7.1

(a) ⎥−L (¬(¬A)) → A

(1)

1. (A→(B→A))

…… {AS1 (A → (B → A)), A = A, B = B}

2. (A → ((B → A) → A)) → ((A → (B → A)) → (A → A))

…… {AS2 (A → (B → C)) → ((A → B) → (A → C))), A = A, B = (B → A), C = A}

3. (A → (B → A)) → (A → A)

…… {From 1 and 2 by MP}

4. (A → A)

…… {From 1 and 3 by MP}

5. ((B → C) → ((A → B) → C)) → ((A → B) → (A → C)))) → (((B → C) → (A → (B → C))) → ((B → C) → ((A → B) → (A → C))))

…… {AS2 (A → (B → C)) → ((A → B) → (A → C))), A = (B → C), B = (A → B), C = C}

6. ((A → (B → C)) → ((A → B) → (A → C))) → ((B → C) → ((A → (B → C) → ((A → B) → (A → C))))

…… {AS1 (A → (B → A)), A = (A → (B → C)) → ((A → B) → (A → C)), B = (B → C)}

7. (A → (B → C)) → ((A → B) → (A → C)))

…… {AS2 (A → (B → C)) → ((A → B) → (A → C))), A = A, B = B, C = C}

8. ((B → C) → ((A → (B → C) → ((A → B) → (A → C))))

…… {From 6 and 7 by MP}

9. ((B → C) → (A → (B → C))) → ((B → C) → ((A → B) → (A → C)))

…… {From 5 and 8 by MP}

10. (B → C) → (A → (B → C))

…… {AS1 (A → (B → A)), A = (B → C), B = A}

11. (B → C) → ((A → B) → (A → C))

…… {From 9 and 10 by MP}

12. (((¬B) → (¬A)) → (A → B)) → (((¬A) → ((¬B) → (¬A)) → ((¬A) → (A → B)))

…… {From 11, A = (¬A), B = ((¬B) → (¬A)), C = (A → B)}

13. (((¬B) → (¬A)) → (A → B))

…… {AS3 (((¬A) → (¬B)) → (B → A)), A = B, B = A}

14. ((¬A) → ((¬B) → (¬A)) → ((¬A) → (A → B))

…… {From 12 and 13 by MP}

15. ((¬A) → ((¬B) → (¬A)))

…… {AS1 (A → (B → A)), A = (¬A), B = (¬B)}

16. (¬A) → (A → B)

…… {From 14 and 15 by MP}

17. ((¬(¬A)) → ((¬(¬A)) → A) → (((¬(¬A)) → (¬(¬A))) → ((¬(¬A)) → A))

…… {AS2 (A → (B → C)) → ((A → B) → (A → C))), A = (¬(¬A)), B = (¬(¬A)), C = A}

18. (((¬A) → (¬(¬(¬A)))) → ((¬(¬A)) → A)) → (((¬(¬A)) → ((¬A) → (¬(¬(¬A))))) → ((¬(¬A)) → ((¬(¬A)) → A)))

…… {From 11, A = (¬(¬A)), B = (¬A) → (¬(¬(¬A))), C = ((¬(¬A)) → A) }

19. (((¬A) → (¬(¬(¬A)))) → ((¬(¬A)) → (¬A)))

…… {AS3 (((¬A) → (¬B)) → (B → A)), A = (¬A), B = (¬(¬(¬A)))}

20. ((¬(¬A)) → ((¬A) → (¬(¬(¬A))))) → ((¬(¬A)) → ((¬(¬A)) → A))

…… {From 18 and 19 by MP}

21. ((¬(¬A)) → ((¬(¬A)) → (¬(¬(¬A))))

…… {From 16, A = (¬(¬A)), B = (¬(¬(¬A)))}

22. (¬(¬A)) → ((¬(¬A)) → A)

…… {From 20 and 21 by MP}

23. ((¬(¬A)) → (¬(¬A))) → ((¬(¬A)) → A)

…… {From 17 and 22 by MP}

24. ((¬(¬A)) → (¬(¬A)))

…… {From 4, A = (¬(¬A))}

25. (¬(¬A)) → A

…… {From 23 and 24 by MP}

(2)

Show that {(¬(¬A))} ⎥−L A

1. (¬(¬A))

…… {Premise}

2. (¬(¬A)) → (A → (¬(¬A)))

