

Discrete Mathematics for Computer Science

Lecture 1a: Course Content and Information

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What is This Course About?

What is Discrete Mathematics?

Discrete mathematics is the study of mathematical structures that are fundamentally **discrete** rather than continuous.

- I.e., the part of mathematics devoted to the study of **discrete objects**.
- E.g., integers, graphs, statements in logic, ...

Thus, discrete mathematics excludes topics in “continuous mathematics”, e.g., real numbers, calculus, ...

Problems Solved Using Discrete Mathematics



Problems Solved Using Discrete Mathematics

Counting: How many ways are there to choose a valid password on a computer system?

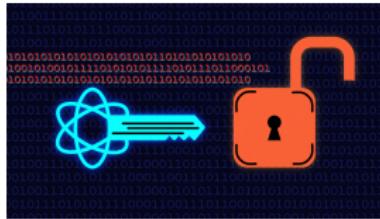


Problems Solved Using Discrete Mathematics

Counting: How many ways are there to choose a valid password on a computer system?



Number Theory and Cryptography: How can I encrypt a message so that no unintended recipient can read it?



Problems Solved Using Discrete Mathematics

Discrete Probability: What is the probability of winning a lottery?



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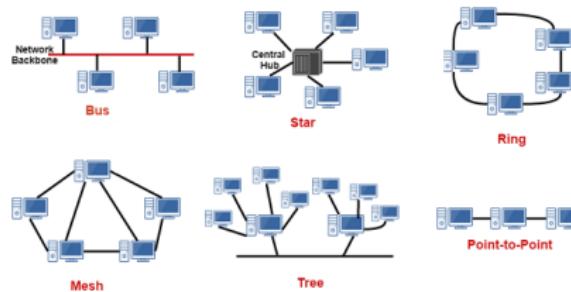


How can I identify spam e-mail messages?



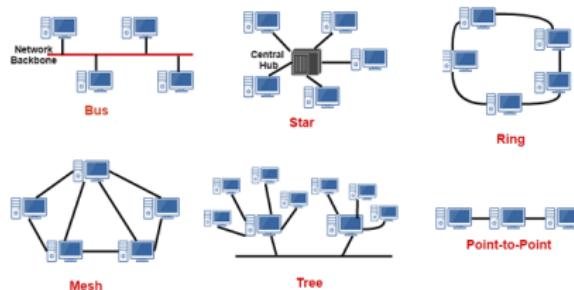
Problems Solved Using Discrete Mathematics

Graph: Is there a link between two computers in a network?



Problems Solved Using Discrete Mathematics

Graph: Is there a link between two computers in a network?



What is the shortest path between two locations?



Problems Solved Using Discrete Mathematics

Algorithm and Complexity: How can a list of integers be sorted so that the integers are in increasing order?



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How many steps are required to do such a sorting?

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Algorithm and Complexity: How can a list of integers be sorted so that the integers are in increasing order?



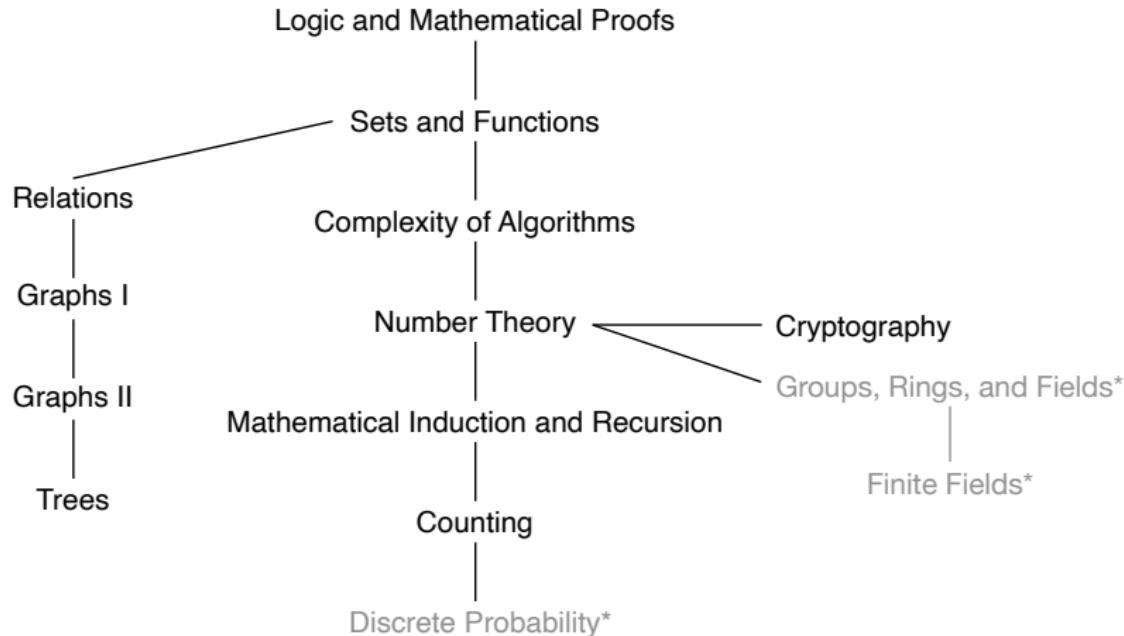
How many steps are required to do such a sorting?

