

Practice Session 9

Type I, Type II, or Neither?

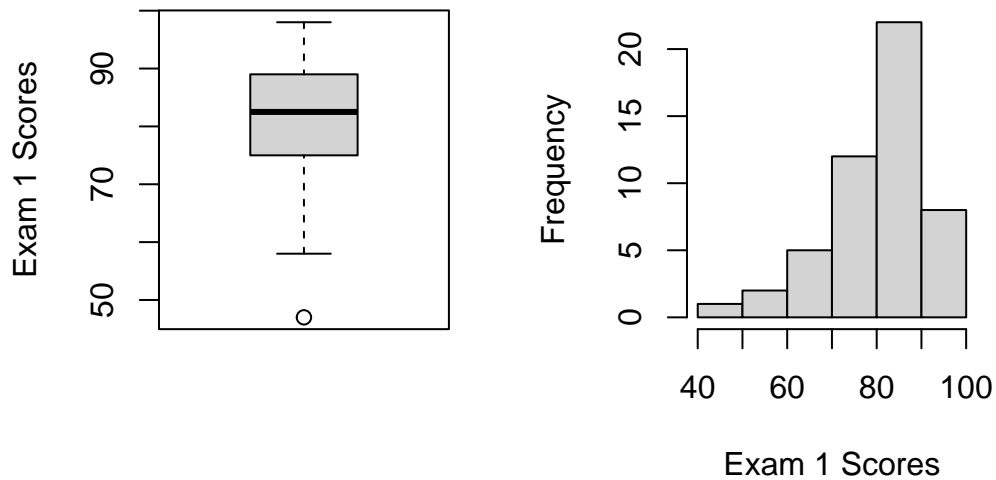
- a) Type I Error
- b) No error
- c)
 - No error
 - Type I Error
 - Type II Error
 - No error

If the disease is dangerous, then a Type II error (saying a patient doesn't have the disease when they really do) can be a big issue. We would want to make the Type II error lower. However, this may make the Type I error larger as a result.

Single Sample T Test

- a)

```
library(Lock5Data)
par(mfrow = c(1, 2))
boxplot(StatGrades$Exam1, main = "", ylab = "Exam 1 Scores")
hist(StatGrades$Exam1, main = "", xlab = "Exam 1 Scores")
```



There appears to be one outlier. We might want to investigate this score further.

b)

- $H_0 : \mu = 80$
- $H_A : \mu \neq 80$

```
n1 = length(StatGrades$Exam1) # get sample size (n = 50)

mean_exam1 = mean(StatGrades$Exam1) # compute sample mean
sd_exam1 = sd(StatGrades$Exam1)
se_exam1 = sd_exam1 / sqrt(n1) # compute standard error

T_stat_exam1 = (mean_exam1 - 80) / se_exam1
T_stat_exam1
```

```
[1] 0.7145975
```

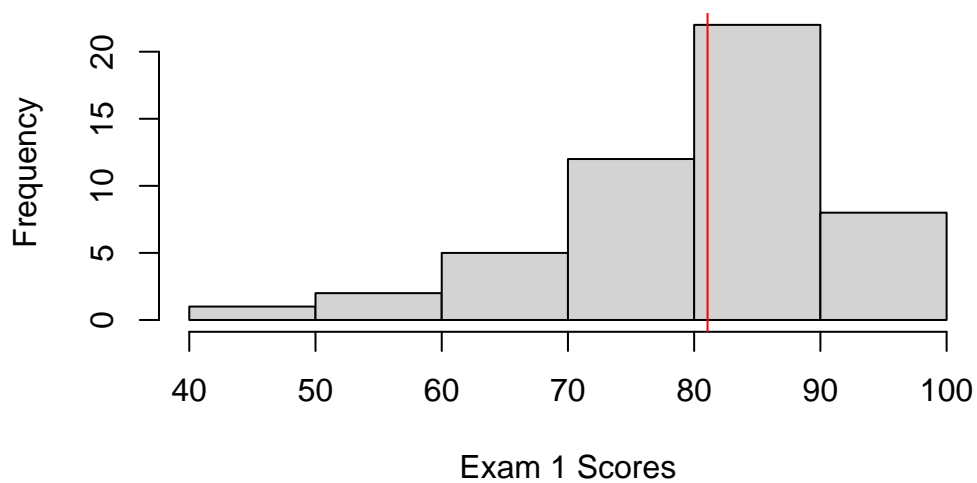
```
# df = 50 - 1 = 49
pt(T_stat_exam1, df = n1 - 1, lower.tail = F)
```

```
[1] 0.2391243
```

Since our p-value is greater than 0.05, we will fail to reject the null hypothesis. We therefore do not have evidence that this year's exam scores are significantly higher than 80.

