In-class exercise for curve fitting

Problem 1. The temperature of the ground at a depth x for surface temperature T_s and initial temperature T_i is given as

$$\boxed{\frac{T - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)}$$

t=60*24*3600
T_s=-15 C
T_i=20 C
T=0 C
α=1.38*10^{-7} m^2/s

How deep should a water main be buried if we want to keep the water from freezing if the surface is at -15 C for 60 days?

Problem 2. Find first two positive values of β that solve this equation for L=4.2m.

$$\boxed{1 + \cosh(\beta L)\cos(\beta L) = 0}$$

For EI=21000 N-m^2 and ρ=0.53 kg/m, calculate the frequencies from

$$\boxed{\omega = \beta^2 \sqrt{\frac{EI}{\rho}}}$$

Problem 3. The following function is linear in the parameters a_1 and a_2.

$$y(x) = a_1 + a_2 \ln x$$

Use least-squares regression with the following data to estimate the values of a_1 and a_2. Use the curve t to estimate the values of y at x=2.5 and at x=11.

x	1	2	3	4	5	6	7	8	9	10
y	10	14	16	18	19	20	21	22	23	23

Problem 4. Chemists and engineers must be able to predict the changes in chemical concentration in a reaction. A model used many single-reactant processes is

$$\text{Rate of change of concentration} = -kC^n$$

where C is the chemical concentration and k is the rate constant. The following data describe the reaction

$$(CH_3)_3CBr + H_2O \rightarrow (CH_3)_3COH + HBr \qquad \frac{dC}{dt} = -kC$$

Use these data to obtain a least-squares fit to estimate the value of k

Time t (h)	C(mol of $(CH3)_3$ CBr/L)
0	0.1039
3.15	0.0896
6.20	0.0776
10.0	0.0639
18.3	0.0353
30.8	0.0207
43.8	0.0101

Problem 5. The following represents pressure samples, in pounds per square inch (psi), taken in a fuel line once every second for 10 sec.

Time (sec)	Pressure (psi)	Time (sec)	Pressure (psi)
1	26.1	6	30.6
2	27.0	7	31.1
3	28.2	8	31.3
4	29.0	9	31.0
5	29.8	10	30.5

a. Fit a first-degree polynomial, a second-degree polynomial, and a third-degree polynomial to these data. Plot the curve fits along with the data points

b. Use the results from part a to predict the pressure at $t = 11$ sec. Explain which curve fit gives the most reliable prediction. Consider the coefficients of determination and the residuals for each fit in making your decision.