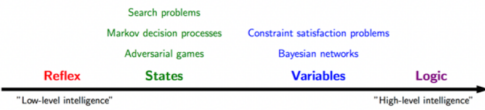


Goal-based agents 有目标的搜索

- **Reflex agents:** use a mapping from states to actions (lookup).
- **Goal-based agents:** problem solving agents or planning agents.



- Agents work towards a **goal**.
- Agents consider the impact of **actions** on future **states**, which means that their job is to identify the action or series of actions that lead to the goal.
- Formalized as a **search** through possible solutions.



Problem solving as search

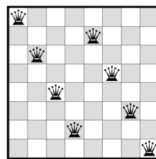
1. **Define the problem through:**
 - (a) Goal formulation
 - (b) Problem formulation
2. **Solving the problem as a 2-stage process:**
 - (a) Search: "mental" or "offline" exploration of several possibilities
 - (b) Execute the solution found

Problem formulation

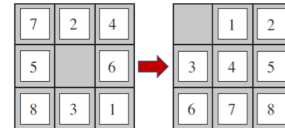
- **Initial state:** the state in which the agent starts.
- **States:** all states reachable from the initial by any sequence of actions. (State space)
- **Actions:** possible actions available to the agent. At a state s , $Actions(s)$ returns the set of actions that can be executed in state s . (Action space)
- **Transition model:** a description of what each action does $Results(s, a)$.
- **Goal test:** determine if a given state is a goal state.
- **Path cost:** function that assigns a numeric cost to each path w.r.t. performance measure.

Formulation: 8-queen problem

- **States:** all arrangements of 8 queens on the board.
- **Initial state:** no queen on the board.
- **Actions:** add a queen to any empty square.
- **Transition:** updated board.
- **Goal test:** 8 queens on board without attacked?



- **States:** any configuration of the 8 tiles on the 3x3 grid
- **Initial state:** any state (e.g., the configuration of the Left).
- **Actions:** move Left, Right, Up or Down.
- **Transition:** Given a state and an action, returns resulting state.
- **Goal state:** the configuration of the Right?
- **Path cost:** #moves.



Formulation: Routing problem

- **States:** In City where $City \in \{\text{New York, San Francisco, Denver, ...}\}$.
- **Initial state:** In Boston
- **Actions:** walk to the adjacent city.
- **Transition:** new city.
- **Goal test:** In Denver
- **Path cost:** path length in kilometers.



State space vs. Search space

- **State space**: a *physical* configuration
- **Search space**: an *abstract* configuration represented by a search tree or graph of possible solutions.
- **Search tree models the sequence of actions.**
 - Root: initial state
 - Branches: actions
 - Nodes: results from actions. A node has: parent, children, depth, path cost, associated state in the state space.
- **Expand**: A function that given a node, creates all children nodes

Search Space Regions

- The search space is divided into three regions:
 - 1. **Explored** (a.k.a. Closed List, Visited Set)
 - 2. **Frontier** (a.k.a. Ready list, Open List, the Fringe)
 - 3. **Unexplored**.
- The **essence of search** is moving nodes from regions (3) to (2) to (1), and the essence of search strategy is deciding the order of such moves.

搜索空间有三部分:

①已搜索 ②前沿子 ③未搜索

Tree search

```
function TREE-SEARCH(initialState, goalTest)
  returns SUCCESS or FAILURE :

  initialize frontier with initialState

  while not frontier.isEmpty():
    state = frontier.remove()

    if goalTest(state):
      return SUCCESS(state)

    for neighbor in state.neighbors():
      frontier.add(neighbor)

  return FAILURE
```

Graph Search

```
function GRAPH-SEARCH(initialState, goalTest)
  returns SUCCESS or FAILURE :

  initialize frontier with initialState
  explored = Set.new()

  while not frontier.isEmpty():
    state = frontier.remove()
    explored.add(state)

    if goalTest(state):
      return SUCCESS(state)

    for neighbor in state.neighbors():
      if neighbor not in frontier ∪ explored:
        frontier.add(neighbor)

  return FAILURE
```

能避免重复搜索同一个节点
(即重复状态)

Search strategies

- A strategy is defined by picking the **order of node expansion**
- Strategies are evaluated along the following dimensions:
 - **Completeness**: Does it always find a solution if it exists?
 - **Time complexity**: # nodes generated/expanded.
 - **Space complexity**: maximum # nodes in memory.
 - **Optimality**: Does it always find the least-cost solution?