c)

```
par(mfrow = c(1, 1))  
hist(StatGrades$Exam1, main = "", xlab = "Exam 1 Scores")  
abline(v = mean_exam1, col = "red")
```

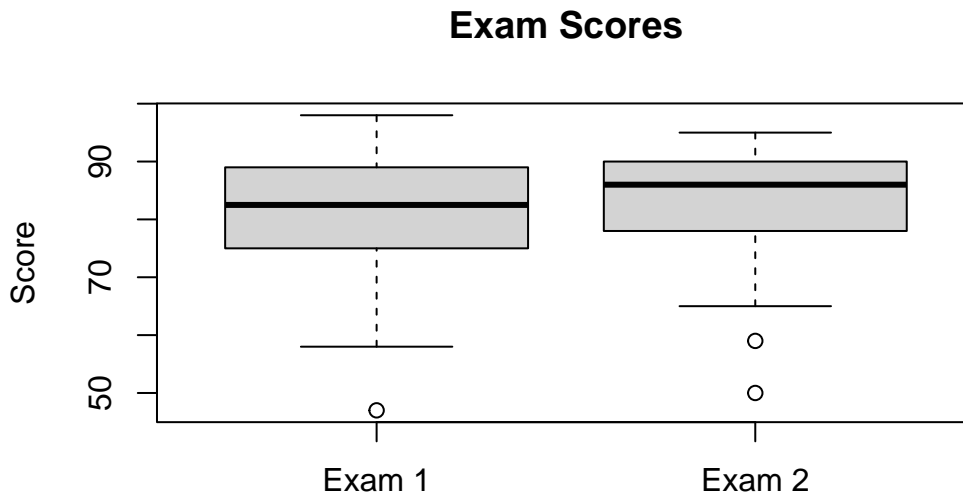


It seems that the sample mean wasn't very far from the null value of 80, which supports the lack of significance in our test.

Two Sample T Test

a)

```
boxplot(StatGrades$Exam1, StatGrades$Exam2,  
        names = c("Exam 1", "Exam 2"),  
        main = "Exam Scores",  
        ylab = "Score")
```



The median scores appear to be similar for both exams. Exam 1 scores appear to have more variability than exam 2 scores. There are two outliers in the exam 2 scores.

b)

- $H_0 : \mu_1 = \mu_2$
- $H_A : \mu_1 \neq \mu_2$

```
n2 = length(StatGrades$Exam2) # get sample size (n = 50)

mean_exam2 = mean(StatGrades$Exam2) # compute sample mean
sd_exam2 = sd(StatGrades$Exam2)

# Compute standard error for two means
se_two_means = sqrt(sd_exam1^2/n1 + sd_exam2^2/n2)
T_stat_two_means = (mean_exam2 - mean_exam1) / se_two_means
T_stat_two_means
```

```
[1] 1.026024
```

Since we are doing a two-sided test, we need to multiply the p-value by 2.

```
2 * pt(T_stat_two_means, df = 50 - 1, lower.tail = F)
```

```
[1] 0.3099189
```

Since our p-value is greater than 0.05, we will fail to reject the null hypothesis. We therefore do not have evidence that this year's students performed differently on the two exams.

c)

- $H_0 : \mu_1 = \mu_{final}$
- $H_A : \mu_1 < \mu_{final}$

Note that we are doing a one-sided test, with the alternative being improvement on the final compared to the first exam.

```
n_final = length(StatGrades$Final) # get sample size (n = 50)

mean_final = mean(StatGrades$Final) # compute sample mean
sd_final = sd(StatGrades$Final)

# Compute standard error for two means
se_one_final = sqrt(sd_exam1^2/n1 + sd_final^2/n_final)
T_stat_one_final = (mean_final - mean_exam1) / se_one_final
T_stat_one_final
```

```
[1] 2.264897
```

```
pt(T_stat_one_final, df = 50 - 1, lower.tail = F)
```

```
[1] 0.01398826
```

Since our p-value is less than 0.05, we will reject the null hypothesis. We therefore have evidence that student scores improved on the final compared to the first exam.

Paired T Test

Let D be the true difference between the exam 1 and exam 2 scores. We will perform the following two-sided test:

- $H_0 : D = 0$
- $H_A : D \neq 0$

```
diff_exams = StatGrades$Exam2 - StatGrades$Exam1
n_diff = length(diff_exams)

mean_diff = mean(diff_exams)
se_diff = sd(diff_exams) / sqrt(n_diff)

T_stat_diff = mean_diff / se_diff
```

```
2 * pt(T_stat_diff, df = n_diff - 1, lower.tail = F)
```

```
[1] 0.01411695
```

Since our p-value is less than 0.05, we will reject the null hypothesis. We therefore have evidence that this year's students performed differently on the two exams.