Report of Midterm Project
ME112: Introduction to Matlab
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ME112-INTRODUCTION TO MATLAB

TRAPPING A CHARGED MICRO DROPLET WITH LINEAR QUADRUPOLE

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ABSTRACT

Electro-dynamic balance (EDB) is widely used in mass spectrometry. In EDB, a linear quadrupole set is built and the electric field is genrated. By controlling different AC voltage and frequency applied on the linear quadrupole set, the response time of trapping the droplet differs.

Keywords: quadrupole, AC voltage, frequency

NOMENCLATURE

Roman letters

V_{AC}	The AC voltage
V_{DC}	The DC voltage
C_d	The drag coefficient
d	The particle diameter

Greek letters

ω	the frequency of the AC voltage

1. INTRODUCTION

The electro-dynamic balance (EDB), which is capable of levitating droplet diameters of a few tens or micrometers (Shaw, 2000; Davis, 1980; Agnes, 2002), is desirable for investigating both diffusion-rate controlled and kinetically controlled evaporation regime (Davis, 1980). To use EDB, the droplet would have to carry electric charges, which requires integrating the capability of adding electric charges to droplets with droplet generation. The combination of EDB and ES provides an effective tool suited for the study of the evaporation and burning characteristics of fuel nanofluid droplets. Droplets thus generated carry static electric charges and, when placed in a properly design electric field as that using EDB, can be suspended immediately followed by experimental probing prior to significant evaporation (this is especially notable as the D2-law indicates that smaller droplets evaporate faster than larger droplets).

2. PRINCIPLES AND MATERIALS

EDB is developed based on the quadrupole mass filter that is proposed by Paul and Steinwedel (1953). A charged particles suspended by means of the electrical fields. The device can trap particles with the size range of nanometer to hundreds of micrometer by adjusting the AC frequency and magnitude.

2.1 MATERIALS

The linear quadruple with AC voltage applied on it (see figure 1) can trap charged micro droplets. Two rods opposite each other have the same voltage, which is Vcos(wt), the other two opposing rods have the same magnitude of voltage but differ in polarity (-Vcos(wt)).

FIGURE 1: LINEAR QUADRUPOLE SETUP

2.2 PRINCIPLES

In the expression of electric field vectors in the vertical and axial directions, the electric field generated by ac voltage V_{AC} applied on the quadrupole is expressed as

$$E_x = -\frac{2x}{r_0^2} V_{AC} \cos(\omega t) \quad (1)$$

$$E_y = \frac{2y}{r_0^2} V_{AC} \cos(\omega t) \quad (2)$$

where ω is the frequency of the AC voltage, r_0 is the distance from center to rod surface (see figure 2).

The electric field that is produced by a DC voltage V_{DC}, in the EDB device is

$$E_{DC} = \frac{V_{DC}}{r_0} \vec{k} \quad (3)$$

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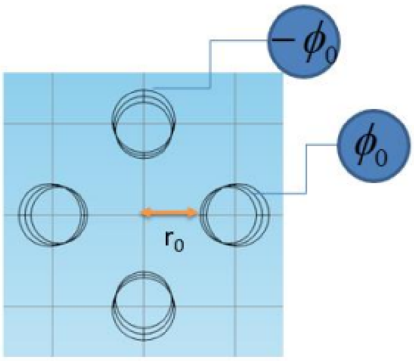

FIGURE 2: SIDE VIEW OF LINEAR QUADRUPOLE AND VOLTAGE ON IT

Suppose a charged particle is injected vertically from the center of the top endcap to a dynamic field. The trajectory of a charged particle with the mass of m and charge of q in the electrical filed are described as

$$m\frac{d^2y}{dt^2} = -C_d\frac{dy}{dt} + qE_y - mg + qE_{DC} \tag{4}$$

$$m\frac{d^2x}{dt^2} = -C_d\frac{dx}{dt} + qE_x \tag{5}$$

where C_d is the drag coefficient and for a spherical particle it can be defined by

$$C_d = 3\pi d\upsilon \tag{6}$$

where d is the particle diameter, and υ is the viscosity of the gas phase.
By substitution, we can get

$$m\frac{d^2y}{dt^2} = \frac{2y}{r_0^2}V_{AC}\cos(\omega t) - q\frac{V_{DC}}{r_0} - 3\pi d\upsilon\frac{dy}{dt} - mg \tag{7}$$

$$m\frac{d^2x}{dt^2} = -\frac{2x}{r_0^2}V_{AC}\cos(\omega t) - 3\pi d\upsilon\frac{dx}{dt} \tag{8}$$