…… {AS1 (A → (B → A)), A = (¬(¬A)), B = A}

3. A → (¬(¬A))

…… {From 1 and 2 by MP}

4. (¬(¬A)) → A

…… {From 3, A = (¬(¬A))}

5. A

…… {From 1 and 4 by MP}

By using the deduction theorem, we have:

⎥−L A → (¬(¬A))

(b) ⎥−L A → (¬(¬A))

(1)

1. (¬(¬(¬A))) → (¬A)

…… {From (a), A = (¬A)}

2. ((¬(¬(¬A))) → (¬A)) → (A → (¬(¬A)))

…… {AS3 (((¬A) → (¬B)) → (B → A)), A = (¬(¬A)), B = (¬A)}

3. A → (¬(¬A))

…… {From 1 and 2 by MP}

(2)

Show that {A} ⎥−L (¬(¬A))

1. A

…… {Premise}

2. A → ((¬(¬A)) → A)

…… {AS1 (A → (B → A)), A =A, B = (¬A)}

3. (¬(¬A)) → A

…… {From 1 and 2 by MP}

4. A → (¬(¬A))

…… {From 3, A = (¬(¬A))}

5. (¬(¬A))

…… {From 1 and 4 by MP}

By using the deduction theorem, we have:

⎥−L A → (¬(¬A))

(c) ⎥−L (¬B) → (B → C)

(1)

1. (¬B) → ((¬C) → (¬B))

…… {AS1 (A → (B → A)), A = (¬B), B = (¬C)}

2. ((¬C) → (¬B)) → (B → C)

…… {AS3 ((¬A) → (¬B)) → (B → A), A = C, B = B}

3. (((¬C) → (¬B)) → (B → C)) → ((¬B) → (((¬C) → (¬B)) → (B → C)))

…… {AS1 (A → (B → A)), A = …… {AS1 (A → (B → A)), A = ((¬C) → (¬B)) → (B → C), B = (¬B)}

4. ((¬B) → (((¬C) → (¬B)) → (B → C)))

…… {From 2 and 3 by MP}

5. ((¬B) → (((¬C) → (¬B)) → (B → C))) → (((¬B) → ((¬C) → (¬B))) → ((¬B) → (B → C)))

…… {AS2 (A → (B → C)) → ((A → B) → (A → C))), A = (¬B), B = ((¬C) → (¬B)), C = (B → C)}

6. ((¬B) → ((¬C) → (¬B))) → ((¬B) → (B → C))

…… {From 4 and 5 by MP}

7. (¬B) → (B → C)

…… {From 1 and 6 by MP}

(2)

Show that {(¬B), B} ⎥−L C

1. (¬B)

…… {Premise}

2. B

…… {Premise}

3. (¬B) → ((¬C) → (¬B))

…… {AS1 (A → (B → A)), A = (¬B), B = (¬C)}

4. (¬C) → (¬B)

…… {From 1 and 3 by MP}

5. ((¬C) → (¬B)) → (B → C)

…… {AS3 ((¬A) → (¬B)) → (B → A), A = C, B = B}

6. B → C

…… {From 4 and 5 by MP}

7. C

…… {From 2 and 6 by MP}

By using the deduction theorem, we have:

⎥−L (¬B) → (B → C)

(d) ⎥−L ((¬C) → (¬B)) → (B → C)

(1)

1. ((¬C) → (¬B)) → (B → C)

…… {AS3 ((¬A) → (¬B)) → (B → A), A = C, B = B}

(2)

Show that {((¬C) → (¬B)), B} ⎥−HB C

1. (¬C) → (¬B)

…… {Premise}

2. B

…… {Premise}

4. (¬(¬A)) → A

…… {From (a)}

5. ((¬C) → (¬B)) → (B → C)

…… {AS3 ((¬A) → (¬B)) → (B → A), A = C, B = B}

6. B → C

…… {From 1 and 5 by MP}

7. C

…… {From 2 and 6 by MP}

By using the deduction theorem, we have:

⎥−L ((¬C) → (¬B)) → (B → C)

(e) ⎥−L (B → C) → ((¬C) → (¬B))

(1)

1. (¬(¬A)) → A

…… {From (a)}

2. (B → C) → ((¬C) → (¬B))

…… {AS3 ((¬A) → (¬B)) → (B → A), A = (¬B), B = (¬C)}

(2)

Show that {(B → C), (¬C)} ⎥−HB (¬B)

1. B → C

…… {Premise}

2. ¬C

…… {Premise}

4. (¬(¬A)) → A

…… {From (a)}

5. (B → C) → ((¬C) → (¬B))