How can it be proved that a sorting algorithm correctly sorts a list?

Application Areas

- Computer science: algorithms, programming languages, cryptography, artificial intelligence, ...
- Electronic engineering: digital communications, signal processing, information theory, coding theory, ...

Topics of This Course



Lecture Schedule (Tentatively)

- 1 Logic (2 lectures)
- 2 Mathematical Proofs (1 lecture)
- 3 Sets and Functions (1 lecture)
- 4 Complexity of Algorithms (1 lecture)
- 5 Number Theory (2.5 lecture)
- 6 Groups, Rings and Fields*
- 7 Cryptography (0.5 lecture)
- 8 Midterm Exam
- 9 Mathematical Induction (1 lecture)
- 10 Recursion (1 lecture)
- 11 Counting (2 lectures)
- 12 Discrete Probability*
- 13 Relations (2 lectures)
- 14 Graph I and II (3 lectures)
- 15 Trees and Review Lecture (1 lecture)

Learning Objectives

- Be able to read, understand, and construct mathematical arguments and proofs
- Understand the formulation of common problems in several areas of discrete mathematics, including counting, graphs, number theory, cryptography, logic and proof, recursions, probability theory, etc.
- Learn a number of discrete mathematical tools
- Apply discrete mathematical tools to solve certain problems in computer science and electronic engineering

Problem I: Mathematical Proof

Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.

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Proof I (direct proof):

Note that $3n + 3$ is even, and so is $n + 1$. It then follows that n is odd.

Problem I: Mathematical Proof

Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.

Proof II (proof by contrapositive):

It is equivalent to prove that “If n is even, then $3n + 2$ is even.” Why?

W.l.o.g., suppose that $n = 2k$ for some integer k , then $3n + 2 = 6k + 2$; which is even.

Problem I: Mathematical Proof

Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.

Proof III (proof by contradiction): Assume to the contrary that there exists an integer n such that $3n + 2$ is odd and n is even. Since n is even, so is $3n$. Then 2 must be odd, leading to a contradiction.

