



# CSE5014 CRYPTOGRAPHY AND NETWORK SECURITY

Dr. QI WANG

Department of Computer Science and Engineering

Office: Room413, CoE South Tower

Email: [wangqi@sustech.edu.cn](mailto:wangqi@sustech.edu.cn)

- A *PRG* is an *efficient*, *deterministic* algorithm that expands a *short, uniform seed* into a *longer, pseudorandom* output

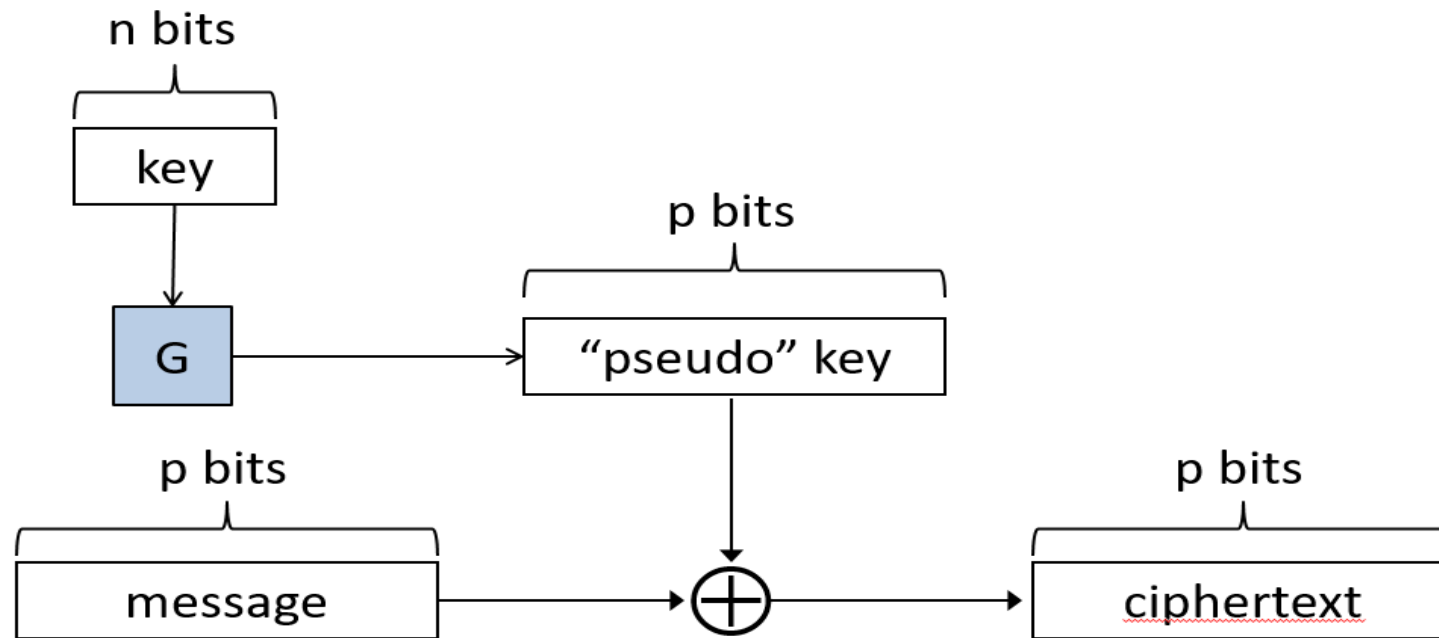
Let  $G$  be a deterministic, poly-time algorithm that is *expanding*, i.e.,  $|G(x)| = p(|x|) > |x|$ .

- For all *efficient* distinguishers  $A$ , there is a *negligible* function  $\epsilon$  such that

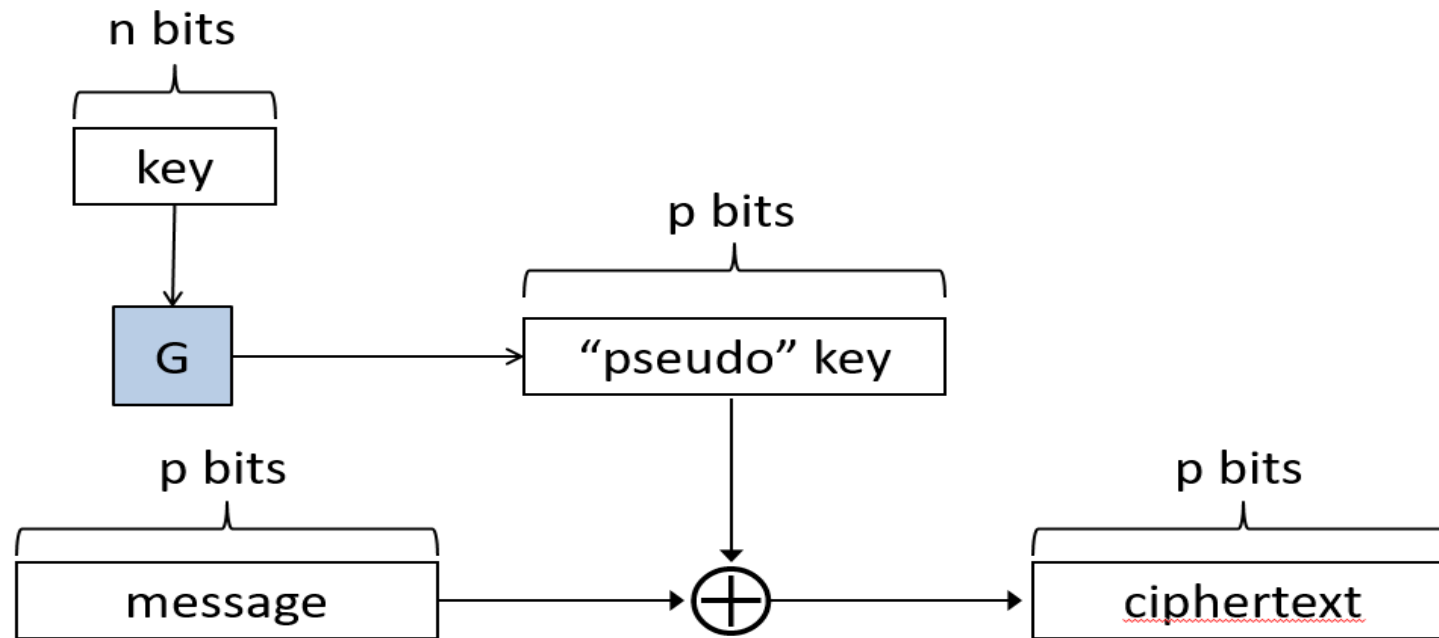
$$|\Pr_{x \leftarrow U_n}[A(G(x)) = 1] - \Pr_{y \leftarrow U_{p(n)}}[A(y) = 1]| \leq \epsilon(n)$$

**No** efficient  $A$  can distinguish whether it is given  $G(x)$  (for uniform  $x$ ) or a uniform string  $y$ !

# Pseudo one-time pad



# Pseudo one-time pad



- **Theorem 3.3** If  $G$  is a pseudorandom generator (PRG), then the pseudo one-time pad (pseudo-OTP)  $\Pi$  is *EAV-secure* (i.e., *computationally secure*)

# CPA-security

## ■ Fix $\Pi$ , $A$

Define a randomized experiment  $\text{PrivKCPA}_{A,\Pi}(n)$ :

1.  $k \leftarrow \text{Gen}(1^n)$
2.  $A(1^n)$  **interacts** with an **encryption oracle**  $\text{Enc}_k(\cdot)$ , and then outputs  $m_0, m_1$  of the same length
3.  $b \leftarrow \{0, 1\}$ ,  $c \leftarrow \text{Enc}_k(m_b)$ , give  $c$  to  $A$
4.  $A$  can **continue** to interact with  $\text{Enc}_k(\cdot)$
5.  $A$  outputs  $b'$ ;  $A$  succeeds if  $b = b'$ , and experiment evaluates to 1 in this case

# CPA-security

## ■ Fix $\Pi$ , $A$

Define a randomized experiment  $\text{PrivKCPA}_{A,\Pi}(n)$ :

1.  $k \leftarrow \text{Gen}(1^n)$
2.  $A(1^n)$  **interacts** with an **encryption oracle**  $\text{Enc}_k(\cdot)$ , and then outputs  $m_0, m_1$  of the same length
3.  $b \leftarrow \{0, 1\}$ ,  $c \leftarrow \text{Enc}_k(m_b)$ , give  $c$  to  $A$
4.  $A$  can **continue** to interact with  $\text{Enc}_k(\cdot)$
5.  $A$  outputs  $b'$ ;  $A$  succeeds if  $b = b'$ , and experiment evaluates to 1 in this case

