Department of
Computer Science and Engineering
计算机科学与工程系

CSE5014 CRYPTOGRAPHY AND NETWORK SECURITY

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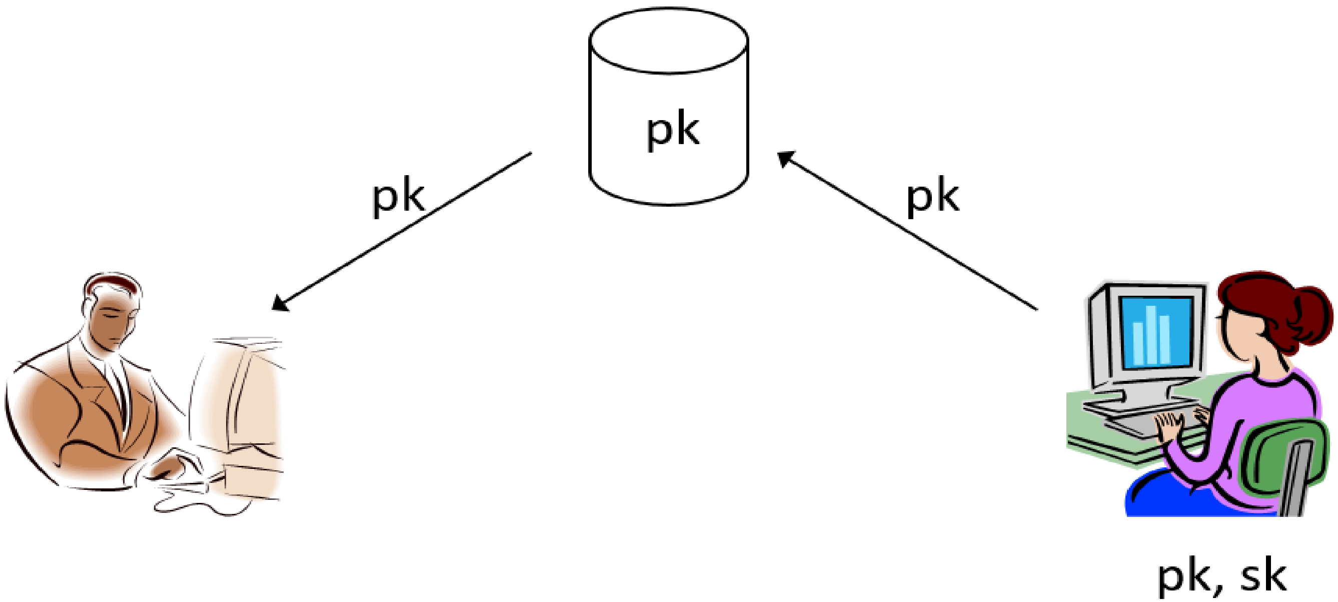
The public-key setting

- A party generates a *pair* of keys: a *public* key *pk* and a *private* key *sk*
  - Public key is widely disseminated
  - Private key is kept secret, and shared with no one
- Private key used by the party who generated it; public key used by everyone else
  - Also called *asymmetric cryptography*
- Security must hold even if the attacker knows *pk*

