

In-class exercise for curve fitting

Problem 1. The temperature of the ground at a depth x for surface temperature T_s and initial temperature T_i is given as

$$\frac{T - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$t = 60 \times 24 \times 3600$$

$$T_s = -15 \text{ C}$$

$$T_i = 20 \text{ C}$$

$$T = 0 \text{ C}$$

$$\alpha = 1.38 \times 10^{-7} \text{ m}^2/\text{s}$$

How deep should a water main be buried if we want to keep the water from freezing if the surface is at -15 C for 60 days?

Problem 2. Find first two positive values of β that solve this equation for $L=4.2\text{m}$.

$$1 + \cosh(\beta L) \cos(\beta L) = 0$$

For $EI=21000 \text{ N}\cdot\text{m}^2$ and $\rho=0.53 \text{ kg/m}$, calculate the frequencies from

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho}}$$

Problem 3. The following function is linear in the parameters a_1 and a_2 .

$$y(x) = a_1 + a_2 \ln x$$

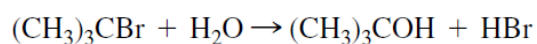
Use least-squares regression with the following data to estimate the values of a_1 and a_2 . Use the curve t to estimate the values of y at $x=2.5$ and at $x=11$.

x	1	2	3	4	5	6	7	8	9	10
y	10	14	16	18	19	20	21	22	23	23

Problem 4. Chemists and engineers must be able to predict the changes in chemical concentration in a reaction. A model used many single-reactant processes is

$$\text{Rate of change of concentration} = -kC^n$$

where C is the chemical concentration and k is the rate constant. The following data describe the reaction



$$\frac{dC}{dt} = -kC$$

Use these data to obtain a least-squares fit to estimate the value of k

Time t (h)	C (mol of $(\text{CH}_3)_3\text{CBr/L}$)
0	0.1039
3.15	0.0896
6.20	0.0776
10.0	0.0639
18.3	0.0353
30.8	0.0207
43.8	0.0101

Problem 5. The following represents pressure samples, in pounds per square inch (psi), taken in a fuel line once every second for 10 sec.

Time (sec)	Pressure (psi)	Time (sec)	Pressure (psi)
1	26.1	6	30.6
2	27.0	7	31.1
3	28.2	8	31.3
4	29.0	9	31.0
5	29.8	10	30.5

- Fit a first-degree polynomial, a second-degree polynomial, and a third-degree polynomial to these data. Plot the curve fits along with the data points
- Use the results from part a to predict the pressure at $t = 11$ sec. Explain which curve fit gives the most reliable prediction. Consider the coefficients of determination and the residuals for each fit in making your decision.