Without external forces, we have

$$-q\frac{V_{DC}}{r_0} = mg \tag{9}$$

That is

$$m\frac{d^2y}{dt^2} = \frac{2y}{r_0^2}V_{AC}\cos(\omega t) - 3\pi d\upsilon\frac{dy}{dt} \tag{10}$$

3. RESULTS AND DISCUSSION

3.1 ONLY x DIRECTION OR ONLY y DIRECTION

At the first time, we considered the motion is only on x direction ($y = 0$) or on y direction ($x = 0$). With initial position and velocity as $x = 1mm$ and $\upsilon_x = 1mm/s$ (or $y = 1mm$ and $\upsilon_y = 1mm/s$), we tested for $\omega = 900, 1200, 1500, 1800$ with $V_{AC} = 1000, 2000$. The MATLAB script is shown in Appendix A and the result is shown as below.

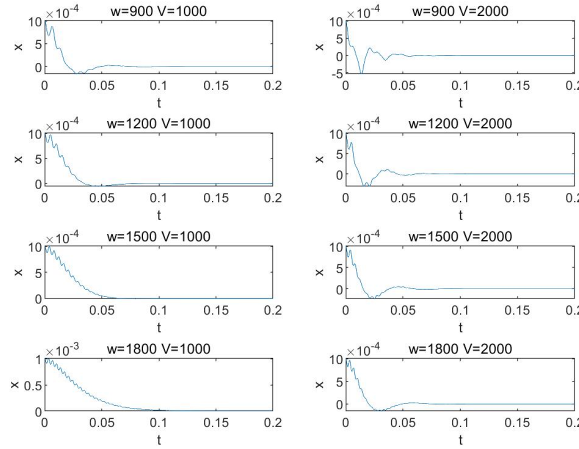

FIGURE 3: TRAJECTORY OF DROPLET IN THE X DIRECTION

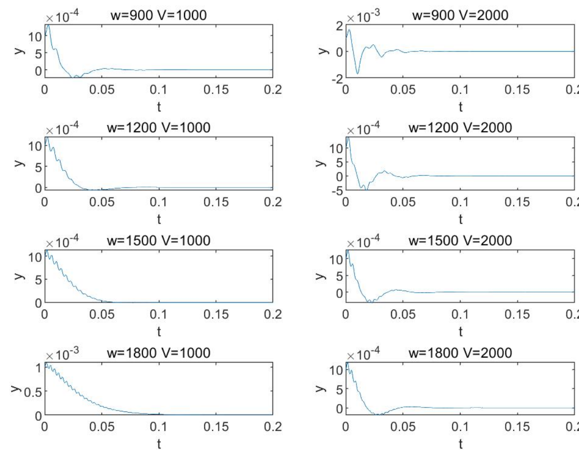

FIGURE 4: TRAJECTORY OF DROPLET IN THE Y DIRECTION

In the same column, V_{AC} is fixed and ω varies; in the same row, ω is fixed and V_{AC} varies.
According to the plot result shown as above, both x and y converge to 0 along their direction, respectively. But the response time is different with different AC voltage or frequency. As ω increases, the vibration frequency and the amplitude both decrease. As V_{AC} increases, the vibration frequency increases but the amplitude decreases.

3.2 x AND y DIRECTION

Then, we considered the combination of both x and y direction motions, but still regardless the evaporation. With initial position and velocity as $x = y = 1mm$ and $\upsilon_x = \upsilon_y = 1mm/s$, we tested for $\omega = 900, 1200, 1500$ with $V_{AC} = 1000, 2000$. The result is shown as below.

