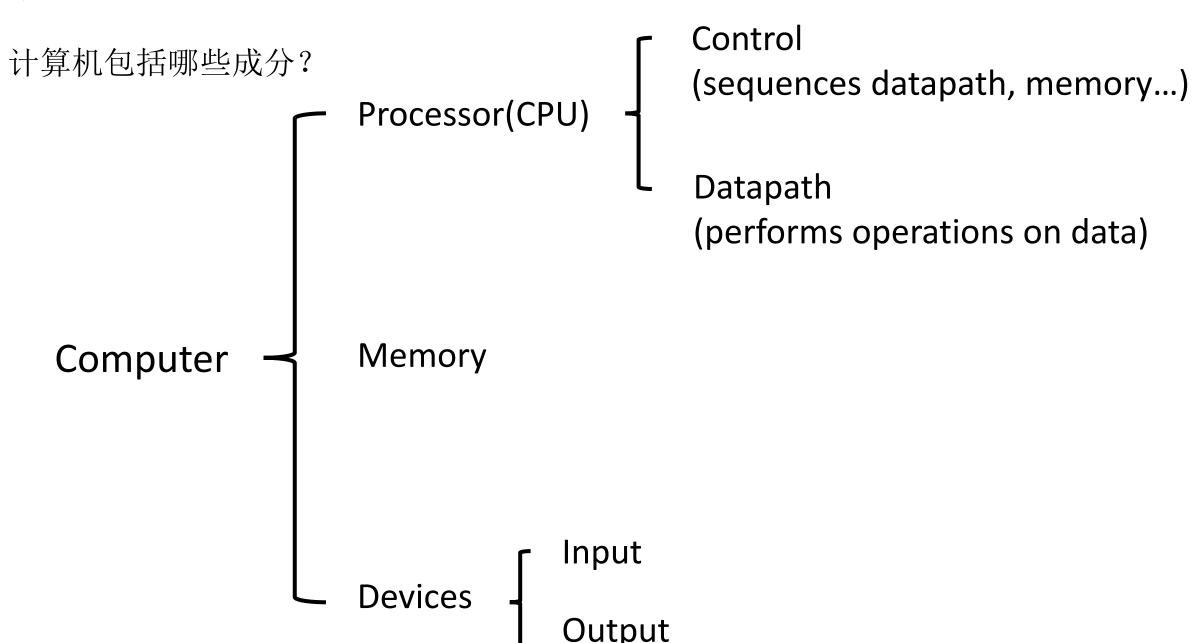
计算机组成原理

Chapter 1 复习

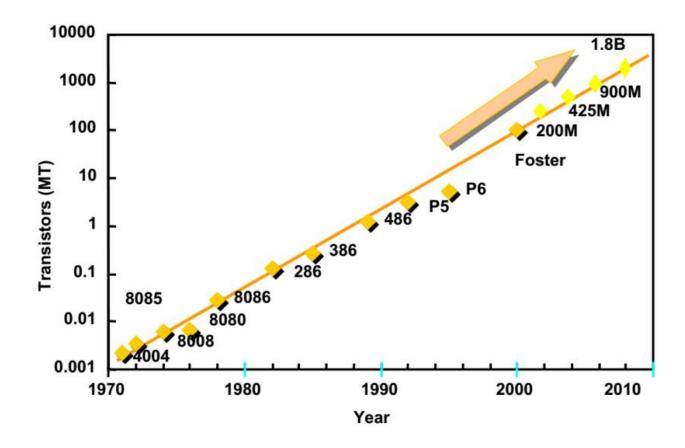
第一个知识点:



第二个知识点:

Moore's Law

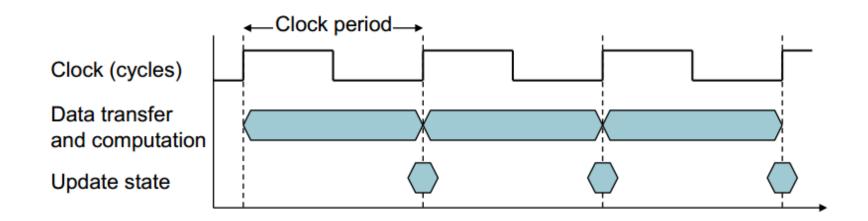
The number of transistors that can be integrated on a die would double every 18 to 24 months.



第三个知识点:

CPU clocking

Operation of digital hardware governed by a constant-rate clock



$$CPU Time = No. of Clock Cycles \times Clock Period$$

$$= \frac{No. of Clock Cycles}{Clock Rate}$$

第四个知识点:

Instruction Count and CPI

 $\begin{aligned} & \text{Clock Cycles} = \text{Instruction Count} \times \text{Cycles per Instruction} \\ & \text{CPU Time} = \text{Instruction Count} \times \text{CPI} \times \text{Clock Cycle Time} \\ & = \frac{\text{Instruction Count} \times \text{CPI}}{\text{Clock Rate}} \end{aligned}$

- Instruction Count for a program
 - Determined by program, ISA and compiler
- Average cycles per instruction
 - Determined by CPU hardware
 - If different instructions have different CPI
 - Average CPI affected by instruction mix

 If different instruction classes take different numbers of cycles

$$Clock\ Cycles = \sum_{i=1}^{n} (CPI_{i} \times Instruction\ Count_{i})$$

Weighted average CPI

$$CPI = \frac{Clock \ Cycles}{Instruction \ Count} = \sum_{i=1}^{n} \left(CPI_i \times \frac{Instruction \ Count_i}{Instruction \ Count} \right)$$
Relative frequency

Performance Summary

$$CPUTime = \frac{Instructions}{Program} \times \frac{Clock\ cycles}{Instruction} \times \frac{Seconds}{Clock\ cycle}$$

- Performance depends on
 - Algorithm: affects IC, possibly CPI
 - Programming language: affects IC, CPI
 - Compiler: affects IC, CPI
 - Instruction set architecture: affects IC, CPI, T_c

第五个知识点:

Dynamic Power

In CMOS IC technology

Power = Capacitive load×Voltage²×Frequency

- The power wall
 - We can't reduce voltage further
 - We can't remove more heat

第六个知识点:

Multiprocessors

- Multicore microprocessors
 - More than one processor per chip
- Requires explicitly parallel programming
 - Compare with instruction level parallelism
 - Hardware executes multiple instructions at once
 - Hidden from the programmer
 - Hard to do
 - Programming for performance
 - Load balancing
 - Optimizing communication and synchronization

第七个知识点:

Amdahl's Law

- Architecture design is very bottleneck-driven make the common case fast, do not waste resources on a component that has little impact on overall performance/power
- Amdahl's Law: performance improvements through an enhancement is limited by the fraction of time the enhancement comes into play

改进后的执行时间 = 受改进影响的执行时间/改进量 + 不受影响的执行时间

1.6 Consider two different implementations of the same instruction set architecture. The instructions can be divided into four classes according to their CPI (class A, B, C, and D). P1 with a clock rate of 2.5 GHz and CPIs of 1, 2, 3, and 3, and P2 with a clock rate of 3 GHz and CPIs of 2, 2, 2, and 2.

Given a program with a dynamic instruction count of 1.0E6 instructions divided into classes as follows: 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster?

- a. What is the global CPI for each implementation?
- b. Find the clock cycles required in both cases.
- a. Global CPI= clock cycles/count of instructions = \sum (Number of instructions × CPI)/ count of instructions
- b. Clock cycles= \sum (Number of instructions \times CPI)

1.8 The Pentium 4 Prescott processor, released in 2004, had a clock rate of 3.6 GHz and voltage of 1.25 V. Assume that, on average, it consumed 10 W of static power and 90 W of dynamic power.

The Core i5 Ivy Bridge, released in 2012, had a clock rate of 3.4 GHz and voltage of 0.9 V. Assume that, on average, it consumed 30 W of static power and 40 W of dynamic power.

- 1.8.1 For each processor find the average capacitive loads.
- 1.8.2 Find the percentage of the total dissipated power comprised by static power and the ratio of static power to dynamic power for each technology.

```
1.8.1

Dy 1.8.2

So t Pentium 4:

the percentage of the total dissipated power comprised by static power

Pen =10/100=10%

Cor the ratio of static power to dynamic power
=10/90=11.11%

or: Core i5 Ivy Bridge:
```

switched

the percentage of the total dissipated power comprised by static power

So t =30/70=42.86%

the ratio of static power to dynamic power

Pen =30/40=75%

Core i5 Ivy Bridge: C=40/(0.9² × 3.4 × 10⁹)= 1.5 × 10⁻⁸F

1.8.3 If the total dissipated power is to be reduced by 10%, how much should the voltage be reduced to maintain the same leakage current? Note: power is defined as the product of voltage and current.