…… {AS3 ((¬A) → (¬B)) → (B → A), A = ¬B, B = ¬C}

6. (¬C) → (¬B)

…… {From 1 and 5 by MP}

7. ¬B

…… {From 2 and 6 by MP}

By using the deduction theorem, we have:

⎥−L (B → C) → ((¬C) → (¬B))

(f) ⎥−L B → ((¬C) → (¬(B → C)))

(1)

(2)

Show that {B, (¬C)} ⎥−L (¬(B → C))

1. ¬C

…… {Premise}

2. B

…… {Premise}

3. ((B → C) → C) → ((¬C) → (¬(B → C)))

…… {AS3 ((¬A) → (¬B)) → (B → A), A = ¬ (B → C), B = ¬C}

4. B → ((B → C) → B)

…… {AS1 (A → (B → A)), A = B, B = (B → C)}

5. (B → C) → B

…… {From 2 and 4 by MP}

6. ((B → C) → (B → C)) → (((B → C) → B) → ((B → C) → C))

…… {AS2 ((A → (B → C)) → ((A → B) → (A → C))), A = (B → C), B = B, C= C)}

7. (X→(B→X))

…… {AS1 (A → (B → A)), A = X, B = B}

8. (X → ((B → X) → X)) → ((X → (B → X)) → (X → X))

…… {AS2 (A → (B → C)) → ((A → B) → (A → C))), A = X, B = (B → X), C = X}

9. (X → (B → X)) → (X → X)

…… {From 7 and 8 by MP}

10. (X → X)

…… {From 7 and 9 by MP}

11. (B → C) → (B → C)

…… {From 10, X = (B → C)}

12. ((B → C) → B) → ((B → C) → C)

…… {From 6 and 11 by MP}

13. (B → C) → C

…… {From 5 and 12 by MP}

14. (¬C) → (¬(B → C))

…… {From 3 and 13 by MP}

15. ¬(B → C)

…… {From 1 and 14 by MP}

By using the deduction theorem, we have:

⎥−L B → ((¬C) → (¬(B → C)))

(g) ⎥−L (B → C) → (((¬B) → C) → C)

(1)

(2)

Show that {((¬B) → C), (B → C)} ⎥−L C

1. ((¬B) → C)

…… {Premise}

2. (B → C)

…… {Premise}

3. (¬B) → ((¬C) → (¬B))

…… {AS1 (A → (B → A)), A = (¬B), B = (¬C)}

4. ((¬C) → (¬B)) → (B → C)

…… {AS3 ((¬A) → (¬B)) → (B → A), A = C, B = B}

5. (¬C) → (¬B)

…… {From 1 and 3 by MP}

6. ((¬C) → (¬B)) → (B → C)

…… {AS3 ((¬A) → (¬B)) → (B → A), A = C, B = B}

7. B → C

…… {From 4 and 5 by MP}

8. C

…… {From 2 and 6 by MP}

By using the deduction theorem, we have:

⎥−L (B → C) → (((¬B) → C) → C)

7.2

(a) ⎥−HB A → (B → (A ∧ B))

(1)

1.

(2)

Show that {A , B} ⎥−HB (A ∧ B)

1. A

…… {Premise}

2. B

…… {Premise}

3. A → (A → A)

…… {AS (A → (B → A)), A = A, B = A}

4. A → A

…… {From 1 and 3 by MP}

5. (A → A) → ((A → B) → (A → (A ∧ B))

…… {AS (A → B) → ((A → C) → (A → (B ∧ C)), A = A, B = A, C = B}

6. (A → B) → (A → (A ∧ B))

…… {From 4 and 5 by MP}

7. B → (A → B)

…… {AS (A → (B → A)), A = B, B = A}

8. A → B

…… {From 2 and 7 by MP}

9. A → (A ∧ B)

…… {From 6 and 8 by MP}

10. (A ∧ B)

…… {From 1 and 9 by MP}

By using the deduction theorem, we have:

⎥−HB A → (B → (A ∧ B))

(b) ⎥−HB ((A ∧ B) ↔ (B ∧ A))

(1)

(2)

(i) Show that ⎥−HB ((A ∧ B) → (B ∧ A))

1. (A ∧ B)