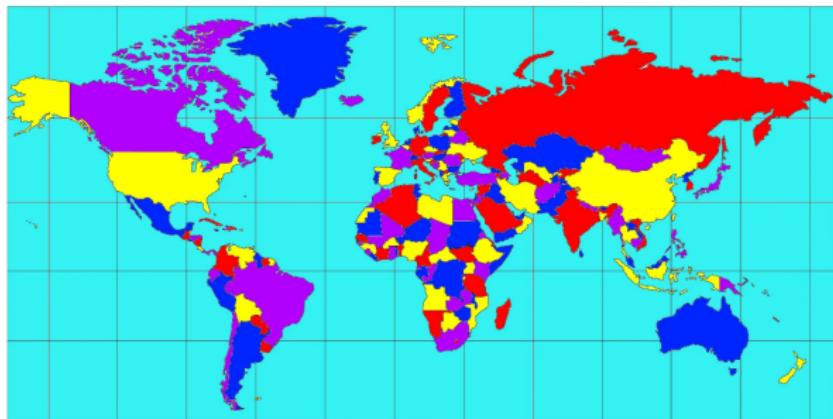
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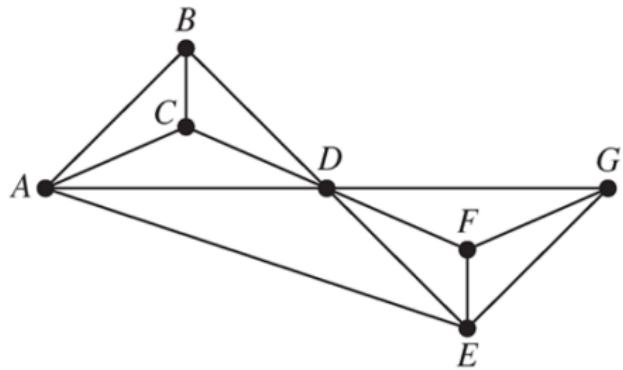
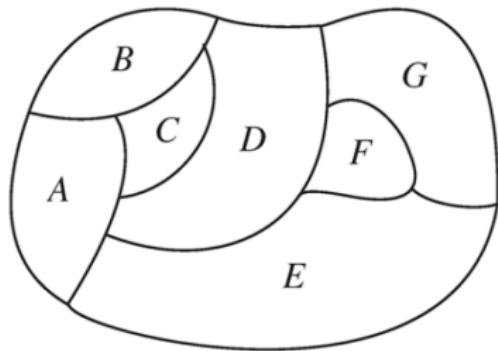
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Problem II: Four-Color Theorem

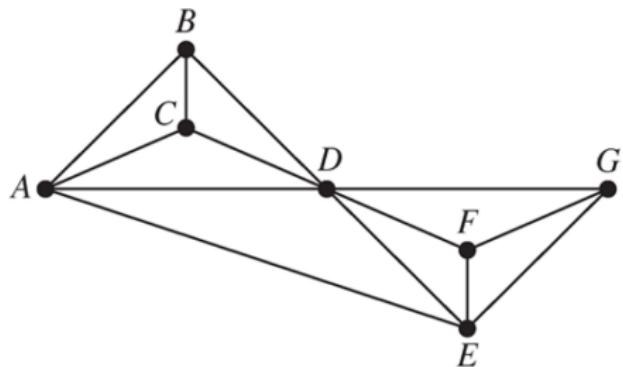
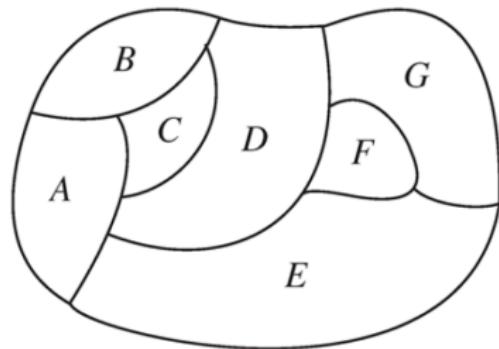
Given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color.



Problem II: Four-Color Theorem



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Four-color theorem holds for all planar graph.

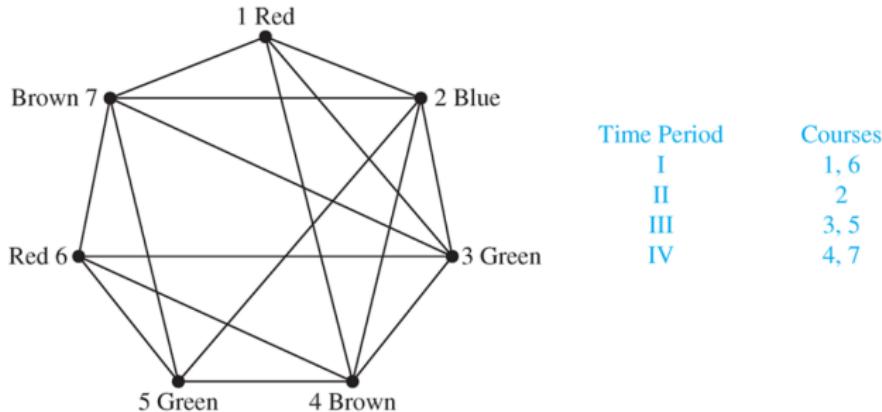
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Applications of graph colorings: Scheduling Final Exams

