Computer Organization and Design

Homework 3

2.19 Assume the following register contents:

$$\$t0 = 0xAAAAAAAA, \quad \$t1 = 0x12345678$$

2.19.1 For the register values shown above, what is the value of $t2 for the following sequence of instructions?

$$\text{sll } \$t2, \$t0, 4$$
$$\text{or } \$t2, \$t2, \$t1$$

2.19.2 For the register values shown above, what is the value of $t2 for the following sequence of instructions?

$$\text{sll } \$t2, \$t0, 4$$
$$\text{andi } \$t2, \$t2, -1$$

2.19.3 For the register values shown above, what is the value of $t2 for the following sequence of instructions?

$$\text{srl } \$t2, \$t0, 3$$
$$\text{andi } \$t2, \$t2, 0xFFEF$$

Solution:

2.19.1

$$0xAAAAAAAA = 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010_2$$
$$0x12345678 = 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111\ 1000_2$$

$\text{sll } \$t2, \$t0, 4 \ \rightarrow \quad \$t2 = \ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 0000_2$

$\text{or } \$t2, \$t2, \$t1 \ \rightarrow$

$$\$t2 = \ 1011\ 1010\ 1011\ 1110\ 1111\ 1110\ 1111\ 1000_2 = 0xBABEFEF8$$

2.19.2

$$-1_{10} = 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_2$$

$\text{sll } \$t2, \$t0, 4 \ \rightarrow \quad \$t2 = \ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 0000_2$

$\text{andi } \$t2, \$t2, -1 \ \rightarrow$

$$\$t2 = 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 0000 = \ 0xAAAAAAA0$$

2.19.3

$$0xFFEF = 0000\ 0000\ 0000\ 0000\ 1111\ 1111\ 1110\ 1111_2$$

$\text{srl } \$t2, \$t0, 3 \ \rightarrow \ \$t2 = 0001\ 0101\ 0101\ 0101\ 0101\ 0101\ 0101\ 0101_2$

$\text{andi } \$t2, \$t2, 0xFFEF \ \rightarrow$

$$\$t2 = 0000\ 0000\ 0000\ 0000\ 0101\ 0101\ 0100\ 0101 = 0x00005545$$

2.26 Consider the following MIPS loop:

```
LOOP: slt $t2, $0, $t1
      beq $t2, $0, DONE
      subi $t1, $t1, 1
      addi $s2, $s2, 2
      j LOOP
DONE:
```

2.26.1 Assume that the register \$t1 is initialized to the value 10. What is the value in register \$s2 assuming \$s2 is initially zero?

2.26.2 For each of the loops above, write the equivalent C code routine. Assume that the registers \$s1, \$s2, \$t1, and \$t2 are integers A, B, i, and temp, respectively.

2.26.3 For the loops written in MIPS assembly above, assume that the register \$t1 is initialized to the value N. How many MIPS instructions are executed?

Solution:

2.26.1 The loop will be executed 10 times, so the result is $2 \times 10 = 20$

2.26.2

```
i=10
do {
    B += 2;
    i = i -1;
} while (i > 0)
```

2.26.3

$5 \times \mathrm{N} + 2$

2.31 Implement the following C code in MIPS assembly. What is the total number of MIPS instructions needed to execute the function?

```
int fib(int n){
    if (n==0)
        return 0;
    else if (n == 1)
        return 1;
    else
        return fib(n−1) + fib(n−2);
```

Solution:

```
fib: addi $sp, $sp, -12      # make room on stack
     sw $ra, 8($sp)          # push $ra
     sw $s0, 4($sp)          # push $s0
     sw $a0, 0($sp)          # push $a0 (N)
     bgt $a0, $0, test2      # if n>0, test if n=1
     add $v0, $0, $0         # else fib(0) = 0
     j rtn                   #
```

```
test2: addi $t0, $0, 1          #
       bne $t0, $a0, gen        # if n>1, gen
       add $v0, $0, $t0         # else fib(1) = 1
       j rtn
gen: subi $a0, $a0,1            # n-1
     jal fib                    # call fib(n-1)
     add $s0, $v0, $0           # copy fib(n-1)
     sub $a0, $a0,1             # n-2
     jal fib                    # call fib(n-2)
     add $v0, $v0, $s0          # fib(n-1)+fib(n-2)
 rtn: lw $a0, 0($sp)            # pop $a0
     lw $s0, 4($sp)             # pop $s0
     lw $ra, 8($sp)             # pop $ra
     addi $sp, $sp, 12          # restore sp
     jr $ra
```

When N=0, 12 instructions, when N=1, 14 instructions.

When N⩾2, $\mathrm{f(N)} = \mathrm{f(N-1)} + \mathrm{f(N-2)} + 18$ instructions, we can solve it:

$$\mathrm{f(N)} = \left(30 - \frac{17+15\sqrt{5}}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^N + \left(\frac{17+15\sqrt{5}}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^N - 18$$

So when N⩾2, $\left(30 - \frac{17+15\sqrt{5}}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^N + \left(\frac{17+15\sqrt{5}}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^N - 18$ instructions

2 二阶非齐次线性递推数列的通项公式

定理2 若二阶非齐次线性递推数列的递推关系为$a_{n+1}=pa_n+qa_{n-1}+A$，其中$p\neq 0, q\neq 0, A\neq 0$，则有：

1) 若$p+q=1$，则当$q=-1$时，$a_n=a_1+(n-1)(a_2-a_1)+\frac{1}{2}(n-1)(n-2)A$；当$q\neq -1$时，$a_n=a_1+\left(a_2-a_1-\frac{A}{1+q}\right)\cdot\frac{1-(-q)^{n-1}}{1+q}+(n-1)\cdot\frac{A}{1-q}$.

2) 若$p+q\neq 1$，则当$p^2+4q=0$时，$a_n=(a_1+\lambda)\beta^{n-1}+(n-1)[a_2+\lambda-\beta(a_1+\lambda)]\beta^{n-2}-\lambda$，其中$\beta=\frac{p}{2}$，$\lambda=\frac{A}{p+q-1}$.

当$p^2+4q>0$时，则有

$$a_n=\left[a_1+\lambda-\frac{a_2+\lambda-\alpha(a_1+\lambda)}{\beta-\alpha}\right]\cdot\alpha^{n-1}+\frac{a_2+\lambda-\alpha(a_1+\lambda)}{\beta-\alpha}\cdot\beta^{n-1}-\lambda,$$

其中$\alpha=\frac{p-\sqrt{p^2+4q}}{2}$，$\beta=\frac{p+\sqrt{p^2+4q}}{2}$，$\lambda=\frac{A}{p+q-1}$.

当$p^2+4q<0$时，则有

$$a_n=\left[a_1+\lambda-\frac{a_2+\lambda-\alpha(a_1+\lambda)}{\beta-\alpha}\right]\cdot\alpha^{n-1}+\frac{a_2+\lambda-\alpha(a_1+\lambda)}{\beta-\alpha}\cdot\beta^{n-1}-\lambda,$$

其中$\alpha=\frac{p-i\sqrt{-p^2-4q}}{2}$，$\beta=\frac{p+i\sqrt{-p^2-4q}}{2}$，$\lambda=\frac{A}{p+q-1}$.