Public-key distribution I, II



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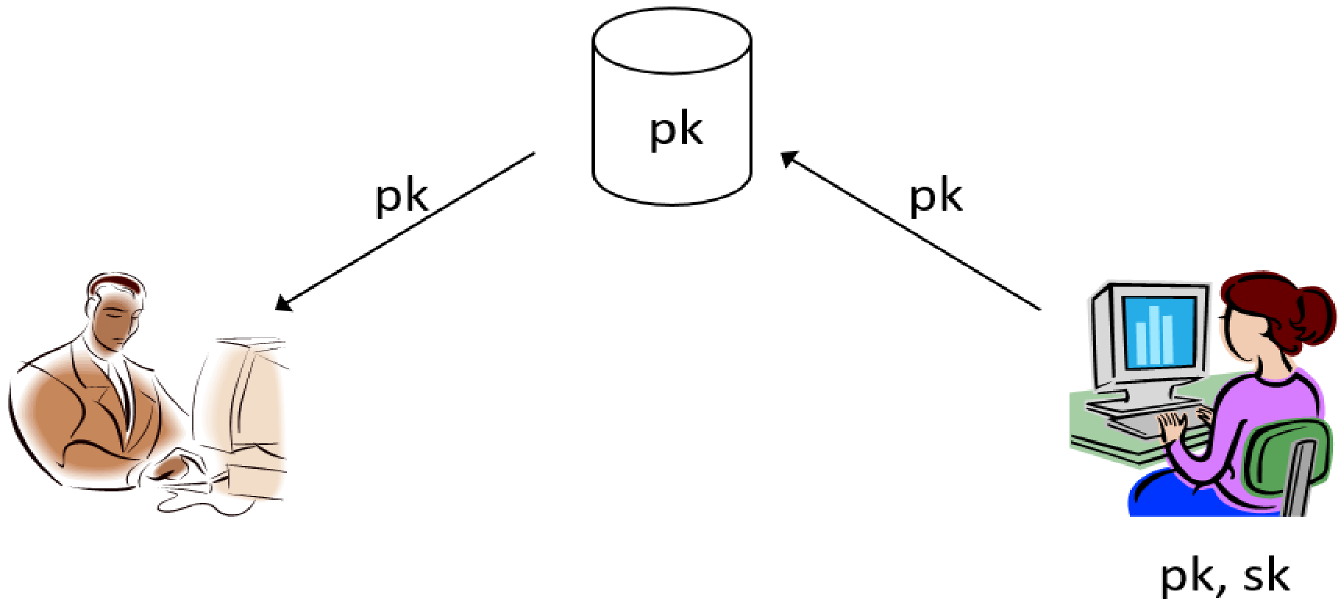

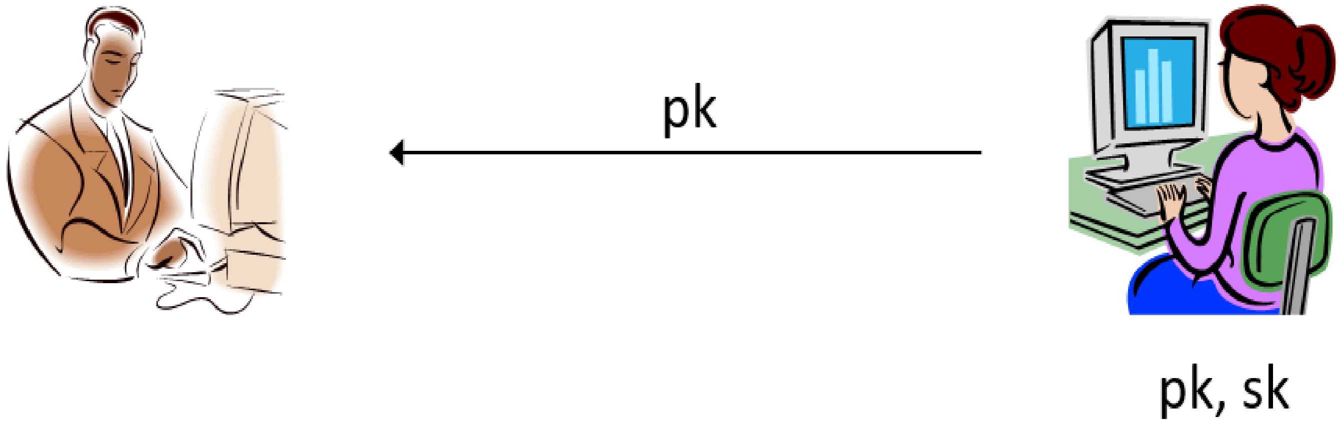

Public-key distribution

- Previous figures (implicitly) assume parties are able to obtain correct cipies of each others' public keys
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	Private-key setting	Public-key setting
Secrecy	Private-key encryption	Public-key encryption
Integrity	Message authentication codes	Digital signature schemes

Addressing drawbacks of private-key crypto

- Key distribution
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- Key management in large systems of N users
  - Each user stores 1 private key and $N - 1$ public keys; only N keys overall
  - Public keys can be stored in a central directory
- Applicability in "open systems"
  - Even parties who have no prior relationship can find each others' public keys and use them

Why study private-key crypto?

