Assignment for Numerical Integration

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```

Problem 1

```
clear all; clc;
syms x
expr = (x+1/x)^2;
I = int(expr, [1,2]);
x1 = 1:1:2; x2 = 1:0.5:2; x3 = 1:1/3:2; x4 = 1:0.25:2;
y1 = (x1+1./x1).^2; y2 = (x2+1./x2).^2; y3 = (x3+1./x3).^2; y4 = (x4+1./x4).^2;
I1 = trapz(x1,y1); I2 = trapz(x2,y2); I3 = trapz(x3,y3); I4 = trapz(x4,y4);
fprintf("n = 1, I1 = %.2f; \nn = 2, I2 = %.2f; \n",I1,I2);
```

```
n = 1, I1 = 5.12;
n = 2, I2 = 4.91;
```

```
fprintf("n = 3, I3 = %.2f; \nn = 4, I4 = %.2f \n",I3,I4);
```

```
n = 3, I3 = 4.87;
n = 4, I4 = 4.85
```

```
fprintf("with analytical solution %.2f, the percent relative errors are: \n", I);
```

```
with analytical solution 4.83, the percent relative errors are:
```

```
fprintf("n = 1, e1 = %.4f%%; \nn = 2, e2 = %.4f%%; \n",(I-I1)/I,(I-I2)/I);
fprintf("n = 3, e3 = %.4f%%; \nn = 4, e4 = %.4f%% \n",(I-I3)/I,(I-I4)/I);
```

```
n = 1, e1 = -0.0603%;
n = 2, e2 = -0.0158%;
n = 3, e3 = -0.0071%;
n = 4, e4 = -0.0040%
```

Problem 2

```
clear all; clc;
% analytical
syms x
expr = x^2*exp(x);
I = int(expr, [0,3]);
% numerical
x = 0:3/4:3; y = x.^2.*exp(x);
I1 = trapz(x,y); I2 = simpint('f',0,3,4);
fprintf("analytical: %.2f; trapezoidal: %.2f; simpson: %.2f \n", I,I1,I2);
```

```
analytical: 98.43; trapezoidal: 112.27; simpson: 99.46
```

```
fprintf("the percent relative errors are: \n");
```

```
the percent relative errors are:
```

```
fprintf("e-trapezoidal = %.4f%%, e-simpson = %.4f%% \n",(I-I1)/I,(I-I2)/I);
```

```
e-trapezoidal = -0.1406%, e-simpson = -0.0105%
```

Problem 3

```
clear all; clc;
y = [0 1 3 5 7 8 9 10]; H = [0 1 1.5 3 3.5 3.2 2 0]; U = [0 0.1 0.12 0.2 0.25 0.3 0.15 0];
% a
d = trapz(y,H)/10;
fprintf("the average depth is %.2f \n",d);
```

```
the average depth is 2.10
```

```
% b
A = trapz(y,H);
fprintf("the cross-sectional area is %.2f \n",A);
```

```
the cross-sectional area is 20.95
```

```
% c
v = trapz(y,U)/10;
fprintf("the average velocity is %.2f \n",v);
```

```
the average velocity is 0.16
```

```
% d
Q = trapz(y,H.*U);
fprintf("the flow rate is %.2f \n",Q);
```

```
the flow rate is 4.28
```

Problem 4

```
clear all; clc;
t = [0 1 5.5 10 12 14 16 18 20 24]; c = [1 1.5 2.3 2.1 4 5 5.5 5 3 1.2];
Q = 20+10*sin(2*pi/24*(t-10));
r = trapz(t,Q.*c)/trapz(t,Q);
fprintf("the flow-weighted average concentration is: %.2f \n", r);
```

```
the flow-weighted average concentration is: 3.44
```

Problem 5

```
clear all; clc;
t = [0 15 30 45 75 105]; R = [18 24 26 20 18 9];
% a
total = trapz(t,R/4);
fprintf("The total number of cars is: %.f \n",total);
```

```
The total number of cars is: 502
```

```
% b
fprintf("The rate is: %.2f cars/minutes \n", total/105);
```

```
The rate is: 4.79 cars/minutes
```

function

```
function y = f(x)
y = x^2*exp(x);
end
function val = simpint(f,a,b,n)
h = (b-a)/n;
s1=0; s2=0;
for k=1:2:(n-1)
    x=a+h*k;
    s1=s1+feval(f,x);
end
for k=2:2:(n-2)
    x=a+h*k;
    s2=s2+feval(f,x);
end
val=h*(feval(f,a)+feval(f,b)+4*s1+2*s2)/3;
end
```