Analysis:

$$P_{total} \downarrow 10\% \longrightarrow \frac{P_{\text{new}}}{P_{old}} = 0.9$$
 $P_{total} = static\ power + dynamic\ power$

 $static\ power = V \times I$

$$dynamic\ power = \left(\frac{1}{2}\right)C \times V^2 \times f$$

Pentium 4: 1.18V

Without 1/2

Core i5 Ivy Bridge: 0.83V

1.15 When a program is adapted to run on multiple processors in a multiprocessor system, the execution time on each processor is comprised of computing time and the overhead time required for locked critical sections and/or to send data from one processor to another.

Assume a program requires t = 100 s of execution time on one processor. When run p processors, each processor requires t/p s, as well as an additional 4 s of overhead, irrespective of the number of processors. Compute the per-processor execution time for 2, 4, 8, 16, 32, 64, and 128 processors. For each case, list the corresponding speedup relative to a single processor and the ratio between actual speedup versus ideal speedup (speedup if there was no overhead).

Solution:

p=1, execution time per processor = 100 s.

For p>1, execution time per processor =100/p s

while the total time = execution time per processor + overhead = 100/p + 4 s

Therefore, when p=2, 4, 8, 16, 32, 64, and 128

Therefore, when p 2, 4, 6, 16, 32, 64, and 126					
Exec.time/	Time	Speedup	Actual speedup/		
processor	w/overhead		ideal speedup		
100					
50	54	100/54=1.85	1.85/2=0.93		
25	29	100/29=3.45	3.45/4=0.86		
12.5	16.5	100/16.5=6.06	6.06/8=0.76		
6.25	10.25	100/10.25=9.76	9.76/16=0.61		
3.125	7.125	100/7.125=14.03	14.03/32=0.44		
1.5625	5.5625	100/5.5625=17.98	17.98/64=0.41		
0.78125	4.78125	100/4.78125=20.91	20.91/128=0.16		
	Exec.time/ processor 100 50 25 12.5 6.25 3.125 1.5625	Exec.time/ processor w/overhead 100 50 54 25 29 12.5 16.5 6.25 3.125 7.125 1.5625 5.5625	Exec.time/ processor Time w/overhead Speedup 50 54 100/54=1.85 25 29 100/29=3.45 12.5 16.5 100/16.5=6.06 6.25 10.25 100/10.25=9.76 3.125 7.125 100/7.125=14.03 1.5625 5.5625 100/5.5625=17.98		

计算机组成原理

Chapter 2

复习

Α

第一个知识点:

MIPS design principle

1. Simplicity favors regularity

2. Smaller is faster

3. Make the common case fast

4. Good design demands good compromises

第二个知识点:

Registers

- The MIPS ISA has 32 registers (x86 has 8 registers) –
 Why not more? Why not less?
- Each register is 32-bit wide (modern 64-bit architectures have 64-bit wide registers)
- A 32-bit entity (4 bytes) is referred to as a word

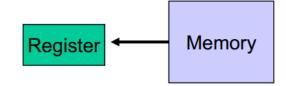
32个寄存器的作用...

第三个知识点:

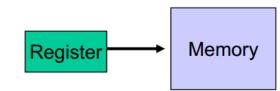
Memory Operands

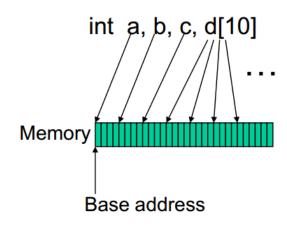
 Values must be fetched from memory before (add and sub) instructions can operate on them

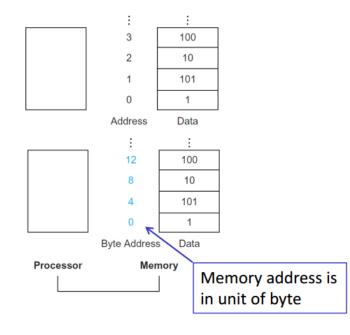
Load word lw \$t0, memory-address

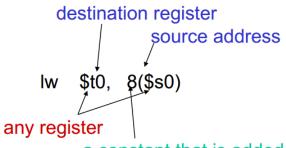


Store word sw \$t0, memory-address









a constant that is added to the register in brackets

第四个知识点:

Numeric Representations

- Decimal 35₁₀
- Binary 00100011₂
- Hexadecimal (compact representation)
 0x 23 or 23_{hex}

0-15 (decimal) \rightarrow 0-9, a-f (hex)

Sign Extension: 正数前补0 负数前补1 无符号数

$$x = x_{n-1} 2^{n-1} + x_{n-2} 2^{n-2} + \dots + x_1 2^1 + x_0 2^0$$

Range: 0 to +2ⁿ-1

有符号数 负数: 取反+1

$$x = -x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \dots + x_12^1 + x_02^0$$

Range: -2^{n-1} to $+2^{n-1}-1$

Complement and add 1

Complement means 1 → 0, 0 → 1

$$x + \overline{x} = 1111...111_2 = -1$$

 $\overline{x} + 1 = -x$

第五个知识点:

Instruction Formats

```
$t0, $s1, $s2
R-type instruction
                    add
        10001 10010 01000 00000
000000
                                     100000
6 bits
         5 bits 5 bits 5 bits 6 bits
                              shamt funct
                 rt
                        rd
          rs
 op
               source dest shift amt function
opcode
        source
```

```
I-type instruction6 bits5 bits5 bits6 bits6 bits7 bits8 bits9 bits16 bits16 bits17 constant
```

第五个知识点:

Instruction Formats

opcode

The field that denotes the operation and format of an instruction.

rs

The first register source operand.

rt

The second register source operand.

rd

The register destination operand. It gets the result of the operation.

shamt

Shif amount.

funct

Function. This feld, ofen called the function code, selects the specifc variant of the operation in the op field.



FMT/FT

FUNC!

11/11/--/0

11/10/--/y

11/10/--/3 11/11/--/3

11/10/--/2 11/11/--/2 11/10/--/1 11/11/--/1

10 /0/--/0

(2) 3d/--/--/-

A CALLS

N.A.

No

No

Yes

Yes Yes Yes

{F[ft],F[ft+1]}

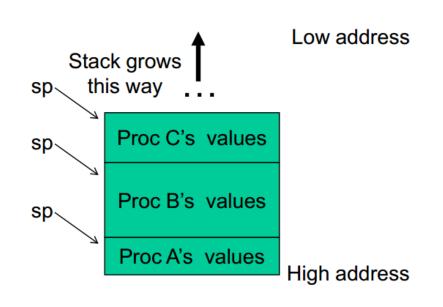
第六个知识点:

Control Instructions

Instruction	Effect
beq Rs, Rt, label	if (Rs == Rt) PC ← label
bne Rs, Rt, label	if (Rs != Rt) PC ← label
bltz Rs, label	if (Rs < 0) PC ← label
blez Rs, label	if (Rs <= 0) PC ← label
bgtz Rs, label	if (Rs > 0) PC ← label
bgez Rs, label	if (Rs >= 0) PC ← label
j jlabel	PC ← jlabel
jr Rs	PC ← Rs
jal jlabel	\$ra ← PC+4, PC ← jlabel
jalr Rs	\$ra ← PC+4, PC ← Rs

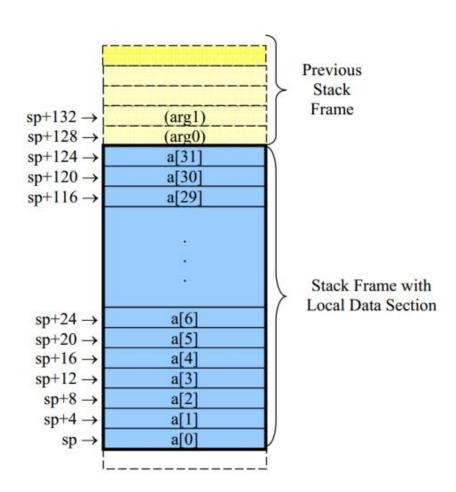
第七个知识点:

Stack



Proc A

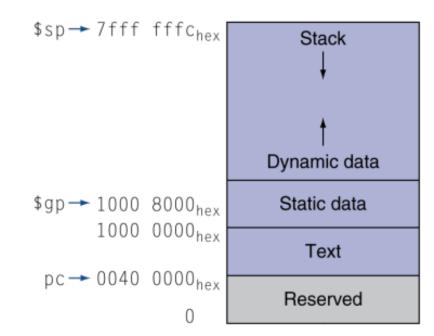
call Proc B
...
call Proc C
...
return
return
return



第八个知识点:

Memory Layout

- Text: program code
- Static data: global variables
 - e.g., static variables in C, constant arrays and strings
 - \$gp initialized to address allowing ±offsets into this segment
- Dynamic data: heap
 - E.g., malloc in C, new in Java
- Stack: automatic storage



2.1 For the following C statement, what is the corresponding MIPS assembly code? Assume that the variables f, g, h, and i are given and could be considered 32-bit integers as declared in a C program. Use a minimal number of MIPS assembly instructions.

$$f = g + (h - 5)$$

addi f, h, -5 add f, f, g

2.3 For the following C statement, what is the corresponding MIPS assembly code? Assume that the variables f, g, h, i, and j are assigned to registers \$s0, \$s1, \$s2, \$s3, and \$s4, respectively. Assume that the base address of the arrays A and B are in registers \$s6 and \$s7, respectively.

$$B[8] = A[i - j]$$

sub \$t0, \$s3, \$s4 sll \$t0, \$t0, 2 add \$t0, \$t0, \$s6 lw \$t1, 0(\$t0) sw \$t1, 32(\$s7)



2.6 The table below shows 32-bit values of an array stored in memory.

Address	Data
24	2
28	4
32	3
36	6
40	1

- 2.6.1 For the memory locations in the table above, write C code to sort the data from lowest to highest, placing the lowest value in the smallest memory location shown in the figure. Assume that the data shown represents the C variable called *Array*, which is an array of type *int*, and that the first number in the array shown is the first element in the array. Assume that this particular machine is a byte-addressable machine and a word consists of four bytes.
- 2.6.2 For the memory locations in the table above, write MIPS code to sort the data from lowest to highest, placing the lowest value in the smallest memory location. Use a minimum number of MIPS instructions. Assume the base address of *Array* is stored in register \$s6.