- Private-key cryptography is more suitable for certain applications
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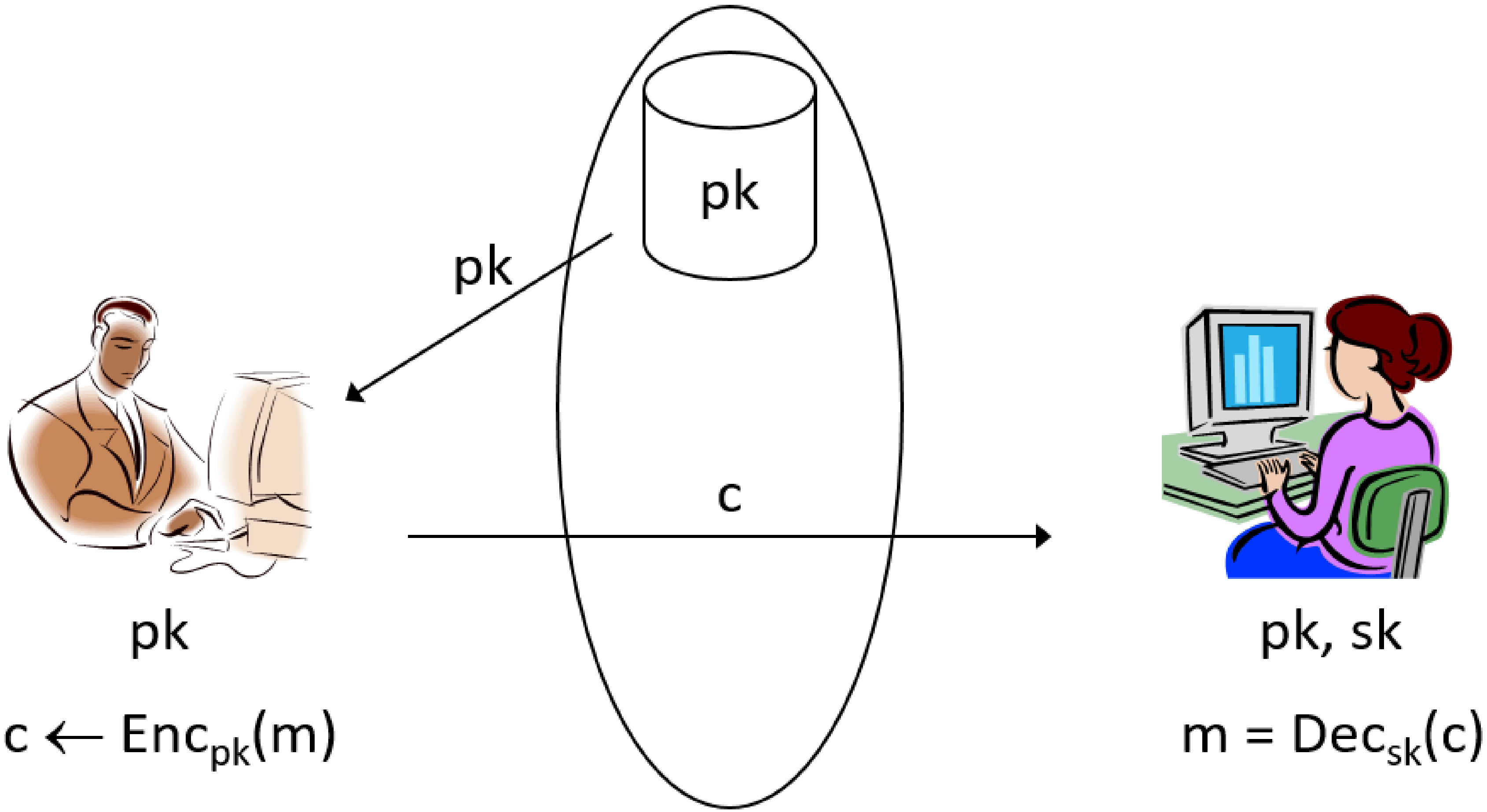
- Public-key crypto is roughly $2 - 3$ orders of magnitude *slower* than private-key crypto
  - If private-key crypto is an option, use it!
  - Private-key crypto is used for *efficiency* even in the public-key setting

Public-key encryption



Public-key encryption

- **Theorem 12.2** A *public-key encryption* scheme is composed of three PPT algorithms:
  - *Gen*: *key-generation algorithm* that on input 1^n outputs pk, sk
  - *Enc*: *encryption algorithm* that on input pk and a message m outputs a ciphertext c
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  For all m and pk, sk output by Gen,

$$Dec_{sk}(Enc_{pk}(m)) = m$$

CPA-security

- Fix a public-key encryption scheme Π and an adversary A
- Define experiment $PubK\text{-}CPA_{A,\Pi}(n)$:
  – Run $Gen(1^n)$ to get keys pk, sk
  – Give pk to A, who outputs m_0, m_1 of same length
  – Choose uniform $b \in \{0, 1\}$ and compute the ciphertext $c \leftarrow Enc_{pk}(m_b)$; give c to A
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- **Theorem 12.3** Public-key encryption scheme Π is *CPA-secure* if for all PPT adversaries A:

$$\Pr[PubK\text{-}CPA_{A,\Pi}(n) = 1] \leq 1/2 + negl(n)$$

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$\Rightarrow$ No *perfectly secret* public-key encryption

$\Rightarrow$ No *deterministic* public-key encryption can be CPA-secure

$\Rightarrow$ CPA-security implies security for encryption multiple messages as in the private-key case

Perfectly secret public-key encryption

- **Definition 12.4** A public-key encryption scheme is *perfectly secret* if for all public keys pk, all messages m_0, m_1, all ciphertexts c, and all algorithms A, we have:
  $\Pr[A(pk, c) = 0 | c \leftarrow Enc_{pk}(m_0)] = \Pr[A(pk, c) = 0 | c \leftarrow Enc_{pk}(m_1)]$

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Theorem 12.5 No public-key encryption scheme is *prefectly secret*.

Proof.

Recall: plain RSA

- Choose random, equal-length primes p, q
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- *RSA assumption*: given N, e only, it is *hard* to compute the e^{th} root of a uniform $c \in \mathbb{Z}_N^*$

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- *RSA assumption* only refers to hardness of computing the e^{th} roots *in its entirety*
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- Plain RSA should *never* be used!

PKCS #1 v1.5

- Standard issued by RSA labs in 1993
- Idea: add *random padding*
  - To encrypt m, choose random r
  - $c = [(r|m)^e \bmod N]$

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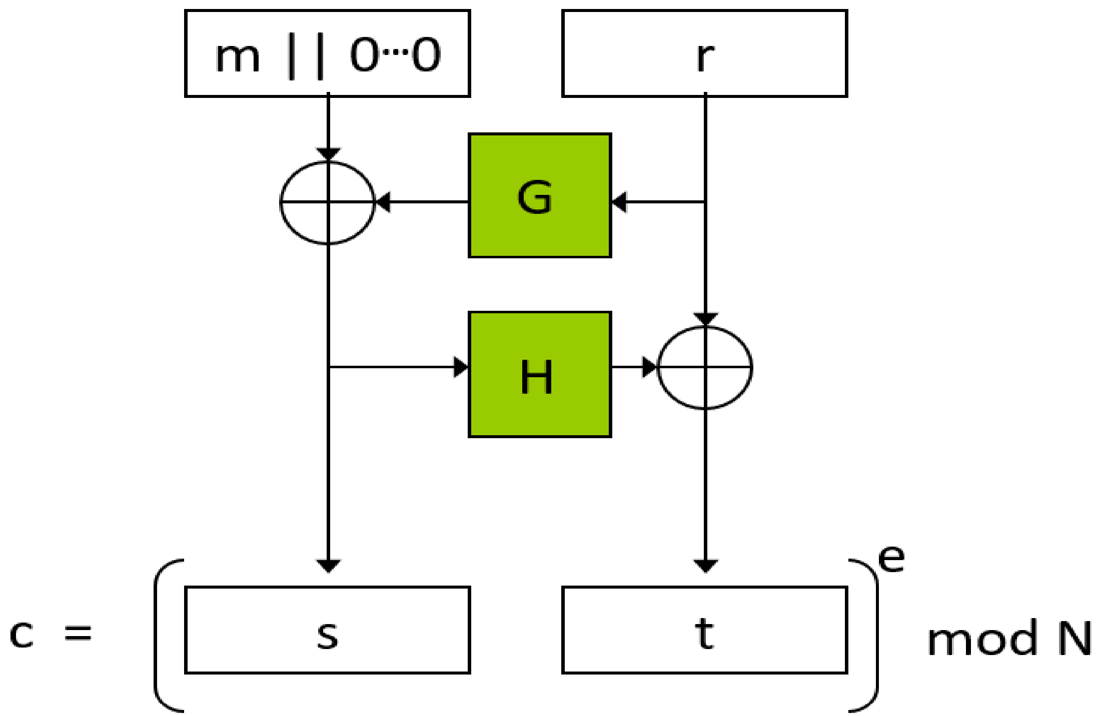
- Idea: add *random padding*
  - To encrypt m, choose random r
  - $c = [(r|m)^e \bmod N]$
- Issues:
  - No proof of *CPA-security* (unless m is very short)
  - *Chosen-plaintext attacks* known if r is too short
  - *Chosen-ciphertext attacks* possible

PKCS #1 v2.0

- *Optimal asymmetric encryption padding*
  - (OAEP) applied to message first

- This padding introduces *redundancy*, so that not every $c \in \mathbb{Z}_N^*$ is a valid ciphertext
  - Need to check for proper format upon decryption
  - Return error if not properly formatted

OAEP



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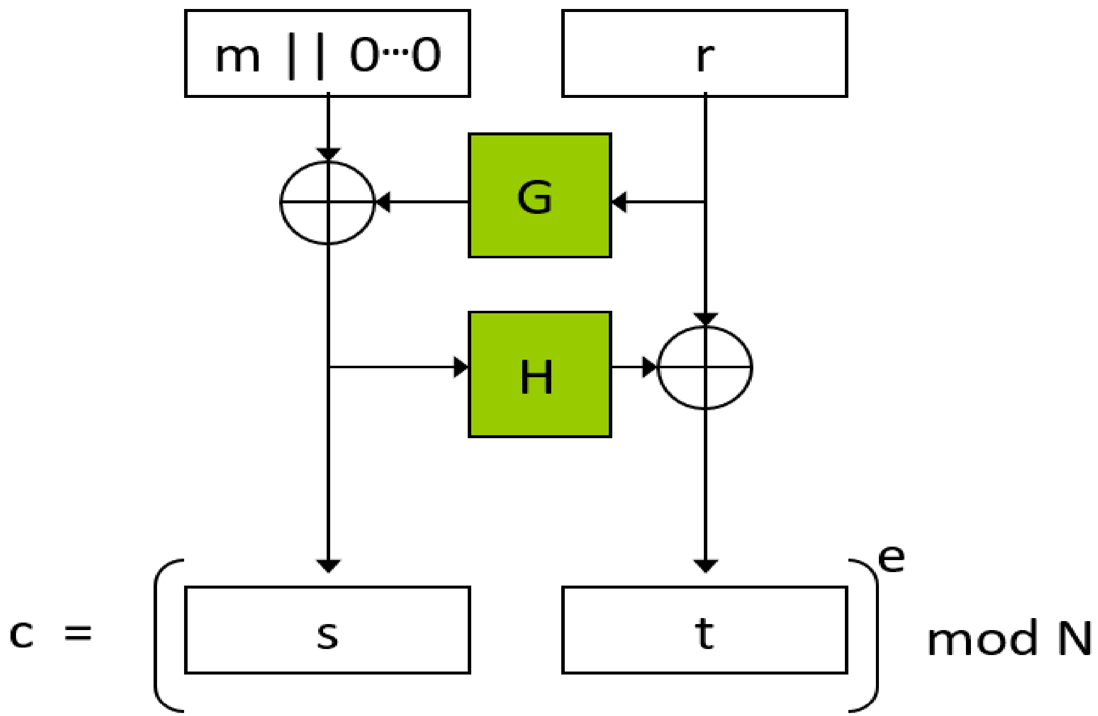

- RSA-OAEP can be proven *CCA-secure* under the *RSA assumption*, if G and H are modeled as *random oracles*

Chosen-ciphertext attacks

pk

$c \leftarrow \mathsf{Enc}_{pk}(m)$

c

pk, sk

c'

m'

Chosen-ciphertext attacks

- *Chosen-ciphertext attacks* are arguably even a greater concern in the public-key setting
  - Attacker might be a legitimate sender
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- Related concern: *malleability*
  - I.e., given a ciphertext c that is the encryption of an unknown message m, might be possible to produce ciphertext c' that decrypts to a related message m'
  - This is also undesirable in the public-key setting

