**5.14** *Huffman code*. Find the **(a)** *binary* and **(b)** *ternary* Huffman codes for the random variable $X$ with probabilities

$$p = \left(\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}\right).$$

**(c)** Calculate $L = \sum p_i l_i$ in each case.

**5.15** *Huffman codes*

**(a)** Construct a binary Huffman code for the following distribution on five symbols: $\mathbf{p} = (0.3, 0.3, 0.2, 0.1, 0.1)$. What is the average length of this code?

**(b)** Construct a probability distribution $\mathbf{p}'$ on five symbols for which the code that you constructed in part (a) has an average length (under $\mathbf{p}'$) equal to its entropy $H(\mathbf{p}')$.

**5.16** *Huffman codes*. Consider a random variable $X$ that takes six values $\{A, B, C, D, E, F\}$ with probabilities 0.5, 0.25, 0.1, 0.05, 0.05, and 0.05, respectively.

**(a)** Construct a binary Huffman code for this random variable. What is its average length?

**(b)** Construct a quaternary Huffman code for this random variable [i.e., a code over an alphabet of four symbols (call them $a$, $b$, $c$ and $d$)]. What is the average length of this code?

**(c)** One way to construct a binary code for the random variable is to start with a quaternary code and convert the symbols into binary using the mapping $a \to 00$, $b \to 01$, $c \to 10$, and $d \to 11$. What is the average length of the binary code for the random variable above constructed by this process?

**(d)** For any random variable $X$, let $L_H$ be the average length of the binary Huffman code for the random variable, and let $L_{QB}$ be the average length code constructed by first building a quaternary Huffman code and converting it to binary. Show that

$$L_H \le L_{QB} < L_H + 2. \qquad (5.146)$$

**(e)** The lower bound in the example is tight. Give an example where the code constructed by converting an optimal quaternary code is also the optimal binary code.

**(f)** The upper bound (i.e., $L_{QB} < L_H + 2$) is not tight. In fact, a better bound is $L_{QB} \le L_H + 1$. Prove this bound, and provide an example where this bound is tight.