**Definition 4.1**  $\Pi$  is **secure against chosen-plaintext attacks** (**CPA-secure**) if for **all PPT** attackers  $A$ , there is a **negligible** function  $\epsilon$  such that

$$\Pr[\text{PrivKCPA}_{A,\Pi}(n) = 1] \leq 1/2 + \epsilon(n)$$

- The number of functions in  $Func_n$  is  $2^{n \cdot 2^n}$



- The number of functions in  $Func_n$  is  $2^{n \cdot 2^n}$
- $\{F_k\}_{k \in \{0,1\}^n}$  is a subset of  $Func_n$ 
  - The number of functions in  $\{F_k\}_{k \in \{0,1\}^n}$  is **at most  $2^n$**

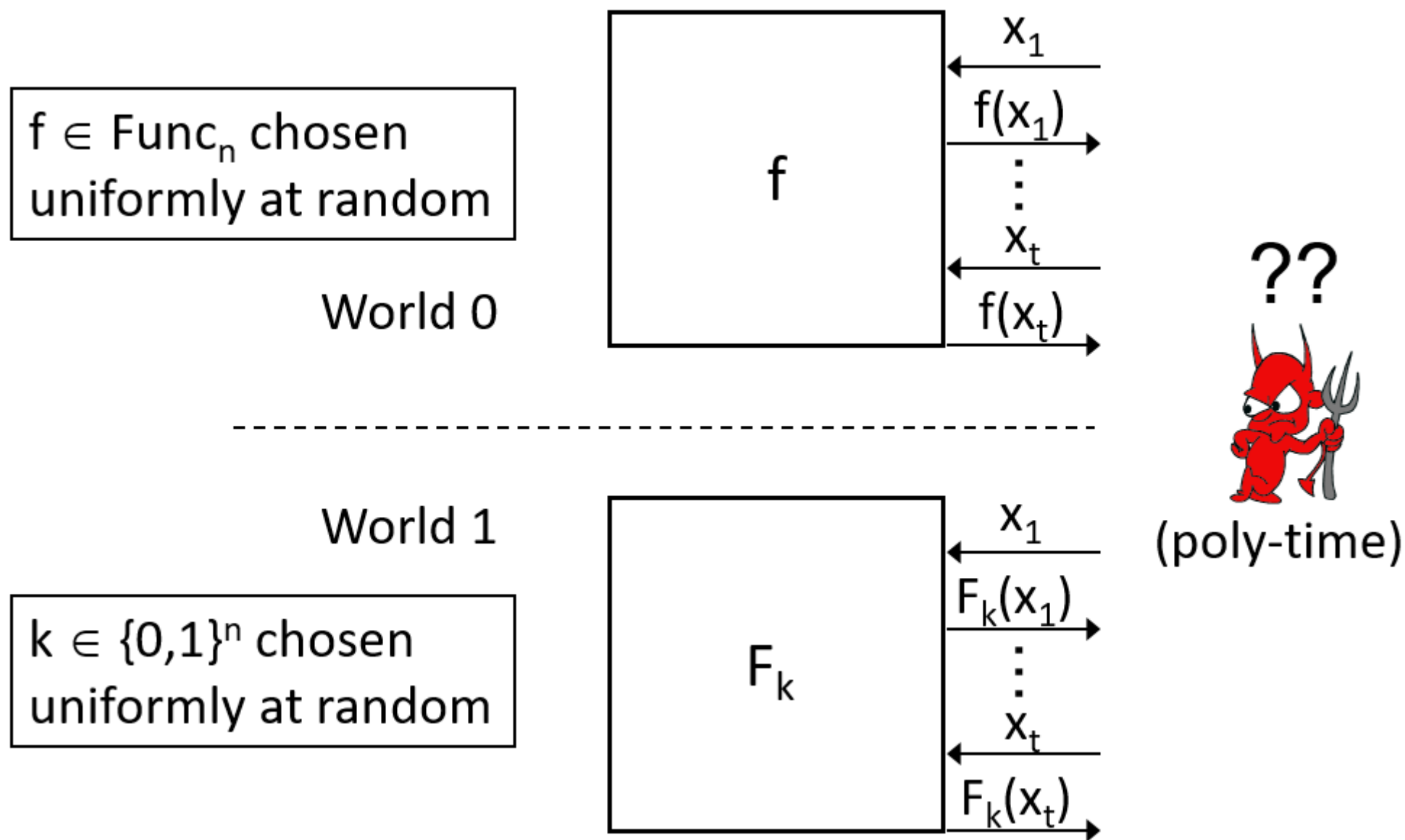


- The number of functions in  $Func_n$  is  $2^{n \cdot 2^n}$
- $\{F_k\}_{k \in \{0,1\}^n}$  is a subset of  $Func_n$ 
  - The number of functions in  $\{F_k\}_{k \in \{0,1\}^n}$  is **at most  $2^n$**

**Definition 4.2**  $F$  is a *pseudorandom function* if  $F_k$ , for uniform  $k \in \{0,1\}^n$  is **indistinguishable** from a uniform function  $f \in Func_n$ .  
Formally, for **all** poly-time distinguishers  $D$ :

$$|\Pr_{k \leftarrow \{0,1\}^n}[D^{F_k(\cdot)}(1^n) = 1] - \Pr_{f \leftarrow Func_n}[D^{f(\cdot)}(1^n) = 1]| \leq \epsilon(n)$$

# PRFs



# Pseudorandom permutations (PRPs)

- Let  $f \in \text{Func}_n$



# Pseudorandom permutations (PRPs)

- Let  $f \in \text{Func}_n$   
 $f$  is a *permutation* if it is a bijection
  - This means that the *inverse*  $f^{-1}$  exists

# Pseudorandom permutations (PRPs)

- Let  $f \in \text{Func}_n$   
 $f$  is a *permutation* if it is a bijection
  - This means that the *inverse*  $f^{-1}$  exists
- Let  $\text{Perm}_n \subset \text{Func}_n$  be the set of permutations
  - What is  $|\text{Perm}_n|$ ?

# Pseudorandom permutations

- Let  $F$  be a length-preserving, keyed function



# Pseudorandom permutations

- Let  $F$  be a length-preserving, keyed function
- $F$  is a *keyed permutation* if
  - $F_k$  is a permutation for every  $k$
  - $F_k^{-1}$  is *efficiently computable* (where  $F_k^{-1}(F_k(x)) = x$ )



# Pseudorandom permutations

- Let  $F$  be a length-preserving, keyed function
- $F$  is a *keyed permutation* if
  - $F_k$  is a permutation for every  $k$
  - $F_k^{-1}$  is *efficiently computable* (where  $F_k^{-1}(F_k(x)) = x$ )
- **Definition 4.3**  $F$  is a *pseudorandom permutation* if  $F_k$ , for **uniform** key  $k \in \{0, 1\}^n$ , is **indistinguishable** from a uniform permutation  $f \in \text{Perm}_n$



# Pseudorandom permutations

- Let  $F$  be a length-preserving, keyed function
- $F$  is a *keyed permutation* if
  - $F_k$  is a permutation for every  $k$
  - $F_k^{-1}$  is *efficiently computable* (where  $F_k^{-1}(F_k(x)) = x$ )
- **Definition 4.3**  $F$  is a *pseudorandom permutation* if  $F_k$ , for **uniform** key  $k \in \{0, 1\}^n$ , is **indistinguishable** from a uniform permutation  $f \in \text{Perm}_n$
- For large enough  $n$ , a random permutation is **indistinguishable** from a random function.
  - In practice, PRPs are also good PRFs

# PRFs vs. PRGs

- PRF  $F$  immediately implies a PRG  $G$ :
  - Define  $G(k) = F_k(0 \dots 0) | F_k(0 \dots 1)$
  - I.e.,  $G(k) = F_k(\langle 0 \rangle) | F_k(\langle 1 \rangle) | F_k(\langle 2 \rangle) | \dots$ ,  
where  $\langle i \rangle$  denotes the  $n$ -bit encoding of  $i$



# PRFs vs. PRGs

- PRF  $F$  immediately implies a PRG  $G$ :
  - Define  $G(k) = F_k(0 \dots 0) | F_k(0 \dots 1)$
  - I.e.,  $G(k) = F_k(\langle 0 \rangle) | F_k(\langle 1 \rangle) | F_k(\langle 2 \rangle) | \dots$ ,  
where  $\langle i \rangle$  denotes the  $n$ -bit encoding of  $i$
- PRF can be viewed as a PRG with random access to **exponentially** long output
  - The function  $F_k$  can be viewed as the  $n2^n$ -bit string  $F_k(0 \dots 0) | \dots | F_k(1 \dots 1)$

# Do PRFs/PRPs exist?

- They are a stronger primitive than PRGs
  - though can be built from PRGs



# Do PRFs/PRPs exist?

- They are a stronger primitive than PRGs
  - though can be built from PRGs

**Theorem** (Goldreich, Goldwasser, Micali 1984)

If the PRG Axiom is **true**, then there exist PRFs.