Address	Data
24	2
28	4
32	3
36	6
40	1

```
temp = Array [0];
temp2 = Array [1];
Array [0] = Array [4];
Array [1] = temp;
Array [4] = Array [3];
Array [3] = temp2;
```

```
lw $t0, 0($s6)
lw $t1, 4($s6)
lw $t2, 16($s6)
sw $t2, 0($s6)
sw $t0, 4($s6)
lw $t0, 12($s6)
sw $t0, 16($s6)
sw $t1, 12($s6)
```

- 2.12 Assume that registers \$s0 and \$s1 hold the values 0x80000000 and 0xD0000000, respectively.
- 2.12.1 What is the value of \$t0 for the following assembly code?

add \$t0, \$s0, \$s1

- 2.12.2 Is the result in \$t0 the desired result, or has there been overflow?
- 2.12.3 For the contents of registers \$s0 and \$s1 as specified above, what is the value of \$t0 for the following assembly code?

sub \$t0, \$s0, \$s1

- 2.12.4 Is the result in \$t0 the desired result, or has there been overflow?
- 2.12.5 For the contents of registers \$s0 and \$s1 as specified above, what is the value of \$t0 for the following assembly code?

add \$t0, \$s0, \$s1

add \$t0, \$t0, \$s0

2.12.6 Is the result in \$t0 the desired result, or has there been overflow?

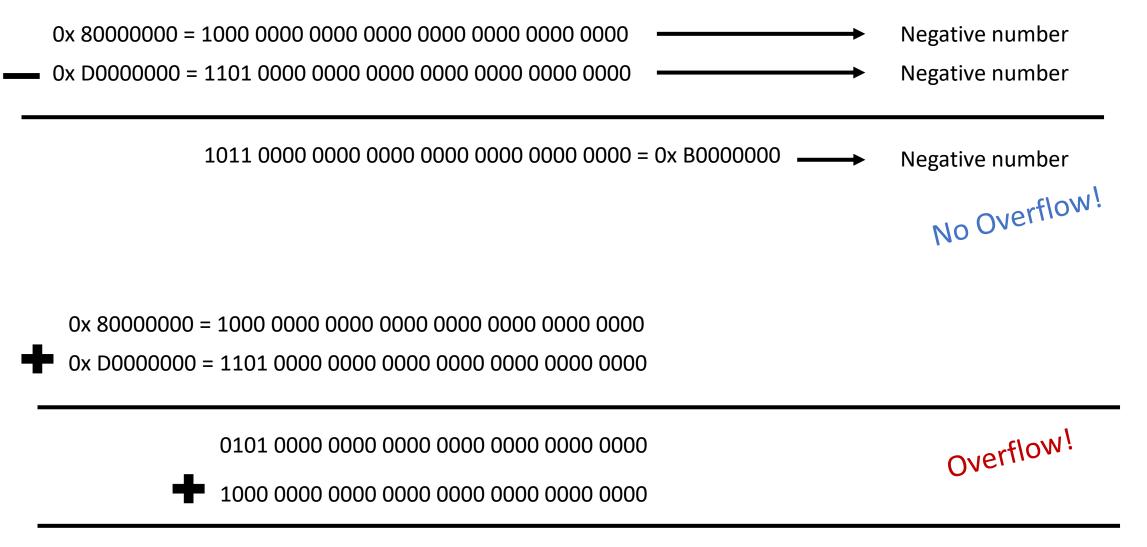
Negative number

Negative number

Positive number

Overflow!

Overflow is usually when the sign of the result doesn't make sense negative numbers. When you add two negative numbers back. But instead you get 0x50000000, which



2.16 Provide the type, assembly language instruction, and binary representation of instruction described by the following MIPS fields: op=0, rs=3, rt=2, rd=3, shamt=0, funct=34

Type: r-type

funct =
$$34 = 32 + 2 = 22_{hex}$$

Assembly language instruction: sub \$v1, \$v1, \$v0

Binary representation:

0x00621822

计算机组成原理

Chapter 2 复习

B

第九个知识点:

Procedure call

Leaf procedure

Non-leaf procedure

- Caller saved: Temp registers \$t0-\$t9 (the callee won't bother saving these, so save them if you care), \$ra (it's about to get over-written), \$a0-\$a3 (so you can put in new arguments)
- Callee saved: \$s0-\$s7 (these typically contain "valuable" data)
- For nested call, caller needs to save on the stack:
 - Its return address
 - Any arguments and temporaries needed after the call
- Restore from the stack after the call

第十个知识点:

MIPS addressing mode

Immediate addressing

Example: addi \$s0, \$s1, 2

op rs rt immediate
6 bits 5 bits 5 bits 16 bits

Register addressing

Example: add \$s0, \$s1, \$s2

 op
 rs
 rt
 rd
 shampt
 funct

 6 bits
 5 bits
 5 bits
 5 bits
 6 bits

Base/Displacement addressing

Example: lw \$s0, 0(s1)

op rs rt immediate
6 bits 5 bits 5 bits 16 bits

Branch addressing (PC-relative addressing)

Example: bne \$s0, \$s1, EXIT

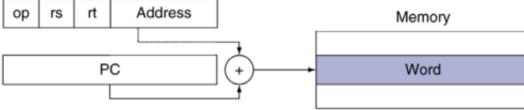
op rs rt constant or address
6 bits 5 bits 5 bits 16 bits

Jump addressing (Pseudo-direct addressing)

Example: jal FACT

op address
6 bits 26 bits

1. Immediate addressing op rt Immediate rs 2. Register addressing rd funct op rs rt 3. Base addressing rt Address op rs Register Byte Halfword 4. PC-relative addressing



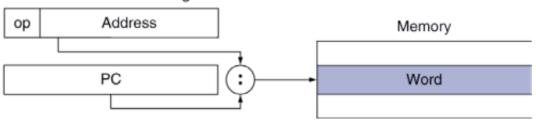
Registers

Register

Memory

Word

5. Pseudodirect addressing



第十一个知识点:

Assembler and Linker

Role of assembler:

- 1. Convert pseudo-instruction into actual hardware instructions
- 2. Convert assembly instructions into machine instructions

Role of linker:

- Stitches different object files into a single executable
 - patch internal and external references
 - determine addresses of data and instruction labels
 - organize code and data modules in memory
- Some libraries (DLLs) are dynamically linked the executable points to dummy routines – these dummy routines call the dynamic linker-loader so they can update the executable to jump to the correct routine

2.19 Assume the following register contents:

$$t0 = 0xAAAAAAAA$$
, $t1 = 0x12345678$

2.19.1 For the register values shown above, what is the value of \$12 for the following sequence of instructions?

sll \$t2, \$t0, 4 or \$t2, \$t2, \$t1

2.19.2 For the register values shown above, what is the value of \$12 for the following sequence of instructions?

sll \$t2, \$t0, 4 andi \$t2, \$t2, -1

2.19.3 For the register values shown above, what is the value of \$t2 for the following sequence of instructions?

srl \$t2, \$t0, 3 andi \$t2, \$t2, 0xFFEF $0xAAAAAAA = 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010$ $0x12345678 = 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111\ 1000_2$

```
2.19.1

sll $t2,$t0,4 \rightarrow $t2 = 1010 1010 1010 1010 1010 1010 1010 0000<sub>2</sub>

0001 0010 0011 0100 0101 0110 0111 1000<sub>2</sub>

or $t2,$t1 \rightarrow

$t2 = 1011 1010 1011 1110 1111 1110 1111 1000<sub>2</sub>
```

 $0xAAAAAAA = 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010$ $0x12345678 = 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111\ 1000_2$

2.19.2

andi \$t2, \$t2, $-1 \rightarrow$ \$t2 = 1010 1010 1010 1010 1010 1010 1010 0000

= 0xAAAAAAA0

 $0xAAAAAAA = 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010\ 1010$ $0x12345678 = 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111\ 1000_2$

2.19.3

 $0xFFEF = 0000\ 0000\ 0000\ 0000\ 1111\ 1111\ 1110\ 1111_2$

 $srl $t2, $t0, 3 \rightarrow $t2 = 0001 \ 0101 \ 0101 \ 0101 \ 0101 \ 0101 \ 0101 \ 0101 \ 0101 \ 0$

andi \$t2, \$t2, $0xFFEF \rightarrow$

t2 = 0000 0000 0000 0000 0101 0101 0100 0101

= 0x00005545

2.26 Consider the following MIPS loop:

```
LOOP: slt $t2, $0, $t1
beq $t2, $0, DONE
subi $t1, $t1, 1
addi $s2, $s2, 2
j LOOP
```

DONE:

- 2.26.1 Assume that the register \$t1 is initialized to the value 10. What is the value in register \$s2 assuming \$s2 is initially zero?
- 2.26.2 For each of the loops above, write the equivalent C code routine. Assume that the registers \$\$1, \$\$2, \$\$1, and \$\$12 are integers A, B, i, and temp, respectively. 2.26.3 For the loops written in MIPS assembly above, assume that the register \$\$11 is initialized to the value N. How many MIPS instructions are executed?