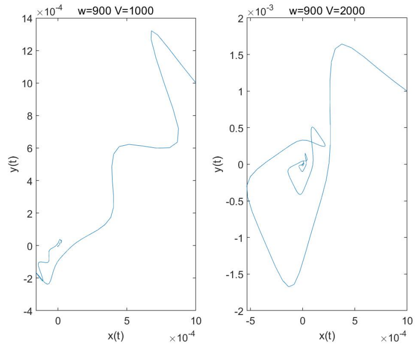

FIGURE 5: COMBINATION OF X AND Y DIRECTION MOTION

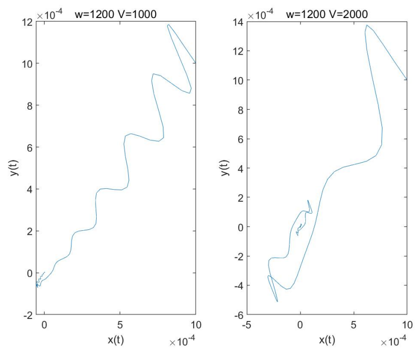

FIGURE 6: COMBINATION OF X AND Y DIRECTION MOTION

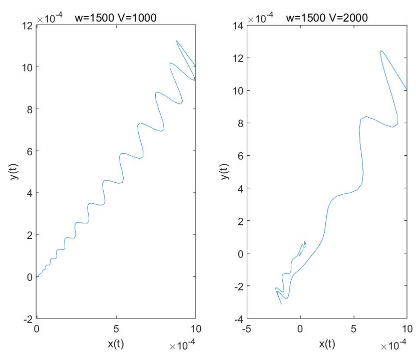

FIGURE 7: COMBINATION OF X AND Y DIRECTION MOTION

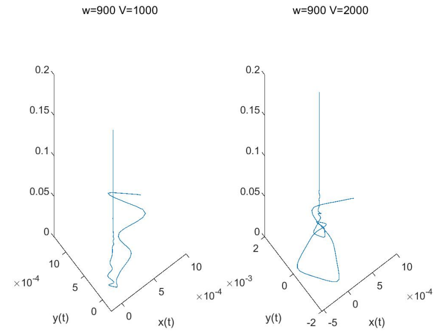

FIGURE 8: COMBINATION OF X AND Y DIRECTION MOTION

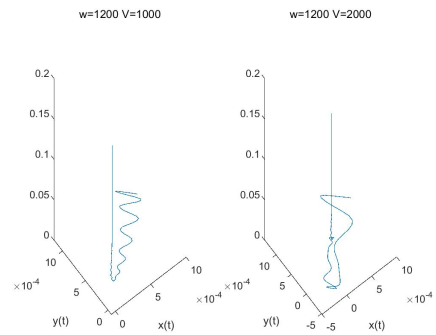

FIGURE 9: COMBINATION OF X AND Y DIRECTION MOTION

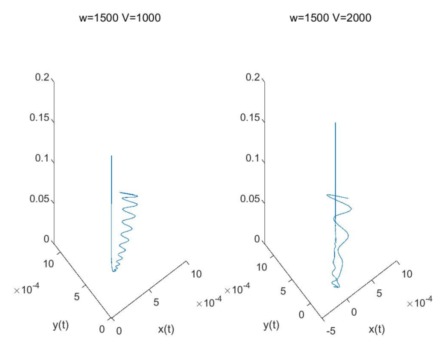

FIGURE 10: COMBINATION OF X AND Y DIRECTION MOTION

If we plot them in 3 axes with $x(t)$, $y(t)$ and t, it is more obvious that the droplet is finally trapped successfully.

3.3 VARY INITIAL VELOCITY

During the exploration on the above, it is clear that the initial velocity is effective on the motion.
For certain AC voltage V_{AC}, frequency ω and initial position $(x0, y0)$, how the initial velocity affects the motion are shown as below. Figure 11 fixed $\omega = 900, V = 1000$ and Figure 12 fixed $\omega = 1500, V = 2000$. The MATLAB script is shown in Appendix C.

As the initial velocity increases, the maximum displacement increases.

3.4 VARY INITIAL POSITION

After considering different initial velocity, it is time to see how the initial position affects. Figure 13 fixed $\omega = 900, V = 1000$ and Figure 14 fixed $\omega = 1500, V = 2000$. The MATLAB script is shown in Appendix D.

However, it is not so obvious that the initial position will change the processing or result of the motion.