## How to Construct Random Functions

ODED GOLDREICH, SHAFI GOLDWASSER,  
AND SILVIO MICALI

*Massachusetts Institute of Technology, Cambridge, Massachusetts*

**Abstract.** A constructive theory of randomness for functions, based on computational complexity, is developed, and a pseudorandom function generator is presented. This generator is a deterministic polynomial-time algorithm that transforms pairs  $(g, r)$ , where  $g$  is *any* one-way function and  $r$  is a random  $k$ -bit string, to polynomial-time computable functions  $f_r: \{1, \dots, 2^k\} \rightarrow \{1, \dots, 2^k\}$ . These  $f_r$ 's cannot be distinguished from *random* functions by any probabilistic polynomial-time algorithm that asks and receives the value of a function at arguments of its choice. The result has applications in cryptography, random constructions, and complexity theory.

Categories and Subject Descriptors: F.0 [Theory of Computation]: General; F.1.1 [Computation by Abstract Devices]: Models of Computation—*computability theory*; G.0 [Mathematics of Computing]: General; G.3 [Mathematics of Computing]: Probability and Statistics—*probabilistic algorithms; random number generation*

General Terms: Algorithms, Security, Theory

Additional Key Words and Phrases: Cryptography, one-way functions, prediction problems, randomness

*I have set up on a Manchester computer a small programme using only 1000 units of storage, whereby the machine supplied with one sixteen figure number replies with another within two seconds. I would defy anyone to learn from these replies sufficient about the programme to be able to predict any replies to untried values.*

A. TURING

# Do PRFs/PRPs exist?

- They are a stronger primitive than PRGs
  - though can be built from PRGs
- In practice, **block ciphers** are used



# Block ciphers

- Block ciphers are practical constructions of *pseudorandom permutations* (PRPs)



# Block ciphers

- Block ciphers are practical constructions of *pseudorandom permutations* (PRPs)

$$F : \{0, 1\}^n \times \{0, 1\}^m \rightarrow \{0, 1\}^m$$

- $n$  = “key length”
- $m$  = “block length”





# Block ciphers

- Block ciphers are practical constructions of *pseudorandom permutations* (PRPs)

$$F : \{0, 1\}^n \times \{0, 1\}^m \rightarrow \{0, 1\}^m$$

- $n$  = “key length”
- $m$  = “block length”

Hard to distinguish  $F_k$  from **uniform**  $f \in \text{Perm}_m$



- *Advanced encryption standard* (AES)
  - Standardized by NIST in 2000 based on a public, worldwide competition lasting over 3 years
  - Block length = 128 bits
  - Key length = 128, 192 or 256 bits



- *Advanced encryption standard* (AES)
  - Standardized by NIST in 2000 based on a public, worldwide competition lasting over 3 years
  - Block length = 128 bits
  - Key length = 128, 192 or 256 bits
- Will discuss details later in the course
  - *Rijndael* named after Vincent Rijmen and Joan Daemen

# History of Block Cipher

- 1972: **NIST** (then **NBS**) called for encryption standard proposals

1976: IBM responded: “**Lucifer**”

NSA tweaked Lucifer to get ***Data Encryption Standard (DES)*** and approved it

The key length is “short”: 56 bits

By late 90's, most commercial applications used **3DES**: three applications of DES with independent keys

It had been used as a standard for encryption until 2000. DES was subject to exhaustive key search attacks.



# History of Block Cipher

- 1997: **NIST** issued call for new ciphers (use for  $\geq 30$  years, protect  $\geq 100$  years)  
1998: 15 candidates accepted in June  
1999: 5 of them were shortlisted in August  
2000: **Rijndael** was selected as the **AES** in October (Daeman, Rijmen)  
2001: issued as FIPS PUB 197 standard in November



# History of Block Cipher

- 1997: **NIST** issued call for new ciphers (use for  $\geq 30$  years, protect  $\geq 100$  years)  
1998: 15 candidates accepted in June  
1999: 5 of them were shortlisted in August  
2000: *Rijndael* was selected as the **AES** in October (Daeman, Rijmen)  
2001: issued as FIPS PUB 197 standard in November



# History of Block Cipher

- 1997: **NIST** issued call for new ciphers (use for  $\geq 30$  years, protect  $\geq 100$  years)
- 1998: 15 candidates accepted in June
- 1999: 5 of them were shortlisted in August
- 2000: **Rijndael** was selected as the **AES** in October (Daeman, Rijmen)
- 2001: issued as FIPS PUB 197 standard in November
- Block length: **128** bits, key length: **128/192/256** bits
- Stronger and faster than 3DES
- Efficient** in both software and hardware
- Simple in design, suitable for smart cards (memory requirement)



# CPA-secure encryption

- Let  $F$  be a length-preserving, keyed function





# CPA-secure encryption

- Let  $F$  be a length-preserving, keyed function

$Gen(1^n)$ : choose a uniform key  $k \in \{0, 1\}^n$



# CPA-secure encryption

- Let  $F$  be a length-preserving, keyed function

$Gen(1^n)$ : choose a uniform key  $k \in \{0, 1\}^n$

$Enc_k(m)$ , for  $|m| = |k|$

- Choose **uniform**  $r \in \{0, 1\}^n$  (*nonce/ initialization vector*)
- Output ciphertext  $\langle r, F_k(r) \oplus m \rangle$



# CPA-secure encryption

- Let  $F$  be a length-preserving, keyed function

$Gen(1^n)$ : choose a uniform key  $k \in \{0, 1\}^n$

$Enc_k(m)$ , for  $|m| = |k|$

- Choose **uniform**  $r \in \{0, 1\}^n$  (*nonce/ initialization vector*)
- Output ciphertext  $\langle r, F_k(r) \oplus m \rangle$

$Dec_k(c_1, c_2)$ : output  $c_2 \oplus F_k(c_1)$

# CPA-secure encryption

- Let  $F$  be a length-preserving, keyed function

$Gen(1^n)$ : choose a uniform key  $k \in \{0, 1\}^n$

$Enc_k(m)$ , for  $|m| = |k|$

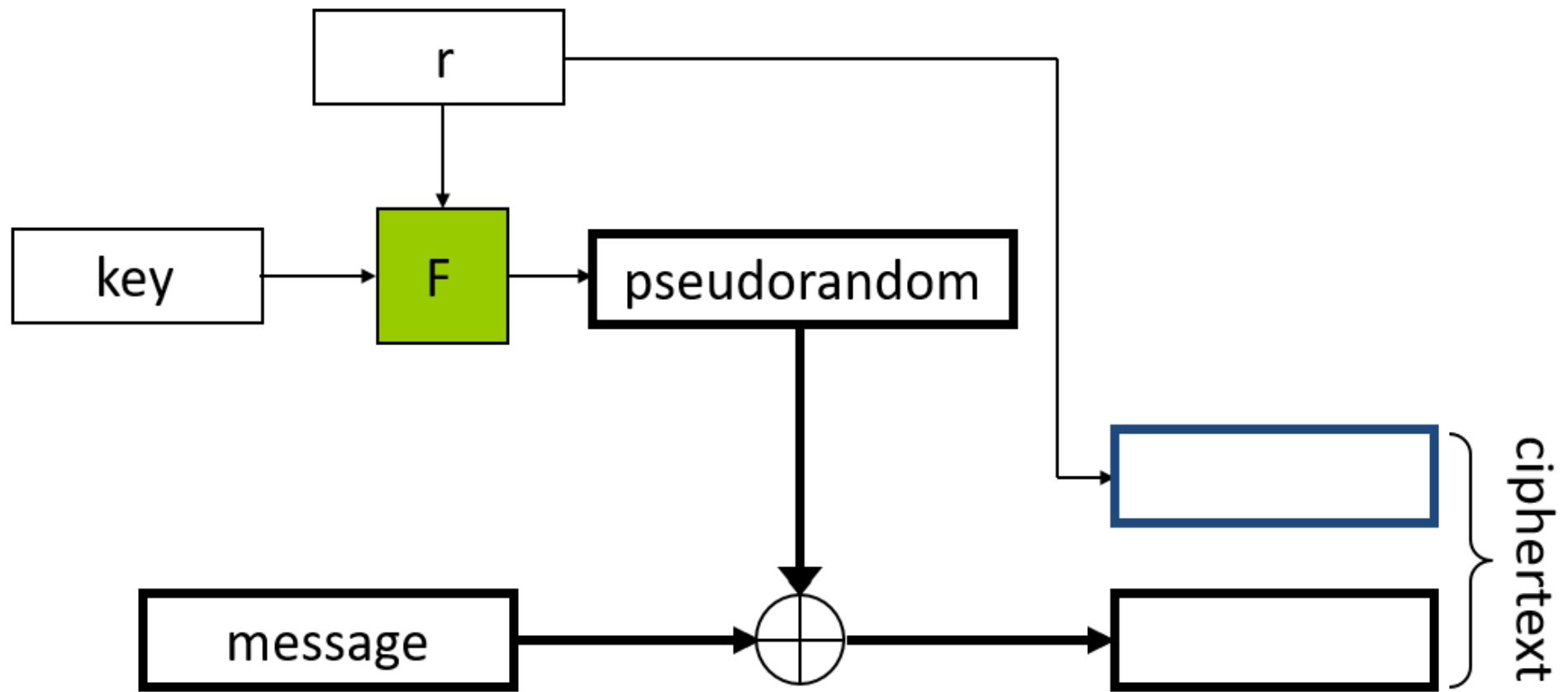
- Choose **uniform**  $r \in \{0, 1\}^n$  (*nonce/ initialization vector*)
- Output ciphertext  $\langle r, F_k(r) \oplus m \rangle$