…… {Premise}

2. (A ∧ B) → B

…… {AS (A ∧ B) → B}

3. (A ∧ B) → A

…… {AS (A ∧ B) → A}

4. B

…… {From 1 and 2 by MP}

5. A

…… {From 1 and 3 by MP}

6. B → (A → B)

…… {AS A → (B → A), A = B, B = A}

7. A → B

…… {From 4 and 6 by MP}

8. A → (A → A)

…… {AS (A → (B → A)), A = A, B = A}

9. A → A

…… {From 5 and 8 by MP}

10. (A → B) → ((A → A) → (A → (B ∧ A))

…… {AS (A → B) → ((A → C) → (A → (B ∧ C)), A = A, B = B, C = A}

11. (A → A) → (A → (B ∧ A)

…… {From 7 and 10 by MP}

12. A → (B ∧ A)

…… {From 9 and 11 by MP}

13. (B ∧ A)

…… {From 5 and 12 by MP}

By using the deduction theorem, we have:

⎥−HB ((A ∧ B) → (B ∧ A))

(ii) Show that ⎥−HB ((B ∧ A) → (A ∧ B))

1. (B ∧ A)

…… {Premise}

2. (B ∧ A) → B

…… {AS (B ∧ A) → B}

3. (B ∧ A) → A

…… {AS (B ∧ A) → A}

4. B

…… {From 1 and 2 by MP}

5. A

…… {From 1 and 3 by MP}

6. B → (A → B)

…… {AS A → (B → A), A = B, B = A}

7. A → B

…… {From 4 and 6 by MP}

8. A → (A → A)

…… {AS (A → (B → A)), A = A, B = A}

9. A → A

…… {From 5 and 8 by MP}

10. (A → A) → ((A → B) → (A → (A ∧ B))

…… {AS (A → B) → ((A → C) → (A → (B ∧ C)), A = A, B = A, C = B}

11. (A → B) → (A → (A ∧ B)

…… {From 9 and 10 by MP}

12. A → (A ∧ B)

…… {From 7 and 11 by MP}

13. (A ∧ B)

…… {From 5 and 12 by MP}

By using the deduction theorem, we have:

⎥−HB ((B ∧ A) → (A ∧ B))

(iii) Show that ⎥−HB ((A ∧ B) ↔ (B ∧ A))

1. (A ∧ B) → (B ∧ A)

…… {From (i)}

2. (B ∧ A) → (A ∧ B)

…… {From (ii)}

3. ((A ∧ B) → (B ∧ A)) → (((B ∧ A) → (A ∧ B)) → ((A ∧ B) ↔ (B ∧ A)))

…… {AS (A → B) → ((B → A) → (A ↔ B)), A = A ∧ B, B = B ∧ A}

4. ((B ∧ A) → (A ∧ B)) → ((A ∧ B) ↔ (B ∧ A))

…… {From 1 and 3 by MP}

5. (A ∧ B) ↔ (B ∧ A)

…… {From 2 and 4 by MP}

Above all, ⎥−HB ((A ∧ B) ↔ (B ∧ A))

(c) ⎥−HB (A ∨ B) ↔ (B ∨ A)

(1)

(2)

(i) Show that {(A ∨ B)}⎥−HB (B ∨ A)

1. (A ∨ B)

…… {Premise}

2. A → (B ∨ A)

…… {AS B → (A ∨ B), A = B, B = A}

3. B → (B ∨ A)

…… {AS A → (A ∨ B), A = B, B = A}

4. (A → (B ∨ A)) → ((B → (B ∨ A)) → ((A ∨ B) → (B ∨ A)))

…… {AS (A → C) → ((B → C) → ((A ∨ B) → C)), A = A, B = B, C = B ∨ A }

5. (B → (B ∨ A)) → ((A ∨ B) → (B ∨ A))

…… {From 2 and 4 by MP}

6. (A ∨ B) → (B ∨ A)

…… {From 3 and 5 by MP}

7. (B ∨ A)

…… {From 1 and 6 by MP}

By using the deduction theorem, we have:

⎥−HB ((A ∨ B) → (B ∨ A))

(ii) Show that {(B ∨ A)}⎥−HB (A ∨ B)

1. (B ∨ A)

…… {Premise}

2. A → (A ∨ B)

…… {AS A → (A ∨ B), A = A, B = B}

3. B → (A ∨ B)

…… {AS B → (A ∨ B), A = A, B = B}

4. (A → (A ∨ B)) → ((B → (A ∨ B)) → ((B ∨ A) → (A ∨ B)))

…… {AS (A → C) → ((B → C) → ((A ∨ B) → C)), A = A, B = B, C = A ∨ B}

5. (B → (A ∨ B)) → ((B ∨ A) → (A ∨ B))

