Assignment for Linear equations

Problem 1.

Solve the following equations:

$$7x + 9y - 9z = 22$$
$$3x + 2y - 4z = 12$$
$$x + 5y - z = -2$$

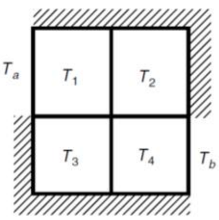
Problem 2.

a. Use MATLAB to nd the coef cients of the quadratic polynomial $y = ax^2 + bx + c$ that passes through the three points $(x, y) = (1, 4)$, $(4, 73)$, $(5, 120)$.

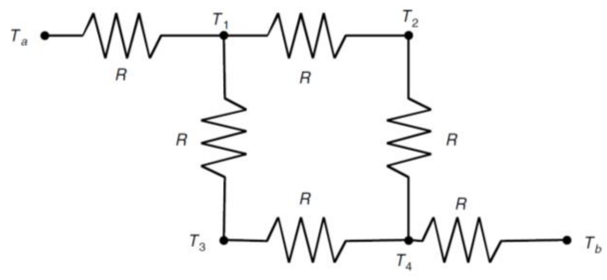
b. Use MATLAB to nd the coef cients of the cubic polynomial $y = ax^3 + bx^2 + cx + d$ that passes through the three points given in part *a*.

Problem 3.

The concept of thermal resistance described in problem 6 in the previous assignment for linear equations can be used to calculate the temperature distribution in the square plate shown in the following graph



(a)



(b)

The plate's edges are insulated so that no heat can escape, except at two points where the edge temperature is heated to T_a and T_b, respectively. The temperature varies through the plate, so no single point can describe the plate's temperature. One way to estimate the temperature distribution is to imagine that the plate consists of four subsquares and to compute the temperature in each subsquare. Let R be the thermal resistance of the material between the centers of adjacent subsquares. Then we can think of the problem as a network of electric resistors, as shown in part (b) of the gure. Let q_{ij} be the heat ow rate between the points whose temperatures are T_i and T_j. If T_a and T_b remain constant for some time, then the heat energy stored in each subsquare is constant also, and the heat ow rate between each subsquare is constant. Under these conditions, conservation of energy says that the heat ow into a subsquare equals the heat ow out. Applying this principle to each subsquare gives the following equations.

$$
\begin{aligned}
q_{a1} &= q_{12} + q_{13} \\
q_{12} &= q_{24} \\
q_{13} &= q_{34} \\
q_{34} + q_{24} &= q_{4b}
\end{aligned}
$$

Substituting $q = (T_i - T_j)/R$, we nd that R can be canceled out of every equation, and they can be rearranged as follows:

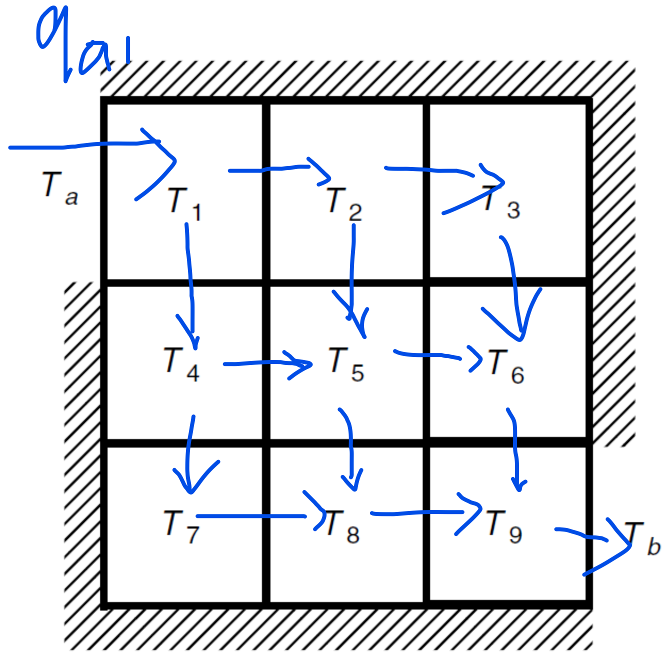
$$
\begin{aligned}
T_1 &= \frac{1}{3}(T_a + T_2 + T_3) \\
T_2 &= \frac{1}{2}(T_1 + T_4) \\
T_3 &= \frac{1}{2}(T_1 + T_4) \\
T_4 &= \frac{1}{3}(T_2 + T_3 + T_{\cancel{3}\,b})
\end{aligned}
$$

These equations tell us that the temperature of each subsquare is the average of the temperatures in the adjacent subsquares!

Solve these equations for the case where $T_a = 150°\text{C}$ and $T_b = 20°\text{C}$.

Problem 4.

Use the averaging principle developed in Problem 2 to calculate the temperature distribution of the plate shown below, using the 3×3 grid and the given values $T_a = 150℃$ and $T_b = 20℃$.



Problem 5.

The following table shows how many hours in process reactors A and B are required to produce 1 ton each of chemical products 1, 2, and 3. The two reactors are available for 35 and 40 hrs per week, respectively.

Hours	Product 1	Product 2	Product 3
Reactor A	6	2	10
Reactor B	3	5	2

Let x, y, and z be the number of tons each of products 1, 2, and 3 that can be produced in one week.

a. Use the data in the table to write two equations in terms of x, y, and z. Determine whether a unique solution exists. If not, use MATLAB to nd the relations between x, y, and z.

b. Note that negative values x, y, and z have no meaning here. Find the allowable ranges for x, y, and z.

c. Suppose the pro ts for each product are \$200, \$300, and \$100 for products 1, 2, and 3, respectively. Find the values of x, y, and z to maximize the pro t.

d. Suppose the pro ts for each product are \$200, \$500, and \$100 for products 1, 2, and 3, respectively. Find the values of x, y, and z to maximize the pro t.

Problem 6.

See Figure P13. Assume that no vehicles stop within the network. A traf c engineer wants to know if the traf c ows $f_1, f_2, \ldots, f_7$ (in vehicles per hour) can be computed given the measured ows shown in the gure. If not, then determine how many more traf c sensors need to be installed, and obtain the expressions for the other traf c ows in terms of the measured quantities.

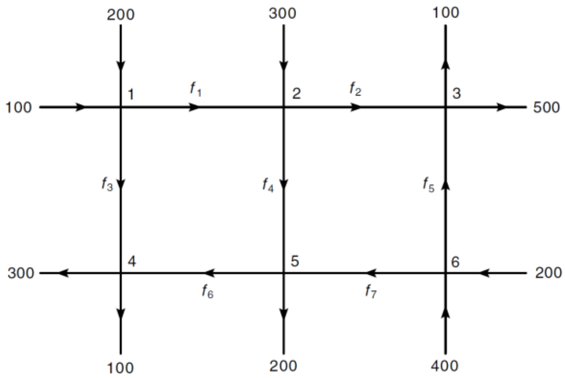


Figure P13