$Dec_k(c_1, c_2)$ : output  $c_2 \oplus F_k(c_1)$

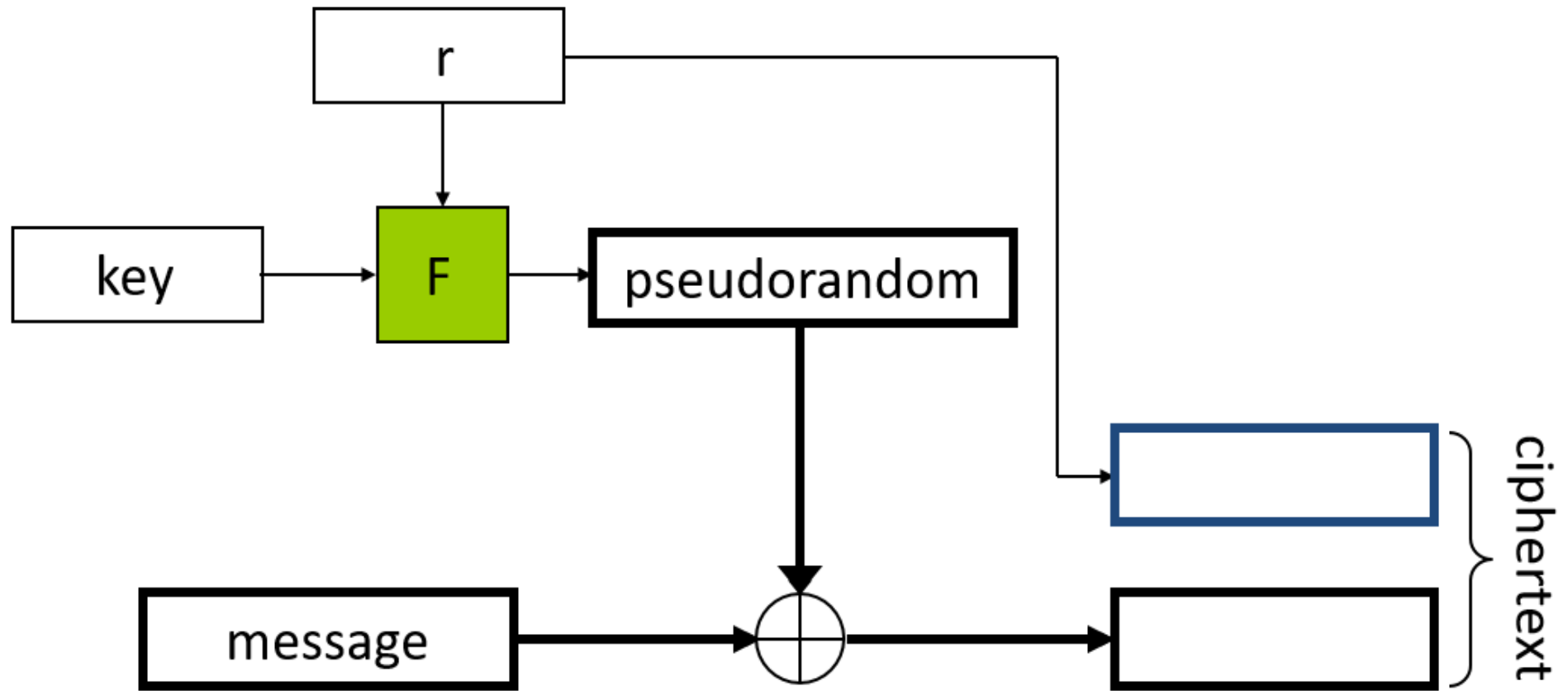
Correctness is immediate



# CPA-secure encryption



# CPA-secure encryption



**Theorem 5.1** If  $F$  is a pseudorandom function, then this scheme is *CPA-secure*.

# Note

- The key may be as long as the message
- But the same key can be used to safely encrypt *multiple* messages

# Security?

- **Theorem 5.1** If  $F$  is a pseudorandom function, then this scheme is *CPA-secure*.

**Proof** by reduction

Let  $\Pi$  denote the scheme



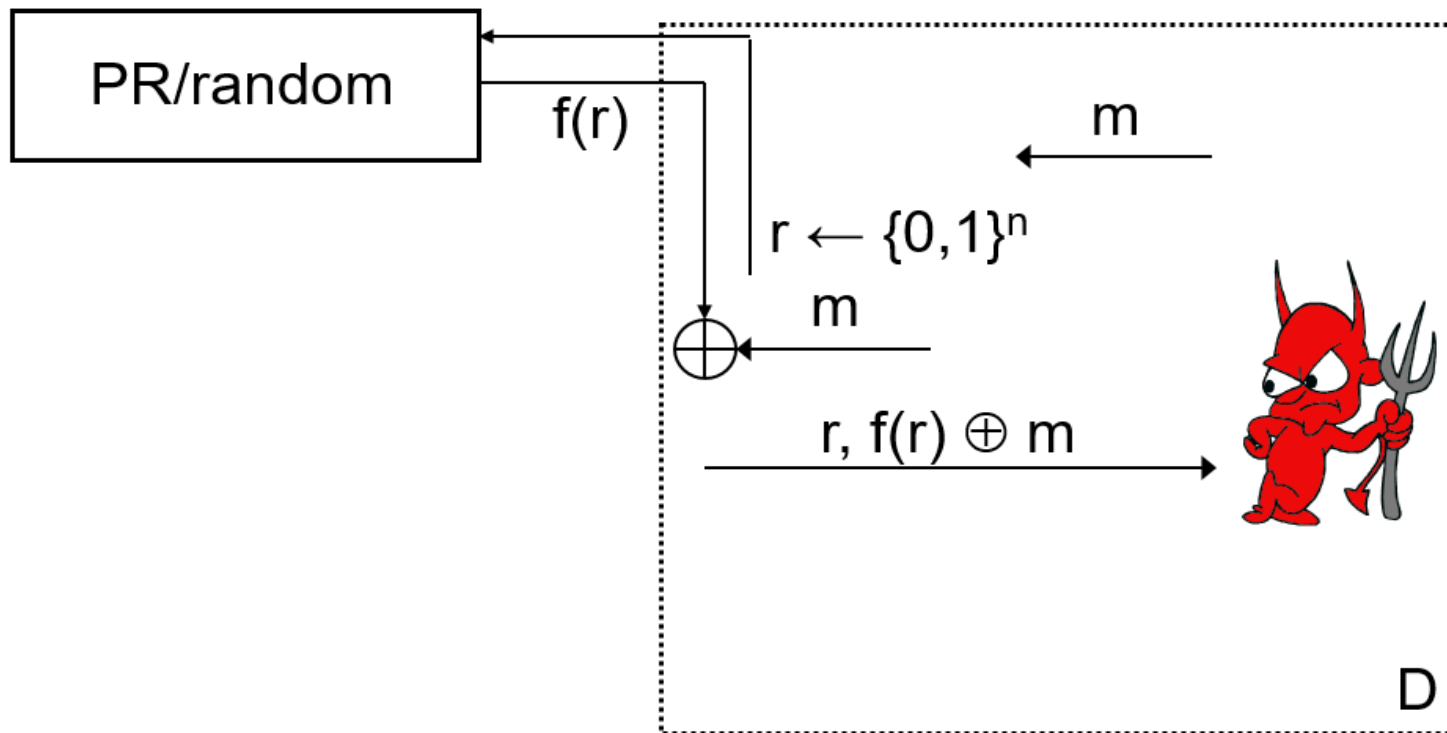


# Security?

- **Theorem 5.1** If  $F$  is a pseudorandom function, then this scheme is *CPA-secure*.