2.26.1 The loop will be executed 10 times, so the result is $2 \times 10 = 20$

```
2.26.2
i=10
do {
 B += 2;
 i = i - 1;
} while (i > 0)
2.26.3
               LOOP: slt $t2, $0, $t1
                      beq $t2, $0, DONE
                      subi $t1, $t1, 1
                      addi $s2, $s2, 2
                      j LOOP
               DONE:
```

2.31 Implement the following C code in MIPS assembly. What is the total number of MIPS instructions needed to execute the function?

```
int fib(int n){
  if (n==0)
    return 0;
  else if (n == 1)
    return 1;
  else
    return fib(n-1) + fib(n-2);
```

```
int fib(int n){
                     bgt $a0, $zero, ELSE1
 if (n==0)
                     add $v0, $zero, $zero
  return 0;
                          addi $t0, $t0, 1
 else if (n == 1)
                          bne $t0, $a0, ELSE2
  return 1;
                          add $v0, $zero, $t0
 else
  return fib(n-1) + fib(n-2);
                         addi $a0, $a0, -1
 addi $a0, $a0, -1
 jal fib
                         jal fib
                         add $v0, $v0, $s0
 add $s0, $zero, $v0
 需要存起来的寄存器?
                                    → $ra, $s0, $a0
```

```
addi $sp, $sp, -12
  fib:
        sw $ra, 8($sp)
        sw $s0, 4($sp)
        sw $a0, 0($sp)
        bgt $a0, $zero, ELSE1
        add $v0, $zero, $zero
        j rtn
        addi $t0, $t0, 1
ELSE1:
        bne $t0, $a0, ELSE2
        add $v0, $zero, $t0
         j rtn
ELSE2:
        addi $a0, $a0, -1
        jal fib
         add $s0, $zero, $v0
         addi $a0, $a0, -1
         ial fib
         add $v0, $v0, $s0
  rtn:
         lw $a0, 0($sp)
         lw $s0, 4($sp)
         lw $ra, 8($sp)
         addi $sp, $sp, 12
        jr $ra
```

```
fib: addi $sp, $sp, -12
                                    When N=0, 12 instructions, when N=1, 14 instructions.
       sw $ra, 8($sp)
      sw $s0, 4($sp)
       sw $a0, 0($sp)
                                观察$a0的变化:
      bgt $a0, $zero, ELSE1
       add $v0, $zero, $zero
      j rtn
                                N=2
                                $a0: 2102
      addi $t0, $t0, 1
ELSE1:
      bne $t0, $a0, ELSE2
       add $v0, $zero, $t0
                                N=3:
       j rtn
ELSE2: addi $a0, $a0, -1
                                N=4:
       jal fib
                                $a0: 4321021321024
       add $s0, $zero, $v0
       addi $a0, $a0, -1
                                          fib(3)
                                                    fib(2)
                                N=5
       ial fib
                                $a0: 5432102132102432102135
       add $v0, $v0, $s0
  rtn:
       Iw $a0, 0($sp)
                                                fib(4)
                                                                fib(3)
                                N=6
       lw $s0, 4($sp)
                                $a0: 6543210213210243210213543210213210246
       Iw $ra, 8($sp)
       addi $sp, $sp, 12
       jr $ra
                                                        fib(5)
                                                                                    fib(4)
```

fib: addi \$sp, \$sp, -12 sw \$ra, 8(\$sp) sw \$s0, 4(\$sp) sw \$a0, 0(\$sp) bgt \$a0, \$zero, ELSE1 add \$v0, \$zero, \$zero j rtn ELSE1: addi \$t0, \$t0, 1 bne \$t0, \$a0, ELSE2 add \$v0, \$zero, \$t0 j rtn ELSE2: addi \$a0, \$a0, -1 jal fib add \$s0, \$zero, \$v0 addi \$a0, \$a0, -1 ial fib add \$v0, \$v0, \$s0 rtn: Iw \$a0, 0(\$sp) Iw \$s0, 4(\$sp) Iw \$ra, 8(\$sp) addi \$sp, \$sp, 12 jr \$ra

When N=0, 12 instructions, when N=1, 14 instructions.

When $N\geq 2$, f(N)=f(N-1)+f(N-2)+18 instructions

N=2: 44 instructions

N=3: 76 instructions

N=4: 138 instructions

N=5: 232 instructions

N=6: 388 instructions

$$f(N) = f(N-1) + f(N-2) + 18$$

2 二阶非齐次线性递推数列的通项公式

定理² 若二阶非齐次线性递推数列的递推关系为 $a_{n+1}=pa_n+qa_{n-1}+A$,其中 $p\neq 0$, $q\neq 0$, $A\neq 0$,则有:

1) 若
$$p+q=1$$
, 则当 $q=-1$ 时, $a_n=a_1+(n-1)$ $(a_2-a_1)+\frac{1}{2}$ $(n-1)$ $(n-2)$ A ; 当 $q\neq -1$ 时, $a_n=a_1+(a_2-a_1-\frac{A}{1+q})$ ·

$$\frac{1-(-q)^{n-1}}{1+q}+(n-1)\cdot\frac{A}{1-q}$$
.

2) 若
$$p+q \neq 1$$
,则当 $p^2+4q=0$ 时, $a_n=(a_1+\lambda)\beta^{n-1}+(n-1)[a_2+\lambda-\beta(a_1+\lambda)]\beta^{n-2}-\lambda$,其中 $\beta=\frac{p}{2}$, $\lambda=\frac{A}{p+q-1}$.

当 $p^2+4q>0$ 时,则有

$$a_n = \left[a_1 + \lambda - \frac{a_2 + \lambda - \alpha \left(a_1 + \lambda\right)}{\beta - \alpha}\right] \cdot \alpha^{n-1} + \frac{a_2 + \lambda - \alpha \left(a_1 + \lambda\right)}{\beta - \alpha} \cdot \beta^{n-1} - \lambda,$$

其中
$$\alpha$$
= $\frac{p-\sqrt{p^2+4q}}{2}$, β = $\frac{p+\sqrt{p^2+4q}}{2}$, λ = $\frac{A}{p+q-1}$.

当 $p^2+4q<0$ 时,则有

$$a_n = \left[a_1 + \lambda - \frac{a_2 + \lambda - \alpha \left(a_1 + \lambda\right)}{\beta - \alpha}\right] \cdot \alpha^{n-1} + \frac{a_2 + \lambda - \alpha \left(a_1 + \lambda\right)}{\beta - \alpha} \cdot \beta^{n-1} - \lambda,$$

其中
$$\alpha$$
= $\frac{p-i\sqrt{-p^2-4q}}{2}$, β = $\frac{p+i\sqrt{-p^2-4q}}{2}$, λ = $\frac{A}{p+q-1}$.

$$f(N) = \left(30 - \frac{17 + 15\sqrt{5}}{\sqrt{5}}\right) \left(\frac{1 - \sqrt{5}}{2}\right)^N + \left(\frac{17 + 15\sqrt{5}}{\sqrt{5}}\right) \left(\frac{1 + \sqrt{5}}{2}\right)^N - 18$$

fib: addi \$sp, \$sp, -12 sw \$ra, 8(\$sp) sw \$s0, 4(\$sp) sw \$a0, 0(\$sp) bgt \$a0, \$zero, ELSE1 add \$v0. \$zero. \$zero j rtn ELSE1: addi \$t0, \$t0, 1 bne \$t0, \$a0, ELSE2 add \$v0, \$zero, \$t0 j rtn ELSE2: addi \$a0, \$a0, -1 jal fib add \$s0, \$zero, \$v0 addi \$a0, \$a0, -1 ial fib add \$v0, \$v0, \$s0 rtn: lw \$a0, 0(\$sp) Iw \$s0, 4(\$sp) Iw \$ra, 8(\$sp) addi \$sp, \$sp, 12 jr \$ra

When N=0, 12 instructions, when N=1, 14 instructions.