In particular, time and space complexity are measured regarding:

- **b** – maximum branching factor of the search tree (actions per state).
- **d** – depth of the solution.
- **m** – maximum depth of the state space (may be ∞) (also noted sometimes **D**).

Two kinds of search: **Uninformed** and **Informed**.

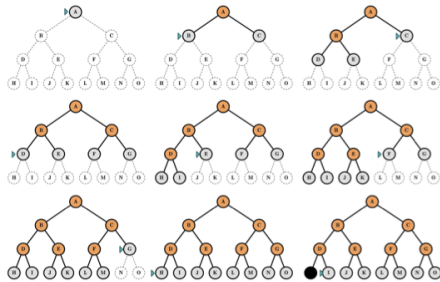
Uninformed Search

- Use **no** domain knowledge.
- **Strategies**:
 1. Breadth-first search (BFS): Expand shallowest node
 2. Depth-first search (DFS): Expand deepest node
 3. Depth-limited search (DLS): Depth first with depth limit
 4. Iterative-deepening search (IDS): DLS with increasing limit
 5. Uniform-cost search (UCS): Expand least cost node

无任何领域知识

Breadth-first search (BFS)

- BFS: Expand **shallowest** first.



Pseudo-code

```
function GRAPH-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE:

    initialize frontier with initialState
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.remove()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.add(neighbor)

    return FAILURE
```

```
function BREADTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE:

    frontier = Queue.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.dequeue()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.enqueue(neighbor)

    return FAILURE
```

BFS: PF Metrics

- **Complete**: Yes (if b is finite)
- **Time**: $1 + b + b^2 + b^3 + \dots + b^d = O(b^d)$
- **Space**: $O(b^d)$
- **Optimal**: Yes (if cost = 1 per step).
- **Implementation**: frontier: FIFO (Queue)

BFS: PF Metrics

How bad is BFS?

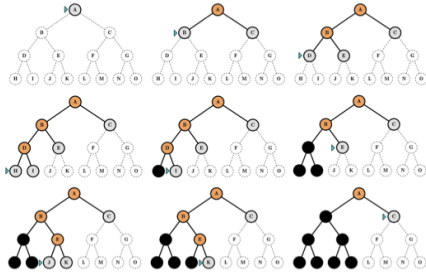
Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Time and Memory requirements for breadth-first search for a branching factor $b=10$; 1 million nodes per second; 1,000 bytes per node.

Memory requirement + exponential time complexity are the biggest handicaps of BFS!

Depth-first Search (DFS)

- DFS: Expand **deepest** first.



Note: Once a node is expanded, it is removed from memory asap all its children are explored.

Pseudo-code

```
function BREADTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE:
    frontier = Queue.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.dequeue()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.enqueue(neighbor)

    return FAILURE
```

```
function DEPTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE:
    frontier = Stack.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.pop()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.push(neighbor)

    return FAILURE
```

DFS: PF Metrics

Complete:

No: fails in infinite-depth spaces, spaces with loops.
Modify to avoid repeated states along path: complete in finite spaces

Time: $1 + b + b^2 + b^3 + \dots + b^m = O(b^m)$

bad if m is much larger than d
but if solutions are dense, may be much faster than BFS.

Space: $O(bm)$ linear space complexity! (needs to store only a single path from the root to a leaf node, along with the remaining unexpanded sibling nodes for each node on the path, hence the m factor.)

Optimal: No

Implementation: fringe: LIFO (Stack)

How bad is DFS?

Time and Memory requirements for breadth-first search for a branching factor $b=10$: 1 million nodes per second, 1,000 bytes per node.

Recall for BFS...

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Depth = 16.

We go down from 10 exabytes in BFS to 156 kilobytes in DFS!

Depth-limited Search (DLS)

- DFS with depth limit L (nodes at level L has no successors).
- Select some limit in depth to explore with DFS
- If we know some knowledge about the problem, may be we don't need to go to a full depth.



Idea: any city can be reached from another city in at most L steps with $L < 36$.

限定DFS搜索深度

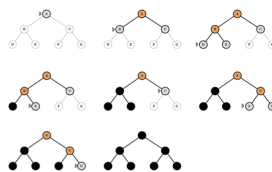
Iterative deepening Search (IDS)

- Combines the benefits of BFS and DFS.
- Idea: Iteratively increase the search limit until the depth of the shallowest solution d is reached.
- Applies **DLS with increasing limits**.
- The algorithm will stop if a solution is found or if DLS returns a failure (no solution).
- Because most of the nodes are on the bottom of the search tree, it not a big waste to iteratively re-generate the top
- Let's take an example with a depth limit between 0 and 3.