**Proof** by reduction

Let  $\Pi$  denote the scheme

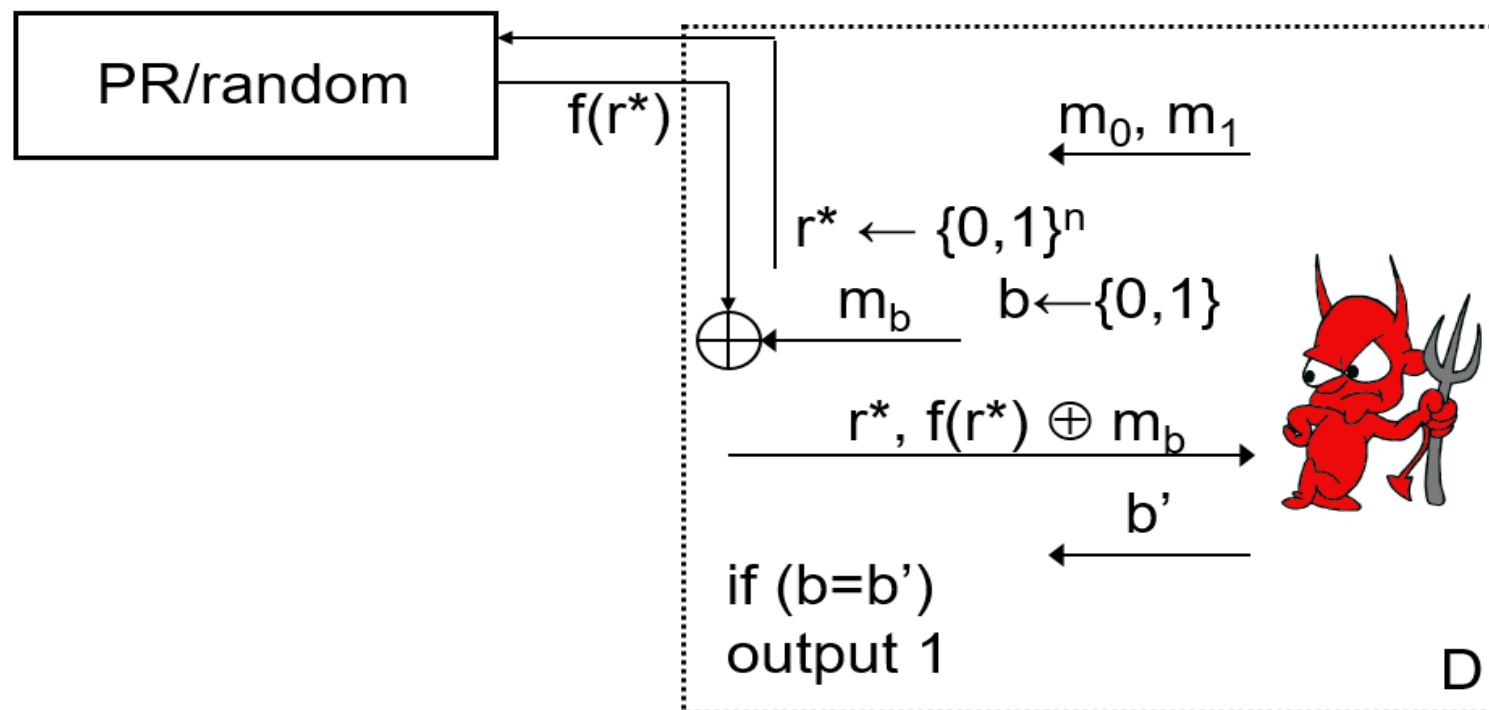


# Security?

- **Theorem 5.1** If  $F$  is a pseudorandom function, then this scheme is *CPA-secure*.

**Proof** by reduction

Let  $\Pi$  denote the scheme



# Analysis

- Let  $\mu(n) = \Pr[\text{PrivCPA}_{Adv, \Pi}(n) = 1]$   
Let  $q(n)$  be a bound on the number of encryption queries made by attacker



# Analysis

- Let  $\mu(n) = \Pr[PrivCPA_{Adv, \Pi}(n) = 1]$   
Let  $q(n)$  be a bound on the number of encryption queries made by attacker

If  $f = F_k$  for **uniform**  $k$ , then the view of Adv is exactly as in  $PrivCPA_{Adv, \Pi}(n)$

$\Rightarrow$

$$\Pr_{k \leftarrow \{0,1\}^n}[D^{F_k(\cdot)} = 1] = \Pr[PrivCPA_{Adv, \Pi}(n) = 1] = \mu(n)$$



# Analysis

- If  $f$  is **uniform**, there are two subcases
  - $r^*$  was used for some other ciphertext  
(call this event *Repeat*)
  - $r^*$  was **not** used for some other ciphertext



# Analysis

- If  $f$  is **uniform**, there are two subcases
  - $r^*$  was used for some other ciphertext  
(call this event *Repeat*)
  - $r^*$  was **not** used for some other ciphertext
- $\Pr_f[D^{f(\cdot)} = 1] \leq \Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] + \Pr[\text{Repeat}]$ 
  - $\Pr[\text{Repeat}] \leq q(n)/2^n$
  - $\Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] = 1/2$

# Analysis

- If  $f$  is **uniform**, there are two subcases
  - $r^*$  was used for some other ciphertext  
(call this event *Repeat*)
  - $r^*$  was **not** used for some other ciphertext
- $\Pr_f[D^{f(\cdot)} = 1] \leq \Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] + \Pr[\text{Repeat}]$ 
  - $\Pr[\text{Repeat}] \leq q(n)/2^n$
  - $\Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] = 1/2$
- Since  $F$  is **pseudorandom**  
 $\Rightarrow |\mu(n) - \Pr_f[D^{f(\cdot)} = 1]| \leq \epsilon(n)$

# Analysis

- If  $f$  is **uniform**, there are two subcases
  - $r^*$  was used for some other ciphertext  
(call this event *Repeat*)
  - $r^*$  was **not** used for some other ciphertext
- $\Pr_f[D^{f(\cdot)} = 1] \leq \Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] + \Pr[\text{Repeat}]$ 
  - $\Pr[\text{Repeat}] \leq q(n)/2^n$
  - $\Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] = 1/2$
- Since  $F$  is **pseudorandom**
  - $\Rightarrow |\mu(n) - \Pr_f[D^{f(\cdot)} = 1]| \leq \epsilon(n)$
  - $\Rightarrow \mu(n) \leq \Pr_f[D^{f(\cdot)} = 1] + \epsilon(n) \leq 1/2 + q(n)/2^n + \epsilon(n)$



# Analysis

- If  $f$  is **uniform**, there are two subcases
    - $r^*$  was used for some other ciphertext  
(call this event *Repeat*)
    - $r^*$  was **not** used for some other ciphertext
  - $\Pr_f[D^{f(\cdot)} = 1] \leq \Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] + \Pr[\text{Repeat}]$ 
    - $\Pr[\text{Repeat}] \leq q(n)/2^n$
    - $\Pr_f[D^{f(\cdot)} = 1 | \neg \text{Repeat}] = 1/2$
  - Since  $F$  is **pseudorandom**
    - $\Rightarrow |\mu(n) - \Pr_f[D^{f(\cdot)} = 1]| \leq \epsilon(n)$
    - $\Rightarrow \mu(n) \leq \Pr_f[D^{f(\cdot)} = 1] + \epsilon(n) \leq 1/2 + q(n)/2^n + \epsilon(n)$
- Note:**  $q(n)/2^n + \epsilon(n) = \epsilon'(n)$  is **negligible**

# Real-world security?

- The security bound we proved is *tight*



# Real-world security?

- The security bound we proved is *tight*
- What happens if a nonce  $r$  is *ever reused*?
- What is the probability that the nonce used in some challenge ciphertext is also used for some other ciphertext?
- What happens to the bound if the nonce is chosen *non-uniformly*?