When $N\geq 2$, f(N)=f(N-1)+f(N-2)+18 instructions

$$f(N) = \left(30 - \frac{17 + 15\sqrt{5}}{\sqrt{5}}\right) \left(\frac{1 - \sqrt{5}}{2}\right)^N + \left(\frac{17 + 15\sqrt{5}}{\sqrt{5}}\right) \left(\frac{1 + \sqrt{5}}{2}\right)^N - 18$$

So when N≥2,

$$\left(30 - \frac{17 + 15\sqrt{5}}{\sqrt{5}}\right) \left(\frac{1 - \sqrt{5}}{2}\right)^{N} + \left(\frac{17 + 15\sqrt{5}}{\sqrt{5}}\right) \left(\frac{1 + \sqrt{5}}{2}\right)^{N} - 18$$

instructions

计算机组成原理

Chapter 3

复习

第一个知识点:

Addition and subtraction

Overflow if result out of range

- Adding +ve and –ve operands, no overflow
- Adding two +ve operands
 - Overflow if result sign is 1
- Adding two –ve operands
 - Overflow if result sign is 0

Overflow if result out of range

- Subtracting two +ve or two –ve operands, no overflow
- Subtracting +ve from –ve operand
 - Overflow if result sign is 0
- Subtracting –ve from +ve operand
 - Overflow if result sign is 1

Dealing with overflow

- Add (add), add immediate (addi), and subtract (Sub) cause exceptions on overflow.
- Add unsigned (addu), add immediate unsigned (addiu), and subtract unsigned (subu) do not cause exceptions on overflow.

Note: addiu: "u" means it doesn't generate overflow exception, but the immediate can be a signed number

Example:
$$7 - 6 = 7 + (-6)$$

+7: 0000 0000 ... 0000 0111

<u>-6: 1111 1111 ... 1111 1010</u>

+1: 0000 0000 ... 0000 0001

第二个知识点:

Saturation arithmetic

Saturation arithmetic is a version of arithmetic in which all operations such as addition and multiplication are limited to a fixed range between a minimum and maximum value.

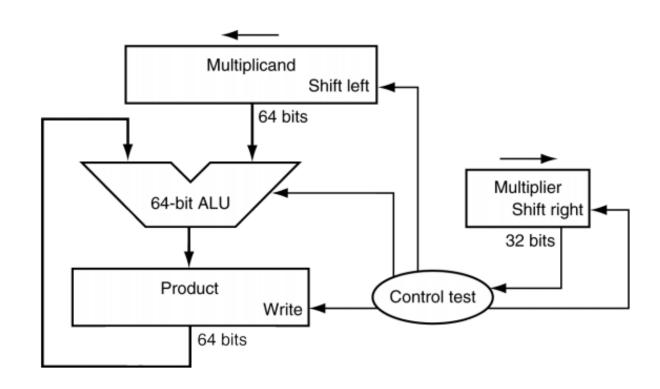
For example, if the valid range of values is from 0 to 100, the following operations produce the following values:

$$60 + 30 = 90$$

 $60 + 43 = 100$
 $(60 + 43) - (75 + 75) = 0$
 $10 \times 11 = 100$
 $99 \times 99 = 100$
 $30 \times (5 - 1) = 100$
 $30 \times 5 - 30 \times 1 = 70$

第三个知识点:

Multiplication



In every step:

- ✓ multiplicand is shifted
- next bit of multiplier is examined (also a shifting step)
- ✓ if this bit is 1, shifted multiplicand is added to the product

Length = n

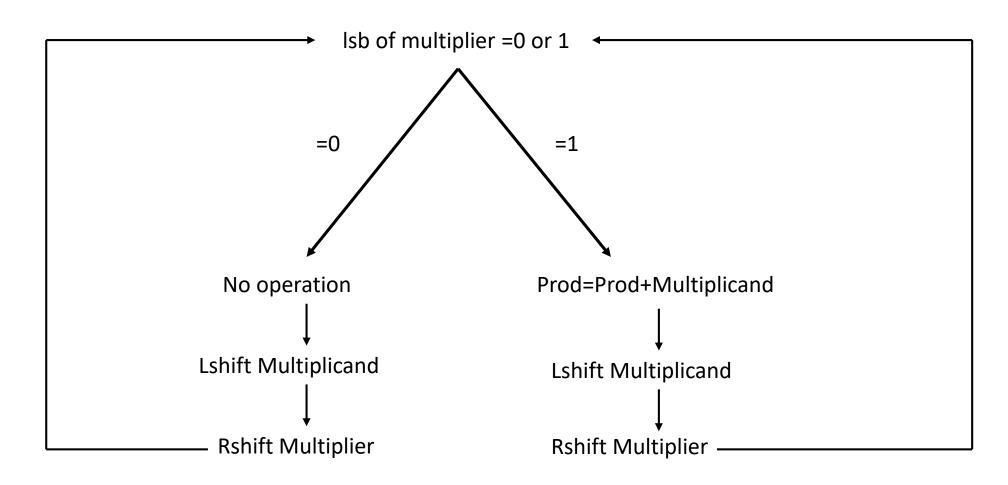
Length = 2n

Length = 2n

Initial values — Multiplier=multiplier

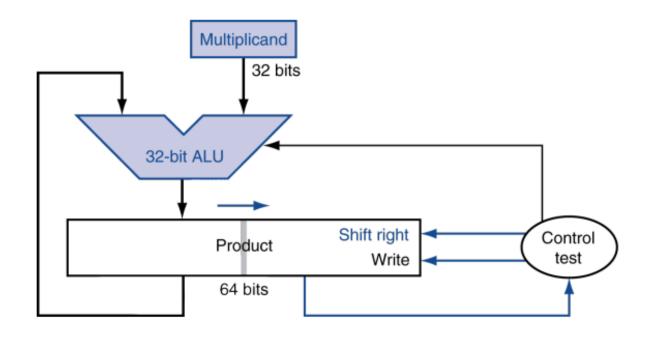
Multiplicand = 0...0 Multiplicand

Product= 0...0



Iteration	Step	Multiplier	Multiplicand	Product
0	Initial values	0011	0000 0010	0000 0000
1	1a: 1 ⇒ Prod = Prod + Mcand	0011	0000 0010	0000 0010
	2: Shift left Multiplicand	0011	0000 0100	0000 0010
	3: Shift right Multiplier	0001	0000 0100	0000 0010
2	1a: 1 ⇒ Prod = Prod + Mcand	0001	0000 0100	0000 0110
	2: Shift left Multiplicand	0001	0000 1000	0000 0110
	3: Shift right Multiplier	0000	0000 1000	0000 0110
3	1: 0 ⇒ No operation	0000	0000 1000	0000 0110
	2: Shift left Multiplicand	0000	0001 0000	0000 0110
	3: Shift right Multiplier	0000	0001 0000	0000 0110
4	1: 0 ⇒ No operation	0000	0001 0000	0000 0110
	2: Shift left Multiplicand	0000	0010 0000	0000 0110
	3: Shift right Multiplier	0000	0010 0000	0000 0110

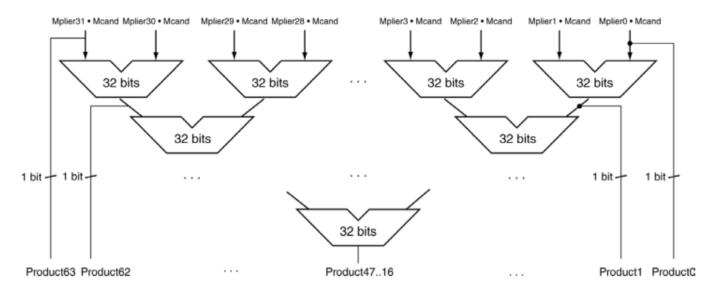
Optimized Multiplier



- ✓ 64-bit "Product" stores product and multiplier
- ✓ the sum keeps shifting right
- ✓ at every step, number of bits in product + multiplier = 64, hence, they share a single 64-bit register
- ✓ 32-bit ALU and multiplicand are used instead of 64-bit ones.