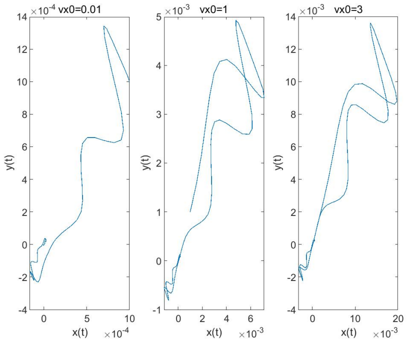

FIGURE 11: ω **= 900,** V **= 1000**

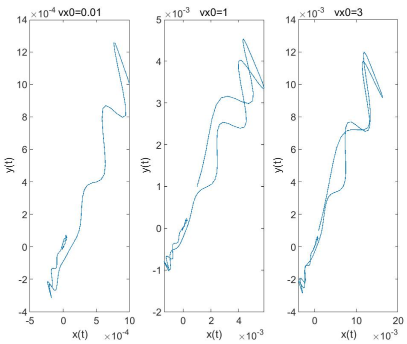

FIGURE 12: ω **= 1500,** V **= 2000**

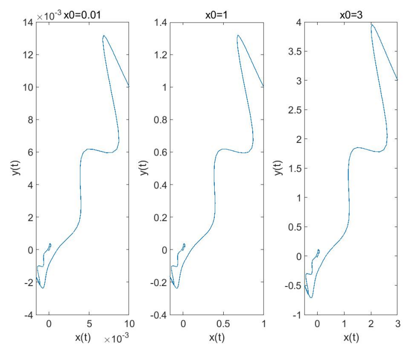

FIGURE 13: ω **= 900,** V **= 1000**

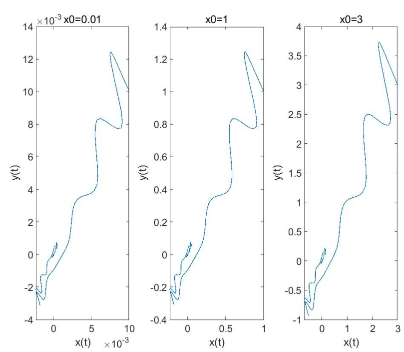

FIGURE 14: ω **= 1500,** V **= 2000**

4. CONCLUSION

Trapping a charged droplet with linear quadrupole is a complex process with several parameters, such as AC voltage V_{AC}, frequency ω, initial velocity $v0$ and so on. It is pity that many other properties are not covered in the model such as the evaporation of the droplet and the charge mass ratio. The model taken above is a simplified model. When it comes to the real operation process, it needs to be corrected by experience and actual measurements.

ACKNOWLEDGMENTS

Thanks to the instruction of Dr. Wei, teaching me the principles of this experiment. And thanks for classmate in the course group. This work is supported by their selfless help.

APPENDIX A. MATLAB SCRIPT FOR ONLY A DIRECTION

```
x0 = 1e-3; %initial position
vx0 = 1e-3; %initial velocity along x

d = 50e-6; %diameter

EPS0 = 8.85e-12;
surface_tension = 0.0728;
R1 = 8*pi*sqrt(EPS0*surface_tension*(d/2)
    ^3); %Rayleigh
    limit
q = 0.3*R1; %droplet charge
%1
figure;
k = 1;
for w = 900:300:1800
    for V = [1000 2000]
        [t, x] = ode45(@(t,x) fun1(t,x,w,V
            ,q), [0 0.2], [x0 vx0]);
        subplot(4,2,k);
        plot(t, x(:,1));
        title("w="+int2str(w)+" "+"V="+
            int2str(V));
        xlabel('t'); ylabel('x');
        k = k + 1;
    end
end

figure;
k = 1;
for w = 900:300:1800
    for V = [1000 2000]
        [t, y] = ode45(@(t,y) fun2(t,y,w,V
            ,q), [0 0.2], [x0 vx0]);
        subplot(4,2,k);
        plot(t, y(:,1));
        title("w="+int2str(w)+" "+"V="+
            int2str(V));
        xlabel('t'); ylabel('y');
        k = k + 1;
    end
end
```

