EE411 · Information Theory and Coding

Assignment 5

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October 31, 2023

1 1

1. Since $l_i = \lceil \log \frac{1}{p_i} \rceil$, we have

$$\log \frac{1}{p_i} \le l_i < \log \frac{1}{p_i} + 1$$

then we have

$$\sum_i p_i \log \frac{1}{p_i} \le \sum_i p_i l_i < \sum_i p_i \log \frac{1}{p_i} + 1$$

i.e.

$$H(X) \le L < H(X) + 1$$

Since $l_i = \lceil \log \frac{1}{p_i} \rceil$, we have

$$2^{-l_i} \le p_i < 2^{-(l_i - 1)}$$

i.e.

$$\sum_{k=1}^{i-1} 2^{-l_k} \le F_i = \sum_{k=1}^{i-1} p_k < \sum_{k=1}^{i-1} 2^{-(l_k - 1)}$$

So we have

$$F_j - F_i \ge F_{i+1} - F_i > 2^{-l_i} - \sum_{k=1}^{i-1} 2^{-l_k} \ge 2^{-l_i}$$

where $j > i$. Thus, the codeword of j differs from the codeword of i for at least one bit in the first l_i. Therefore, the code is prefix-free.

2. The code is constructed as below.

X	Pr	codeword
1	0.5	0
2	0.25	10
3	0.125	110
4	0.125	111

2 2

To find a good lower bound of D, the codeword must be prefix code. Thus, by Kraft's inequality, we have

$$\sum_{i=1}^{6} D^{-l_i} \le 1$$

When $D = 2$, $\sum_{i=1}^{6} D^{-l_i} = \frac{7}{4} > 1$; when $D = 3$, $\sum_{i=1}^{6} D^{-l_i} = \frac{26}{27} < 1$. Therefore, a good lower bound on D is 3.