…… {From 2 and 4 by MP}

6. (B ∨ A) → (A ∨ B)

…… {From 3 and 5 by MP}

7. (A ∨ B)

…… {From 1 and 6 by MP}

By using the deduction theorem, we have:

⎥−HB ((B ∨ A) → (A ∨ B))

(iii) Show that ⎥−HB ((A ∨ B) ↔ (B ∨ A))

1. (A ∨ B) → (B ∨ A)

…… {From (i)}

2. (B ∨ A) → (A ∨ B)

…… {From (ii)}

3. ((B ∨ A) → (A ∨ B)) → (((A ∨ B) → (B ∨ A)) → ((B ∨ A) ↔ (A ∨ B)))

…… {AS (A → B) → ((B → A) → (A ↔ B)), A = B ∨ A, B = A ∨ B}

4. ((A ∨ B) → (B ∨ A)) → ((B ∨ A) ↔ (A ∨ B))

…… {From 2 and 3 by MP}

5. (B ∨ A) ↔ (A ∨ B)

…… {From 1 and 4 by MP}

Above all, ⎥−HB ((A ∨ B) ↔ (B ∨ A))

(d) ⎥−HB (A → B) → ((C ∨ A) → (C ∨ B))

(1)

1.

(2)

Show that {(A → B) , (C ∨ A)} ⎥−HB (C ∨ B)

1. (A → B)

…… {Premise}

2. (C ∨ A)

…… {Premise}

3. (A → B) → ((B → (C ∨ B)) → (A → (C ∨ B)))

…… {AS (A → B) → ((B → C) → (A → C)), A = A, B = B, C = C ∨ B}

4. ((B → (C ∨ B)) → (A → (C ∨ B)))

…… {From 1 and 3 by MP}

5. B → (C ∨ B)

…… {AS B → (A ∨ B), A = C, B = B}

6. A → (C ∨ B)

…… {From 4 and 5 by MP}

7. (C → (C ∨ B)) → ((A → (C ∨ B)) → ((C ∨ A) → (C ∨ B)))

…… {AS (A → C) → ((B → C) → ((A ∨ B) → C)), A = C, B = A, C = C ∨ B }

8. C → (C ∨ B)

…… {AS A → (A ∨ B), A = C, B = B}

9. (A → (C ∨ B)) → ((C ∨ A) → (C ∨ B))

…… {From 7 and 8 by MP}

10. (C ∨ A) → (C ∨ B)

…… {From 6 and 9 by MP}

11. (C ∨ B)

…… {From 2 and 10 by MP}

By using the deduction theorem, we have:

⎥−HB (A → B) → ((C ∨ A) → (C ∨ B))

(e) ⎥−HB (A ∨ (B ∨ C)) ↔ ((A ∨ B) ∨ C))

(1)

1.

(2)

(f) ⎥−HB (A ∨ (B ∧ C)) ↔ ((A ∨ B) ∧ (A ∨ C))

(1)

1.

(2)

(g) ⎥−HB (A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (A ∧ C))

(1)

1.

(2)

(i) Show that {(A ∧ (B ∨ C))} ⎥−HB ((A ∧ B) ∨ (A ∧ C))

1. (A ∧ (B ∨ C))

…… {Premise}

2. (A ∧ (B ∨ C)) → A

…… {AS (A ∧ B) → A, A = A, B = B ∨ C}

3. (A ∧ (B ∨ C)) → (B ∨ C)

…… {AS (A ∧ B) → B, A = A, B = B ∨ C}

4. A

…… {From 1 and 2 by MP}

5. (B ∨ C)

…… {From 1 and 3 by MP}

6. A → (B → A)

…… {AS A → (B → A), A = A, B = B}

7. A → (C → A)

…… {AS A → (B → A), A = A, B = C}

8. (B → A)

…… {From 4 and 6 by MP}

9. (C → A)

…… {From 4 and 7 by MP}

10. B → (B → B)

…… {AS A → (B → A), A = B, B = B}

11. (B → (B → B)) → (B → B)

…… {AS (A → (A → B)) → (A → B), A = B, B = B}

12. (B → B)

…… {From 10 and 11 by MP}

13. C → (C → C)

…… {AS A → (B → A), A = C, B = C}

14. (C → (C → C)) → (C → C)

…… {AS (A → (A → B)) → (A → B), A = C, B = C}

15. (C → C)

