

(m) ((∀y)(∃x)T(x, y, g(x, y))) ∨ ¬(∀x)Q(y, f(x))

Free: y in Q(y, f(x))

Bound: x and y in T(x, y, g(x, y)); x in Q(y, f(x))