EE411 · Information Theory and Coding

Assignment 10

刘乐奇 / 12011327

December 18, 2023

1.

$$\begin{aligned}
h(X) &= -\int_0^\infty \lambda e^{-\lambda x}(\ln\lambda - \lambda x)dx \\
&= -\lambda\ln\lambda\int_0^\infty e^{-\lambda x}dx + \lambda\int_0^\infty \lambda x e^{-\lambda x}dx \\
&= -\lambda\ln\lambda\cdot\frac{-1}{\lambda}e^{-\lambda x}\Big|_0^\infty - (x+1)e^{-x}\Big|_0^\infty \\
&= -\ln\lambda + 1 \\
&= \log\frac{e}{\lambda} \quad bits
\end{aligned}$$

2.

$$\begin{aligned}
h(X) &= -\int_{-\infty}^\infty \frac{1}{2}\lambda e^{-\lambda|x|}(-\ln 2 + \ln\lambda - \lambda|x|)dx \\
&= \frac{1}{2}\lambda(\ln 2 - \ln\lambda)\int_{-\infty}^\infty e^{-\lambda|x|}dx + \frac{1}{2}\lambda\int_{-\infty}^\infty \lambda|x|e^{-\lambda|x|}dx \\
&= \lambda(\ln 2 - \ln\lambda)\cdot\frac{-1}{\lambda}e^{-\lambda x}\Big|_0^\infty - (x+1)e^{-x}\Big|_0^\infty \\
&= \ln 2 - \ln\lambda + 1 \\
&= \log\frac{2e}{\lambda} \quad bits
\end{aligned}$$

3. The sum of two normal distribution variables is also normal, and $X_1 + X_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Then its differential entropy is

$$h(X) = \frac{1}{2}\log 2\pi e(\sigma_1^2 + \sigma_2^2) \quad bits$$