EE411 · Information Theory and Coding

Assignment 6

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1 5.14

1. Following is the binary Huffman codes. Then we have

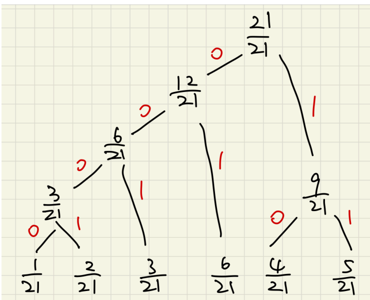

Figure 1: Binary Huffman Tree

X	Pr	C(X)
1	$\frac{1}{21}$	0000
2	$\frac{2}{21}$	0001
3	$\frac{3}{21}$	001
4	$\frac{4}{21}$	10
5	$\frac{5}{21}$	11
6	$\frac{6}{21}$	01

Figure 2: Binary Huffman Code

$$L = \sum p_i l_i = \frac{1}{21} \cdot 4 + \frac{2}{21} \cdot 4 + \frac{3}{21} \cdot 3 + \frac{4}{21} \cdot 2 + \frac{5}{21} \cdot 2 + \frac{6}{21} \cdot 2 = \frac{48}{21} \, bits$$

2. We need to add a dummy symbol with possibility 0. Following is the ternary Huffman codes. Then

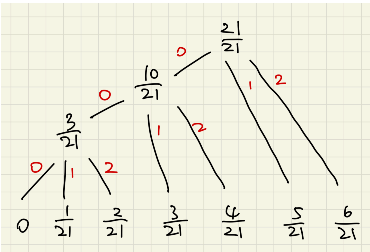

Figure 3: Ternary Huffman Tree

X	Pr	C(X)
1	$\frac{1}{21}$	001
2	$\frac{2}{21}$	002
3	$\frac{3}{21}$	01
4	$\frac{4}{21}$	02
5	$\frac{5}{21}$	1
6	$\frac{6}{21}$	2

Figure 4: Ternary Huffman Code

we have

$$L = \sum p_i l_i = \frac{1}{21} \cdot 3 + \frac{2}{21} \cdot 3 + \frac{3}{21} \cdot 2 + \frac{4}{21} \cdot 2 + \frac{5}{21} \cdot 1 + \frac{6}{21} \cdot 1 = \frac{34}{21} \, ternaryBits$$

2 5.15

1. Following is the binary Huffman code. Then we have

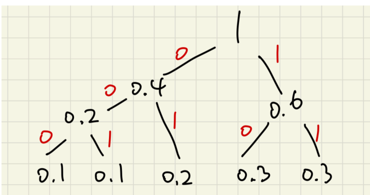

Figure 5: Binary Huffman Tree

X	Pr	C(X)
1	0.3	11
2	0.3	10
3	0.2	01
4	0.1	001
5	0.1	000

Figure 6: Binary Huffman Code

$$L = \sum p_i l_i = 0.3 \cdot 2 + 0.3 \cdot 2 + 0.2 \cdot 2 + 0.1 \cdot 3 + 0.1 \cdot 3 = 2.2 \, bits$$

2. To achieve the request, we need to set $p'_i = 2^{-l_i}$ for $i = 1, 2, 3, 4, 5$, i.e., the probability distribution $\mathbf{p}' = (0.25, 0.25, 0.25, 0.125, 0.125)$.

3 5.16

1. Following is the binary Huffman code.

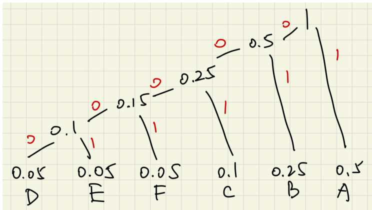

Figure 7: Binary Huffman Tree

X	Pr	C(X)
A	0.5	1
B	0.25	01
C	0.1	001
D	0.05	00000
E	0.05	00001
F	0.05	0001

Figure 8: Binary Huffman Code

The average length is

$$L = \sum p_i l_i = 0.5 \cdot 1 + 0.25 \cdot 2 + 0.1 \cdot 3 + 0.05 \cdot 5 + 0.05 \cdot 5 + 0.05 \cdot 4 = 2\,bits$$

2. We need to add a dummy symbol with possibility 0. Following is the quaternary Huffman code.

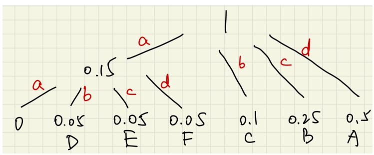

Figure 9: Quaternary Huffman Tree

X	Pr	C(X)
A	0.5	d
B	0.25	c
C	0.1	b
D	0.05	ab
E	0.05	ac
F	0.05	ad

Figure 10: Quaternary Huffman Code

The average length is

$$L = \sum p_i l_i = 0.5 \cdot 1 + 0.25 \cdot 1 + 0.1 \cdot 1 + 0.05 \cdot 2 + 0.05 \cdot 2 + 0.05 \cdot 2 = 1.15\,quaternaryBits$$

3. By this process, we have

X	Pr	$C_4(X)$	$C_2(X)$
A	0.5	d	11
B	0.25	c	10
C	0.1	b	01
D	0.05	ab	0001
E	0.05	ac	0010
F	0.05	ad	0011

Table 1: Quaternary Huffman Code to Binary

The average length is

$$L = \sum p_i l_i = 0.5 \cdot 2 + 0.25 \cdot 2 + 0.1 \cdot 2 + 0.05 \cdot 4 + 0.05 \cdot 4 + 0.05 \cdot 4 = 2.3\,bits$$

4. Since binary Huffman code is an optimal code, its length is shortest, i.e., $L_H \leq L_{QB}$. By the theorem of Shannon Codes, we have

$$H_4(X) \leq L_Q < H_4(X) + 1$$

Since the code word of each symbol in the quaternary code is converted into 2 bits, we have $L_{QB} = 2L_Q$. By the property of entropy, we have $H_2(X) = 2H_4(X)$. So we have

$$H_2(X) \leq L_{QB} < H_2(X) + 2$$

Since $H_2(X) \leq L_H$, we have $L_{QB} < L_H + 2$. Therefore, we have

$$L_H \leq L_{QB} < L_H + 2$$

5. For a random variable that has 4 values with equal probability. Then its quaternary Huffman code is 1 quaternary symbol for each source symbol. The average length $L_{QB} = 2$ bits. Besides, the binary Huffman code also has an average length $L_H = 2$ bits, i.e., $L_H = L_{QB}$.

6. Suppose there is a binary Huffman code for a random variable X and its average length is L_H. Append a 0 to each of the codewords of odd length, then convert all the codewords using the mapping $00 \rightarrow a, 01 \rightarrow b, 10 \rightarrow c, 11 \rightarrow d$. Let L_{BQ} denote the average length of this codeword. As a consequence, we have

$$L_{BQ} = \frac{L_H + \sum_{l_i \text{ } is \text{ } odd} 1 \cdot p_i}{2} < \frac{L_H + 1}{2}$$

And since $L_Q \leq L_{BQ}$, we have

$$L_{QB} = 2L_Q \leq 2L_{BQ} < L_H + 1$$

We can take the example as $X = \{x_1, x_2\}$, then $L_H = 1$ and $L_{QB} = 2$.