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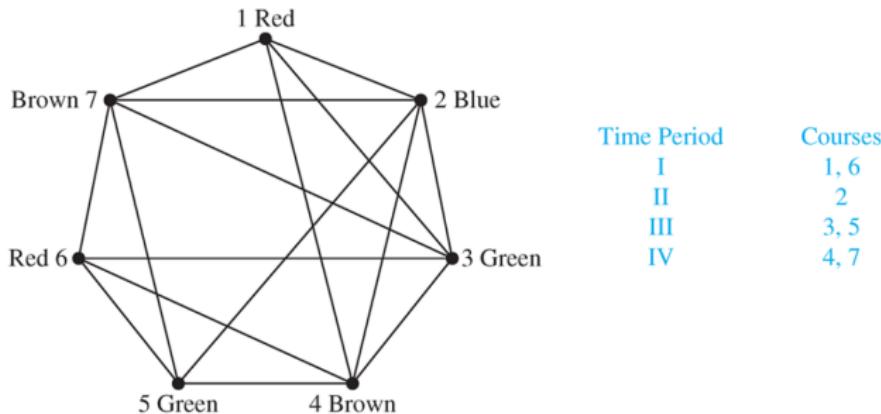
Vertices represent courses. There is an edge between two vertices if there exists at least one student taking both courses.



Problem II: Four-Color Theorem

Applications of graph colorings: Scheduling Final Exams

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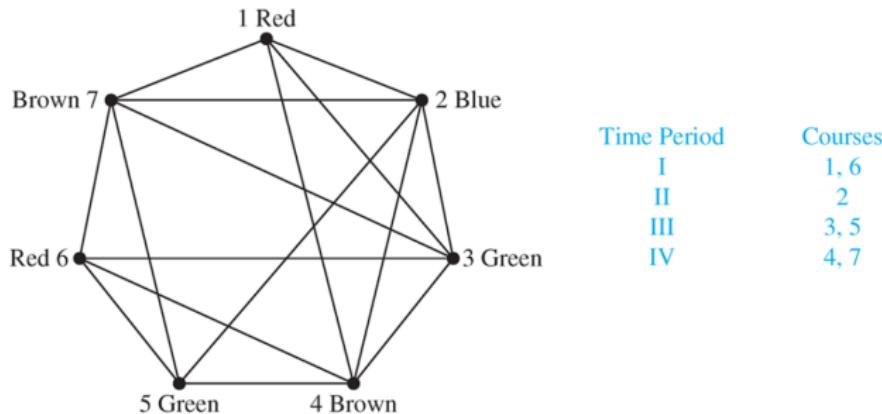


Graph coloring is computationally hard!!!

Problem II: Four-Color Theorem

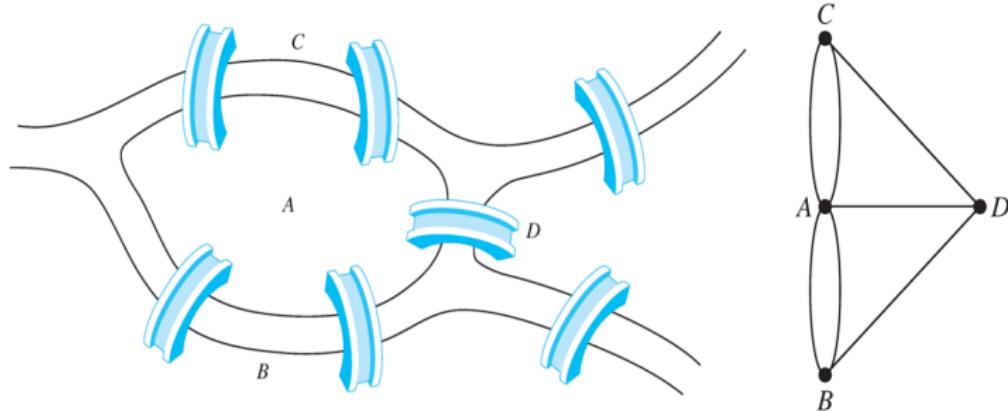
Applications of graph colorings: Scheduling Final Exams