…… {From 13 and 14 by MP}

16. (B → A) → ((B → B) → (B → (A ∧ B)))

…… {AS (A → B) → ((A → C) → (A → (B ∧ C))), A = B, B = A, C = B}

17. (C → A) → ((C → C) → (C → (A ∧ C)))

…… {AS (A → B) → ((A → C) → (A → (B ∧ C))), A = C, B = A, C = C}

18. (B → B) → (B → (A ∧ B))

…… {From 8 and 16 by MP}

19. (C → C) → (C → (A ∧ C))

…… {From 9 and 17 by MP}

20. B → (A ∧ B)

…… {From 12 and 18 by MP}

21. C → (A ∧ C)

…… {From 15 and 19 by MP}

22. a. (A ∧ B)

…… {From 5 and 20 by MP}

or b. (A ∧ C)

…… {From 5 and 21 by MP}

23. (A ∧ B) → ((A ∧ B) ∨ (A ∧ C))

…… {AS (A → (A ∨ B)), A = A ∧ B, B = A ∧ C}

24. (A ∧ C) → ((A ∧ B) ∨ (A ∧ C))

…… {AS (B → (A ∨ B)), A = A ∧ B, B = A ∧ C}

25. (A ∧ B) ∨ (A ∧ C)

…… {From 22.a and 23 by MP}

or (A ∧ B) ∨ (A ∧ C)

…… {From 22.b and 23 by MP}

By using the deduction theorem, we have:

⎥−HB ((A ∧ (B ∨ C)) → ((A ∧ B) ∨ (A ∧ C)))

(ii) Show that {((A ∧ B) ∨ (A ∧ C))} ⎥−HB (A ∧ (B ∨ C))

① First we can show that {(A ∧ B)}⎥−HB (B ∨ C)

1. (A ∧ B)

…… {premise}

2. (A ∧ B) → B

…… {AS (A ∧ B) → B, A = A, B = B}

3. B

…… {From 1 and 2 by MP}

4. B → (B ∨ C)

…… {AS (A → (A ∨ B)), A = B, B = C}

5. (B ∨ C)

…… {From 3 and 4 by MP}

By using the deduction theorem, we have:

⎥−HB ((A ∧ B) → (B ∨ C))

② Then we can show that {((A ∧ B) ∨ (A ∧ C))} ⎥−HB (A ∧ (B ∨ C))

1. (A ∧ B) → (B ∨ C)

…… {From ①}

2. ((A ∧ B) → A) → (((A ∧ B) → (B ∨ C)) → ((A ∧ B) → (A ∧ (B ∨ C))))

…… {AS (A → B) → ((A → C) → (A → (B ∧ C))), A = A ∧ B, B = A, C = B ∨ C}

3. (A ∧ B) → A

…… {AS (A ∧ B) → A, A = A, B = B}

4. ((A ∧ B) → (B ∨ C)) → ((A ∧ B) → (A ∧ (B ∨ C)))

…… {From 2 and 3 by MP}

5. (A ∧ B) → (A ∧ (B ∨ C))

…… {From 1 and 4 by MP}

6. (A ∧ (B ∨ C))

…… {From 3 and 5 by MP}

By using the deduction theorem, we have:

⎥−HB (((A ∧ B) ∨ (A ∧ C)) → (A ∧ (B ∨ C)))

(iii) Show that ⎥−HB (A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (A ∧ C))

1. ((A ∧ (B ∨ C)) → ((A ∧ B) ∨ (A ∧ C)))

…… {From (i)}

2. (((A ∧ B) ∨ (A ∧ C)) → (A ∧ (B ∨ C)))

…… {From (ii)}

3. ((A ∧ (B ∨ C)) → ((A ∧ B) ∨ (A ∧ C))) → ((((A ∧ B) ∨ (A ∧ C)) → (A ∧ (B ∨ C))) → ((A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (A ∧ C)))

…… {AS (A → B) → ((B → A) → (A ↔ B)), A = ((A ∧ (B ∨ C)) → ((A ∧ B) ∨ (A ∧ C))), B = (((A ∧ B) ∨ (A ∧ C)) → (A ∧ (B ∨ C)))}

4. ((((A ∧ B) ∨ (A ∧ C)) → (A ∧ (B ∨ C))) → ((A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (A ∧ C)))

…… {From 1 and 3 by MP}

5. (A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (A ∧ C))

…… {From 2 and 4 by MP}

Above all, ⎥−HB (A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (A ∧ C))

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