Diffie-Hellman key exchange

$$\xrightarrow{\quad G,\ q,\ g,\ h_1 \quad}$$

$$\xleftarrow{\quad h_2 \quad}$$

$(G, q, g) \leftarrow \mathcal{G}(1^n)$

$x \leftarrow \mathbb{Z}_q$

$h_1 = g^x$

$y \leftarrow \mathbb{Z}_q$

$h_2 = g^y$

$k = (h_1)^y$

El Gamal encryption

$$G, q, g, h_1 \longrightarrow$$

$$h_2 \longleftarrow$$

$$c_2 = k \cdot m \longleftarrow$$

$$(G, q, g) \leftarrow \mathcal{G}(1^n)$$
$$x \leftarrow \mathbb{Z}_q$$
$$h_1 = g^x$$

$$k = (h_2)^x$$
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El Gamal encryption

Public key

G, q, g, h_1 $\longrightarrow$

$h_2, h_1^y \cdot m$ $\longleftarrow$

$(G, q, g) \leftarrow \mathcal{G}(1^n)$
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El Gamal encryption

- $Gen(1^n)$
  – Run $\mathcal{G}(1^n)$ to obtain G, q, g. Choose uniform $x \in \mathbb{Z}_q$. The *public key* is (G, q, g, g^x) and the *private key* is x

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- $Dec_{sk}(c_1, c_2)$
  - Output c_2 / c_1^x

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- *Dlog assumption* alone is not enough here

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  we have $\quad c_1, c_2' = g^y, h^y \cdot (\alpha m)$
  – I.e., encryption of m becomes an encryption of αm!

Chosen-ciphertext attacks security

- Use *key derivation* coupled with *CCA-secure* private-key encryption scheme
  - I.e., ciphertext is

$$g^y, Enc'_k(m),$$

where $k = H(h^y)$ and Enc' is a CCA-secure scheme

- Can be proved *CCA-secure* under appropriate assumptions, if H is modeled as a random oracle.

CPA-secure public key encryption

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  - To encrypt $x \in \{0,1\}^n$, choose $r \leftarrow_R \{0,1\}^n$ and compute $f(r), H(r) \oplus x$.
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- **Theorem 5.1** (CPA security from PRFs)
  Suppose that F is a length-preserving, keyed PRF, then the following is a *CPA-secure encryption scheme*:

$$Enc_k(m) = \langle r, F_k(r) \oplus m \rangle$$
$$Dec_k(c_1, c_2) = c_2 \oplus F_k(c_1)$$

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  Proof. For public key encryption, CPA security means that an adversary A that gets as input the encryption key $f(\cdot)$ cannot distinguish $Enc(x_1)$ and $Enc(x_2)$ for every x_1, x_2, since encryption is public. In the random oracle model, A has access to the random oracle $H(\cdot)$.

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  However, in this experiment, the only infomation A gets about r^* is $f(r^*)$. Thus, if it queries $H(\cdot)$ the value r^*, then it inverted the *trapdoor permutation*, which is almost impossible!

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  However, A gets no information about x^* and will not be able to guess if it is equal to x_1 or x_2 with probability greater than $1/2$.

CCA Secure Public Key Encryption

- **Construction 13.3**: Construct an encryption scheme (using two independent random oracles) as follows:
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  At each step i of the attack, for every string $w \in \{0,1\}^n$, we define ${H'}_i^{-1}(w)$ as: if the oracle H was queried before with x, r and returned w, then ${H'}_i^{-1}(w) = (x, r)$; otherwise, ${H'}_i^{-1}(w) = \perp$.

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Observation: a pair x, r completely determines a ciphertext y, z, w, and y, z completely determine x, r.

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  The only difference happens if A managed to ask the oracle a query y, z, w satisfying the following:
  - $w \neq w^*$.
  - w was not returned as the answer of any previous query x, r to $H'(\cdot)$ by A.
  - If we let x, r be the values determined by y, z, then $H'(x, r) = w$. However, since (x, r) was not asked before, the probability that this happens is only 2^{-n}.

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  Basically A has no use for the decryption box and hence it would be sufficient to prove that the scheme is just *CPA-secure*.

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Review

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  RSA trapdoor function, signature

Next Lecture

- digital signature ...