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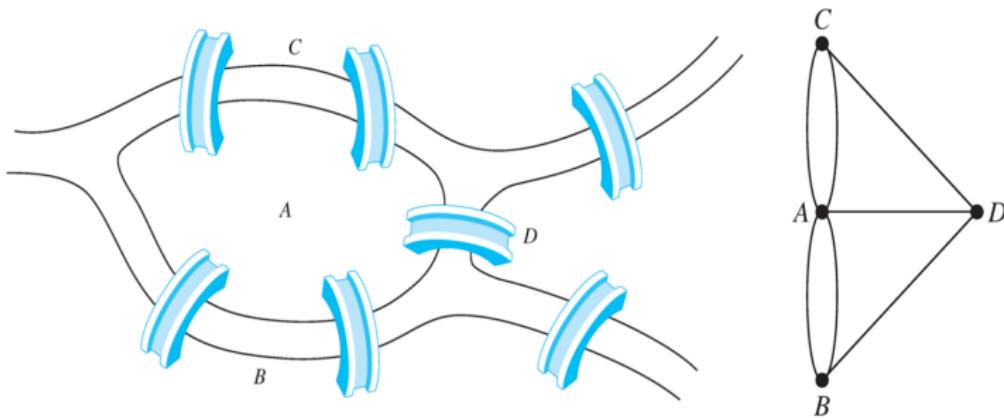
Problem III: Königsberg seven-bridge problem

People wondered whether it was possible to start at some location in the town, travel across all the bridges once without crossing any bridge twice, and return to the starting point.



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Have such a solution if and only if each of its vertices has even degree.

Problem IV: Fibonacci Number – Population of Rabbits

Reproducing pairs (at least two months old)	Young pairs (less than two months old)	Month	Reproducing pairs	Young pairs	Total pairs
		0	0	1	1
		1	0	1	1
		2	1	1	2
		3	1	2	3
		4	2	3	5

- A young pair of rabbits (one of each sex) is placed on an island.
- A pair of rabbits does not breed until they are 2 months old.
- After they are 2 months old, each pair of rabbits produces another pair each month. Assuming that no rabbits ever die.

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$$\Rightarrow F_0 = 1, F_1 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2$$

\Rightarrow What is the closed-form expression of F_n ?

Ready to Start?

Before We Start: Course Information

About This Course

- Instructor:

Dr. Ming Tang

Department of Computer Science and Engineering

Office: Room 613, South Tower, CoE Building

Email: tangm3@sustech.edu.cn

- Course Webpage:

sakai.sustech.edu.cn → CS 201 01 spring2022

QQ Chat Group: 924408091

About This Course

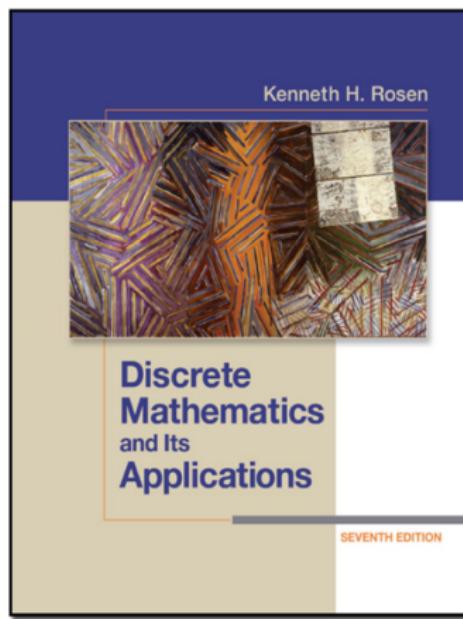
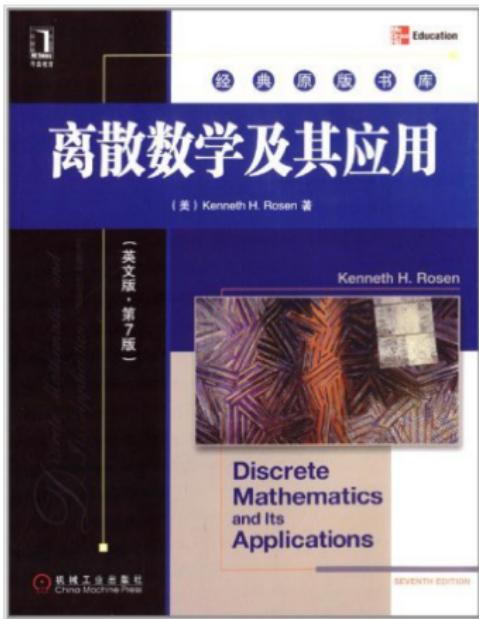
- Lectures:

- ▶ Online at Tencent Classs
- ▶ (Or Room 101 in Liyuan Building 1)
- ▶ Monday 10:20–12:10 (every week)
- ▶ Wednesday 16:20–18:10 (every two weeks)

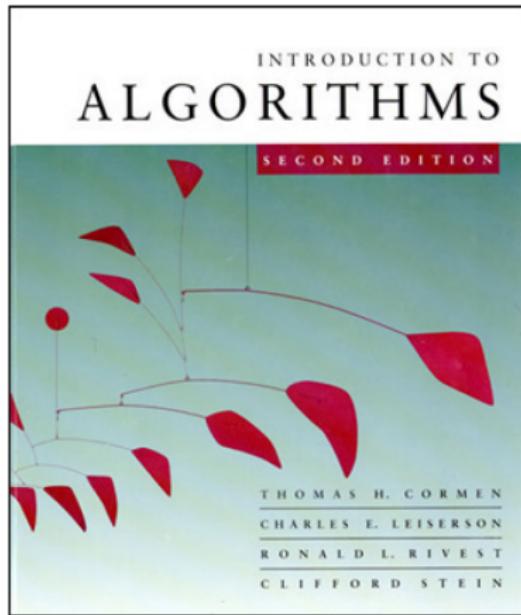
- Office hours:

- ▶ Online at Tencent Classroom
- ▶ Thursday 16:00–18:00 (tentative)
- ▶ Or send an email for appointment

Textbook



Reference Books



Marking Scheme

- Quiz in class (10%)
- Homework assignments (20%)
 - ▶ Assigned in class and posted on the course webpage
 - ▶ Must be submitted online before due date
 - ▶ No extension policy
- Midterm exam (30%)
 - ▶ Covers the first half of the course materials
- Final exam (40%)
 - ▶ Cover the entire semester's materials
- Project (5%, optional)

Important Messages to Students

Main ideas will be covered in class, but some details might be skipped.
You are responsible for **all materials** in the assigned materials:

- E.g., assigned sections in textbook, ...

About Plagiarism:

- Homework assignments should be worked on and written up **individually**, though group discussions are allowed.
- Any unintellectual behavior and cheating on homework assignments and exams will be dealt with **severely**.
- If you get the main idea for a solution from someone else or a website, you **must acknowledge** that source in your submission.

Plagiarism Policy

Plagiarism in an assignment or a quiz:

- For the first time: the score of the assignment or quiz will be **zero**
- For the second time: the score of the course will be **zero**

When two assignments are nearly identical, it may be difficult to tell who actually wrote it. Thus, the policy will apply to **BOTH** students, unless one confesses having copied without the knowledge of the other.

What is OK and what is not OK?

- It is OK to discuss an assignment with a friend and share ideas. At the time of actual writing, you should **write it alone**.
- It is OK to get the main idea for a solution from the web, as long as you **acknowledge** the source. At the time of actual writing, you should **write answers on your own** instead of copying from the web.
- It is OK to show your assignment to friends to explain the logic, as long as the friends write their assignment **on their own later**.
- It is OK to help friends debug their programs. You will probably learn a lot by doing so.

It is **NOT OK** to take the assignment of a friend, make a few cosmetic changes (variable names), and pass it as your work.

Assignment Declaration Form



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

计算机科学与工程系
Department of Computer Science and Engineering

Undergraduate Students Assignment Declaration Form

This is _____ (student ID: _____), who has enrolled in _____ course, originated the Department of Computer Science and Engineering. I have read and understood the regulations on plagiarism in assignments and theses according to "Regulations on Academic Misconduct in Assignments for Undergraduate Students in the SUSTech Department of Computer Science and Engineering". I promise that I will follow these regulations during the study of this course.

Signature:

Date:



Other Important Messages

- You are encouraged to **ask questions** in class:
 - ▶ If you're having trouble understanding something, then at least 50% of the class is also having trouble: they'll be happy if you ask.
- You are welcome to **provide suggestions and comments**:
 - ▶ If you find mistakes or just think that something's confusing, please email me. Your classmates and future students will thank you.

Acknowledgements

- Most slides were prepared based on the lecture notes and slides from Prof. Qi Wang in CS Department at SUSTech
- Some slides were prepared based on lecture materials used in the following institutions:

