EE411 · Information Theory and Coding

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Assignment 9

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Since we have

$$\sum_{\forall C} = \sum_{\forall x_1^n} \sum_{C:x_C^n(1)=x_1^n}$$

$$Pr[C] = \prod_{i=1}^{2^{nR}} Pr[x_C^n(i)]$$

and

$$\sum_{C:x_C^n(1)=x_1^n} \prod_{i=2}^{2^{nR}} Pr[x_C^n(i)] = \sum_{\forall x_2^n} Pr[x_C^n(2)] \sum_{\forall x_3^n} Pr[x_C^n(3)] \dots \sum_{\forall x_{2^{nR}}^n} Pr[x_C^n(2^{nR})] = 1$$

Then

$$\begin{aligned}
\sum_{\forall C} Pr[C]Pr[e_i(C)|w=1] &= \sum_{\forall x_1^n} \sum_{C:x_C^n(1)=x_1^n} \prod_{i=1}^{2^{nR}} Pr[x_C^n(i)]Pr[e_i(C)|w=1] \\
&= \sum_{\forall x_1^n} Pr[x_C^n(1)]Pr[e_i(C)|w=1] \sum_{C:x_C^n(1)=x_1^n} \prod_{i=2}^{2^{nR}} Pr[x_C^n(i)] \\
&= \sum_{\forall x_1^n} Pr[x_C^n(1)]Pr[e_i(C)|w=1] \\
&= \sum_{\forall x_1^n} Pr[x_1^n]Pr[\{(x_1^n, Y_1^n) \in A_\epsilon^{(n)}\}|w=1] \\
&= Pr[\{(X_1^n, Y_1^n) \in A_\epsilon^{(n)}\}|w=1]
\end{aligned}$$

By joint AEP, we have

$$\sum_{\forall C} Pr[C]Pr[e_i(C)|w=1] = Pr[\{(X_1^n, Y_1^n) \in A_\epsilon^{(n)}\}|w=1] \le 2^{-n(I(X;Y)-3\epsilon)}$$