Faster Multiplier

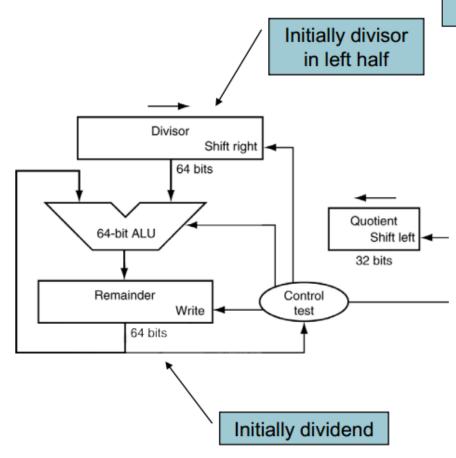
- Uses multiple pipelined adders
 - Cost/performance tradeoff

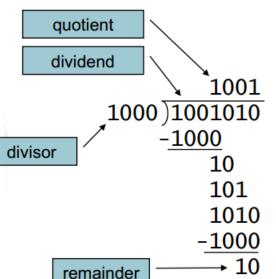


- Can be pipelined
 - Several multiplication performed in parallel

第四个知识点:

Division





- Check for 0 divisor
- Long division approach
 - If divisor ≤ dividend bits
 - 1 bit in quotient, subtract
 - Otherwise
 - 0 bit in quotient, bring down next dividend bit
- Restoring division
 - Do the subtract, and if remainder goes <
 0, add divisor back
- Signed division
 - Divide using absolute values
 - Adjust sign of quotient and remainder as required

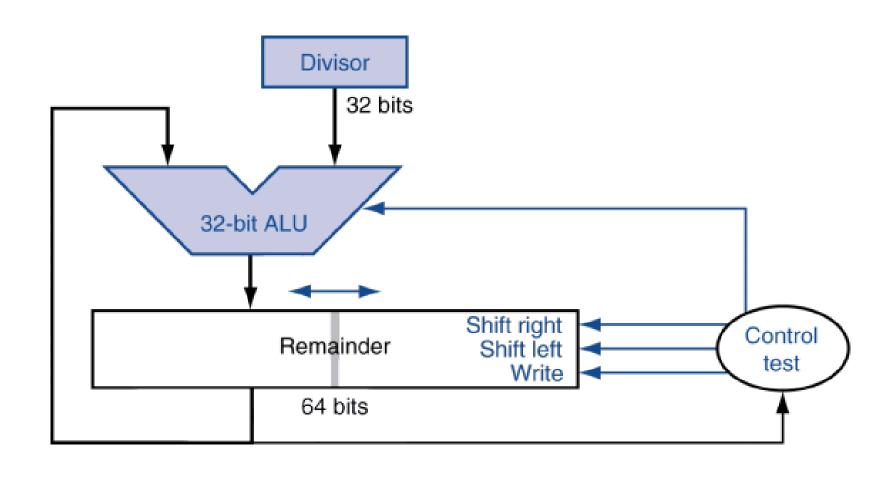
Length = 2n Length = 2n Length = nDivisor = Divisor 0...0 Remainder = 0...0 Dividend Initial values Quotient= 0...0 Rem= Rem-Div Rem≥0 or <0 <0 ≥0

No operation Rem=Rem+Div Lshift Quotient (Q0=0) Lshift Quotient (Q0=1) **Rshift Divisor Rshift Divisor**

Iteration number = n+1

Iteration	Step	Quotient	Divisor	Remainder
0	Initial values	0000	0010 0000	0000 0111
1	1: Rem = Rem - Div	0000	0010 0000	①110 0111
	2b: Rem $< 0 \implies$ +Div, sll Q, Q0 = 0	0000	0010 0000	0000 0111
	3: Shift Div right	0000	0001 0000	0000 0111
	1: Rem = Rem - Div	0000	0001 0000	1111 0111
2	2b: Rem $< 0 \implies$ +Div, sll Q, Q0 = 0	0000	0001 0000	0000 0111
	3: Shift Div right	0000	0000 1000	0000 0111
3	1: Rem = Rem - Div	0000	0000 1000	①111 1111
	2b: Rem $< 0 \implies$ +Div, sll Q, Q0 = 0	0000	0000 1000	0000 0111
	3: Shift Div right	0000	0000 0100	0000 0111
	1: Rem = Rem - Div	0000	0000 0100	@000 0011
4	2a: Rem $\geq 0 \implies$ sll Q, Q0 = 1	0001	0000 0100	0000 0011
	3: Shift Div right	0001	0000 0010	0000 0011
5	1: Rem = Rem - Div	0001	0000 0010	@000 0001
	2a: Rem $\geq 0 \implies$ sll Q, Q0 = 1	0011	0000 0010	0000 0001
	3: Shift Div right	0011	0000 0001	0000 0001

Optimized Divider



第五个知识点:

Represent Floating Point

Single precision (32-bit)

Double precision (64-bit)

 $\pm 1.xxxxxxx_2 \times 2^{yyyy}$ single: 8 bits single: 23 bits double: 11 bits double: 52 bits

S Exponent (yyyy+Bias) Fraction (xxxx) $x = (-1)^S \times (1 + Fraction) \times 2^{(Exponent-Bias)}$

Normalize significant: 1.0 ≤ |significant| < 2.0

Single: Bias = 127;

Double: Bias = 1023

-0.875

$$0.875 \times 2 = 1.75$$
 1

$$0.75 \times 2 = 1.5$$
 1

$$0.5 \times 2 = 1$$
 1

$$-0.111 \times 2^0 = -1.11 \times 2^{-1}$$

$$-0.875 = (-1)^1 \times 1.11_2 \times 2^{-1}$$

Fraction =
$$1100 ... 00_2$$

Exponent =
$$-1$$
 + Bias

- Single: $-1 + 127 = 126 = 011111110_2$
- Double: -1 + 1023 = 1022 = 011111111110₂

Single: 1011111101100...00

Double: 10111111111101100...00

第六个知识点:

Floating Point Range

Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
 - Exponent: 00000001 \Rightarrow actual exponent = 1 - 127 = -126
 - Fraction: $000...00 \Rightarrow \text{significand} = 1.0$
 - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
 - ◆ exponent: 11111110
 ⇒ actual exponent = 254 127 = +127
 - Fraction: 111...11 ⇒ significand ≈ 2.0
 - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

Double-Precision Range

- Exponents 0000...00 and 1111...11 reserved
- Smallest value
 - Exponent: 0000000001
 ⇒ actual exponent = 1 1023 = -1022
 - Fraction: $000...00 \Rightarrow \text{significand} = 1.0$
 - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
 - Exponent: 11111111110
 ⇒ actual exponent = 2046 1023 = +1023
 - Fraction: 111...11 ⇒ significand ≈ 2.0
 - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

第七个知识点:

Floating-Point Precision

Relative precision

- all fraction bits are significant
- $\Delta A/|A| = 2^{-23} \times 2^{exponent}/|1 \times 2^{exponent}| = 2^{-23}$
- ◆ Single: approx 2⁻²³
 - Equivalent to 23 \times log₁₀2 \approx 23 \times 0.3 \approx 6 decimal digits of precision
- ◆ Double: approx 2⁻⁵²
 - Equivalent to $52 \times \log_{10} 2 \approx 52 \times 0.3 \approx 16$ decimal digits of precision

第八个知识点:

Floating-Point Instruction in MIPS

Separate FP registers

- 32 single-precision: \$f0, \$f1, ... \$f31
- Paired for double-precision: \$f0/\$f1, \$f2/\$f3, ...
 - Release 2 of MIPs ISA supports 32 imes 64-bit FP reg's

FP instructions operate only on FP registers

- Programs generally don't do integer ops on FP data, or vice versa
- More registers with minimal code-size impact

FP load and store instructions

- ↑ lwc1, ldc1, swc1, sdc1
 - e.g., ldc1 \$f8, 32(\$sp)

- Single-precision arithmetic
 - add.s, sub.s, mul.s, div.s
 - e.g., add.s \$f0, \$f1, \$f6
- Double-precision arithmetic
 - add.d, sub.d, mul.d, div.d
 - e.g., mul.d \$f4, \$f4, \$f6
- Single- and double-precision comparison
 - c. xx.s, c. xx.d (xx is eq, 1t, 1e, ...)
 - Sets or clears FP condition-code bit
 - e.g. c.lt.s \$f3, \$f4
- Branch on FP condition code true or false
 - bc1t, bc1f
 - e.g., bc1t TargetLabel

3.9 Assume 151 and 214 are signed 8-bit decimal integers stored in two's complement format. Calculate 151 + 214 using saturating arithmetic. The result should be written in decimal. Show your work.

$$151 = 10010111_2 = -(01101001) = -105$$

$$214 = 11010110_2 = -(00101010) = -42$$

signed 8-bit decimal integers \longrightarrow 00000000 \sim 011111111 \rightarrow 0 \sim 127

Range is : $-128 \sim 127$

151 + 214 = -105 + (-42) = -147-147 is out of the range - 128~127 Using saturating arithmetic, the result is -128 3.10 Assume 151 and 214 are signed 8-bit decimal integers stored in two's complement format. Calculate 151–214 using saturating arithmetic. The result should be written in decimal. Show your work.

$$151 = 10010111_2 = -(01101001) = -105$$

$$214 = 11010110_2 = -(00101010) = -42$$

signed 8-bit decimal integers
$$\longrightarrow$$
 00000000 \sim 011111111 \rightarrow 0 \sim 127

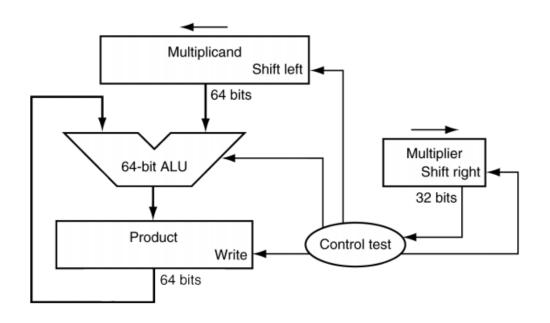
Range is :
$$-128 \sim 127$$

$$151 - 214 = -105 - (-42) = -63$$

$$-63$$
 is in the range $-128\sim127$

Using saturating arithmetic, the result is -63

3.12 Using a table similar to that shown in Figure 3.6, calculate the product of the octal unsigned 6-bit integers 62 and 12 using the hardware described in Figure 3.3. You should show the contents of each register on each step.