# CPA-secure encryption

- We have shown a *CPA-secure* encryption scheme based on any block cipher/ PRF
  - $Enc_k(m) = \langle r, F_k(r) \oplus m \rangle$



# CPA-secure encryption

- We have shown a *CPA-secure* encryption scheme based on any block cipher/ PRF
  - $Enc_k(m) = \langle r, F_k(r) \oplus m \rangle$
- Drawbacks?
  - A 1-block plaintext results in a 2-block ciphertext
  - **Only** defined for encryption of  $n$ -bit messages



# Encrypting long messages?

- Recall that CPA-security  $\Rightarrow$  security for the encryption of multiple messages



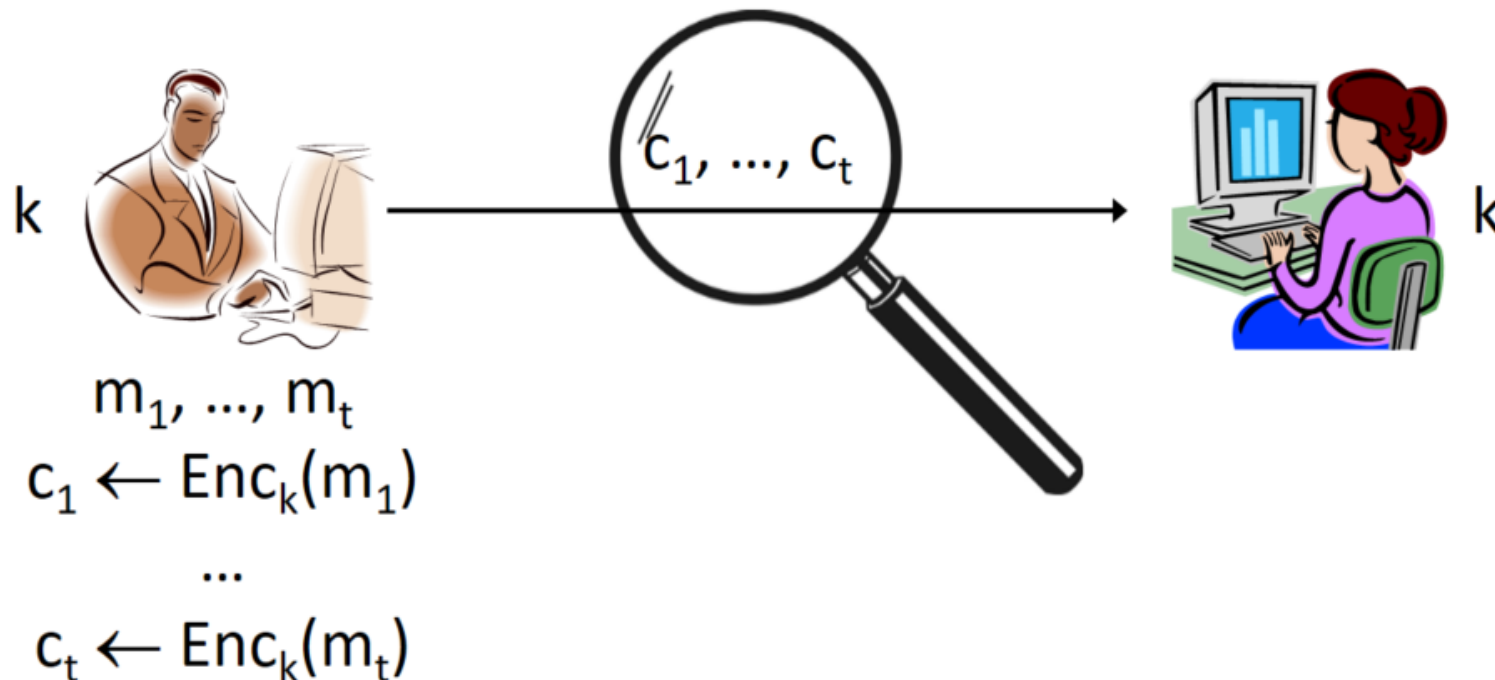
# Encrypting long messages?

- Recall that *CPA-security*  $\Rightarrow$  security for the encryption of *multiple* messages
- So, can encrypt the message  $m_1, \dots, m_t$  as  $Enc_k(m_1), Enc_k(m_2), \dots, Enc_k(m_t)$ 
  - This is also *CPA-secure*!



# Encrypting long messages?

- Recall that **CPA-security**  $\Rightarrow$  security for the encryption of **multiple** messages
- So, can encrypt the message  $m_1, \dots, m_t$  as  $Enc_k(m_1), Enc_k(m_2), \dots, Enc_k(m_t)$ 
  - This is also **CPA-secure**!





# Drawback

- The ciphertext is *twice* the length of the plaintext
  - I.e., ciphertext expansion by a factor of two



# Drawback

- The ciphertext is *twice* the length of the plaintext
  - I.e., ciphertext expansion by a factor of two
- Can we do better?



# Drawback

- The ciphertext is *twice* the length of the plaintext
  - I.e., ciphertext expansion by a factor of two
- Can we do better?
- Modes of operation
  - *Block-cipher* modes of operation
  - *Stream-cipher* modes of operation



# CTR (Counter) mode

- $Enc_k(m_1, \dots, m_t)$  // note:  $t$  is arbitrary
  - Choose  $ctr \leftarrow_R \{0, 1\}^n$ , set  $c_0 = ctr$
  - For  $i = 1$  to  $t$ :
    - $c_i = m_i \oplus F_k(ctr + i)$
  - Output  $c_0, c_1, \dots, c_t$



# CTR (Counter) mode

- $Enc_k(m_1, \dots, m_t)$  // note:  $t$  is arbitrary
  - Choose  $ctr \leftarrow_R \{0, 1\}^n$ , set  $c_0 = ctr$
  - For  $i = 1$  to  $t$ :
    - $c_i = m_i \oplus F_k(ctr + i)$
  - Output  $c_0, c_1, \dots, c_t$
- Decryption?