APPENDIX B. MATLAB SCRIPT FOR X AND Y DIRECTION

```
clear all; clc;

x0 = 1e-3; %initial position
vx0 = 1e-3; %initial velocity along x

d = 50e-6; %diameter

EPS0 = 8.85e-12;
surface_tension = 0.0728;
Rl = 8*pi*sqrt(EPS0*surface_tension*(d/2)^3); %Rayleigh limit
q = 0.3*Rl; %droplet charge
%2
k = 1;
for w = 900:300:1800
    figure;
    for V = [1000 2000]
        [t1, x] = ode45(@(t,x) fun1(t,x,w,V,q), [0 0.2], [x0 vx0]);
        [t2, y] = ode45(@(t,y) fun2(t,y,w,V,q), [0 0.2], [x0 vx0]);
        t = linspace(0, 0.5, size(t1, 1) * 2);
        xvals = interp1(t1, x(:, 1), t);
        yvals = interp1(t2, y(:, 1), t);
        subplot(1,2,k);
        plot(xvals, yvals);
        xlabel('x(t)'); ylabel('y(t)');
        k = k + 1;
    end
    k = 1;
end

k = 1;
for w = 900:300:1800
    figure;
    for V = [1000 2000]
        [t1, x] = ode45(@(t,x) fun1(t,x,w,V,q), [0 0.2], [x0 vx0]);
        [t2, y] = ode45(@(t,y) fun2(t,y,w,V,q), [0 0.2], [x0 vx0]);
        t = linspace(0, 0.5, size(t1, 1) * 2);
        xvals = interp1(t1, x(:, 1), t);
        yvals = interp1(t2, y(:, 1), t);
        subplot(1,2,k);
        plot3(xvals, yvals, t);
        title("w="+int2str(w)+" "+"V="+int2str(V));
        xlabel('x(t)'); ylabel('y(t)');
        k = k + 1;
    end
    k = 1;
end
```

APPENDIX C. MATLAB SCRIPT FOR VERY VELOCITY

```
%3
k = 1;
x0 = 1e-3;
vx0 = [0.01 1 3];
w = [900 1500];
V = [1000 2000];
for j = 1:2
    figure
    for i = 1:3
        [t1, x] = ode45(@(t,x) fun1(t,x,w(j),V(j),q), [0 0.2], [x0 vx0(i)]);
        [t2, y] = ode45(@(t,y) fun2(t,y,w(j),V(j),q), [0 0.2], [x0 vx0(i)]);
        t = linspace(0, 0.5, size(t1, 1) * 2);
        xvals = interp1(t1, x(:, 1), t);
        yvals = interp1(t2, y(:, 1), t);
        subplot(1,3,k);
        plot(xvals, yvals);
        title("vx0="+num2str(vx0(i)));
        xlabel('x(t)'); ylabel('y(t)');
        k = k + 1;
    end
    k = 1;
end
```

APPENDIX D. MATLAB SCRIPT FOR VERY POSITION

```
k = 1;
x0 = [0.01 1 3];
vx0 = 1e-3;
w = [900 1500];
V = [1000 2000];
for j = 1:2
    figure
    for i = 1:3
        [t1, x] = ode45(@(t,x) fun1(t,x,w(j),V(j),q), [0 0.2], [x0(i) vx0]);
        [t2, y] = ode45(@(t,y) fun2(t,y,w(j),V(j),q), [0 0.2], [x0(i) vx0]);
        t = linspace(0, 0.5, size(t1, 1) * 2);
        xvals = interp1(t1, x(:, 1), t);
        yvals = interp1(t2, y(:, 1), t);
        subplot(1,3,k);
        plot(xvals, yvals);
        title("x0="+num2str(x0(i)));
        xlabel('x(t)'); ylabel('y(t)');
        k = k + 1;
    end
    k = 1;
```

```
end
```

APPENDIX E. FUNCTIONS

```
function z = fun1(t, x, w, V, q)
r0 = 0.012; %initial distance

d = 50e-6; %diameter
m = 1e3*4/3*pi*(d/2)^3; %mass

Ex = -2*x(1)/r0^2*V*cos(w*t);

ita = 17.9e-6;

z = zeros(2,1);
z(1) = x(2);
z(2) = 1/m*Ex*q-3*pi*ita*d*1/m*x(2);
end

function z = fun2(t, y, w, V, q)
r0 = 0.012; %initial distance

d = 50e-6; %diameter
m = 1e3*4/3*pi*(d/2)^3; %mass

Ey = 2*y(1)/r0^2*V*cos(w*t);

ita = 17.9e-6;

z = zeros(2,1);
z(1) = y(2);
z(2) = 1/m*Ey*q-3*pi*ita*d*1/m*y(2);
end
```