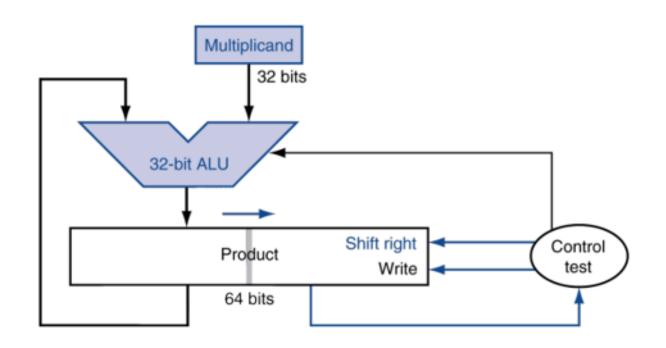


$$62_{oct} = 110\ 010_2$$

$$12_{oct} = 001\ 010_2$$

Step	Action	Multiplier	Multiplicand	Product
0	Initial Vals	001 010	000 000 110 010	000 000 000 000
1	lsb=0. no op	001 010	000 000 110 010	000 000 000 000
	Lshift Mcand	001 010	000 001 100 100	000 000 000 000
	Rshift Mplier	000 101	000 001 100 100	000 000 000 000
2	Prod = Prod + Mcand	000 101	000 001 100 100	000 001 100 100
	Lshift Mcand	000 101	000 011 001 000	000 001 100 100
	Rshift Mplier	000 010	000 011 001 000	000 001 100 100
3	lsb=0. no op	000 010	000 011 001 000	000 001 100 100
	Lshift Mcand	000 010	000 110 010 000	000 001 100 100
	Rshift Mplier	000 001	000 110 010 000	000 001 100 100
4	Prod = Prod + Mcand	000 001	000 110 010 000	000 111 110 100
	Lshift Mcand	000 001	001 100 100 000	000 111 110 100
	Rshift Mplier	000 000	001 100 100 000	000 111 110 100
5	lsb=0. no op	000 000	001 100 100 000	000 111 110 100
	Lshift Mcand	000 000	011 001 000 000	000 111 110 100
	Rshift Mplier	000 000	011 001 000 000	000 111 110 100
	lsb=0. no op	000 000	011 001 000 000	000 111 110 100
6	Lshift Mcand	000 000	110 010 000 000	000 111 110 100
	Rshift Mplier	000 000	110 010 000 000	000 111 110 100

3.13 Using a table similar to that shown in Figure 3.6, calculate the product of the hexadecimal unsigned 8-bit integers 62 and 12 using the hardware described in Figure 3.5. You should show the contents of each register on each step.



$$62_{hex} = 0110\ 0010_2$$

$$12_{hex} = 0001\ 0010_2$$

Step	Action	Multiplicand	Product/Multiplier
0	Initial Vals	0110 0010	0000 0000 0001 0010
4	lsb = 0, no op	0110 0010	0000 0000 0001 0010
1	Rshift Product	0110 0010	`0000 0000 0001
0	Prod = Prod +Mcand	0110 0010	0110 0010 0000 1001
2	Rshift Product	0110 0010	0011 0001 0000 0100
•	lsb = 0, no op	0110 0010	0011 0001 0000 0100
3	Rshift Product	0110 0010	0001 1000 1000 0010
,	lsb = 0, no op	0110 0010	0001 1000 1000 0010
4	Rshift Product	0110 0010	0000 1100 0100 0001
F	Prod = Prod +Mcand	0110 0010	0110 1110 0100 0001
5	Rshift Product	0110 0010	0011 0111 0010 0000
C	lsb = 0, no op	0110 0010	0011 0111 0010 0000
6	Rshift Product	0110 0010	0001 1011 1001 0000
7	lsb = 0, no op	0110 0010	0001 1011 1001 0000
	Rshift Product	0110 0010	0000 1101 1100 1000
8	lsb = 0, no op	0110 0010	0000 1101 1100 1000
	Rshift Product	0110 0010	0000 0110 1110 0100

计算机组成原理

Chapter 4

复习

A

第一个知识点:

A basic MIPS Implementation

For every instruction, the first two steps are identical:

- 1. Send the *program counter* (PC) to the memory that contains the code and fetch the instruction from that memory.
- 2. Read one or two registers, using fields of the instruction to select the registers to read. For the load word instruction, we need to read only one register, but most other instructions require reading two registers.

After these two steps, the actions required to complete the instruction depend on the instruction class. Fortunately, for each of the three instruction classes (memory-reference, arithmetic-logical, and branches), the actions are largely the same, independent of the exact instruction.

第一个知识点:

A basic MIPS Implementation

- basic math (add, sub, and, or, slt)
- memory access (lw and sw)
- branch and jump instructions (beq and j)
 - We need memory
 - to store instructions
 - to store data
 - for now, let's make them separate units
 - We need registers, ALU, and a whole lot of control logic
 - CPU operations common to all instructions:
 - use the program counter (PC) to pull instruction out of instruction memory
 - read register values

第二个知识点:

Clocking Methodology

- Information encoded in binary
 - Low voltage = 0, High voltage = 1
 - One wire per bit
 - Multi-bit data encoded on multi-wire buses
- Combinational element
 - Operate on data
 - Output is a function of input
- State (sequential) elements
 - Store information

第二个知识点:

Combinational Elements

And gateY=A&B

- AdderY = A + B
- A + Y

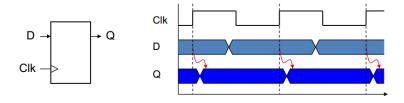
- Multiplexer
 - Y = S ? 1:0
- Arithmetic/Logic Unit
 - Y = F(A, B)



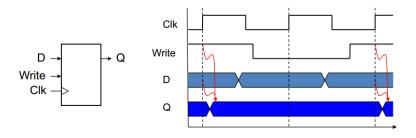


Sequential Elements

- · Register: stores data in a circuit
 - Uses a clock signal to determine when to update the stored value
 - Edge-triggered: update when Clk changes from 0 to 1

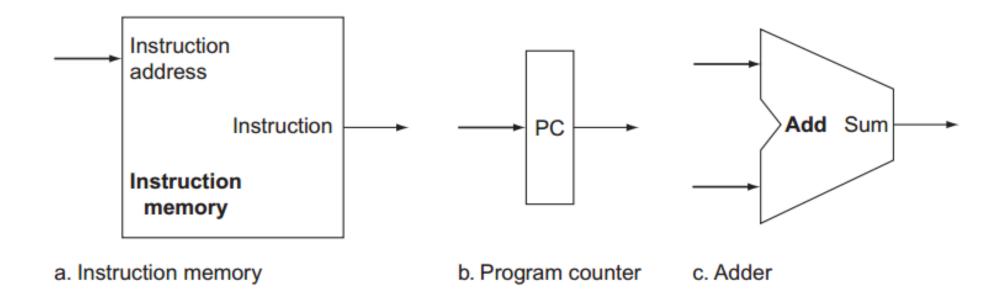


- · Register with write control
 - Only updates on clock edge when write control input is 1
 - Used when stored value is required later



第三个知识点:

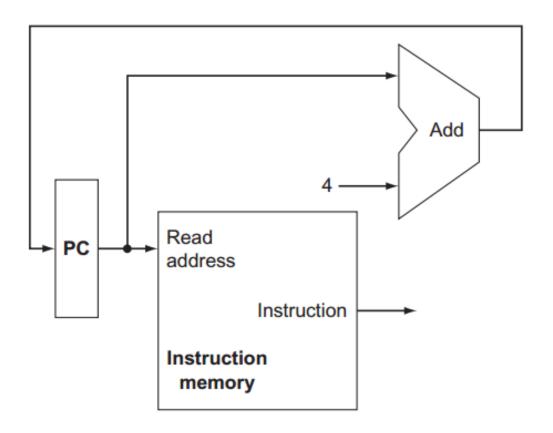
Datapath



Two state elements are needed to store and access instructions, and an adder is needed to compute the next instruction address.