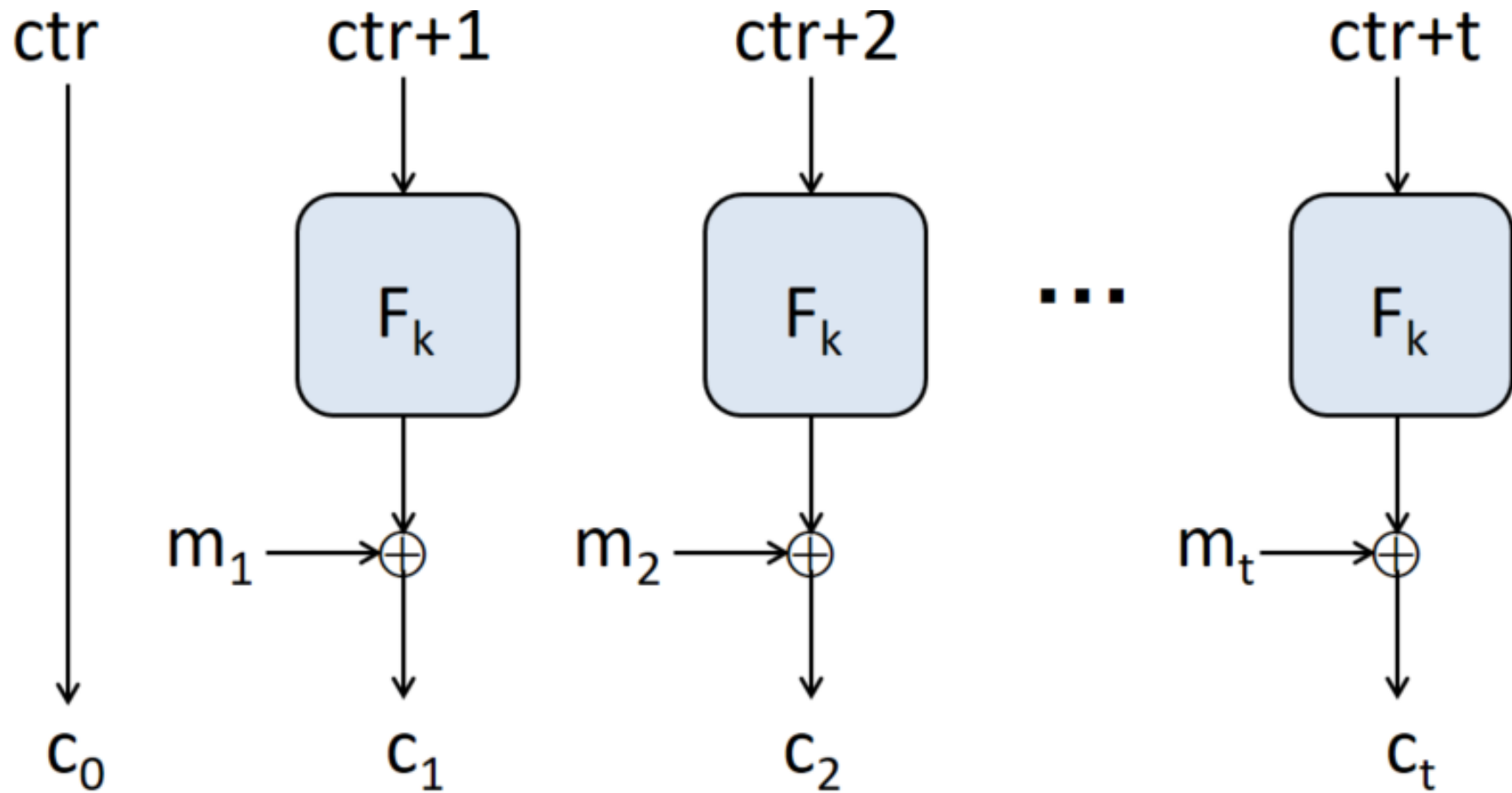


# CTR (Counter) mode

- $Enc_k(m_1, \dots, m_t)$  // note:  $t$  is arbitrary
  - Choose  $ctr \leftarrow_R \{0, 1\}^n$ , set  $c_0 = ctr$
  - For  $i = 1$  to  $t$ :
    - $c_i = m_i \oplus F_k(ctr + i)$
  - Output  $c_0, c_1, \dots, c_t$
- Decryption?
- Ciphertext expansion is just 1 block



# CTR mode



# CTR mode

- **Theorem 5.2** If  $F$  is a pseudorandom function, then CTR mode is *CPA-secure*.





- **Theorem 5.2** If  $F$  is a pseudorandom function, then CTR mode is *CPA-secure*.
- **Proof sketch:**  
The sequences  $F_k(ctr_i + 1), \dots, F_k(ctr_i + t)$  used to encrypt the  $i$ -th message is *pseudorandom*



- **Theorem 5.2** If  $F$  is a pseudorandom function, then CTR mode is *CPA-secure*.
- **Proof sketch:**

The sequences  $F_k(ctr_i + 1), \dots, F_k(ctr_i + t)$  used to encrypt the  $i$ -th message is **pseudorandom**

  - Moreover, it is independent of every other such sequence unless  $ctr_i + j = ctr_{i'} + j'$  for some  $i, j, i', j'$
  - Just need to bound the probability of that event

# CBC (Cipher Block Chaining) mode

- $Enc_k(m_1, \dots, m_t)$  // note:  $t$  is arbitrary
  - Choose  $c_0 \leftarrow_R \{0, 1\}^n$  (also called the  $IV$ )
  - For  $i = 1$  to  $t$ :
    - $c_i = F_k(m_i \oplus c_{i-1})$
  - Output  $c_0, c_1, \dots, c_t$

# CBC (Cipher Block Chaining) mode

- $Enc_k(m_1, \dots, m_t)$  // note:  $t$  is arbitrary
  - Choose  $c_0 \leftarrow_R \{0, 1\}^n$  (also called the *IV*)
  - For  $i = 1$  to  $t$ :
    - $c_i = F_k(m_i \oplus c_{i-1})$
  - Output  $c_0, c_1, \dots, c_t$
- Decryption?
  - Requires  $F$  to be *invertible*

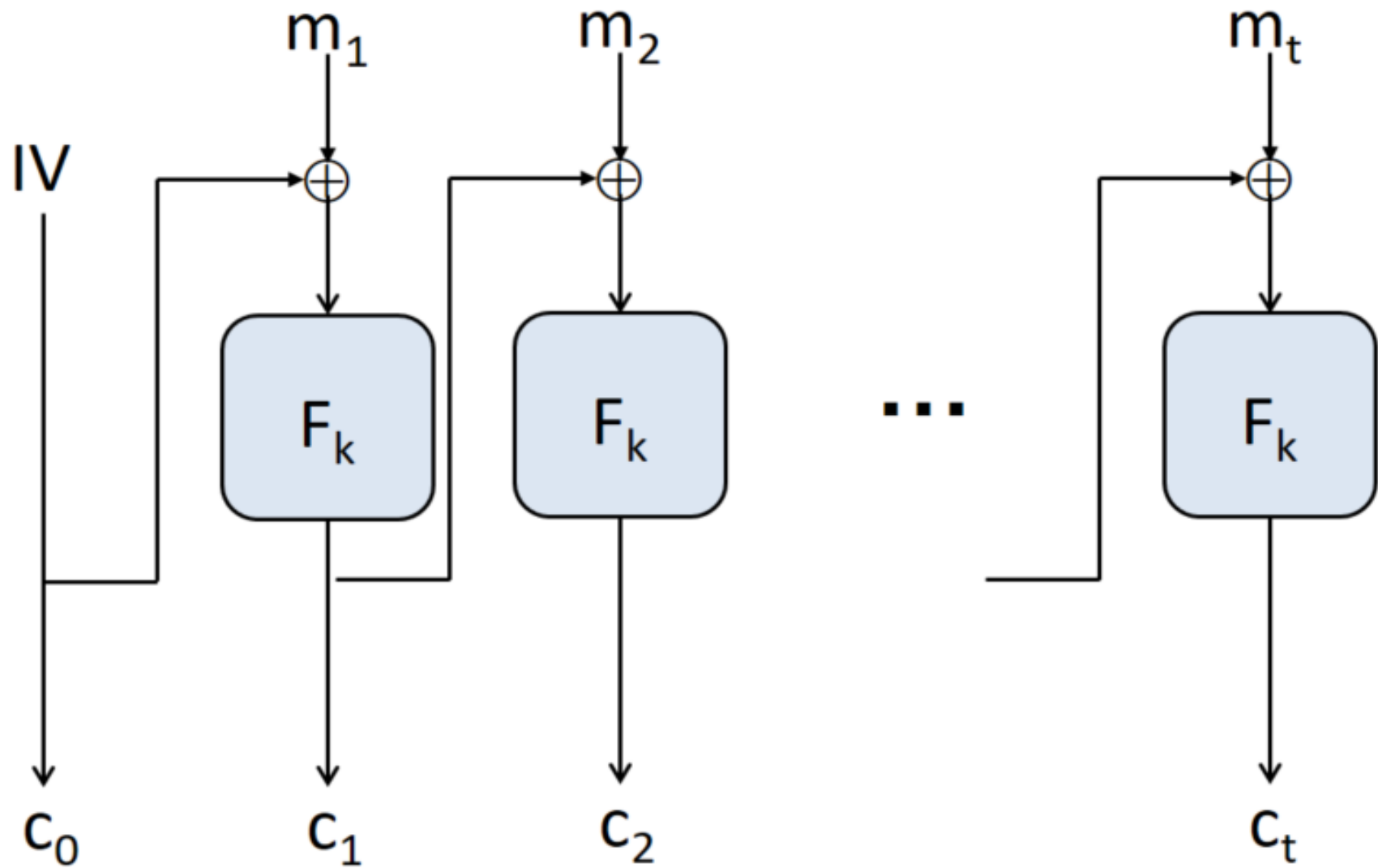


# CBC (Cipher Block Chaining) mode

- $Enc_k(m_1, \dots, m_t)$  // note:  $t$  is arbitrary
  - Choose  $c_0 \leftarrow_R \{0, 1\}^n$  (also called the *IV*)
  - For  $i = 1$  to  $t$ :
    - $c_i = F_k(m_i \oplus c_{i-1})$
  - Output  $c_0, c_1, \dots, c_t$
- Decryption?
  - Requires  $F$  to be *invertible*
- Ciphertext expansion is just 1 block



# CBC mode



# CBC mode

- **Theorem 5.3** If  $F$  is a pseudorandom function, then CBC mode is *CPA-secure*.



# CBC mode

- **Theorem 5.3** If  $F$  is a pseudorandom function, then CBC mode is *CPA-secure*.
- **Proof** is more complicated than for CTR mode





# ECB (Electronic Codebook) mode

- $Enc_k(m_1, \dots, m_t) = F_k(m_1), \dots, F_k(m_t)$



# ECB (Electronic Codebook) mode

- $Enc_k(m_1, \dots, m_t) = F_k(m_1), \dots, F_k(m_t)$
- Deterministic
  - Not CPA-secure!
  - Efficient: online computation



# ECB (Electronic Codebook) mode

- $Enc_k(m_1, \dots, m_t) = F_k(m_1), \dots, F_k(m_t)$
- Deterministic
  - Not CPA-secure!
  - Efficient: online computation
- Can tell from the ciphertext whether  $m_i = m_j$ 
  - Not even EAV-secure!