Limit = 0



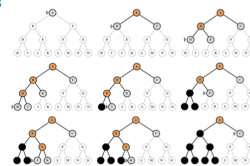
Limit = 2



Limit = 1



Limit = 3



不断加深DFS深度

Uniform-cost Search (UCS)

- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
- We want the **cheapest** not shallowest solution.
- Modify BFS: Prioritize by cost not depth → Expand node n with the lowest path cost $g(n)$
- Explores increasing costs.



限定只搜索几个节点
使cost g(n)最小

UCS: Pseudo-code

```
function BREADTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE:
        frontier = Queue.new(initialState)
        explored = Set.new()

        while not frontier.isEmpty():
            state = frontier.dequeue()
            explored.add(state)

            if goalTest(state):
                return SUCCESS(state)

            for neighbor in state.neighbors():
                if neighbor not in frontier ∪ explored:
                    frontier.enqueue(neighbor)

        return FAILURE

function UNIFORM-COST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE: /* Cost: f(n) = g(n) */
        frontier = Heap.new(initialState)
        explored = Set.new()

        while not frontier.isEmpty():
            state = frontier.deleteMin()
            explored.add(state)

            if goalTest(state):
                return SUCCESS(state)

            for neighbor in state.neighbors():
                if neighbor not in frontier ∪ explored:
                    frontier.insert(neighbor)
                else if neighbor in frontier:
                    frontier.decreaseKey(neighbor)

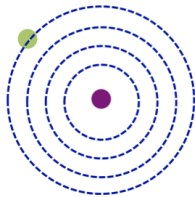
        return FAILURE
```

Cost: $f(n) = \text{depth}(n)$ 即BFS

Cost: $f(n) = -\text{depth}(n)$ 即DFS

UCS: PF Metrics

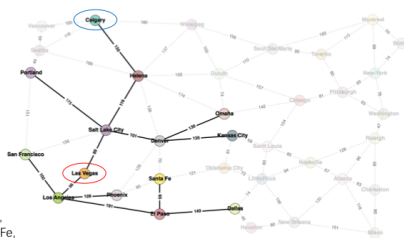
- Complete:** Yes, if solution has a finite cost.
- Time:**
 - Suppose C^* : cost of the optimal solution.
 - Every action costs at least ϵ (bound on the cost).
 - The effective depth is roughly C^*/ϵ (how deep the cheapest solution could be).
 - $O(b^{C^*/\epsilon})$
- Space:** # of nodes with $g \leq$ cost of optimal solution, $O(b^{C^*/\epsilon})$
- Optimal:** Yes.
- Implementation:** frontier = queue ordered by path cost $g(n)$, lowest first = Heap!
- While complete and optimal, UCS explores the space in every direction because no information is provided about the goal!**



Example using the map

BFS

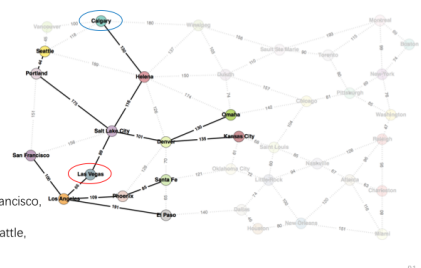
Start: Las Vegas
Goal: Calgary



Order of Visit: Las Vegas, Los Angeles, Salt Lake City, El Paso, Phoenix, San Francisco, Denver, Helena, Portland, Dallas, Santa Fe, Kansas City, Omaha, Calgary.

UCS

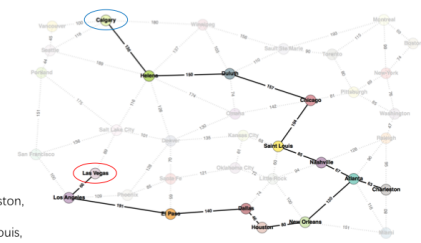
Start: Las Vegas
Goal: Calgary



Order of Visit: Las Vegas, Los Angeles, Salt Lake City, San Francisco, Phoenix, Denver, Helena, El Paso, Santa Fe, Portland, Seattle, Omaha, Kansas City, Calgary.

DFS

Start: Las Vegas
Goal: Calgary



Order of Visit: Las Vegas, Los Angeles, El Paso, Dallas, Houston, New Orleans, Atlanta, Charleston, Nashville, Saint Louis, Chicago, Duluth, Helena, Calgary.