第三个知识点:

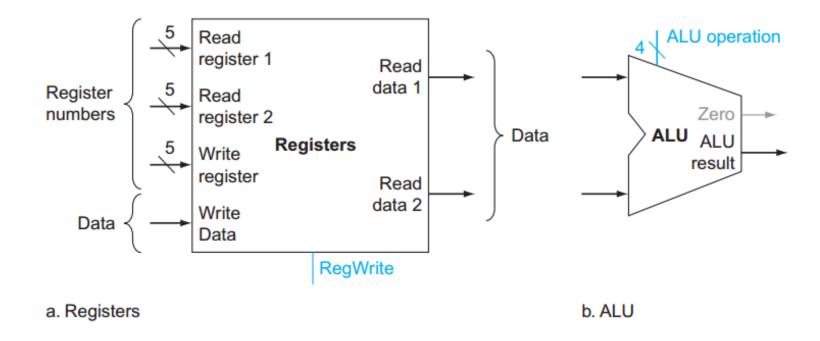
Datapath



A portion of the datapath used for fetching instructions and incrementing the program counter.

第三个知识点:

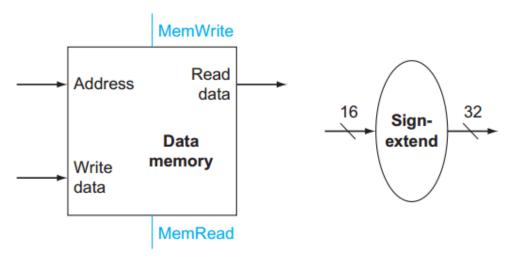
R Type



The two elements needed to implement R-format ALU operations are the register file and the ALU

I Type

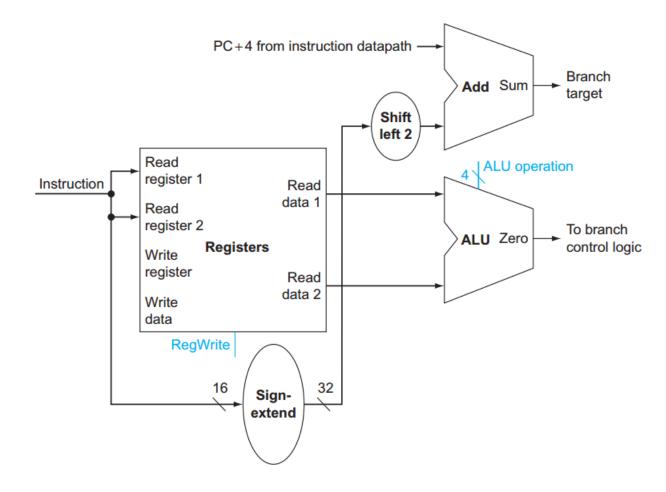
- The instruction set architecture specifes that the base for the branch address calculation is the address of the instruction following the branch. Since we compute PC + 4 (the address of the next instruction) in the instruction fetch datapath, it is easy to use this value as the base for computing the branch target address.
- The architecture also states that the offset field is shifed left 2 bits so that it is a word offset; this shift increases the effective range of the offset field by a factor of 4.



a. Data memory unit

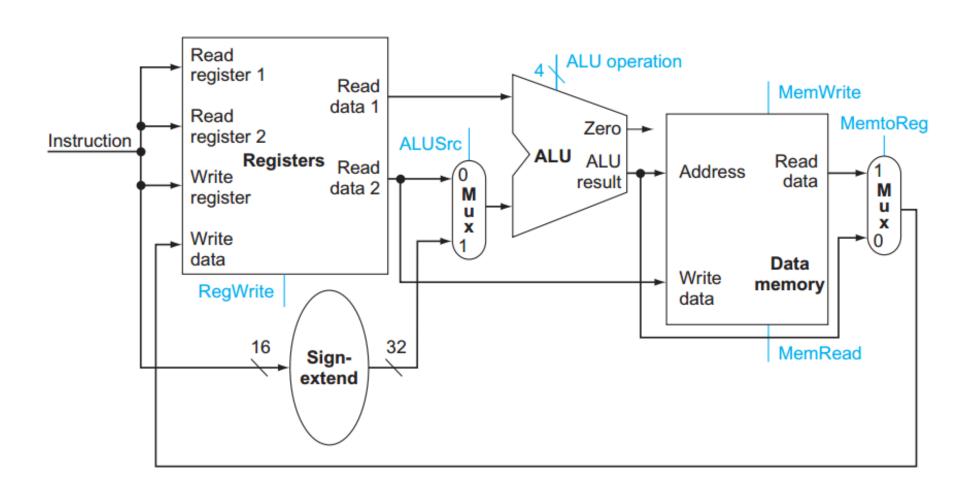
b. Sign extension unit

I Type

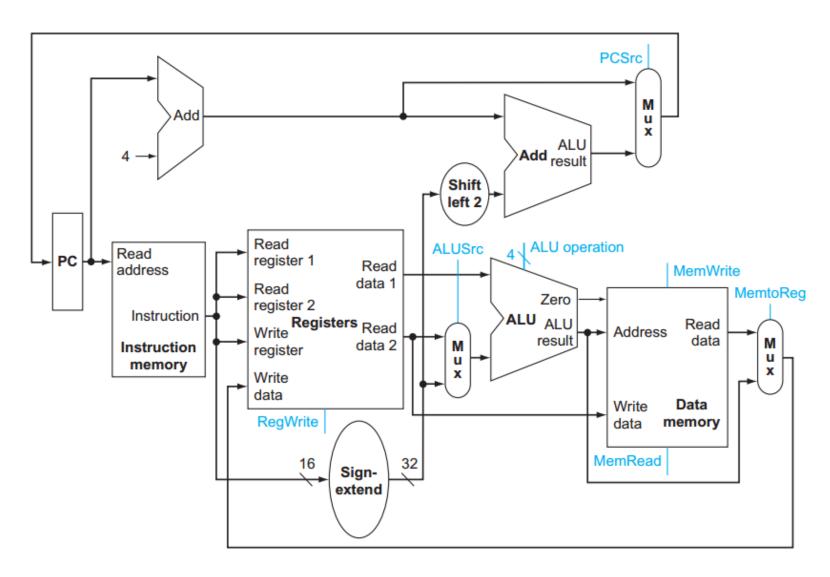


The datapath for a branch uses the ALU to evaluate the branch condition and a separate adder to compute the branch target as the sum of the incremented PC and the sign-extended, lower 16 bits of the instruction (the branch displacement), shifted left 2 bits.

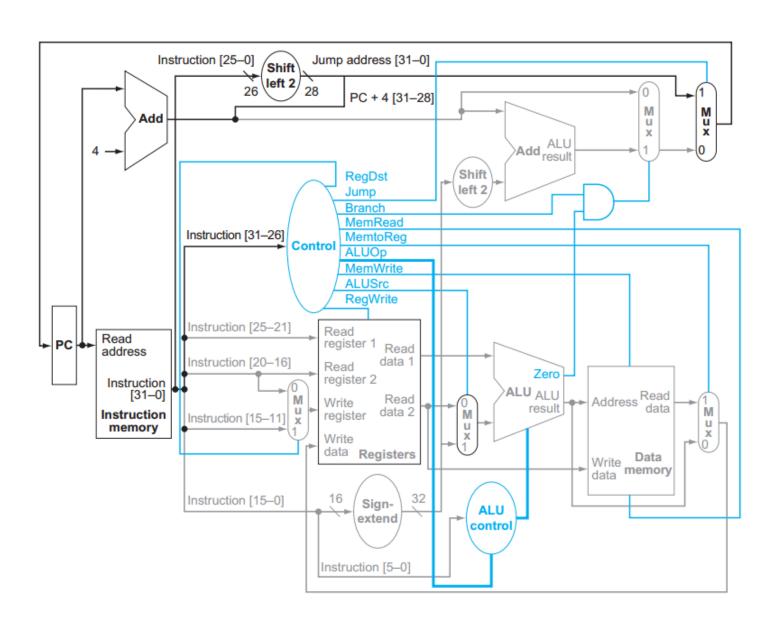
The datapath for the memory instructions and the R-type instructions.



The simple datapath for the core MIPS architecture combines the elements required by different instruction classes.



J Type and Control



Control

Instruction	RegDst	ALUSrc	Memto- Reg		Mem- Read		Branch	ALUOp1	ALUOp0
R-format	1	0	0	1	0	0	0	1	0
1w	0	1	1	1	1	0	0	0	0
SW	X	1	X	0	0	1	0	0	0
beq	X	0	X	0	0	0	1	0	1

Instruction opcode	ALUOp	Instruction operation	Funct field	Desired ALU action	ALU control input
LW	00	load word	XXXXXX	add	0010
SW	00	store word	XXXXXX	add	0010
Branch equal	01	branch equal	XXXXXX	subtract	0110
R-type	10	add	100000	add	0010
R-type	10	subtract	100010	subtract	0110
R-type	10	AND	100100	AND	0000
R-type	10	OR	100101	OR	0001
R-type	10	set on less than	101010	set on less than	0111