# ECB (Electronic Codebook) mode

- $Enc_k(m_1, \dots, m_t) = F_k(m_1), \dots, F_k(m_t)$
- Deterministic
  - **Not** CPA-secure!
  - **Efficient**: online computation
- Can tell from the ciphertext whether  $m_i = m_j$ 
  - **Not** even EAV-secure!



# Stream ciphers

- As we defined, PRGs are *limited*
  - They have fixed-length output
  - They produce output in “one shot”
- In practice, PRGs are based on *stream ciphers*
  - Can be viewed as producing an “infinite” stream of pseudorandom bits, on demand
  - More flexible, more efficient



# Stream ciphers

- Pair of efficient, deterministic algorithms (**Init**, **GetBits**)



# Stream ciphers

- Pair of efficient, deterministic algorithms (**Init**, **GetBits**)
  - **Init** takes a seed  $s_0$  (and optional  $IV$ ), and outputs initial state  $st_0$
  - **GetBits** takes the current state  $st$  and outputs a bit  $y$  along with updated state  $st'$



# Stream ciphers

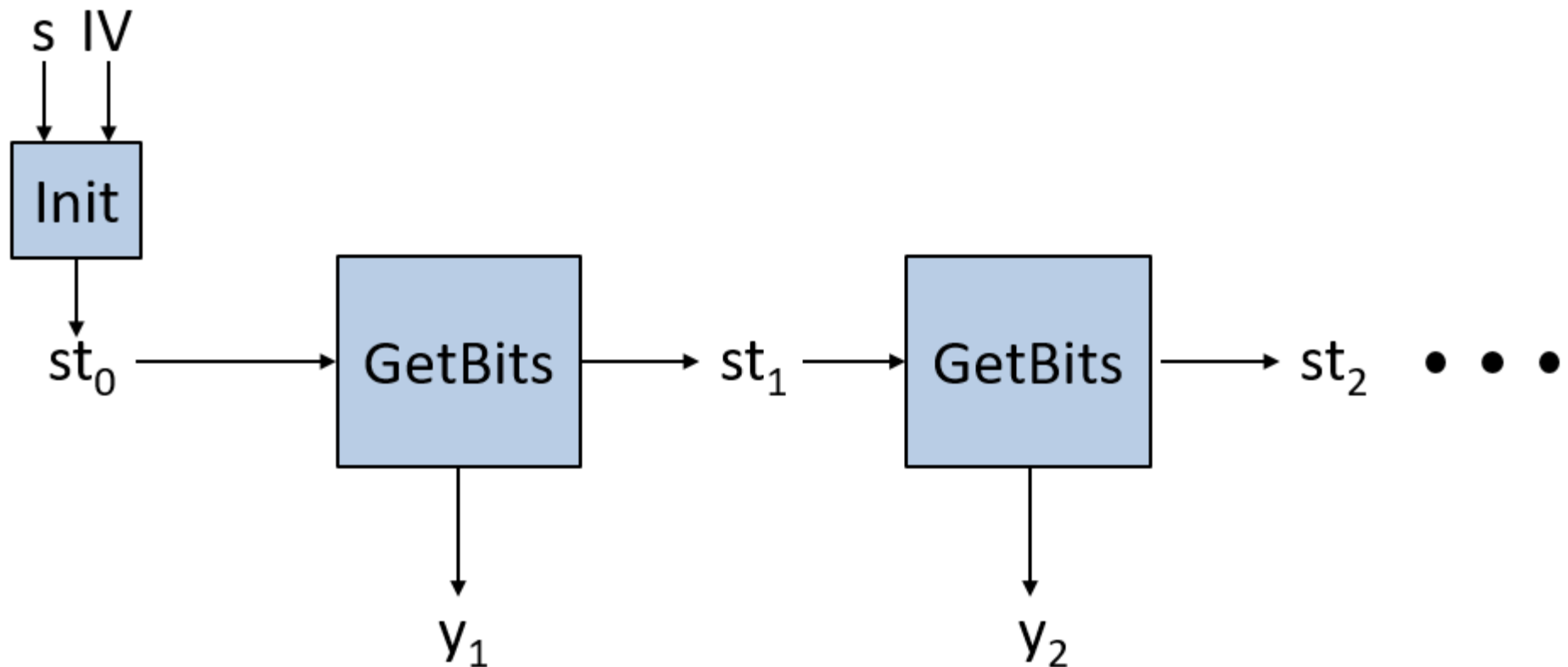
- Pair of efficient, deterministic algorithms (**Init**, **GetBits**)
  - **Init** takes a seed  $s_0$  (and optional  $IV$ ), and outputs initial state  $st_0$
  - **GetBits** takes the current state  $st$  and outputs a bit  $y$  along with updated state  $st'$
  - In practice,  $y$  would be a block rather than a bit





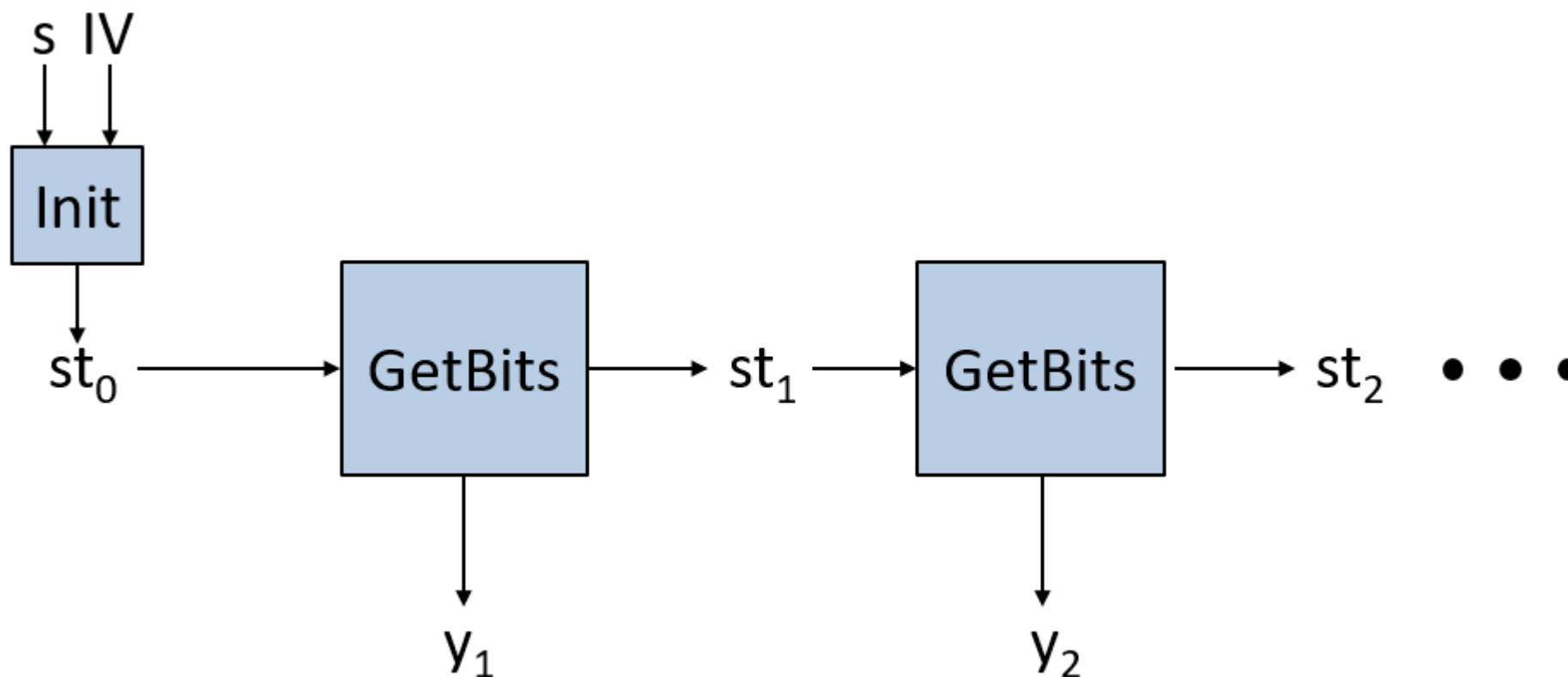
# Stream ciphers

- Can use (**Init**, **GetBits**) to generate **any** desired number of output bits from an initial seed



# Stream ciphers

- A *stream cipher* is *secure* (informally) if the output stream generated from a uniform seed is *pseudorandom*
  - I.e., regardless of how long the output stream is (so long as it is polynomial)



# Next Lecture

- stream cipher, CCA security ...

